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Economic Benefits of Renting Software

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In this article, we analyze the economics of a monopoly firm selling and renting a packaged software product by employing an intertemporal monopoly pricing game to model the firm's pricing strategy. The game models the software product as two versions; the first version is available in the first period and the second, a revised version, is available in the second period. The second version benefits from consumer reports of bugs and requests for additional features. This is modeled using delayed network externalities that take effect only in the second period. We observe that the introduction of the rental product in the first period leads to an increase in profits. We also find that the firm's profits are monotonically increasing with the intensity of the network effect. As the intensity of the network effect becomes stronger, the firm chooses to reduce its prices in the first period to expand the size of its network and later increases prices in the second period. Because many of the customers who choose to rent in the first period subsequently make a purchase in the second period, the firm is able to capture the benefits of network externalities in the first period without reducing sales in the second period. For high levels of network intensity, consumer surplus and social welfare are also higher.

software renting, monopoly, intertemporal monopoly pricing game, network effect, pricing of software products, consumer surplus

1. INTRODUCTION

Rental markets for cars, apartments, and other durable economic goods are well established. Computer hardware rental markets also exist but the same is not true for software products because of a unique characteristic. Unlike other physical goods, it is very easy to copy software, causing enormous problems in enforcing copyright laws. The Business Software Alliance and the Software Publisher's

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Association estimated losses from software piracy to be about $13 billion in 1995 [1]. This problem with intellectual property right enforcement has affected the feasibility and growth of software rental markets. However, several technologies are emerging that would enable software renting on a large scale. For instance, Microsoft's electronic distribution framework lets it implement new pricing and licensing agreements including pay-per-view and software rental [2]. The Electronic Licensing and Security Initiative was formed in May 1996 [3] to build and operate a scalable clearing house infrastructure that will make electronic distribution of software secure, accountable, and quick.

Currently, software is distributed through multiple channels using a variety of media such as floppy disks, CD–ROMs and digital audio tapes. With the advent of the World Wide Web (Web) many vendors are allowing customers to download software directly from their sites.

There are three distinct ways of distributing software. First, the software is sold outright, which means that all rights including copyright for the software change hands. This is usually the case when software consulting companies produce customized software for their clients. Second, packaged software is licensed to a party, wherein the producers allow the licensee to use the software under the terms of the license agreement. The licensee cannot resell, redistribute, modify, or rent the license without the permission of the producer. A typical license is for a specific period of time (e.g., large systems such as Oracle, Kappa, SAP, etc. have annual licenses), but because of the difficulties in the enforcement of such contracts, most mass-marketed software product licenses have no time limitations.

Third, software is rented as a durable good. Software renting is similar to software licensing, but it uses new technologies to enforce a relatively short time limit on the period over which the consumer may use the software [4, 5]. It is illegal to rent software without the copyright owner's permission. Hence, within our definition, software licenses that have a time limitation that is shorter than about half of the product's life are considered to be *rented*, whereas licenses with no time limits or limits that are longer than half of the life of the product are considered to be *licensed*. However, to avoid any confusion with the terminology in common use, we will refer to software licensing as *selling*.

The benefits of renting have been shown in various articles [6]. The goal of this article is to provide insights into the additional benefits that may accrue from renting a software product. One of the key attributes of software products is that it is hard to know how many times software has been copied, but it is easy to know how many times it was run. This difference gives the pay-per-use paradigm for software an advantage over traditional means of selling software. Also, initial fixed costs to produce a software product are high but the marginal cost to produce an additional copy of the software is negligible. This and another important property called *network externality* together suggest that it may be optimal for the producer to increase sales by selling to lower valuation consumers. As it will be explained later, the utility to a consumer from the software product is affected by the number of consumers that use that software product (i.e., the greater the number of users, the greater the utility to each consumer for using the product); this is called network externality or the *network effect*.

The practice of distributing the beta version of a software for free is widespread in the information technology industry. Our model explains this phenomenon as being caused by the fact that the producer would like to have people report bugs, defects, and problems so they can be fixed and hence add value to the first commercial version of the software. This enhances consumers' willingness to pay, thus the higher revenues from the first version exceed the revenue loss from giving the beta version free. Our model shows that it is indeed optimal for the producer to reduce prices of the first version of the software and keep it at a low fixed value when the network effect is strong.

By renting the software, the producer can capture the benefits of the network effect and keep the first period selling prices at a higher level. Hence, the producer can make profits in both periods, and the cumulative profits are higher when it rents the software.

2. RELATED WORK

In economics literature, renting has attracted considerable attention. One of the earliest articles in this area is that of Bulow [7]. In his often cited article, he showed that, although sellers have less monopoly power than renters and nondurable goods monopolists, it is possible that the seller will cause a greater welfare loss than other types of monopolies. The time interval over which a good is consumed is important, because if this period is small, as for perishable goods, then renting it is not viable. Goods that are consumed over a long period of time are referred to as *durable goods.* Examples of durable goods include houses, cars, household appliances, and so forth. We consider software to be a durable good. In the following paragraph, we summarize economic literature related to durable goods monopolies. Durable goods producers can increase profits if they can effectively price discriminate. We provide brief comments about price discrimination in the following section.

Durable goods monopolies have been studied by many researchers. A monopoly firm selling a durable good over multiple periods faces the time inconsistency problem (Coase conjecture, [8]). The time inconsistency problem arises from the following observation about the behavior of the monopoly firm. In a multiperiod game, the monopoly firm can announce the profit maximizing price in the first period. Consumers who value the product more than its price will purchase it in the first period. To sell more products in the second and later periods, the firm must lower the price of the product. Rational consumers with perfect foresight would therefore, prefer to wait until the second period to take advantage of lower prices. Hence, the monopoly competes with itself over current and future periods. The equilibrium solution with the monopoly committing to all future prices is not an equilibrium of the two stage game without commitment. In other words, in equilibrium, consumers expect the firm to charge competitive prices in future periods of the game. Because they can wait for the next offer by incurring small delay cost, they cannot be induced to accept higher prices. Hence, the monopolist ends up charging prices close to the competitive level. Gul and

Wilson [9] verified Coase's conjecture for stationary strategies (i.e., that reducing the period length drives all prices to zero or the least valuation due to intertemporal competition between the firm's product across periods). Ausubel and Deneckere [10] showed that the Coase conjecture fails to hold for nonstationary strategies. Bensaid and Lesne [11] proved that the Coase conjecture does not hold if the product exhibits network externalities and the intensity of the externality is strong.

Introducing software renting allows the monopoly to charge different prices for the sales market and rental market. This causes consumers to self-select into different groups according to their price-product preference. This is called *third-degree price discrimination*. Another instance of such discrimination is the existence of professional, student, and standard versions of software that provide differentiation by varying the nature of the license agreement for the product although these versions have essentially the same functionality.

2.1 Network Externalities

In our article, the benefits from software renting depend on the intensity of the network effect. We explain the network effect, also called network externality, in this section. Consumers' valuation of a product is affected by network externalities associated with the product. A *positive network externality* signifies the fact that the value of a unit of the good increases with the number of units sold. A common example is that of the telephone. Having a telephone provides value only if other people own compatible phones so that one can call them. The value of the phone increases as the number of people you can call increases. Similarly, any communication software such as e-mail software, file transfer, or web browser exhibit network externalities [12]. Almost all durable goods exhibit network externalities in that they draw on common use and repair expertise. This is what Liebowitz and Margolis [13] call *metaphorical network externalities*. Durable goods exhibit at least two other important types of network externalities: the *word of mouth externality* and the *learning by doing externality*. The more the number of initial buyers of a product, the more information regarding its quality is available. This lowers potential customers' search costs. This is called word of mouth externality. The learning by doing externality is like achieving economies of scale. As more products are produced, the producer learns better production and management techniques.

In this paragraph, we mention some related literature on network externalities. Brynjolfsson and Kemerer [14] built a hedonic model to determine the effects of network externalities, standards, intrinsic features, and a time trend on microcomputer spreadsheet software prices. Their findings included increased prices of spreadsheet products under network externalities. Katz and Shapiro [15] analyzed technology adoption in industries where network externalities are significant. In their 1992 article, Katz and Shapiro [16] studied the introduction of a new product in a market with network externalities. They provided conditions under which, in equilibrium, firms tend to converge to new, incompatible technologies. They also analyze the firms' incentives to make their products compatible.

3. MOTIVATION

Short-term rental of durable economic goods is a common everyday occurrence. There is a rich body of literature in finance on the tax related benefits from capital leasing. These papers assume that leasing for time period t is equivalent to purchasing the product, using it for time t and then selling it back. For references and issues related to capital leasing, see Long, Lewellen, and McConnell [17], Miller and Upton [18], and Dill, Myers, and Bautista [19].

An article by Flath [6] advanced several reasons why short-term renting offers economic advantages. He showed that renting reduces transactions costs, including the cost of identifying, assuring, maintaining quality, and costs of search. A commonly cited example is that of automobile renting. The cost of searching and then determining the quality of an automobile is significant; the same is true of the transactions costs in a buy or sell transaction. Consumers who need an automobile for a short duration can reduce these costs by renting. The argument equating renting to buying for a short period of time relies on consumers' ability to sell the used product. However, there cannot exist an organized mass market for used software for reasons outlined in the following paragraph. Therefore, software renting offers much greater economic value as compared to renting of other durable goods.

Typically, firms sell a license to use their software. This may be in the form of an end-user license or a site license. There are numerous restrictions on the sale and transfer of these licenses [20] that reduce the viability of a used software market. The development of a rental market will fill the void caused by the absence of a used software market. In recent years, the development of the Internet and the Web has led to the development of electronic markets featuring dramatic reductions in transactions costs. This can aid a software rental market considerably, especially in view of the high costs of identifying and assuring the quality of used software products in a fast changing information world.

Software products exhibit positive network externalities. For example, the producer learns of bugs as users point them out, and he can then fix some of them in the revised version of the product. He also benefits from reuse of code, design, test cases, documentation, and so forth. Consumers benefit from these externalities when they purchase the upgraded version of the software. Because the second version is sold only in the second period, the first period consumer does not benefit from the (positive) network externalities and only the second period consumers enjoy this benefit. These externalities are called *delayed externalities.* A similar setup for network externalities was first used by Bensaid and Lesne [11].

There are a number of reasons why the consumer may want to rent the product. The consumer of a software product would rent, rather than buy if (a) one uses it for short term projects and would not use it thereafter; (b) one just wants to develop marketable expertise (e.g., learning to use Office 97, SAP, Oracle); (c) one wants to evaluate the software and depending on its usability and performance, would later decide whether to buy it; or (d) one wants to avoid negative externality (i.e., one would rather rent the software than use it in a crowded lab/network).

This article examines the following questions:

- How do prices change with the magnitude of the externality effect?
- What are the extra benefits to the firm from renting the product?
- Do the consumers benefit from the introduction of a rental product? Does the size of the market change?

Although there is a lot of effort to enable the technology for software renting, there is no serious attempt to quantify the benefits of this emerging technology. A number of researchers have examined pricing strategies for a durable product that exhibits network externalities [7, 8, 11, 21]. We are not aware of any work that tries to examine the pricing strategies for renting a software product that exhibits network externalities.

Because the software company observes increasing returns to investment, the possibility of the emergence of short-term natural monopolies is significant. One dominant strategy for the software companies may be to establish a customer base for its product and then make profits by selling upgrades [22]. The passage of the Computer Software Rental Act in 1990 made it illegal to rent software without the copyright owner's specific permission.

4. THE MODEL

We analyze a market for a software product that exhibits network externalities. We study the case when there is a single producer and examine the behavior of a monopoly firm selling and renting a software product. We take prices to be net of marginal cost and assume that the marginal cost of production is constant.

The model is setup as a two period game. In the first period, the firm sells and rents the first version of the software package. It decides on the selling and renting price in the first period and commits to a selling price for the second period. In the second period, the firm sells only the upgraded version. We assume that the marginal revenue from this quality improvement exceeds the marginal cost of higher quality. Consumers decide whether to buy in period one. Those who have not bought in period one may buy in period two; those who buy the second version will benefit from externalities created by the consumers who purchased or rented in the first period.

Let Ω_1 be the software product sold in the first period, Ω_2 be the product rented in the first period, and Ω_3 be the upgraded version of the product sold in the second period. Let q_1 be the first period quantity demanded for Ω_1, q_2 be the quantity demanded for Ω_2, and q_3 be the quantity demanded for Ω_3 in the second period. We assume that there is a continuum of consumers indexed by the reservation prices $h \in (0, 1)$, with h distributed uniformly in the unit interval (we are not introducing randomness in the model; for a particular consumer, h is fixed). The distribution of h across consumers is known to the firm, but the firm does not know the h for a particular consumer. The parameter $e \geq 0$ is defined as the level of network effect and $\delta \in \{0, 1\}$ is the discount factor. The prices are given by p_1, p_2, and p_3 for Ω_1, Ω_2, and Ω_3, respectively.

For the sake of completeness, we cite the following as a proposition from Bensaid and Lesne [11]:

> Proposition 1: When there is no network effect, the firm has an incentive to lower its price in the second period to sell to the residual consumers, which creates the Coase [8] effect. When a network effect exists, and is large enough, the second period price will be higher than the first period price.

This implies that when network externalities are large enough, there is no time inconsistency problem. Therefore, our results hold even if the firm cannot credibly commit to its second period prices.

We look at two cases, one with renting and the other without renting, separately.

4.1 No Renting Case

In this case we use a two period game where the firm does not produce Ω_2. The consumers either buy in the first period or in the second period or do not buy at all. We employ a model similar to the one used by Bensaid and Lesne [11].

The consumer surplus functions are given by:

$$v_1(h, p_1) = h - p_1,$$

if the consumer buys the software product in the first period;

$$v_2(h, p_3, q_1) = \delta(k_2 h - p_3 + eq_1),$$

if the consumer buys the software product in the second period.

Here and throughout this article, h is the utility derived by a customer from consuming Ω_1 (over two periods), which also serves as an index number representing each customer, hence the consumer's surplus for purchasing Ω_1 is $h - p_1$. Similarly the consumer's surplus from consuming Ω_3 is the sum of $\delta(k_2 h - p_3)$ and δeq_1, where δeq_1 is the benefit from delayed network externalities and hence a function of the number of copies sold in the first period and the intensity of the network effect. The parameter $k_2 \in \{0, 1\}$ models the reduced utility from Ω_3 (as compared to Ω_1) because Ω_3 is available only in the second period, although Ω_1 can be consumed in both periods.

Let h_1 be the lowest reservation price at which a consumer is willing to buy Ω_1, and h_3 be the lowest reservation price at which a consumer is willing to buy Ω_3.

$v_1(h, p_1) - v_2(h, p_3, q_1)$ is increasing in h, therefore all consumers with reservation price greater than h_1 will buy in the first period. Also $v_2(h, p_3, q_1)$ is increasing in

h, therefore all consumers with reservation price greater than h_3 will buy in the second period. Hence, all consumers with reservation price in the interval $(h_1, 1)$ will buy in the first period. The consumers with reservation prices in the interval (h_3, h_1) will buy in the second period and all those with reservation prices falling into the interval $(0, h_3)$ will not buy.

4.2 Boundary Cases

When $h_1 = 0$, the market is covered in the first period by the firm (i.e., everyone purchases the first version of the software). If $h_1 = 1$, the firm will not sell anything in the first period, and if $h_3 = h_1$, the firm sells nothing in the second period. Finally, when $h_3 = 0$, the market is covered in both periods by the firm.

4.3 Model Solution

We calculate the reservation price for the consumer indifferent between buying in the first period and buying in the second period. Because h_1 is the lowest reservation price for the consumer who decides to buy in the first period, he or she must be indifferent between consuming in the first period and consuming in the second period. Similarly, h_3 is the lowest reservation price for the consumer who chooses to buy in the second period, hence he or she must be indifferent between consuming in the second period and never buying at all. From the indifference equations:

$$h_1 - p_1 = \delta(k_2 h_1 - p_3 + eq_1); \tag{1}$$

$$\delta(k_2 h_3 - p_3 + eq_1) = 0. \tag{2}$$

Replacing $q_1 = 1 - h_1$ and solving Equations 1 and 2, we get:

$$h_1 = \frac{p_1 + \delta(e - p_3)}{1 + \delta(e - k_2)}; \tag{3}$$

$$h_3 = \frac{e(p_1 + \delta k_2 - 1) + p_3 - \delta k_2 p_3}{k_2(1 + \delta(e - k_2))}. \tag{4}$$

And from the inequalities,

$$h_1 \leq 1,$$
$$h_1 \geq h_3, \text{ and}$$
$$h_3 \geq 0,$$

by substituting Equations 3 and 4, we get the following regions for p_1, p_3:

$$-1 + \delta k_2 + p_1 - \delta p_3 \leq 0, \tag{5}$$

$$(e - \delta e^2 - k_2 + \delta k_2^2)p_1 + \delta e k_2 - e + (1 - \delta k_2 + \delta e)p_3 - \delta e^2 \le 0, \text{ and} \tag{6}$$

$$e(1 - (p_1 + \delta k_2)) - p_3 + \delta k_2 p_3 \le 0. \tag{7}$$

Using Equations 3 and 4, we derive the following demand functions:

$$D_1(p_1, p_3) = q_1 = 1 - h_1 = \frac{1 - \delta k_2 - p_1 + \delta p_3}{1 + \delta(e - k_2)}$$

$$D_2(p_1, p_3) = q_3 = h_1 - h_3 = \frac{p_1 k_2 - e(p_1 - 1) - p_3}{k_2 + \delta k_2(e - k_2)}.$$

Note that these demand functions are valid in the region given by the inequalities (5, 6, 7). The profit function is simply given by $\Pi = p_1 D_1 + \delta p_3 D_2$.

The following figures plot the optimal prices and demand functions for specific parameter values. We observe that the second period price increases as the intensity of network externality increases. This is in line with our expectations given the assumptions in the beginning of this section. From the behavior of the demand functions, we also see that as the externality intensifies, second period demand increases due to the network effect, and the first period demand declines to a fixed level. Because the model is mathematically intractable, we solve this model for reasonable values of k_2 and δ.

Example 1: The parameter k_2 models the consumer's willingness to pay for Ω_3 relative to Ω_1. Setting this parameter to 0.5 would mean that the length of Period 1 is equal to the length of Period 2, hence each is about half of the life of the product. We solve

$$\max \Pi = p_1 D_1 + \delta p_3 D_2$$
$$\text{subject to (ST) inequalities (5, 6, 7),}$$

to get the optimal price and profit functions with parameter values $\delta = 0.9$ and $k_2 = 0.5$. They are given by

$$p_1^* = \begin{cases} \dfrac{0.55 + 0.9e - 0.9e^2}{1.1 - 1.8e + 0.9e^2} & \text{if } e < 1 \\ 0.275 & \text{if } e \ge 1, \end{cases}$$

$$p_3^* = \begin{cases} \dfrac{0.5(-0.55 - 1.45e)}{-1.1 - 1.8e + 0.9e^2} & \text{if } e < 1 \\ 0.5e & \text{if } e \ge 1, \text{ and} \end{cases}$$

$$\pi^* = \begin{cases} \dfrac{0.45(0.61+e)^2}{((0.55+0.9e)(2.49-e)(0.49+e))} & \text{if } e < 1 \\[2ex] \dfrac{0.20(0.61+e)^2}{0.55+0.9e} & \text{if } e \geq 1. \end{cases}$$

The plots of prices and demands are given below (see Figures 1 through 4).

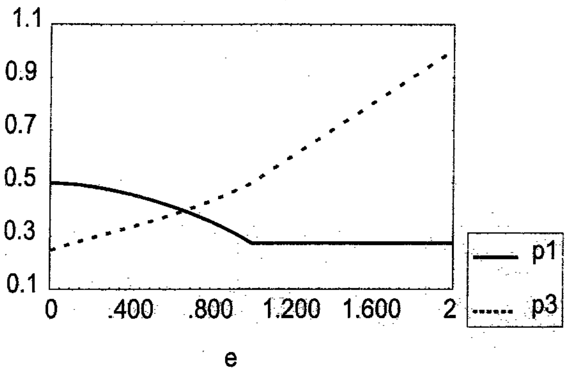

Figure 1. Optimal prices plotted with respect to the intensity of the network effect. Parameter values are: $\delta = 0.9$, $k_2 = 0.5$.

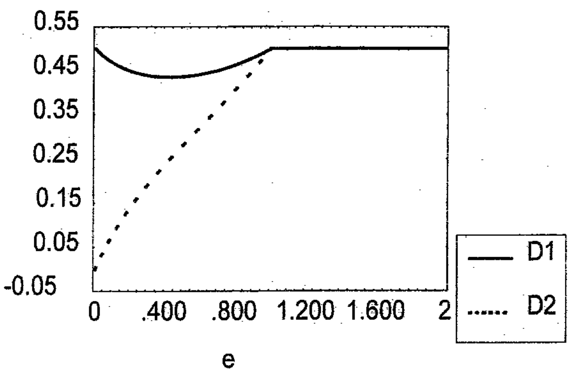

Figure 2. Demand functions calculated for parameter values $\delta = 0.9$, $k_2 = 0.5$. For large values of e, demand lines overlap.

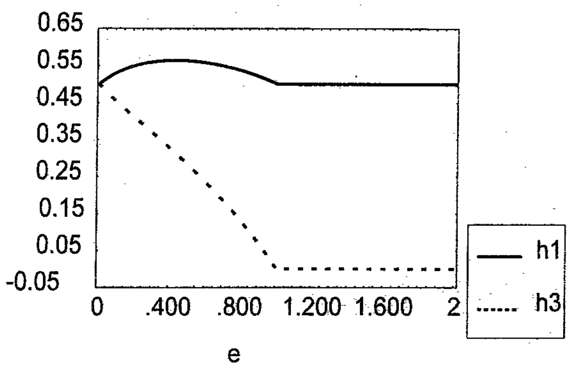

Figure 3. Reservation price levels for no renting case with parameter values $\delta = 0.9$, $k_2 = 0.5$.

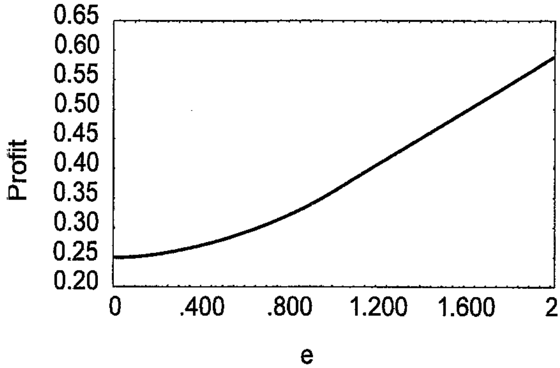

Figure 4. Profit in the no renting case. Parameter values are $\delta = 0.9$, $k_2 = 0.5$.

As the intensity of the network effect increases, the firm reduces p_1 but increases p_3. The price of Ω_1 falls until the market is fully covered. As a result, the total demand increases until $e = 1$. The higher the value of e, the less elastic the demand D_2, hence we observe higher second period monopoly price. When e is sufficiently large, second period consumers enjoy a large positive network effect and hence extra surplus generated by the network effect. This result is in line with Bensaid and Lesne's results [11].

4.4 Renting

Here we employ a two stage game with the firm selling and renting in the first period and only selling in the second period. In the first period of the game, the consumers decide to buy or rent the software product or stay out of the market. If a consumer did not buy in the first period he or she would decide whether to buy the product in the second period. If he or she bought the product in the first period, he or she would do nothing in the second period. The rental market exists only in the first period. Depending on the first period prices, some consumers would either choose to wait until the second period to take advantage of the network externalities or would rent in the first period.

The consumer surplus is given by:

$$\begin{aligned} v_1^B(h, p_1) &= h - p_1 \text{ if the consumer buys the software product,} \\ v_1^R(h, p_2) &= k_1 h - p_2 \text{ if the consumer rents the product, and} \\ &= 0 \text{ otherwise.} \end{aligned}$$

Note that v_1^B is the consumer surplus from consuming the product over two periods.

The present value of the consumer surplus from second period consumption is given by:

$$\begin{aligned} v_2(h, p_3, q_1, q_2) &= \delta(k_2 h - p_3 + e(q_1 + q_2)) \text{ if the consumer buys the software product,} \\ &= 0 \text{ otherwise.} \end{aligned}$$

v_is defined above depend on the parameters $k_1, k_2 \in \{0, 1\}$. These parameters lay out the differences among the three types of products ($\Omega_1, \Omega_2, \Omega_3$) that we are analyzing. The reservation price without the network effect for Ω_2 is k_1h and that for Ω_3 is $\delta k_2 h$. We assume that the sum of the reservation prices of Ω_2 and Ω_3 will be less than the reservation price of Ω_1, namely $(k_1 + \delta k_2)h < h$, and hence $(k_1 + \delta k_2) < 1$.

Let h_1 be the lowest reservation price among consumers who choose to buy Ω_1, h_2 be the lowest reservation price among consumers who choose to rent Ω_2, h_3 be the lowest reservation price among consumers who choose to buy Ω_3, and h_4 be the lowest reservation price among those consumers who choose to rent first period and buy in the second period.

The set of choices available to the consumer can be represented using the notation (x_1, x_2, x_3) where $x_i \in \{0, 1\}$, x_i represents whether or not the consumer chose to buy Ω_1 in the first period, x_2 represents his or her choice regarding Ω_2 in the first period, and x_3 represents his or her choice regarding Ω_3 in the second period. So, for example, (0, 1, 1) represents the action of renting in the first period and buying in the second period whereas (0, 0, 1) represents the action of buying in the second period only. The domain of the consumer's choices is characterized by the space {(0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1), (1, 0, 0)}. The strategy (1, 0, 1) is dominated by the strategy (0, 1, 1) because the second strategy costs less while providing the same benefits; therefore, we do not mention the strategy (1, 0, 1) in the domain. Depending on the values of the parameters k_1, k_2, δ, we have two interesting cases.

4.5 Case 1: $\delta k_2 > k_1$

In this case, the reservation price of Ω_3 without the network effect is higher than the reservation price of Ω_2. Under the condition for this case (i.e., $\delta k_2 > k_1$), the strategy (0, 0, 1) clearly dominates the strategy (0, 1, 0)—see Proposition 9. The domain of the consumers' choices is characterized by the space {(0, 0, 0), (0, 0, 1), (0, 1, 1), (1, 0, 0)}. The points correspond to the strategies in which the consumer can choose (a) not to buy or rent, (b) to buy in the second period, (c) rent in the first period and buy in the second period, and finally (d) buy in the first period, respectively. The strategies can be represented as given in Table 1.

The reservation price in the interval $\{h_1, 1\}$ represents high reservation price consumers who buy in the first period. The reservation price in the interval $\{h_4, h_1\}$ represents the medium reservation price a consumer who would rather rent

Table 1
Strategies for Case 1

	Period 1			*Period 2*	
Strategy	*Do Nothing*	*Buy*	*Rent*	*Do Nothing*	*Buy*
(0, 0, 0)	x			x	
(1, 0, 0)		x		x	
(0, 0, 1)	x				x
(0, 1, 1)			x		x

in the first period and buy, if he or she is satisfied, in the second period. The reservation price in the interval $\{h_3, h_4\}$ would represent a consumer who would buy in the second period. Finally, if the reservation price is in the interval $\{0, h_3\}$, the consumer would neither buy nor rent.

We prove in the Appendix that (see Propositions 8 and 10):

$$1 \geq h_1 \geq h_4 \geq h_3 \geq 0. \tag{8}$$

The demand functions are given by

$$D_1(p_1, p_2, p_3) = q_1 = 1 - h_1,$$
$$D_2(p_1, p_2, p_3) = q_2 = h_1 - h_4, \text{ and}$$
$$D_3(p_1, p_2, p_3) = q_3 = h_1 - h_3.$$

The consumer with index h_1 who is indifferent between the strategies (1, 0, 0) and (0, 1, 1) will have equal surplus from each. This leads to Equation 9 in the following. The consumer with index h_4 who is indifferent between the strategies (0, 0, 1) and (0, 0, 0) will have equal surplus from each. This leads to Equation 10 in the following. Finally, the consumer with index h_3 who is indifferent between the strategies (0, 1, 1) and (0, 0, 1) will have equal surplus from each. This leads to Equation 11 in the following. Thus, we have this set of equations:

$$h_1 - p_1 = \delta(k_2 h_1 - p_3 + e(1 - h_4)) + k_1 h_1 - p_2, \tag{9}$$

$$\delta(k_2 h_3 - p_3 + e(1 - h_4)) = 0, \text{ and} \tag{10}$$

$$k_1 h_4 - p_2 = 0. \tag{11}$$

Solving Equations 9, 10, and 11 for h_1, h_3 and h_4 the following solution is obtained:

$$h_1 = \frac{\delta e + p_1 - p_2 - \frac{\delta e p_2}{k_1} - \delta p_3}{1 - k_1 - \delta k_2},$$

$$h_3 = \frac{-ek_1 + ep_2 + k_1 p_3}{k_1 k_2}, \text{ and}$$

$$h_4 = \frac{p_2}{k_1}.$$

And from the inequalities 8 (see Propositions 8, 9, and 10 in the Appendix for proof):

$$h_1 \leq 1,$$
$$h_1 \geq h_4,$$
$$h_4 \geq h_3, \text{ and}$$
$$h_3 \geq 0,$$

We get the following regions for p_1, p_2, p_3:

$$\delta e + p_1 - p_2 - \frac{\delta e p_2}{k_1} - \delta p_3 - 1 + k_1 + \delta k_2 \leq 0, \tag{12}$$

$$-ek_1 + ep_2 + k_1 p_3 - k_2 p_2 \leq 0, \tag{13}$$

$$p_2 - p_2 \delta k_2 - k_1 \left(\delta e + p_1 - \frac{\delta e p_2}{k_1} - \delta_3 \right) \leq 0, \text{ and} \tag{14}$$

$$ek_1 - ep_2 - k_1 p_3 \leq 0. \tag{15}$$

From the solution to the constrained optimization problem:

$$\max \Pi = p_1 D_1 + p_2 D_2 + \delta p_3 D_3$$
$$\text{ST inequalities (12, 13, 14, 15)}$$

we obtain the following propositions. The proofs as well as the analytical solutions are provided in the Appendix.

Example 2: The parameter k_1 models the consumer's willingness to pay for the rental product relative to Ω_1. Setting this parameter to 0.2 is analogous to short-term renting since the smaller the value of k_1, the lesser the consumer's willingness to pay, and therefore the rental period must have been shorter. The parameter k_2 has the same purpose for Ω_3. Setting this parameter to 0.5 would mean that the length of Period 1 is equal to the length of Period 2, hence each is about half of the life of the product. For $k_1 = 0.5$, $k_2 = 0.2$, $\delta = 0.9$, the optimal prices and profit are given analytically in the Appendix. The plots corresponding to these results are given in Figures 5–8.

We prove the following propositions for the given parameter values $\delta = 0.9$, $k_1 = 0.2$, and $k_2 = 0.5$ in the Appendix. The plots of prices, reservation prices, demands, and profit with respect to e are provided. Complete solutions are also provided in the Appendix.

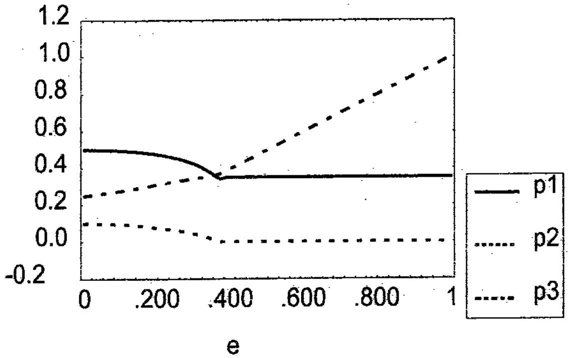

Figure 5. Optimal prices for Case 1 with parameter values $\delta = 0.9$, $k_1 = 0.2$, $k_2 = 0.5$.

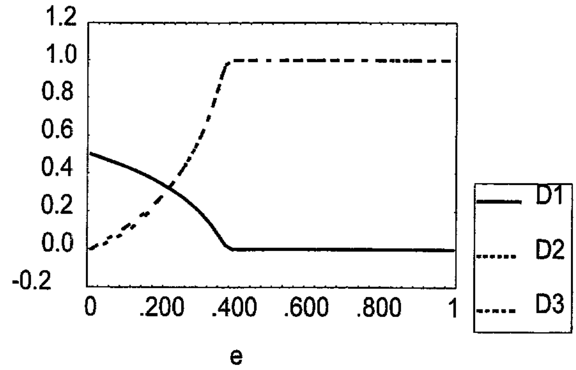

Figure 6. Demand functions calculated for Case 1 with parameter values set at $\delta = 0.9$, $k_1 = 0.2$, $k_2 = 0.5$. Here, D_2 and D_3 are overlapping as e gets higher.

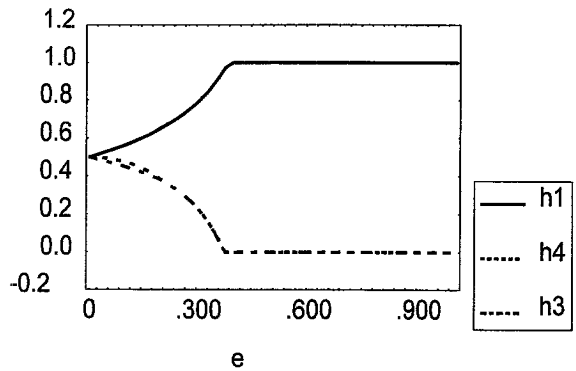

Figure 7. Reservation price levels for Case 1 with parameter values $\delta = 0.9$, $k_1 = 0.2$, $k_2 = 0.5$. Here, h_4 and h_3 coincide as e gets higher.

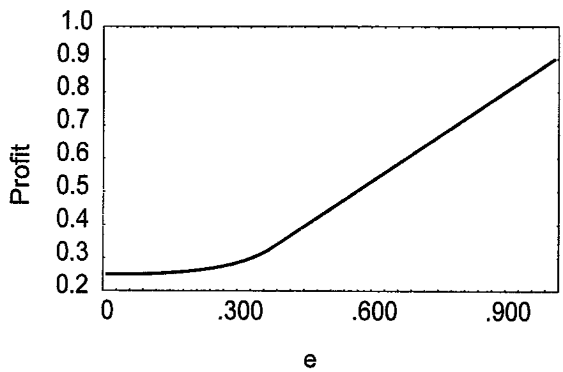

Figure 8. Optimal profit for Case 1. Parameter values are $\delta = 0.9$, $k_1 = 0.2$, $k_2 = 0.5$.

It is to the benefit of the firm to take advantage of the effect of network externalities to increase its profits. We see this behavior in the following propositions.

> Proposition 2: The second period equilibrium price in the renting case is higher, independent of the level of the network effect, than the one in the no renting case.

The network externalities raise the utility of the consumers and therefore their willingness to pay in the second period. The firm takes advantage of this and charges a higher price in the second period.

Proposition 3: The second period equilibrium quantity demanded in the renting case is higher for large enough network intensity than the one in the no renting case.

Another effect of the network externalities is seen in this proposition. Despite higher prices, the firm also has higher sales in the second period. The increase in the number of consumers due to the network effect adds to the already existing demand and therefore increases the quantity demanded. This expansion in the size of the network causes the consumers' willingness to pay to increase further.

Proposition 4: The firm's equilibrium profits are higher when the firm is renting as well as selling its product.

This proposition is intuitive and the logical result of the previous propositions.

Proposition 5: Total consumer surplus is higher, for high enough network intensity, in the renting case than when the firm is not renting.

This along with the previous proposition implies that the net increase in social welfare is shared between the firm and the consumers. Hence, the firm makes higher profits and the consumers enjoy a higher surplus. Therefore, social welfare is improved by introducing software renting.

Proposition 6: Market is covered if $e \geq 0.37$.

The strengthening of the network effect causes increase in overall demand. If the network effect is strong enough then all the consumers participate and the entire market is covered. The consumers can participate in the market by renting the product or purchasing, or both.

4.6 Case 2: $\delta k_2 < k_1$

In this case, the domain of the consumers' choices is characterized by the space $\{(0,0,0), (0,1,0), (0,1,1), (1,0,0)\}$. We prove that the strategy $(0, 0, 1)$ is dominated by $(0, 1, 0)$—(Appendix, Proposition 11). The points correspond to the strategies in which the consumer can choose (a) not to buy or rent, (b) rent in the first period, (c) rent in the first period and buy in the second period, and finally (d) buy in the first period, respectively. We can represent the strategies as given in Table 2.

We prove in Propositions 8 and 12 in the Appendix that:

$$1 \geq h_1 \geq h_4 \geq h_2 \geq 0. \tag{16}$$

The reservation price in the interval $\{h_1, 1\}$ represents high reservation price consumers who buy in the first period. The reservation price in the interval $\{h_4, h_1\}$ represents the medium reservation price consumer who would rather rent in

Table 2
Strategies for Case 2

	Period 1			*Period 2*	
Strategy	*Do Nothing*	*Buy*	*Rent*	*Do Nothing*	*Buy*
(0, 0, 0)	x			x	
(1, 0, 0)		x		x	
(0, 1, 0)			x	x	
(0, 1, 1)			x		x

the first period and buy, if he or she is satisfied, in the second period. The reservation price in the interval $\{h_2, h_4\}$ would represent a consumer who would rent in the first period but would not buy in the second period. Finally, if the reservation price is in the interval $\{0, h_2\}$, the consumer would neither buy nor rent.

When $h_1 = 1$, there are no sales in the first period; when $h_1 = h_4$, there are no consumers who rent in the first period and buy in the second period; when $h_2 = h_4$, there are no consumers who rent in the first period; and when $h_2 = 0$, the market is covered by the firm at the end of the two periods.

$$\begin{aligned} D_1 &= q_1 = 1 - h_1, \\ D_2 &= q_2 = h_1 - h_2, \text{ and} \\ D_3 &= q_3 = h_1 - h_4. \end{aligned}$$

The consumer with index h_1 who is indifferent between the strategies (1, 0, 0) and (0, 1, 1) will have equal surplus from each. This leads to Equation 17 in the following. The consumer with index h_4 who is indifferent between the strategies (0, 1, 0) and (0, 0, 0) will have equal surplus from each. This leads to Equation 18 in the following. Finally, the consumer with index h_2 who is indifferent between the strategies (0, 1, 1) and (0, 0, 1) will have equal surplus from each. This leads to Equation 19 in the following. Thus we have this set of equations:

$$h_1 - p_1 = \delta(k_2 h_1 - p_3 + e(1 - h_2)) + k_1 h_1 - p_2, \tag{17}$$

$$\delta(k_2 h_4 - p_3 + e(1 - h_2)) = 0, \text{ and} \tag{18}$$

$$k_1 h_2 - p_2 = 0. \tag{19}$$

From Equations 17, 18, 19, the following solution is obtained:

$$h_1 = \frac{-p_1 + p_2 + \delta p_3 + k_1 e \delta p_2 - e\delta}{-1 + \delta k_2 + k_1},$$

$$h_2 = k_1 p_2, \text{ and}$$

$$h_4 = \frac{-e k_1 + p_3 k_1 + e p_2}{k_1 k_2}.$$

And from the inequalities from Equation 16, we get the following regions for p_1, p_2, p_3:

$$-1 + \delta k + \delta e + p_1 - \delta p_3 - 2\delta e p_2 \leq 0, \tag{20}$$

$$-p_3 + kp_1 + ep_1 - 2ep_2 \leq 0, \tag{21}$$

$$2(e - k)p_2 + 2\delta k^2 p_2 + p_3 - p_3\delta k - ep_1 \leq 0, \text{ and} \tag{22}$$

$$-k_1 p_2 \leq 0. \tag{23}$$

The demand functions are given by:

$$D_1(p_1, p_2, p_3) = 1 - h_1,$$
$$D_2(p_1, p_2, p_3) = h_1 - h_2, \text{ and}$$
$$D_3(p_1, p_2, p_3) = h_1 - h_4.$$

Example 3: We solve the following constrained optimization problem:

$$\max \Pi = p_1 D_1 + p_2(D_2 + D_3) + \delta D_3$$
$$\text{s.t. inequalities (20, 21, 22, 23)}$$

for $k_1 = 0.5$, $k_2 = 0.5$, $\delta = 0.9$, and the optimal prices and profits are presented in Figures 9–12. Complete solution is provided in the Appendix.

In this case, we examine three possible strategies of the consumer, {(1, 0, 0), (0, 1, 0), (0, 1, 1)}, and we solve the model for particular values of δ, k_1, k_2. For this particular example, we find that the strategy (0, 1, 1) becomes dominant for higher values of the network effect (i.e., consumers choose to rent in the first period and then buy in the second period). There are two versions of the software and only the upgraded version sold in Period 2 benefits from the network effect. When the network effects are strong, the consumers prefer to buy Version 2 rather than use Version 1 in Period 2. Given this, the consumer saves money in Period

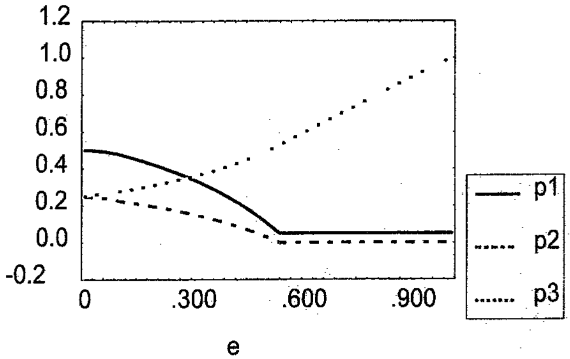

Figure 9. Optimal prices for Case 2. Parameter values are $\delta = 0.9$, $k_1 = 0.5$, $k_2 = 0.5$.

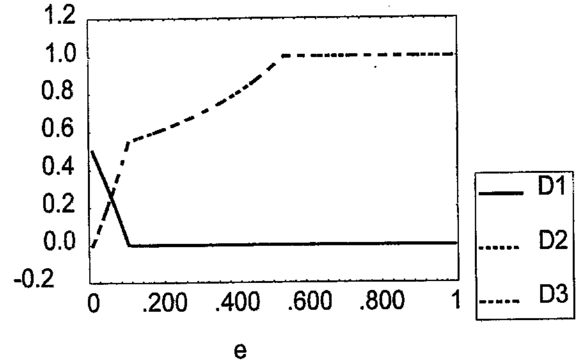

Figure 10. Demand functions for Case 2. Parameter values are $\delta = 0.9$, $k_1 = k_2 = 0.5$. Here, D_2 and D_3 overlap.

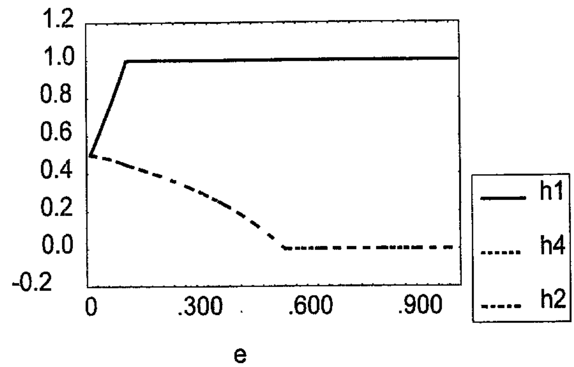

Figure 11. Reservation prices for Case 2. Parameter values are $\delta = 0.9$, $k_1 = k_2 = 0.5$. Here, h_2 and h_4 overlap.

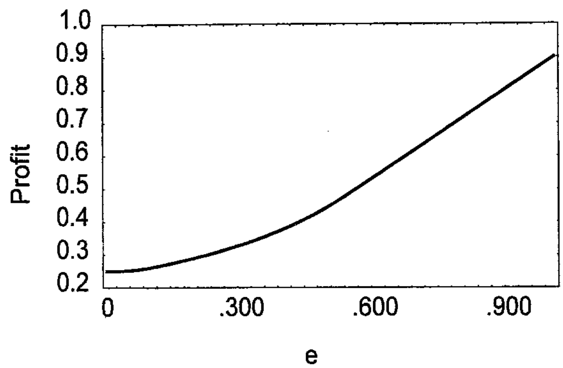

Figure 12. Optimal profits for Case 2.

1 by renting the product instead of purchasing Version 1. For the values used in the example previously we find that no consumers follow the strategy (0, 1, 0). High valuation consumers follow the strategy (1, 0, 0) when the network effects are weak. The pricing strategy of the firm in Case 2 is similar to the strategy in the previous case. The firm increases the price of Ω_3 and decreases the prices of Ω_1 and Ω_2 as the network effect intensifies. The market is covered at $e = 0.58$ and we find that the firm's profits are monotonically increasing.

We feel that the assumption $\delta k_2 < k_1$ reduces the intuitive appeal of this case, as our objective in this article is to focus on short-term renting. We present this case for the completeness of our model.

5. CONCLUSION AND MANAGERIAL INSIGHTS

Software renting presents many new and interesting challenges for interdisciplinary research. The challenges are technical, cultural, and organizational. In this article, we employed economic tools to find optimal pricing strategies for a monopoly firm employing a marketing–sales approach on the cybermarket. From a management point of view, this study is an initial attempt to shed light on the possibility of a software firm's use of renting as a strategic tool to increase its market share. In economics literature, monopoly firms are considered to be economically inefficient and price discrimination is treated as a powerful but socially undesirable tool. From our implementation of a game theoretic model, it turns out that renting is a way to price discriminate in a socially desirable way. It benefits both the firm and the users by providing higher consumer surplus and higher profits. As the extent of network externality increases, the benefits from having a single network seem to outweigh the inefficiencies created by the existence of a monopoly firm. Our results demonstrate how optimal pricing schedules behave over different levels of network intensity. Our model also incorporates previous findings on short-term leasing and monopoly renting.

We have used reservation price based utility functions to come up with the derived demands. Renting for this set of demand functions has a positive effect on the profitability of the firm. The limitation comes from the specific form of the utility functions. However, because our aim is to assess the effects of network externalities on the renting and selling behavior of the firm, this limitation can be tolerated. Other limitations include the absence of an upgrade option and that the rental price is not a function of the duration of the rental. An improvement of the techniques used in this article may be to employ empirical methods to estimate the utility and demand functions and carry out the same analysis under a more realistic setting. The relation of the decision to provide any type of product (be it rental or a try out) to the quality of that product is an important issue by itself. It has been addressed in articles by many researchers in different contexts. In our case, this is an exogenous factor and is not addressed explicitly For issues related to the economics of try-outs, dependence of the decision to rent or try out on the quality levels, cost of increasing it, and the effect of quality uncertainty on demand, see Stahl, Choi, and Whinston [23].

In Section 6, we explain ways to extend this approach as well as our aim for future research.

6. DIRECTIONS FOR FUTURE RESEARCH

We analyzed the pricing problem of a software product in which the decision of renting or selling were implicitly decided on by the firm in the first stage. A more general case might be to look at the firm's choice of renting or selling as a strategic decision.

By looking at monopoly behavior, we simplified the real-world situation to gain an insight into the effect of such a market on the firm's decision and the effect of externalities. The duopoly case in which two firms interact through reaction functions is more realistic and can help answer questions about compatibility and interoperability across competing products. However, as the computational complexity of this problem increases, simulation tools may be needed. We hope to address these issues in a future effort.

The monopoly case also offers many interesting problems such as preemptive pricing behavior. The firm may use its pricing strategy for the rental market to erect barriers to entry; this and the situation under monopolistic competition are important issues for future research.

This model does not incorporate the consumer's ability to upgrade although it is on our current research agenda. Another extension of this article might be to look at issues that arise from compatibility and complementarity of software products. The existing literature on complementarity provides tools to analyze this problem in a broad way.

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A. APPENDIX

In this section, we will prove the necessary propositions we have used in the article as well as provide the analytical solution to Case 1.

A.1 Analytical Solution to the Renting Case 1

The constrained (Kuhn–Tucker) optimization problem solved for the parameter values $k_1 = 0.5$, $k_2 = 0.2$, and $\delta = 0.9$ yields the following results:

$$p_1^* = \begin{cases} \dfrac{1.38 \times 10^{-16}(4.34 - e)(e^2 - .40^2)(e + 3.34 \times 10^{-17})}{(e^2 - .44^2)} & \text{if } e \le 0.19 \\ \dfrac{1.57 \times 10^{-16}(e - 0.5)(e - 0.46)(e + 1.23)(e + 6.36 \times 10^{15})}{(e - 0.54)(e - .05)(e + 2.09)} & \text{if } 0.19 < e < 0.37 \\ 0.35 & \text{if } e \ge 0.37; \end{cases}$$

$$p_2^* = \begin{cases} \dfrac{55963.9(e - 0.31)(e + 3.34 \times 10^{-16})(e + 0.31)}{(-54409.3e + 279819e^3)} & \text{if } e \le 0.19 \\ \dfrac{0.2(e - 0.5)(e - 0.37)(e + 1.53)}{(e - 0.54)(e - 0.5)(e + 2.09)} & \text{if } 0.19 < e < 0.37 \\ 0 & \text{if } e \ge 0.37; \end{cases}$$

$$p_3^* = \begin{cases} \dfrac{-264.5 - 264.5e + 1360.23e^2 - 4.55 \times 10^{-23}e^3}{(5440.93e^2 - 1057.96)} & \text{if } e \le 0.19 \\ \dfrac{3.93 \times 10^{-16}(e - 0.55)(e + 0.57)(e + 2.26 \times 10^{15})}{(e - 0.54)(e + 2.09)} & \text{if } 0.19 < e < 0.37 \\ e & \text{if } e \ge 0.37; \end{cases}$$

$$\pi^* = \begin{cases} \dfrac{P_1(e)}{R_1(e)} & \text{if } e \leq 0.19 \\ \dfrac{P_2(e)}{R_2(e)} & \text{if } 0.19 < e < 0.37, \text{ where} \\ 0.9e & \text{if} \geq 0.37; \end{cases}$$

$$P_1(e) = -(1.60022 \times 10^{-31} (-4.65482 \times 10^{14} + e) (-0.493007 + e) (-8.47645 \times 10^{17} + e) (8.47645 \times 10^{-17} + e) (0.493007 + e) (2.68502 \times 10^{15} + e));$$

$$R_1(e) = (e^2 - 0.440959^2)\, e^2;$$

$$P_2(e) = -(1.24105 \times 10^{-31} (-1.20129 \times 10^8 + e) (-0.722222 + e) (-0.536907 + e) (1.2013 \times 10^8 + e) (2.17138 \times 10^{14} + e) (0.25 - 1.\, e + e^2));$$

$$R_2(e) = ((-0.536907 + e)^2 (2.09246 + e) (-0.5 + e)^2).$$ ■

A.2 Proof of Proposition 2

Proof: Because we have piecewise defined price functions over the domain of e, we need to check various regions. First, we will look at the selling price in the first period for both cases:

(a) for $e \leq 0.19$,

$$p_1^{R*} - p_1^{N*} = \frac{0.4e^2(e + 0.25)^2}{-(e - 2.49)(e^2 - 0.44^2)(e + 0.49)} < 0;$$

(b) for $0.19 < e \leq 0.37$,

$$p_1^{R*} - p_1^{N*} = \frac{10^{-16}(e^2 - 10^{-16})(e + 3.6 \times 10^{15})}{-(e - 2.49)(e + 0.49)(e - 0.54)(e + 2.09)} < 0;$$

(c) for $0.37 < e < 1$,

$$p_1^{R*} - p_1^{N*} = \frac{0.65(e - 0.81)(e + 0.34)}{-(e - 2.49)(e + 0.49)} < 0;$$

and finally (d) for $e \geq 1$,

$$p_1^{R*} - p_1^{N*} = 0.075 > 0,$$

which is always positive for this range of e.

Next, we will look at the second period prices:

(a) for $e \leq 0.19$,

$$p_2^{R*} - p_2^{N*} = \frac{0.25(e+0.25)(e+0.78)e^2}{(e-2.49)(e^2-0.44^2)(e+0.49)} > 0;$$

(b) for $0.19 < e \leq 0.37$,

$$p_2^{R*} - p_2^{N*} = \frac{0.89e(e+0.24)(e^2-1.31e+0.49)}{(e-2.49)(e+0.49)(e-0.54)(e+2.09)} > 0;$$

(c) for $0.37 < e < 1$,

$$p_2^{R*} - p_2^{N*} = \frac{(e-2.12)(e-0.32)(e+0.44)}{(e-2.49)(e+0.49)} > 0;$$

and finally (d) for $e \geq 1$,

$$p_2^{R*} - p_2^{N*} = 0.5e > 0,$$

which is always positive for this range of e. ■

A.3 Proof of Proposition 4

Proof: In the renting case, the equilibrium firm profits are given by:

$$\Pi^{R*} = \begin{cases} \dfrac{P_1(e)}{R_1(e)} & \text{if } e \leq 0.19 \\ \dfrac{P_2(e)}{R_2(e)} & \text{if } 0.19 < e < 0.37 \\ 0.9e & \text{if } e \geq 0.37, \end{cases}$$

where

$$P_1(e) = -(1.60022 \times 10^{-31}\,(-4.65482 \times 10^{14} + e)\,(-0.493007 + e)\,(-8.47645 \times 10^{17} + e)\\ (8.47645 \times 10^{-17} + e)\,(0.493007 + e)\,(2.68502 \times 10^{15} + e));$$

$$R_1(e) = (e^2 - 0.440959^2)\,e^2;$$

$$P_2(e) = -(1.24105 \times 10^{-31}\,(-1.20129 \times 10^{8} + e)\,(-0.722222 + e)\,(-0.536907 + e)\\ (1.2013 \times 10^{8} + e)\,(2.17138 \times 10^{14} + e)\,(0.25 - 1.e + e^2));$$

$$R_2(e) = ((-0.536907 + e)^2\,(2.09246 + e)\,(-0.5 + e)^2).$$

In the no renting case, the equilibrium profits are:

$$\Pi^{N*} = \begin{cases} \dfrac{0.45(0.61+e)^2}{((0.55+0.9e)(2.49-e)(0.49+e))} & \text{if } e < 1 \\ \dfrac{0.20(0.61+e)^2}{0.55+0.9e} & \text{if } e \geq 1. \end{cases}$$

Because we have piecewise defined profit function over the domain of e, we need to check various regions:

(a) for $e \leq 0.19$,

$$\Pi^{R*} - \Pi^{N*} = \frac{\left(\begin{array}{c} -(1.44568 \times 10^{-31}(-4.2642 \times 10^{14} + e)(-0.440959 + e) \\ (-2.1829 \times 10^{-16} + e)(1.29079 \times 10^{-14} + e)(0.440959 + e) \\ (0.611123 + e)(3.2443 \times 10^{15} + e)(0.00712208 - 0.0494427e + e^2) \\ (0.0791112 + 0.549431e + e^2)) \end{array}\right)}{\begin{array}{c}((-0.440959 + e)^2 e^2 (0.440959 + e)^2 (-2.49 + 1.e) \\ (0.49 + 1.e)(0.611111 + 1.e)).\end{array}}$$

Because the denominator is negative for this range of e, the above term is positive only if the numerator is negative, and hence, only if the term in the parentheses in the numerator is positive. Checking the roots of the polynomial equation thus yields the following valid regions:

$$e \in (-\infty, -0.61) \cup (-0.44, -10^{-14}) \cup (10^{-14}, 0.44) \cup (10^{41}, \infty).$$

Hence $\Pi^{R*} - \Pi^{N*} > 0$ for $e \leq 0.19$.

(b) For $0.19 < e \leq 0.37$,

$$\Pi^{R*} - \Pi^{N*} = \frac{\left(\begin{array}{c} -(1.48927 \times 10^{-31}(-1.27251 \times 10^{8} + e) \\ (-0.536907 + e)(-0.5 + e)(-0.5 + e)(0.10399 + e) \\ (0.611117 + e)(2.09246 + e)(1.27251 \times 10^{8} + e) \\ (3.68598 \times 10^{14} + e)(0.0704372 - 0.0774674\, e + e^2)) \end{array}\right)}{\begin{array}{c}((-0.536907 + e)^2 (2.09246 + e)^2 (-2.49 + 1.e)(-0.5 + 1.e)^2 \\ (0.49 + 1.e)(0.611111 + 1.e)).\end{array}}$$

Here also, because the denominator is negative for this range of e, the previous term is positive only if the numerator is negative, and hence, only if the term in the parentheses in the numerator is positive. Checking the roots of the polynomial equation thus yields the following valid regions:

$$e \in (-\infty, -10^{8}) \cup (-2.09, -0.61) \cup (-0.10, 0.5) \cup (0.5, 0.53) \cup (10^{8}, \infty).$$

Hence $\Pi^{R*} - \Pi^{N*} > 0$ for $0.19 < e \leq 0.37$.

(c) For $0.37 < e < 1$,

$$\Pi^{R*} - \Pi^{N*} = \frac{\begin{pmatrix} (3.80648 \times 10^{-16}(-2.22998 + e)(-0.291108 + e) \\ (-0.521091 + e)(0.611108 + e)(2.36439 \times 10^{15} + e)) \end{pmatrix}}{((-2.49 + 1.e)(0.49 + 1.e)(0.611111 + 1.e)).}$$

Here also, because the denominator is negative for this range of e, the above term is positive only if the numerator is negative. Checking the roots of the polynomial equation thus yields the following valid regions:

$$e \in (-\infty, -10^{15}) \cup (-0.61, -0.52) \cup (0.29, 2.23).$$

Hence $\Pi^{R^*} - \Pi^{N^*} > 0$ for $0.37 < e < 1$.

(d) Finally, for $e \geq 1$,

$$\Pi^{R*} - \Pi^{N*} = -0.135309 + 0.677778e,$$

which is always positive for this range of e. ■

A.4 Proof of Proposition 5

Proof: The consumer surplus for Case 1 is given by:

$$CS^R = \int_{h_1}^{1} (h - p_1)dh + \int_{h_3}^{h_1} (k_1 h - p_2)dh + \int_{h_3}^{h_4} \delta(k_2 h - p_3 + e(1 - h_4))dh$$

$$= \begin{cases} \dfrac{(0.177778(0.611111 + e)(0.24 - 0.99e + e^2)(0.24 + 0.90e + e^2)}{(0.19 - 0.88e + e^2)(0.19 + 0.88e + e^2)} & \text{if } e \leq 0.19 \\ \dfrac{(0.442901(-0.5 + e)^2(0.356218 - 0.98645e + e^2))}{((2.09246 + e)^2(0.288269 - 1.07381e + e^2))} & \text{if } 0.19 < e < 0.37 \\ 0.325 & \text{if } e \geq 0.37. \end{cases}$$

The consumer surplus in the no renting case is:

$$CS^N = \int_{h_1}^{1} (h - p_1)dh + \int_{h_3}^{h_1} \delta(k_2 h - p_3 + e(1 - h_1))dh$$

$$= \begin{cases} \dfrac{0.652778(0.286052 + 0.93617\, e + e^2)}{((6.20365 - 4.98142e + e^2)(0.240798 + 0.981424e + e^2))} & \text{if } e \leq 1 \\ \dfrac{0.29375(0.37345 + 1.22221\, e + e^2))}{(0.611111 + 1.e)^2} & \text{if } e > 1. \end{cases}$$

Now, we will compare the above consumer surplus values:

(a) For $e \leq 0.19$,

$$CS^R - CS^N = \frac{-(0.19(e^2 - 0.55^2)(e^2 - 0.33^2))}{(0.19 - 0.88e + e^2)(0.19 + 0.88e + e^2)} < 0,$$

because for the range if e given above, the numerator is negative and the denominator is positive, and hence, the whole term is negative;

(b) for $0.19 < e \leq 0.37$,

$$CS^R - CS^N = \frac{-(0.29(e-0.71)(e-0.5)^2(e-0.35)(e+0.87)(e+3.31))}{((e+2.09)^2(e^2-1.07e+0.29)(e^2-e+0.25))},$$

which is negative for $0.19 < e \leq 0.3544$ and positive for $0.3544 < e \leq 0.37$;

(c) for $0.37 < e < 1$,

$$CS^R - CS^N = \frac{0.03(0.37+1.22e+e^2)(1.23+2.22e+e^2)}{(0.61+e)^2(1.11+e)^2} > 0;$$

and finally (d) for $e \geq 1$,

$$CS^R - CS^N = \frac{\left[\dfrac{0.325(e-3.90)(e-1.07)(e+0.35)(e+0.63)}{(e+2.22e+1.23)}\right]}{(e+1.11)^2(e^2-4.98e+6.20)(e^2+0.98e+0.24)} > 0.$$

■

Proposition 6: The market is covered if $e \geq 0.37$.

A.5 Proof of Proposition 6

Proof: Market is covered when $D_1 + D_3 = 1$ or $D_1 + D_2 = 1$. Solving for e, we get $e = 0.37$. ■

Proposition 7: All consumers who have reservation prices above h_1 will buy in the first period.

A.6 Proof of Proposition 7

Proof: By definition, the net surplus from choosing the strategy (1, 0, 0) over (0, 1, 0) is given by:

$$\begin{aligned} v_1^B(h, p_1) - v_1^R(h, p_2) &= h - p_1 - k_1 h + p_2 \\ &= (1-k_1)h - p_1 + p_2 \end{aligned}$$

Because $k_1 < 1$, this expression is increasing in h. For the net surplus of choosing the strategy (1, 0, 0) over (0, 0, 1), we have:

$$\begin{aligned} v_1^B(h, p_1) - v_2(h, p_3, q_1, q_2) &= h - p_1 - \delta(k_2 h - p_3 + e(q_1+q_2)) \\ &= (1-\delta k_2)h - p_1 + \delta p_3 - \delta e(q_1+q_2), \end{aligned}$$

and because $\delta k_2 < 1$, this expression also increases in h. Finally, the net surplus of choosing the strategy (1, 0, 0) over (0, 1, 1), we have:

$$\begin{aligned} v_1^B(h, p_1) - v_1^R(h, p_2) - v_2(h, p_3, q_1, q_2) &= h - p_1 - k_1 h + p_2 - \delta(k_2 h - p_3 + e(q_1 + q_2)) \\ &= (1 - k_1 - \delta k_2)h - p_1 + p_2 + \delta p_3 - \delta e(q_1 + q_2), \end{aligned}$$

and because $k_1 + \delta k_2 < 1$, this expression also increases in h. Thus we have shown that all consumers who have reservation prices above h_1 will buy in the first period. ■

Proposition 8: The lowest reservation price at which a consumer is willing to rent in period one and buy in period two is less than or equal to the lowest reservation price at which a consumer is willing to buy in period one: $h_4 \leq h_1$.

A.7 Proof of Proposition 8

Proof: Let $S_4(h) = v_1^R(h, p_2) + v_2(h, p_3, q_1, q_2)$. S_4 then represents the total consumer surplus from renting in the first period and buying in the second period, with $v_1^R(h, p_2), v_2(h, p_3, q_1, q_2) \geq 0$.

Now, for $h \geq h_4$:

$$\begin{gathered} v_1^B(h, p_1) - S_4(h) \leq v_1^B(h, p_1) - S_4(h_1) \\ \Rightarrow \begin{pmatrix} (1 - k_1 - \delta k_2)h - p_1 + p_2 \\ -\delta(-p_3 + e(q_1 + q_2)) \end{pmatrix} \leq \begin{pmatrix} (1 - k_1 - \delta k_2)h_1 - p_1 + p_2 \\ -\delta(-p_3 + e(q_1 + q_2)) \end{pmatrix} \\ \Rightarrow h \leq h_1, \end{gathered}$$

because $k_1 + \delta k_2 < 1$ by assumption. ■

Proposition 9: (Case 1) No consumers with choice (0, 1, 0) and with the lowest reservation price of h_2 can exist.

A.8 Proof of Proposition 9

Proof: Assume that there is a consumer with choice (0, 1, 0) and with the lowest reservation price of h_2. Then there exists an $h_0 > h_2$ such that:

$$\begin{gathered} v_1^R(h, p_2) - v_2(h, p_3, q_1, q_2) > v_1^R(h_2, p_2) - v_2(h_2, p_3, q_1, q_2) \\ \Rightarrow (k_1 h - p_2) - \delta(k_2 h - p_3 + e(q_1 + q_2)) > (k_1 h_2 - p_2) - \delta(k_2 h_2 - p_3 + e(q_1 + q_2)) \\ \Rightarrow k_1 h - \delta k_2 h > k_1 h_2 - \delta k_2 h_2 \\ \Rightarrow (h - h_2)(k_1 - \delta k_2) > 0 \\ \Rightarrow k_1 > \delta k_2, \end{gathered}$$

which violates the assumption for Case 1, namely $\delta k_2 > k_1$. ■

Proposition 10 Case 1: The lowest reservation price at which a consumer decides not to rent or buy in period one but buy in period two is less than or equal to the lowest reservation price at which a consumer is willing to rent in period one and buy in period two: $h_3 \leq h_4$.

A.9 Proof of Proposition 10

Proof: For consumers who adopt (0, 0, 1) as a strategy, there must exist $h \geq h_3$ such that:

$$\begin{aligned} v_2(h, p_3, q_1, q_2) - S_4(h) &\geq v_2(h_4, p_3, q_1, q_2) - S_4(h_4) \\ &\Rightarrow -(k_1 h - p_2) \geq -(k_1 h_4 - p_2) \\ &\Rightarrow h \leq h_4 \\ &\Rightarrow h_3 \leq h_4 \end{aligned}$$

■

Proposition 11: (Case 2) No consumers with choice (0, 0, 1) and with the lowest reservation price of h_3 can exist.

A.9 Proof of Proposition 11

Proof: Assume that there is a consumer with choice (0, 0, 1) and with the lowest reservation price of h_3. Then there exists an $h > h_3$ such that:

$$\begin{aligned} v_2(h, p_3, q_1, q_2) - v_1^R(h, p_2) &> v_2(h_3, p_3, q_1, q_2) - v_1^R(h_3, p_2) \\ \Rightarrow \delta(k_2 h - p_3 + e(q_1 + q_2)) - (k_1 h - p_2) &> \delta(k_2 h_3 - p_3 + e(q_1 + q_2)) - (k_1 h_3 - p_2) \\ \Rightarrow (\delta k_2 - k_1)(h - h_3) &> 0 \\ \Rightarrow \delta k_2 &> k_1, \end{aligned}$$

which violates our assumption for Case 2, namely $\delta k_2 < k_1$. ■

Proposition 12: (Case 2) The lowest reservation price at which a consumer is willing to rent in period one but not buy in period two is less than or equal to the lowest reservation price at which a consumer is willing to rent in period one and buy in period two: $h_2 \leq h_4$.

A.10 Proof of Proposition 12 (Case 2)

Proof: For consumers who adopt (0, 1, 0) as a strategy, there must exist $h \geq h_2$ such that:

$$\begin{aligned} v_1^R(h, p_2) - S_4(h) &\geq v_1^R(h_4, p_2) - S_4(h_4) \\ &\Rightarrow -\delta(k_2 h - p_3 + e(q_1 + q_2)) \geq -\delta(k_2 h_4 - p_3 + e(q_1 + q_2)) \\ &\Rightarrow h \leq h_4 \\ &\Rightarrow h_2 \leq h_4. \end{aligned}$$

■