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Interfaces with Other Disciplines

Capacity and entry issues in online exchanges

Subhajyoti Bandyopadhyay [a,*], John M. Barron [b,1], Alok R. Chaturvedi [c,2]

[a] *Warrington School of Business, University of Florida, Gainesville, FL 32611, United States*
[b] *Department of Economics, Krannert Graduate School of Management, Purdue University, West Lafayette, IN 37907, United States*
[c] *Department of MIS, Krannert Graduate School of Management, Purdue University, West Lafayette, IN 37907, United States*

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Abstract

With the advent of open standards and Internet technologies, the number of sellers who can participate in online exchanges is greatly increased. We model the competition between identical sellers vying for the same business, and find that there exists a mixed-strategy equilibrium in prices. The results help us understand the dynamics between a seller's capacity and his motivation to participate in an auction.

Keywords: Auctions; Economics; Game theory; Online exchanges; Supplier competition

1. Introduction

This paper was motivated by the rapid growth of online exchanges (sometimes called business-to-business, or B2B, exchanges) used by firms for their procurement requirements. With the advent of Internet technologies and their underlying open standards, it has become much easier for vendors to bid in response to Requests for Proposal (or RFP's) that they otherwise might not have even come to know of. Institutional buyers thus have a larger universe of potential vendors for their supplies of raw materials and other internal purchases. This arrangement has several advantages like less dependence on a few vendors, and cost savings through the increased competition among the vendors.

Carrier Corp., the world's largest manufacturers of air-conditioners, and one of the pioneers of using online exchanges, has managed to quantify their own benefits of using this mechanism. Just by putting their requirements online through FreeMarkets, and thereby getting a larger number of suppliers to bid for the requirements, Carrier Corp. realized average savings of 15% on the cost of components, amounting to more than $100 million [1] of savings in the year 2000. The company firmly believes that the online exchanges can be

* Corresponding author. Tel.: +1 352 392 5946; fax: +1 352 392 5438.
E-mail addresses: shubho.bandyopadhyay@cba.ufl.edu (S. Bandyopadhyay), barron@mgmt.purdue.edu (J.M. Barron), alok@mgmt.purdue.edu (A.R. Chaturvedi).
1 Tel.: +1 765 494 4451; fax: +1 765 494 1526.
2 Tel.: +1 765 494 9048; fax: +1 765 494 1526.

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a powerful cost-cutting tool when there are several vendors to choose from, especially in the current environment of worldwide sourcing for components in the manufacturing sector. ChemConnect, an online exchange specializing in chemical products, provides the example of a Fortune 500 household products company realizing 15.6% cost savings in raw materials by procuring from 13 different suppliers at the same time, while Sunoco Inc., a leading manufacturer and marketer of petroleum and petrochemical products, expanded its universe of suppliers by two and realized cost savings of 12%.

Among the several advantages of B2B transactions are expanding market reach, lowering buyers' costs and identifying best practices [2]. The focus of this paper is to analyze how prices might reduce from the increased competition between sellers. This is especially true in those online exchanges that are called "buy-centric markets" or "procurement hubs" in industry parlance (all the previously mentioned exchanges are examples of such buy-centric markets). These buy-centric hubs are especially common in the market for components and raw materials (like the auto industry or Carrier Corp.), but any large buyer of relatively homogeneous goods will have significant interests in joining an exchange.[3] Roland Berger, a consulting firm specializing in the auto industry estimates that about 33% of the cost savings in North American auto industry will come from purchase-related savings [3].

Suppliers however have become increasingly discomfited with the process of reverse auctions in these exchanges. While original equipment manufacturer (OEM) buyers might argue that the price competition induces efficient competition that was lacking before, the suppliers fear that this competition might benefit only the buyers, resulting in a disproportionate sharing of the surplus [4].

> [The buyer] talks about the relationship being a partnership and this [the auction] really takes that away. There is not a partnership there at all. What they do is take your existing business that you have worked very hard to achieve and maintain. You work with them to give them cost reductions over the years and they send it out across the board for a competitive bid. I just do not think that is fair.
>
> *Supplier comment* [4]

The Wall Street Journal reports that large retailers like Wal-Mart have been quick to capitalize on this increased competition between sellers to drive down wholesale prices of electrical goods from Chinese manufacturers, calling it the "survival of the cheapest" [5].

As Dai and Kauffman [6] have noted, sellers have reduced information about buyers in open exchanges, which might make the effects of the competition stacked heavily against the sellers. Given the monopsony power of a limited number of large buyers however, sellers have limited options. A recent Forrester Research survey found that 57% of auto suppliers polled said they respond to online RFP's currently, and 70% said they will do so by 2003 [7].

We analyze the case of competition between identical suppliers (we use the terms suppliers and sellers interchangeably throughout the paper) of a single raw material or component vying for business from a single large buyer within a B2B exchange framework. We concentrate on the mechanism of a one-sided, sealed-bid reverse auction. We assume that there is no *combined* capacity constraint as such: the sellers were supplying to the entire demand before the birth of the exchange, and continue to do so after it comes into play. It is conceivable however, that the firms individually cannot supply to the entire market. This means that while there is competition between the firms to be the low-price bidder, it is not as extreme as a Bertrand game that results in prices equal to marginal cost. As the nature of the equilibrium shows however, there remains an incentive to be the low-price bidder and have the "first invitation" to supply a requirement.

There are several objectives of the paper. We explore the nature of the pricing strategy that the suppliers employ in such environments in a game-theoretic framework. A related issue is to consider the conditions under which this competition would be debilitating for the suppliers.

Our experience with bidding in industrial markets suggests that managers resort to mixed-price strategies, and indeed our analysis shows that there exists a support of prices where the sellers randomize their bids. We obtain an explicit expression for the average price paid by the buyers, and show that for some values of the

[3] Wal-Mart for example has its own private exchange for buying the products they sell.

parameters within the model, the exchange promotes competition that results in lower bids[4] in a two-seller model. The nature of the equilibrium indicates the mechanism of entry into the exchange, the second seller being the one that caters to the residual demand. Our results also indicate why larger sellers might be more amenable to joining the exchange than smaller sellers: it is easier for a smaller number of players to induce cooperation for mutual benefit.

While the benefits of the buyers are readily apparent, sellers have to analyze the costs and benefits of joining the exchange. (Indeed, as Baumgartner et al. [8] point out, sellers have been extremely reluctant to join these exchanges in several instances, fearing increased competition.) Dai and Kauffman [6] model the environment where there is one large buyer and two suppliers. The suppliers are identical, realizing the same costs and benefits. By assigning the various costs and benefits of the electronic network and the exchange for both the buyer and the sellers, they develop several simple what-if scenarios that determine whether a supplier prefers an extranet or an electronic network. Our paper continues to investigate this model, and considers the effects when the two sellers (and later generalizing the model to n sellers) have decided to participate in the exchange, and are competing for any business within that framework.

2. Model environment and assumptions

We look at a competitive game that is unique in several respects. To illustrate this, consider a hypothetical example in which a large automaker wants to purchase tires for one of its mass-market models. The number of tire manufacturers is limited, and the automaker has its own set of preferred suppliers, since the cost of analyzing quotations from the entire universe of suppliers (dedicated account management teams for buyers, sales force for sellers, cost of sending individual RFPs to the entire universe of sellers, etc.) might be prohibitively high. The demand for tires can be considered inelastic below a certain reservation price.

Though the demand for the final product (the car) might be elastic, it is unlikely that the demand for the tires, which constitute a very small fraction of the value of the car, is also elastic. Rather, manufacturers have a reservation price for tires – set by industry norms or internal costing data – and buy from their established list of sellers as long as the price is less than this reservation price.[5] To put it in another way, the demand for the end product might go up or down because of its price, but the demand for the raw materials that are used to make the product will not show such a behavior – more likely, the manufacturer of the product would revise its reservation price for the raw material.

With the advent of the online exchange, the buyers put their requirements on the exchange only once, instead of contacting the sellers individually. The sellers – who can be located anywhere around the world – look at this entire requirement, and then decide to quote their prices. The lowest priced seller is first invited to supply the required quantity, and if there is any residual demand, it passes on to the seller with the second lowest price, and so on till the demand is completely satisfied.[6]

We consider the mechanism of a one-sided, sealed bid auction mechanism: the buyer posts a RFP and invites specific suppliers to view and respond to the RFP. The suppliers are also given a date (usually several weeks in the future) by which they are expected to place their bids. The suppliers bid their selling prices, all of which are opened at a later date. The supplier with the lowest bid price gets the first invitation to cater to the demand, followed by the supplier with the second-lowest bid price, and so on, until the entire demand is met.

From the modeling point of view, it is immaterial whether the sellers respond to an aggregate demand of several buyers (which might be an anti-trust issue) or one single demand from a buyer[7] – what is important to

[4] The gain to the buyers will probably be lesser than what these theoretical exercises might suggest, since in practice the buyers would be negotiating better prices from their vendors even before the onset of the online exchanges.

[5] We contacted Roland Berger to confirm this assumption. We found out that auto manufacturers indeed have "target cost" structures for components, which is equivalent to the reservation price in our nomenclature. See also Footnote 6.

[6] A confirmation of this mechanism was obtained from IndiaMarkets.com, India's largest B2B portal. The software used at IndiaMarkets.com is licensed from one of the pioneer B2B portals, Ariba. Fig. 3 (in Appendix A) shows an actual RFP at this site, as presented to suppliers. Identities have been concealed for confidentiality reasons.

[7] The assumption in the case of multiple buyers, of course, is that all of them have the same reservation price. If that is not so, we would have to consider the contract between the buyers that specifies the way they would divide a seller's output between themselves.

note is that the entire requirement is auctioned to the sellers, and for any unfulfilled demand, a lower priced bidder is invited before a higher priced bidder to satisfy it.

It is readily apparent that with unlimited capacity, the sellers respond with a Bertrand competition in prices with the seller or sellers with the lowest marginal cost outbidding the others. This is not to the advantage of the sellers. At the other end of the spectrum, if the total capacity of the sellers is so limited as to be less than the total demand, it is easy to see that the sellers can sell their entire capacities at the buyer's reservation price.

Our model is different from either of these cases. It is realistic to think of sellers having limited capacities so that any one seller cannot meet market demand. However in our setting we add the conditions that the aggregate output of the sellers exceeds total quantity demanded and that a firm sells all it can produce only if it is the low price seller. That is, the lowest priced seller sells to capacity, but a higher priced seller only sells to a residual demand. Sellers are therefore pulled by two opposing "forces" – on one hand, higher prices fetch higher margins, but on the other, higher prices bring about increased chances of being underbid by competition and thus selling less.

Several models that analyze oligopolistic competition exist in literature in which the competitors are capacity-constrained. Kreps and Scheinkman [9] showed that even if two oligopolists compete in a Bertrand fashion (i.e. over prices) in a two-stage game where there is initially a pre-commitment over the quantity to be produced, the equilibrium is the Cournot outcome. Levitan and Shubik [10] show that varying the limited capacity of the two firms can result in a Cournot competition in one extreme to a Bertrand competition in the other. In our model, the demand is known and is fixed, as long as the price is below a reservation price. These assumptions change the equilibrium from a pure strategy to a mixed-strategy outcome. As we pointed out earlier, these assumptions are relevant in the particular setting that we model; crucially, they allow us to extend our initial two-player setting results to that involving n players.

To solve the generalized n-seller model, we start off by first considering a simplified version of the above scenario. The intuition from the two-seller model then helps us to analyze the n-seller model. Thus, we assume that there is a single buyer and two sellers, with the buyer deciding on how much of its requirements are supplied by either seller. The two sellers are identical, with the same (constant) marginal costs c and capacity K. The suppliers have complete information about each other. With the advent of the exchange, the buyer puts its RFP on the exchange, and the two sellers compete for being the low bidder. The low bidder can sell to capacity, but the higher priced bidder cannot. Thus, there is an incentive for a seller not to bid very high, but at the same time, not bid so low as to "leave money on the table".

3. The model

We consider the case when both sellers have equal capacities K which are lower than the cumulative demand, but together are greater than the cumulative demand Q (i.e. $2K > Q > K$). In such a setting, the lower priced seller is invited first to sell the required quantity, and after he has supplied his total capacity K, the other seller can then sell the residual demand $(Q - K)$.

A summary of all the variables used in this paper appears in Table 1 in Appendix A.

Proposition 1. *There can be no symmetric equilibrium in pure strategies.*

Proof. Let there be a price $\hat{p}$ such that $(\hat{p} - c)K = (r - c)(Q - K)$. Since $2K - Q > 0$, $K > (Q - K)$, which implies that $\hat{p} < r$. At these prices, both sellers make the same profits. The low priced seller has a lower margin, but sells more; the high price seller meets residual demand at a price equal to the buyers' reservation value. In this setting, the higher priced seller is content to sell to the residual demand at the reservation price when the lower priced seller sets his own price at $\hat{p}$. However, when one seller sets its price at r, the best response for the other seller is to price his product at $(r - \varepsilon)$ which prompts the first seller to undercut the second by an infinitesimal amount and so on, resulting in the prices spiraling down till one of the sellers reaches the price $\hat{p}$, at which point the other seller responds with a price of r, and the cycle starts once again. Similarly, the best response to a bid at any price p, where $\hat{p} < p < r$, is to undercut by an infinitesimal amount, which results in a similar downward spiral in prices. Thus, there can be no pure strategy equilibrium in prices (and there is no incentive for any seller to undercut below the price $\hat{p}$). (In fact, the existence of a mixed-strategy equilibrium in a game with discontinuous payoffs has been proved by [11] – our paper finds out the specific nature of the solution under the proposed set of demand and cost assumptions.)

In a mixed-strategy equilibrium, each seller chooses a price according to a probability density function $f(p)$. This function therefore effectively defines the strategy of the sellers: when the requirements are put forth by the exchange, the sellers respond with a price p according to its density function $f(p)$. In its choice of pricing strategy, each seller takes the other seller's pricing strategy and the demand behavior of the buyers as given. If its price turns out to be the lower of the two prices, it sells to capacity K and has profits $\Pi_s(p)$. The other seller then sells the residual demand $(Q - K)$ and has profits $\Pi_f(p)$. We analyze the case of a symmetric equilibrium, when both sellers choose the same pricing strategy (which is reasonable, since both the sellers are identical). □

Proposition 2. *The function $f(p) = 0$ when $p > r$ and $p \leqslant c$.*

Proof. Above the reservation price, nothing will be bought. At a price equal to marginal cost, it is better to be undersold and sell to the residual demand and make positive profits. □

Proposition 3. *There is no equilibrium where both sellers charge the same price with positive probability.*

Proof. Similar to that of Proposition 1. The best response of each seller is to undercut the other by ε, so charging the same price cannot be a Nash equilibrium. □

We can therefore concentrate on establishing the nature of a price randomizing solution.

Proposition 4. *There are no point masses in the equilibrium pricing strategies.*

Proof. If the price p were charged with some positive probability, there would be a positive probability of a tie at p. By Proposition 3, that is not possible. □

Since there are no point masses in the equilibrium density function, we can concentrate on a cumulative distribution function that will be continuous on some range (p_1, p_2), such that $c < p_1 < p_2 \leqslant r$. Let $f(p)$ be the cumulative distribution function for $f(p)$; from Proposition 4, we can state that $f(p) = F'(p)$ almost everywhere. In other words, (p_1, p_2) represents the range of the support of the mixed-strategy in prices that the two players engage in. In this respect, the model has some similarities to [12].

Proposition 5. *The maximum of the support of prices is given by $p_2 = r$.*

Proof. If a seller is to sell to the residual demand (since at price p_2 it will be definitely undersold – there cannot be any ties), it is best to sell it off at the highest possible price that he can get, which is r. □

We now proceed to expected profit function of a seller. When a seller charges a price $\hat{p}$, two events are possible. It may be that $\hat{p}$ is the smallest price charged, an event which we term as a success, in which case, the seller sells to capacity with profit $\Pi_s(\hat{p})$. This happens when the other seller charges a price higher than $\hat{p}$, an event that happens with probability $(1 - F(\hat{p}))$. On the other hand, the seller might be undersold (we call that event a *failure*), and in that case, the profit it makes is $\Pi_f(\hat{p})$, an event that occurs with probability $F(\hat{p})$. (Proposition 3 shows that we can leave out the possibility of a tie.) Hence the expected profit of either seller at any price $\hat{p}$ is

$$\Pi_s(\hat{p})(1 - F(\hat{p})) + \Pi_f(\hat{p})F(\hat{p}), \tag{1}$$

where

$$\Pi_s(\hat{p}) = \hat{p}K - cK = (\hat{p} - c)K \tag{2}$$

and

$$\Pi_f(\hat{p}) = \hat{p}(Q - K) - c(Q - K) = (\hat{p} - c)(Q - K). \tag{3}$$

We next prove that the expected profit at any price is constant. Let the strategy space of seller i ($i = 1, 2$) be denoted by S_i. Consider a sequence of finite approximations S_i^n of S_i converging on to S_i for all i. By Nash's existence theorem, each discretized game with strategy sets $x_i S_i^n$ has a mixed-strategy equilibrium. The expected profit at any price in the discrete strategy space has to be equal, for otherwise a player would place higher probabilities for those prices that yield higher payoffs. By the law of weak convergence, there is a subsequence of Nash-equilibrium mixed-strategy profiles, which without loss of generality can be taken to the above sequence itself, which converges to the mixed-strategy $f(p)$ on S_i. Since the expected payoff (i.e., the profit) is equal for any price in the discrete mixed-strategy profiles, the same holds true when the sequence converges onto $f(p)$. Thus, we must have for all prices p

$$\Pi_s(p)(1 - F(p)) + \Pi_f(p)F(p) = \Pi, \tag{4}$$

where Π is the value of the expected profit at any price.

Re-arranging (4), we get

$$F(p) = \frac{\Pi_s - \Pi}{\Pi_s - \Pi_f}. \tag{5}$$

At $p = r$, the seller in question sells to the residual demand, with (expected) profit

$$\Pi_f(r) = (r - c)(Q - K) \tag{6}$$

By the reasoning above, this should be equal to Π.

Similarly, at $p = p_1$, the seller sells to capacity, with (expected) profit

$$\Pi_s(p_1) = (p_1 - c)K = \Pi \tag{7}$$

as before.

Using (2), (3), (5) and (6) we get the expression for $F(p)$

$$F(p) = \frac{(p - c)K - (r - c)(Q - K)}{(p - c)(2K - Q)}, \tag{8}$$

which is of the form

$$F(p) = A - \frac{B}{(p - c)}, \tag{9}$$

where

$$A = \frac{K}{(2K - Q)} \quad \text{and} \quad B = \frac{(r - c)(Q - K)}{(2K - Q)} \tag{10}$$

are constants.

We must have $F(p_1) = 0$. Using (8) above, we get

$$p_1 = \frac{(r - c)(Q - K)}{K} + c. \tag{11}$$

Note that this expression can also be obtained from the fact that $\Pi_s(p_1) = \Pi_f(r) = \Pi$, as well as from the proof of Proposition 1. We note that since $r > c$ and $Q > K > 0$, $p_1 > c$, i.e., the sellers always charge higher than their marginal cost.

Note that at the limit when $2K \to Q$ (i.e., the two sellers can just clear the market), $p_1 \to r$, i.e., the density function tends to a point mass at $p = r$. This is expected since the two firms need not compete to quote the lower price, and can afford to price their quantities at the market-clearing price, which is the reservation price r.

The following proposition defines the nature of $F(p)$.

Proposition 6. *$F(p)$ is strictly increasing in its support.*

Proof. If not, let $p' < \hat{p} < p''$, where p', $\hat{p}$ and p'' lie within the support of $f(p)$, where $F(p') = F(p'') < 1$. Now $\hat{p}$ succeeds in being the lowest price in the same circumstances that p' succeeds in being the lowest price: when the other price is higher than p''. Similarly, $\hat{p}$ fails to be the lowest price when the other seller charges a price less than p', in which case p' also fails to be the lowest price. But in either case, since $\hat{p} > p'$, charging $\hat{p}$ will result in larger profits than charging p'. □

We now have a complete characterization of the equilibrium price density function: $f(p) > 0$ for all p in (p_1, r) and $f(p) = F'(p)$, where $F(p)$ is given by (8) above. We thus get

$$f(p) = F'(p) = \frac{(r-c)(Q-K)}{(2K-Q)(p-c)^2}. \tag{12}$$

From (11) we get

$$f(p_1) = \frac{K^2}{(2K-Q)(r-c)(Q-K)} \tag{13}$$

and

$$f(r) = \frac{(Q-K)}{(2K-Q)(r-c)}. \tag{14}$$

Eqs. (12)–(14) characterize the entire equilibrium price distribution function.

Fig. 1 shows that approximate shape of the graph of $F(p)$.

Fig. 2 shows the approximate nature of the graph of $f(p)$.

Thus, the sellers randomize between prices (p_1, r) with monotonously decreasing probability densities as prices increase.

The reason for this behavior of $f(p)$ can be understood by looking at the nature of the profit function in Eq. (4).

Another way to describe the nature of $f(p)$ would be as follows. Differentiating (4) with respect to p on both sides, we get on re-arranging

$$f(p)[\Pi_s(p) - \Pi_f(p)] + F(p)[\Pi_s(p) - \Pi_f(p)]' = K. \tag{15}$$

The left hand side of Eq. (17) can be seen to be the derivative of $F(p)[\Pi_s(p) - \Pi_f(p)]$, which can be termed as the expected "regret" of being the higher priced seller at any price p. Since this expected regret increases linearly with p (the derivative is a constant), sellers will tend to stack up their equilibrium densities at lower prices, explaining the monotonously decreasing curve of $f(p)$.

It will be of interest to find out the average price paid by the buyers under the equilibrium price density function. The average price is simply

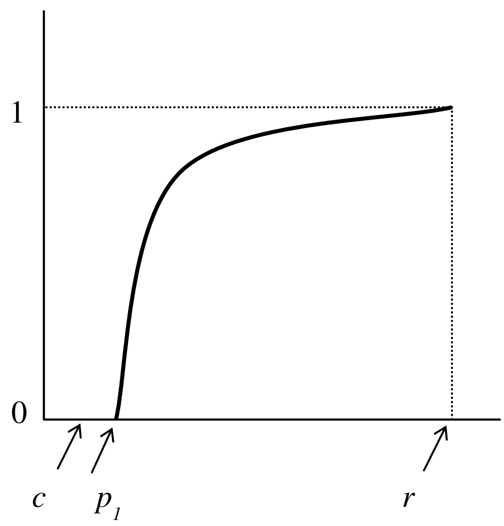

Fig. 1. Graph of $F(p)$.

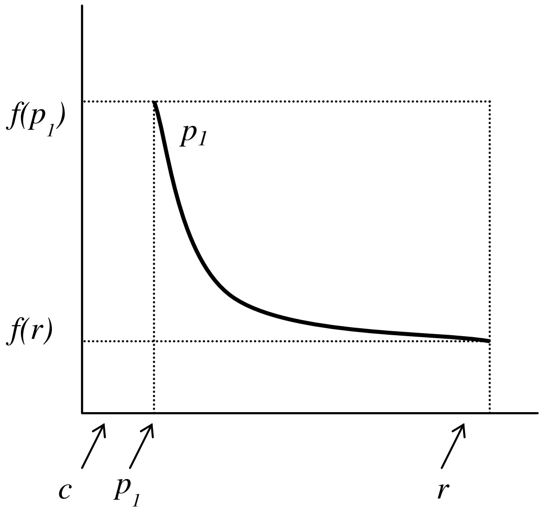

Fig. 2. Graph of $f(p)$.

$$\bar{p} = \int_{p_1}^{r} pf(p)\,\mathrm{d}p. \tag{16}$$

Integrating by substitution gives us

$$\bar{p} = \frac{(r-c)(Q-K)}{(2K-Q)}\left[\ln\left(\frac{r-c}{p_1-c}\right) + c\left(\frac{1}{p_1-c} - \frac{1}{r-c}\right)\right]. \tag{17}$$

One can check that $\frac{\delta F(p;c)}{\delta c} < 0$, and therefore $F(p;c_2)$ first-order stochastically dominates $F(p;c_1)$ for any $c_2 > c_1$. The average price $\bar{p}$ therefore increases in c. We now illustrate the results with a numerical example.

Let each buyer require a quantity 1 of the commodity, and have a reservation price equal to 1. Each seller can supply a maximum quantity of 1.5. They also have a marginal cost of 0.2. That is, $Q = 2,\ K = 1.5,\ r = 1,\ c = 0.2$.

Inserting these values in Eqs. (11) and (17) gives us $p_1 = 0.467$ and $\bar{p} = 0.64$.

The sellers could have (theoretically at least) sold their products to the buyer at the reservation price of $r = 1$. Now, due to the competition induced by the exchange, the average price has fallen by 36%. It must be noted that the average drop in prices as found from the above analysis is probably somewhat higher than what could be expected in real life. It is conceivable that large buyers, even in the absence of competition among sellers, would manage to force their preferred vendors to charge less than their reservation price r, by introducing the realistic threat of taking their business elsewhere otherwise. For an industry with a limited number of buyers, this is a credible threat indeed. However, it is instructive to note the effect of the exchange, whereby a substantial price reduction can be brought about by just the effect of increased competition.

Let us denote the lower quoted price to be $p_{\min}$. Then the density function of $p_{\min}$ is given by

$$f(p_{\min}) = 2(1 - F(p))f(p). \tag{18}$$

Thus the average of the lower quoted price, $\bar{p}_{\min}$, is given by

$$\bar{p}_{\min} = \int_{p_1}^{r} pf(p_{\min})\,\mathrm{d}p. \tag{19}$$

This gives us the value of $\bar{p}_{\min}$ as

$$\bar{p}_{\min} = \frac{2c(r-c)(Q-K)^2}{(2K-Q)^2}\ln\left(\frac{r-c}{p_1-c}\right) + \frac{(r-c)(Q-K)}{(2K-Q)}(r-p_1) - \frac{(r-c)(Q-K)}{(2K-Q)}\left(1-\frac{cK}{(2K-Q)}\right)\left(\frac{1}{r-c}-\frac{1}{p_1-c}\right) - \frac{c(r-c)(Q-K)}{2(2K-Q)}\left(\frac{1}{(r-c)^2}-\frac{1}{(p_1-c)^2}\right). \tag{20}$$

For the numerical values we assumed to calculate p_1 and $\bar{p}$, the value of $\bar{p}_{\min}$ turns out to be 0.55.

Similarly, let us denote the higher quoted price to be $p_{\max}$. Then the density function of $p_{\max}$ is given by

$$f(p_{\max}) = 2F(p)f(p) \tag{21}$$

and the average of the higher quoted price, $\bar{p}_{\max}$, is given by

$$\bar{p}_{\max} = \int_{p_1}^{r} pf(p_{\max})\,\mathrm{d}p. \tag{22}$$

This gives us the value of $\bar{p}_{\max}$ as

$$\bar{p}_{\max} = c\left[\frac{(r-c)(Q-K)}{(2K-Q)}\right]^2\left(\frac{1}{(r-c)^2}-\frac{1}{(p_1-c)^2}\right) + \frac{(r-c)(Q-K)}{(2K-Q)}\left[\frac{(r-c)(Q-K)-cK}{(2K-Q)}\right]\left(\frac{1}{r-c}-\frac{1}{p_1-c}\right) + \frac{K(r-c)(Q-K)}{(2K-Q)}\ln\left(\frac{r-c}{p_1-c}\right). \tag{23}$$

Again, for our numerical values above, we compute $\bar{p}_{\max} = 0.76$.

4. Extending the model to a *n*-seller case

A similar argument can be followed to find the equilibrium density function in the case when there are n sellers. The expressions for the various functions will be more complicated than the two-seller model, but we can get enough insight to comment on the nature of the competition when there are n players.

We first consider the simpler case when all the n sellers are required to supply the entire demand, but however their total capacity exceeds that demand, i.e., $(n-1)$ sellers supply to capacity, and the last seller supplies to the residual demand $(nK > Q) > (n-1)K)$. The demand is fulfilled the same way, with a lower priced seller getting preference over a higher priced seller to supply any residual demand. A seller *fails* if its price turns out to be the highest price among all the sellers (an event that happens with a probability of $(F(p))^{n-1}$) and *succeeds* otherwise (an event that occurs with a probability of $(1-(F(p))^{n-1})$). The profits during these events are signified by $\Pi_{\mathrm{s}}(p)$ and $\Pi_{\mathrm{f}}(p)$ respectively, as before, and the profit at any price on the support is Π. Using similar arguments as before, we get

$$\Pi_{\mathrm{f}}(F(p))^{n-1} + \Pi_{\mathrm{s}}(1-(F(p))^{n-1}) = \Pi, \tag{24}$$

which gives

$$F_n(p) = \left(\frac{\Pi_{\mathrm{s}}-\Pi}{\Pi_{\mathrm{s}}-\Pi_{\mathrm{f}}}\right)^{1/n-1}, \tag{25}$$

where we introduce the subscript n in the last step to differentiate between the current expression and (5) above.

The expression for $\Pi_{\mathrm{s}}(p)$ remains unchanged, while the expression for $\Pi_{\mathrm{f}}(p)$ changes to

$$\Pi_{\mathrm{f}}(p) = (p-c)(Q-(n-1)K). \tag{26}$$

For $n = 2$, the expression in Eq. (27) reduces to (5), as expected. It can also be easily checked that the support of the strategy changes to (p_1^n, r), where p_1^n is given by

$$p_1^n = \frac{(r-c)(Q-(n-1)K)}{K} + c. \tag{27}$$

The expression for $f_n(p)$ can be found to be

$$f_n(p) = \frac{1}{n-1}\left(\frac{(p-c)K-(r-c)(Q-K)}{(p-c)(nK-Q)}\right)^{2-n/n-1}\left(\frac{(r-c)((Q-K)}{(nK-Q)(p-c)^2}\right). \tag{28}$$

There does not seem any easy way to find the expression for the average price from the above expression, and indeed there is no closed-form solution for the average price $\bar{p}_n = \int_{p_1}^{r} pf_n(p)\,\mathrm{d}p$ for a generalized n. For $n = 3$, the average price is given by

$$\begin{aligned}\bar{p}_3 = {} & \sqrt{\frac{K^2}{(5KQ-6K^2-Q^2)(r-c)}}\left(\frac{(Q-2K)(r-c)}{K}\right)^{3/2}\frac{\ln\left(\frac{(Q-2K)(r-c)}{\sqrt{K}}\right)}{2\sqrt{K}} \\ & + ((Q-2K)(r-c))^{3/2}\ln\left(2(r-c)\sqrt{(3K-Q)}\right) + \frac{(4K-Q)(r-c)}{2\sqrt{(3K-Q)(r-c)}}.\end{aligned} \tag{29}$$

When n changes from 2 to 3, the fact that the relative values of Q and K change (our assumption is $nK > Q > (n-1)K$, and if this is to be satisfied, as n goes from 2 to 3, either the total demand increases or the capacities of the sellers decrease) does not allow for a direct comparison between the average prices, but we can nonetheless gain an intuition about its nature from the following observations.

Consider the case when there are two players, with the higher priced seller supplying the residual. The third seller then does not enter the market (as we show in the next section, such an entry would reduce the competition to Bertrand). If the demand increases, or if the capacities of the sellers decrease, such that the two existing sellers cannot supply to the total demand, the third seller enters the market. Let us assume that the quantity of the residual demand is the same as before (so that the profit made by the highest priced seller is the same in both cases). In this scenario, a seller is "successful" as long as he can bid lower than at least one of the two other sellers (i.e. either he is the lowest priced bidder, or he is the second-lowest priced bidder). The higher probability of success by pricing low induces lower average prices as compared to the case when there are two sellers, when a seller is successful only when he is the lowest priced bidder. Following a similar chain of reasoning, we can generalize the result for any n. However, as mentioned before, no generalized observations can be made, as the exact value of the average price will depend on the relative magnitudes of Q and $(n-1)K$.

5. Generalized model

We now extend our model to a more generalized framework – that of n identical suppliers with capacity K each, each competing to get a part of the total requirement Q; however, only m suppliers are needed to supply the entire quantity, with $(m-1)$ suppliers supplying to capacity, and one supplier supplying the residual, i.e.,

$$(m-1)K < Q < mK. \tag{30}$$

The remaining $(n-m)$ suppliers do not supply anything, where $(n-m) \geqslant 1$, to distinguish this scenario with the previous ones.

As in the case for two sellers, we can show that

Proposition 7. *There can be no equilibrium in pure strategies.*

Proof. If there is, then the resulting Nash equilibrium should result in the same payoff for all players, as the players are equivalent. If that is not the case, a player choosing a price (strategy) that gives him a lower payoff is better off choosing a price that gives a higher payoff, and with identical players that employ identical strategies with identical payoffs, that is not possible. To get the same payoff, all players must choose the same price in a pure strategy, but that clearly is not a best response, since the best response by a player to a price p chosen by the other $(n-1)$ players is to select a price $(p-\varepsilon)$. □

We can also show using similar reasoning that there cannot be any equilibrium with a price tied above cost, and therefore there are no point masses in the equilibrium pricing strategies.

Therefore, if there exists a support for prices above marginal cost, there can be only a mixed-strategy equilibrium in that support. Let the highest price in that support be p'. Since only a fraction of the suppliers can cater to the demand, at the price p', the supplier does not sell anything, and therefore his profits are zero. Further, as p' lies on the support, the expected profit at that price should equal the expected profit at any other price, i.e., the expected profit at any price should be zero. Therefore, the seller does not have any incentive to charge p', and since p' can in fact be *any* price above marginal cost, it is apparent that the seller does not have any incentive to charge any price that is above the marginal cost c. Thus, the support of the strategy collapses to a point $p = c$: all suppliers supply at cost.

In fact, the intuition of this result can be gleaned from the expression for $f(p)$ in Eq. (8). If $K \rightarrow Q$, we have two sellers each of who is able to supply the entire demand. Then $F(p) \rightarrow 1$ for any $p > c$, and since no supplier is willing to supply below cost (i.e., $F(p) = 0$ for $p < c$), we have a degenerate probability density function $f(p)$ with point mass at $p = c$.

Thus, the very existence of a situation where only a fraction of the sellers can sell (as opposed to the earlier cases where the seller with the highest price is at least able to sell the residual) is enough to result in a Bertrand competition. In other words, with $n = m + 1$, we have a mixed-strategy with prices above marginal cost. But increasing n to $m + 2$ is enough to reduce the competition to Bertrand.

The implications of the result are clear. It is in the sellers' interest to enter into the auction as long as the total capacity of the bidding sellers is such that at the very least the highest-priced bidder can sell to a residual demand. However, if there is the slightest overcapacity, such that there is one extra seller than required to supply to the required demand, the sellers are reduced to selling at marginal cost. On the other hand, it is in the interest of the buyers to introduce as much competition as possible, so as to drive down the price (since the average price falls with increasing n, and is reduced to marginal cost if $n > m$.

It is very important in the sellers' interest therefore that the reverse auction mechanism lets each prospective bidder know at the very least the total quantity being bid for so far, so that he can decide whether it makes sense for him to bid or not. This is a crucial design requirement, as sellers have been reluctant to the myriad exchanges that have cropped up for every industry, fearing (and as our analysis shows, rightfully so) that it will lead to the buyers extracting the entire surplus from the sellers in a transaction. The exchanges should promote competition, but suitable information sharing will ensure that the competition results in an equitable division of the surplus.

The results help us understand the dynamics of entry in this type of a reverse auction setting. Players will enter as long as they know that they will definitely supply to some unfulfilled demand. However, it is interesting to note that severity of competition depends very much on the relative magnitude of the residual demand as compared to the players' capacity. If the residual demand is such that the highest-priced bidder ends up supplying nearly his entire capacity, the competition is relatively "mild", as the players know that regardless of whether they win or lose, they will end up supplying most of their capacity (the fact that they supply this residual demand at a higher price compared to others also helps). However, if the residual demand is a small fraction of the players' capacity, the price war (and therefore the expected price) is that much more severe, since the consequences of being the highest priced seller are dire.

6. Managerial insights

The inherent attraction of B2B marketplaces lies in their promise of bringing efficiency to the supply chain. Buyers can get better prices from suppliers simply from the effects of increased liquidity and therefore the increased competition among sellers. In our introduction we quoted the comments of a seller who argued that the exchanges hamper any long-term partnership, and it is now easy to see why – since sellers randomize their pricing, they are never sure if they are going to win.

The analysis also shows that industry overcapacities will be severely penalized. Traditionally, each buyer would conceivably have a limited set of suppliers for supplying a particular raw material or component, as the costs of analyzing quotations from a large universe of sellers would have been very expensive. Thus, we can think of scenarios where there is overcapacity as a whole in the industry for that component, but since only a limited number of sellers competed for any business, they could extract economic profits from a transaction. But with an open exchange, the number of competitors for any RFP is greatly increased. As our analysis shows, the presence of just a single "superfluous" supplier reduces the competition to Bertrand. While negotiations for lower prices for raw materials have always been an integral component of the procurement process, online exchanges have helped exacerbate the competition among suppliers. Thus, component manufacturers can expect to be more stringent in dealing with any excess capacity, which in turn means that any demand fluctuations for the end product would swiftly be felt all through the supply chain. Managers will have greater responsibility in forecasting industry downturns or upswings, and scalability (in either direction) in manufacturing capabilities would be a major competitive edge. This will ring even more true as competitive pressures of the online marketplaces are expected to drive commoditizing of the raw materials.

In order to survive such potentially fatal price reductions, suppliers will therefore be needed to innovate and implement new features in their products. However, much of the impetus of doing so will have to be self-motivated, since OEMs might not be willing to try out higher-priced components that raise the cost of the end product. This will possibly mean that suppliers will have to proactively consider the end consumer, in order to educate them of their products' innovations in order to create a 'pull' for these products (Forrester Research provides an example: to get consumers to buy its active head-restraint system through GM's online configurator, Autoliv, a Tier 1 supplier, must reach consumers and educate them on the system's benefits – something Autoliv never had to do when it was just selling to the OEMs [13]).

7. Future directions of research

Some future directions of research would include considering asymmetric costs and capacities. Generalizations to n sellers would be difficult in such cases, but it would be interesting to consider the equilibria where some sellers have inherent cost or capacity advantages. Appendix A.3 refers to the intuition behind the equilibrium in a game between two players who have different marginal costs – the seller with the higher marginal cost would sometimes charge the reservation price r and cater to the residual demand, and at other times engage in random "sales", while the seller with the lower marginal cost will always randomize its bids. In the retailing world, this pricing behavior might help explain why competition in an oligopoly might result in one competitor randomizing its prices and another preferring to have sudden "sales" at different times of the year. Specifically, in the United States, Wal-Mart resorts to "everyday low pricing" without ever resorting to "sales", while its competitors have "sales" during certain times of the year. Wal-Mart, with its arguably lower cost structures can randomize its prices everyday (without ever having "sales"), while its competitors, hampered by their higher costs, on average have higher prices and sometimes engage in "sales".

Another possible direction of research would be the characteristics of the game when the preservation price r is not common knowledge, or might be subject to change with changing buyer requirements.[8]

We are currently trying to implement a B2B reverse auction in a synthetic environment, where semi-intelligent sellers learn over time to converge to an optimum strategy. Some questions that we hope to answer include: can such agents learn over time to converge to an optimal game-theoretic equilibrium? If yes, how fast do they converge to those equilibria? Our initial findings are encouraging – using relatively simple heuristics, we found that artificial software agents converge upon the ideal behavior over time for simple 2-player games [14]. Extensions of such research can have useful implications in the implementation of automated mechanisms by sellers in transactions of this kind.

8. Conclusion

We have shown the effect of increased competition on prices within a B2B marketplace. We analyzed the equilibrium for a two-seller case, and then extended our insights from there to an n-seller case. The generalized model showed the effects of overcapacity, and how the presence of even a single superfluous bidder can reduce the competition to Bertrand. This helps us understand the mechanics of entry in such markets. The results also

[8] We thank an anonymous reviewer for this suggestion.

show why larger suppliers might be more willing to join such exchanges as compared to smaller suppliers, who are inherently at a disadvantageous position as compared to their larger competitors.

The model simplifies real-life conditions, since sellers cannot be expected to have identical costs and capacities in real life. A future direction of research can be to include these changes in the analysis (we outline a proof in Appendix A that shows the nature of the equilibrium when the two players have different marginal costs of production). However, the simplified model does bring out the essential characteristics of such a competition. Industrial marketing experience suggests that sellers would resort to price randomizations, which is what our analysis shows. Further, as expected, the equilibrium density function of prices $f(p)$ decreases monotonously with price. The model captures some of the unique features that we can expect to see in an exchange (limited number of sellers with huge buying power, modified capacity constraints, inelastic demand function below reservation price), and shows how competition in those environments can bring down the overall price paid by the buyers.

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Appendix A

A.1

See Fig. 3.

A.2

See Table 1.

A.3. Nature of the asymmetric equilibrium with two players

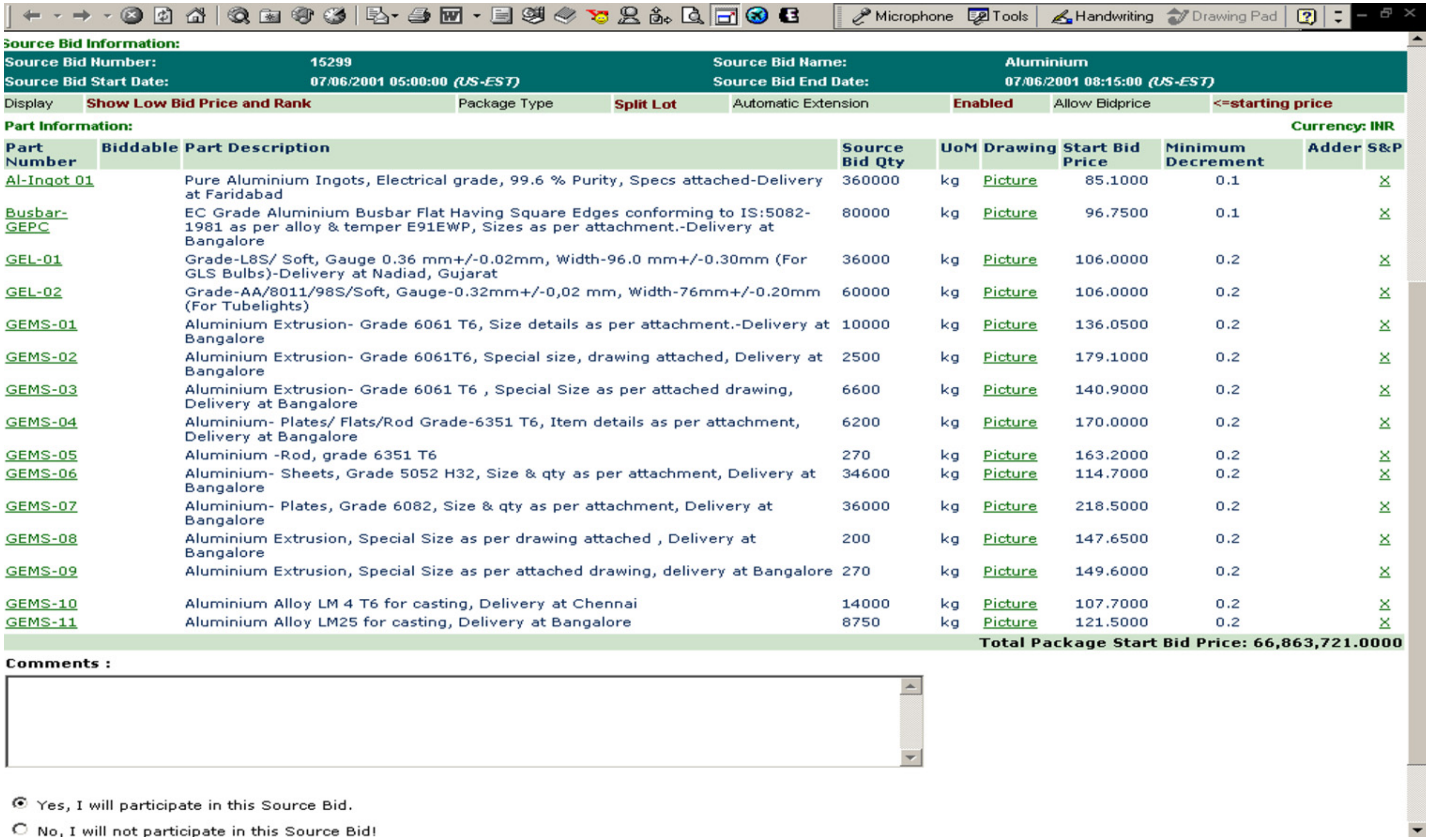
If c_L and c_H are the marginal costs of production of the two suppliers, where $c_L < c_H$, it is easy to see from prior analysis that neither supplier has any incentive to set his price below p_H^*, where

$$p_H^* = \frac{(r - c_H)(Q - K)}{K} + c_H. \tag{A.1}$$

Note that the seller with the inherent cost advantage can *always* win by pricing his bid at p_H^*, which invokes the best response from the other seller to price his bid always at r. However, pricing his bid at p_H^* is not the best response of the lower priced seller (note also that a pure strategy response by both sellers is not possible). In fact, he can do better: he can randomize his bids in the interval (p_H^*, r) in such a fashion so that the best response of the higher priced seller is still the pure strategy of $p = r$. In other words, his best response to the pure strategy of the higher-priced seller (who since he has no other option but to lose, sets his price to the maximum to extract the highest possible surplus) is to randomize his prices in a manner that maximizes his expected profit, and still keep his competition pegged (at least, for some times) at r. Mathematically the problem is as follows:

$$\text{Find } F(p) \text{ that maximizes } \int_{p_H^*}^{r} K(p - c_L) f(p) \tag{A.2}$$

where $f(p) = F'(p)$.



Source Bid Information:

Source Bid Number:	15299	Source Bid Name:	Aluminium
Source Bid Start Date:	07/06/2001 05:00:00 *(US-EST)*	Source Bid End Date:	07/06/2001 08:15:00 *(US-EST)*

Display	Show Low Bid Price and Rank	Package Type	Split Lot	Automatic Extension	Enabled	Allow Bidprice	<=starting price

Part Information: Currency: INR

Part Number	Biddable	Part Description	Source Bid Qty	UoM	Drawing	Start Bid Price	Minimum Decrement	Adder	S&P
Al-Ingot 01		Pure Aluminium Ingots, Electrical grade, 99.6 % Purity, Specs attached-Delivery at Faridabad	360000	kg	Picture	85.1000	0.1		X
Busbar-GEPC		EC Grade Aluminium Busbar Flat Having Square Edges conforming to IS:5082-1981 as per alloy & temper E91EWP, Sizes as per attachment.-Delivery at Bangalore	80000	kg	Picture	96.7500	0.1		X
GEL-01		Grade-L8S/ Soft, Gauge 0.36 mm+/-0.02mm, Width-96.0 mm+/-0.30mm (For GLS Bulbs)-Delivery at Nadiad, Gujarat	36000	kg	Picture	106.0000	0.2		X
GEL-02		Grade-AA/8011/98S/Soft, Gauge-0.32mm+/-0,02 mm, Width-76mm+/-0.20mm (For Tubelights)	60000	kg	Picture	106.0000	0.2		X
GEMS-01		Aluminium Extrusion- Grade 6061 T6, Size details as per attachment.-Delivery at Bangalore	10000	kg	Picture	136.0500	0.2		X
GEMS-02		Aluminium Extrusion- Grade 6061T6, Special size, drawing attached, Delivery at Bangalore	2500	kg	Picture	179.1000	0.2		X
GEMS-03		Aluminium Extrusion- Grade 6061 T6 , Special Size as per attached drawing, Delivery at Bangalore	6600	kg	Picture	140.9000	0.2		X
GEMS-04		Aluminium- Plates/ Flats/Rod Grade-6351 T6, Item details as per attachment, Delivery at Bangalore	6200	kg	Picture	170.0000	0.2		X
GEMS-05		Aluminium -Rod, grade 6351 T6	270	kg	Picture	163.2000	0.2		X
GEMS-06		Aluminium- Sheets, Grade 5052 H32, Size & qty as per attachment, Delivery at Bangalore	34600	kg	Picture	114.7000	0.2		X
GEMS-07		Aluminium- Plates, Grade 6082, Size & qty as per attachment, Delivery at Bangalore	36000	kg	Picture	218.5000	0.2		X
GEMS-08		Aluminium Extrusion, Special Size as per drawing attached , Delivery at Bangalore	200	kg	Picture	147.6500	0.2		X
GEMS-09		Aluminium Extrusion, Special Size as per attached drawing, delivery at Bangalore	270	kg	Picture	149.6000	0.2		X
GEMS-10		Aluminium Alloy LM 4 T6 for casting, Delivery at Chennai	14000	kg	Picture	107.7000	0.2		X
GEMS-11		Aluminium Alloy LM25 for casting, Delivery at Bangalore	8750	kg	Picture	121.5000	0.2		X

Total Package Start Bid Price: 66,863,721.0000

Comments :

Yes, I will participate in this Source Bid.

No, I will not participate in this Source Bid!

Fig. 3. A B2B RFQ at IndiaMarkets.com, India's largest B2B portal.

Table 1
Explanation of variables used in the model

Variable	Explanation
Q	Total quantity demanded
K	Capacity of each seller
C	Constant marginal cost of each seller
r	Reservation price of each buyer
p_1	Lowest price charged by the sellers
p_2	Highest price charged by the sellers
$f(p)$	Sellers' density function of prices
$F(p)$	Sellers' cumulative density function of prices
$\Pi_s(p)$	Profit on selling to capacity
$\Pi_f(p)$	Profit on being the highest priced seller who sells something
Π	Expected profit at any price level p
$\bar{p}$	Average price paid by the buyers
$\bar{p}_{\min}$	Expected minimum price
$\bar{p}_{\max}$	Expected maximum price
p_1^n	Lowest price in the support of the equilibrium when there are n suppliers
$f_n(p)$	Density function of prices when there are n sellers
$F_n(p)$	Cumulative density function of prices when there are n sellers
$\bar{p}_n$	Average price paid by the buyers when there are n sellers
$E(\Pi)$	Expected profits with variable costs
$p = g(c)$	Price p as a function of cost c
(c_1, c_2)	Range of uniformly distributed cost c
(a, b)	Range of uniformly distributed price p corresponding to cost distribution above
M	Number of sellers selling to capacity in the generalized model

Subject to the condition

$$(p - c_H)[K(1 - F(p)) + (Q - K)F(p)] < (Q - K)(r - c_H) \tag{A.2a}$$

with $F(c_H) = 0$ and $F(r) = 1$.

The seller with the higher marginal cost would at times be forced to price his bid at r, but would also randomize his bids at other times between p_H^* and r. In other words, the seller with the higher marginal cost would sometimes get the residual demand by charging r, and at other times have random sales.

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