Homework 6

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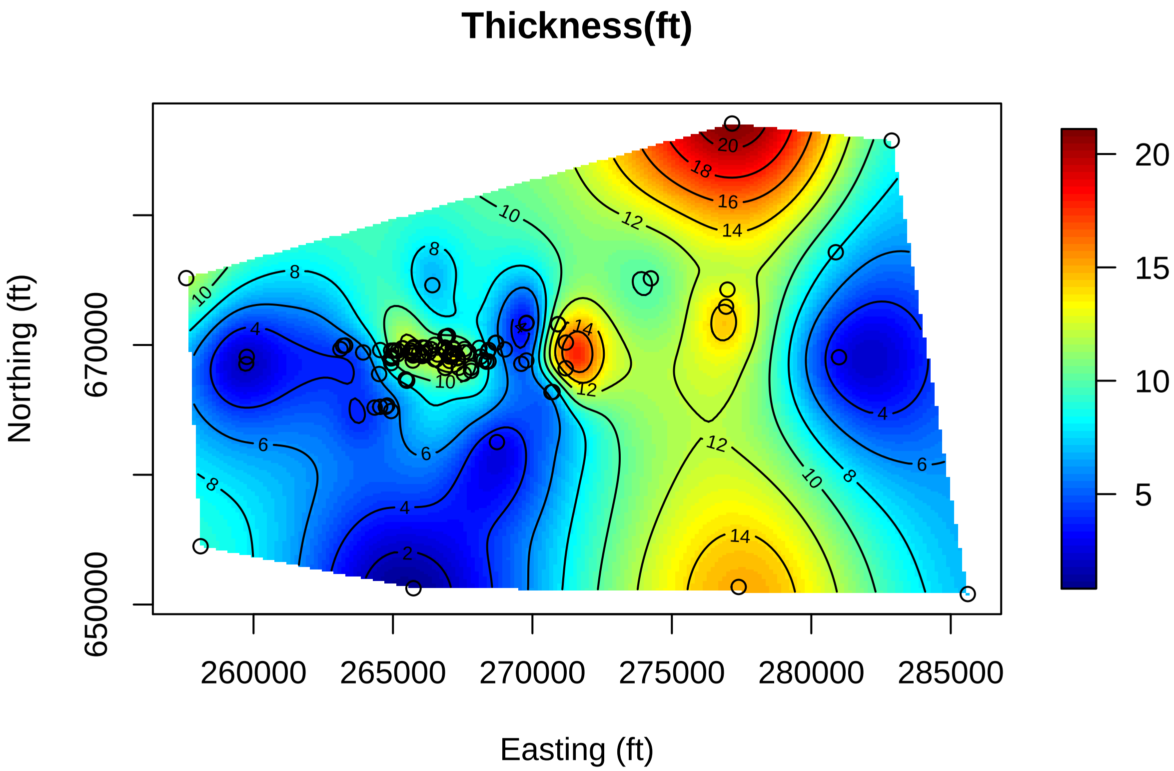
Note: This file is produced by RMarkdown , and the lines start with ## are the outputs of R codes.

Excercise 6

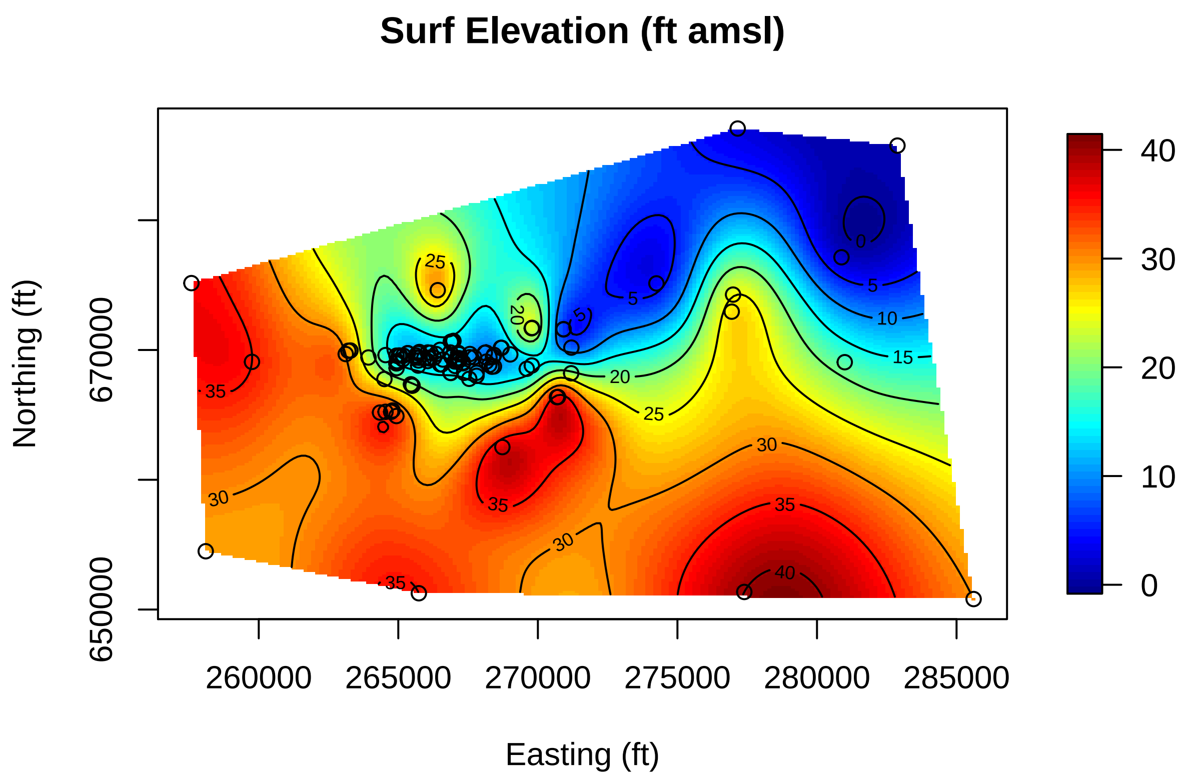
```r
#load libraries
library(spBayes)
library(MBA)
library(geoR)
library(fields)
library(sp)
library(maptools)
library(rgdal)
library(classInt)
library(lattice)
library(dplyr)
```

a)

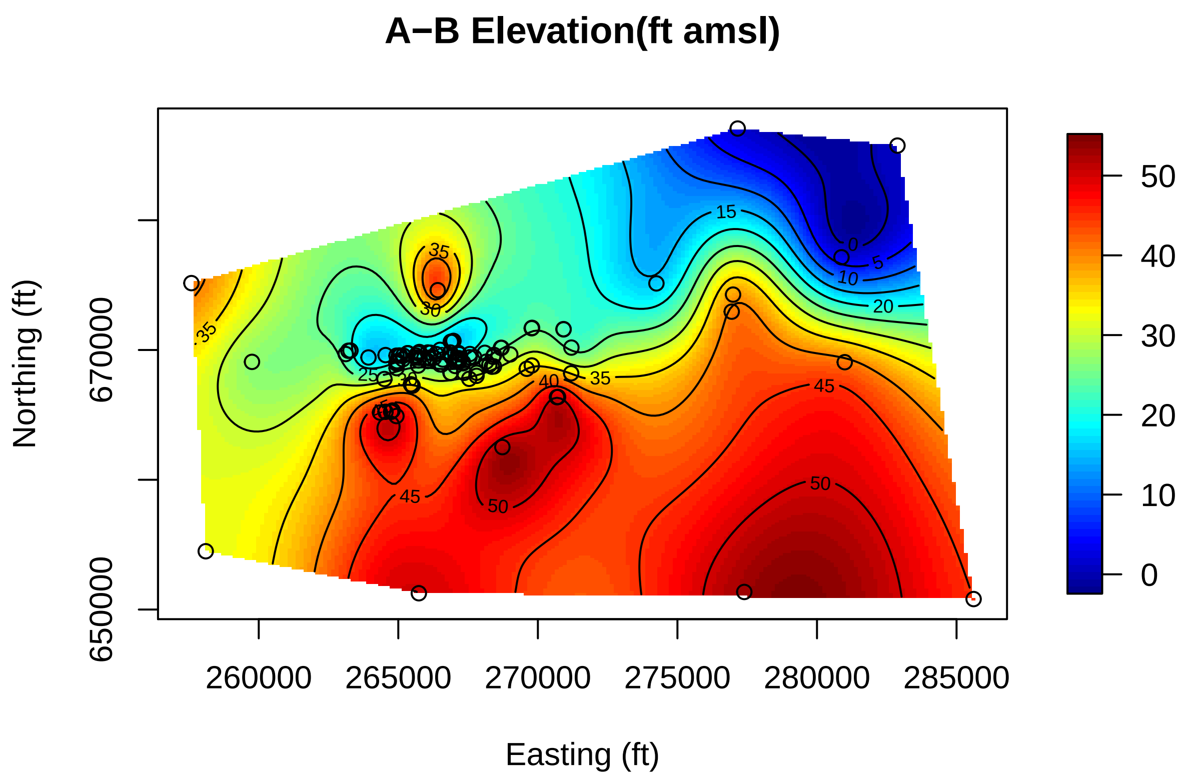
```r
lithology_df <- read.csv("lithology.csv",header = T)
# drop the NA or miss data
lithology_df <- lithology_df %>% filter( !is.na(Thickness_ft) &
                                          !is.na(Surf_Elevation_ft_amsl) &
                                          !is.na(A_B_Elevation_ft_amsl))
lithology_df$Surf_Elevation_ft_amsl <- as.numeric(lithology_df$Surf_Elevation_ft_amsl)
lithology_df$A_B_Elevation_ft_amsl <- as.numeric(lithology_df$A_B_Elevation_ft_amsl)

## Extract the coordinates
coords <- as.matrix(lithology_df[,c("Easting_ft","Northing_ft")])

x.res <- 200; y.res <- 200

surf <- mba.surf(cbind(coords,
                       lithology_df$Thickness_ft),
                 no.X=x.res, no.Y=y.res, h=5,
                 m=2, extend=FALSE)$xyz.est
zlim.Thickness_ft <- range(surf[[3]], na.rm=TRUE)
image.plot(surf, xaxs = "r",
           yaxs = "r", xlab="Easting (ft)",
           ylab="Northing (ft)",
           main="Thickness(ft)" )
points(coords)
contour(surf,add = T)
```




```
surf <- mba.surf(cbind(coords,
                       lithology_df$Surf_Elevation_ft_amsl),
                 no.X=x.res, no.Y=y.res, h=5,
                 m=2, extend=FALSE)$xyz.est
zlim.Surf_Elevation_ft_amsl <- range(surf[[3]], na.rm=TRUE)
image.plot(surf, xaxs = "r",
           yaxs = "r", xlab="Easting (ft)",
           ylab="Northing (ft)",
           main="Surf Elevation (ft amsl)")
points(coords)
contour(surf,add = T)
```




```r
surf <- mba.surf(cbind(coords,
                       lithology_df$A_B_Elevation_ft_amsl),
                 no.X=x.res, no.Y=y.res, h=5,
                 m=2, extend=FALSE)$xyz.est
zlim.A_B_Elevation_ft_amsl <- range(surf[[3]], na.rm=TRUE)
image.plot(surf, xaxs = "r",
           yaxs = "r", xlab="Easting (ft)",
           ylab="Northing (ft)",
           main="A-B Elevation(ft amsl)")
points(coords)
contour(surf,add = T)
```



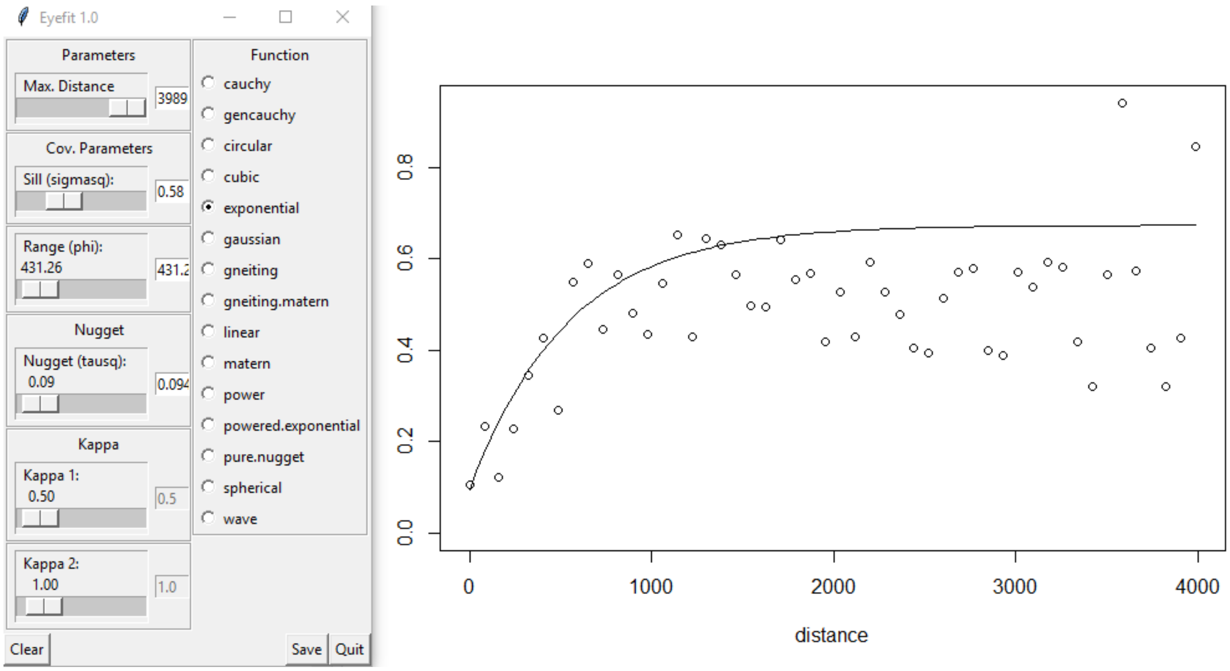
As we can see from the above results, the thickness IQR range about 5 feet to 20 feet whereas Surf Elevation is about 10 to 40 feet. Most of the points are located around Easting 265000 feet and Northing 67000 feet.

b)

```
# compute the variogram for exponential
log.thickness <- log(lithology_df$Thickness_ft)
bins = 50
max.dist <- 0.1*max(iDist(coords))
log.thick.vario <- variog(coords = coords, data = log.thickness,
                          uvec = (seq(0, max.dist, length = bins)))
```

```
## variog: computing omnidirectional variogram
```

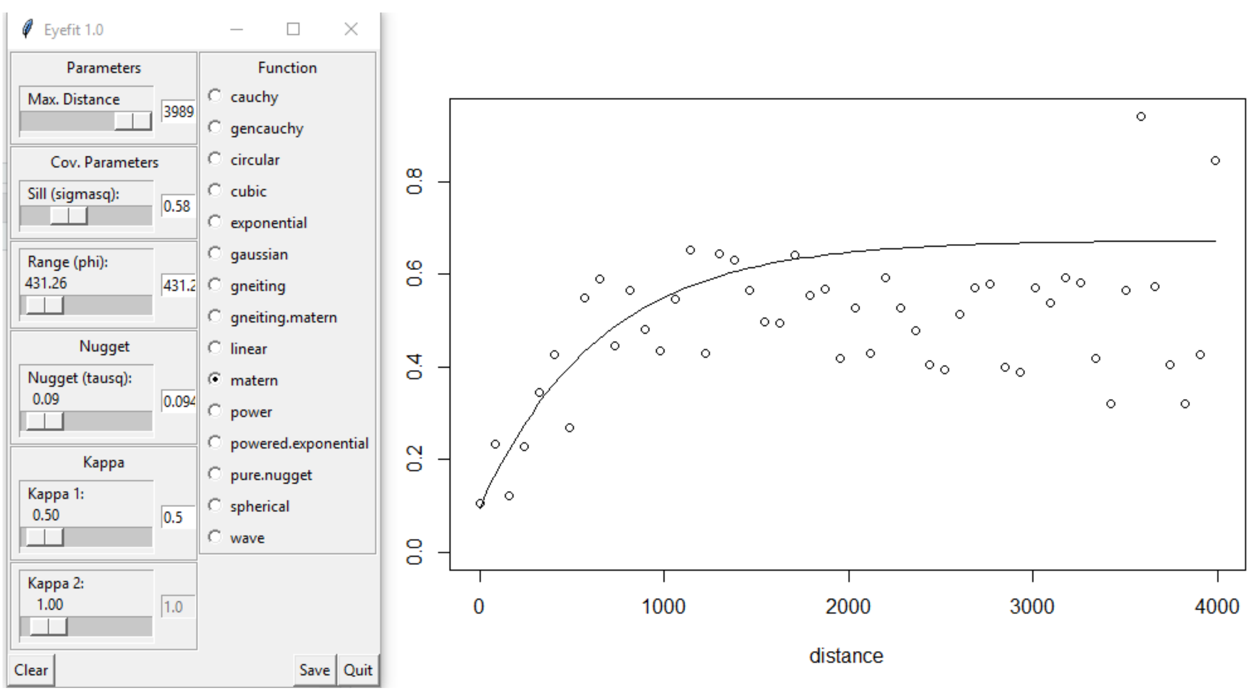
```
plot(log.thick.vario)
eyefit(log.thick.vario,silent=TRUE)
```




```
# compute the variogram for matern
log.thick.vario <- variog(coords = coords, data = log.thickness,
                          uvec = (seq(0, max.dist, length = bins)))
```

```
## variog: computing omnidirectional variogram
```

```
plot(log.thick.vario)
eyefit(log.thick.vario,silent=TRUE)
```




```
log.thickness <- log(lithology_df$Thickness_ft)

p <- 3 ## This is the number of columns in the design matrix
```

```r
## Set the prior mean and precision for the regression
beta.prior.mean <- as.matrix(rep(0, times=p))
beta.prior.precision <- matrix(0, nrow=p, ncol=p)

## For use with bayesGeostatExact, do the following
phi <- 0.014 ## Set the spatial range (from the variogram)
alpha <- 0.016/0.08 ## Set the nugget/partial-sill ratio
sigma.sq.prior.shape <- 0.1 ## Set IG shape for sigma.sq (partial sill)
sigma.sq.prior.rate <- 0.1 ## Set IG scale for sigma.sq (partial sill)

## Run bayesGeostatExact to deliver exact posterior samples
sp.exact <- bayesGeostatExact(
  log.thickness ~ Surf_Elevation_ft_amsl + A_B_Elevation_ft_amsl,
  data=lithology_df, coords=coords, n.samples=1000,
  beta.prior.mean=beta.prior.mean,
  beta.prior.precision=beta.prior.precision,
  cov.model="exponential",
  phi=phi, alpha=alpha,
  sigma.sq.prior.shape=sigma.sq.prior.shape,
  sigma.sq.prior.rate=sigma.sq.prior.rate,
  sp.effects=FALSE)
```

```
## -------------------------------------------------
##  General model description
## -------------------------------------------------
## Model fit with 113 observations.
## Number of covariates 3 (including intercept if specified).
## Using the exponential spatial correlation model.
##
## -------------------------------------------------
##      Sampling
## -------------------------------------------------
## Sampled: 1000 of 1000, 100%
```

```r
##Produce the posterior summaries
round(summary(sp.exact$p.samples)$quantiles,3)
```

```
##                          2.5%    25%    50%    75%  97.5%
## (Intercept)             2.317  2.517  2.623  2.727  2.942
## Surf_Elevation_ft_amsl -0.050 -0.037 -0.031 -0.025 -0.013
## A_B_Elevation_ft_amsl  -0.014 -0.005 -0.001  0.004  0.013
## sigma.sq                0.269  0.326  0.355  0.387  0.460
## tau.sq                  0.054  0.065  0.071  0.077  0.092
```

```r
phi <- 1/1100 ## Set the spatial range (from the variogram)
alpha <- 0.094/0.58 ## Set the nugget/partial-sill ratio
nu <- 0.5

## Run bayesGeostatExact to deliver exact posterior samples
sp.exact <- bayesGeostatExact(
  log.thickness ~ Surf_Elevation_ft_amsl + A_B_Elevation_ft_amsl,
  data=lithology_df, coords=coords, n.samples=1000,
  beta.prior.mean=beta.prior.mean,
  beta.prior.precision=beta.prior.precision,
  cov.model="matern",
```

```r
  phi=phi, alpha=alpha,
  nu=nu,
  sigma.sq.prior.shape=sigma.sq.prior.shape,
  sigma.sq.prior.rate=sigma.sq.prior.rate,
  sp.effects=FALSE)
```

```
## ----------------------------------------------------
##  General model description
## ----------------------------------------------------
## Model fit with 113 observations.
## Number of covariates 3 (including intercept if specified).
## Using the matern spatial correlation model.
##
## ----------------------------------------------------
##      Sampling
## ----------------------------------------------------
## Sampled: 1000 of 1000, 100%
```

```r
round(summary(sp.exact$p.samples)$quantiles,3)
```

```
##                            2.5%    25%    50%    75% 97.5%
## (Intercept)               1.892  2.360  2.566  2.780 3.211
## Surf_Elevation_ft_amsl -0.041 -0.023 -0.014 -0.002 0.015
## A_B_Elevation_ft_amsl  -0.031 -0.019 -0.013 -0.008 0.002
## sigma.sq                  0.534  0.630  0.692  0.761 0.899
## tau.sq                    0.087  0.102  0.112  0.123 0.146
```

As we can see from the above results, the log Thickness has a negative correlation. For exponential, the model can draw as $log.thickness = 2.315 - 0.048 * Surf\_Elevation\_ft\_amsl - 0.015 * A\_B\_Elevation\_ft\_amsl$, and $\sigma^2 = 0.276$, $\tau^2 = 0.055$. For exponential, the model can draw as $log.thickness = 1.926 - 0.039 * Surf\_Elevation\_ft\_amsl - 0.028 * A\_B\_Elevation\_ft\_amsl$, and $\sigma^2 = 0.544$, $\tau^2 = 0.088$.

c)

```r
## Run spLM to deliver MCMC samples from marginal posterior distributions
n.samples <- 1000
thickness.sp <- spLM(log.thickness ~ Surf_Elevation_ft_amsl + A_B_Elevation_ft_amsl,
               data=lithology_df, coords=coords,
               starting=list("phi"=3/1100,"sigma.sq"=0.08,"tau.sq"=0.02),
               tuning=list("phi"=0.1, "sigma.sq"=0.05, "tau.sq"=0.05),
               priors=list("phi.Unif"=c(3/1500, 3/50),
                           "sigma.sq.IG"=c(0.1,0.1),"tau.sq.IG"=c(0.1, 0.1)),
               cov.model="exponential",n.samples=n.samples)
```

```
## ----------------------------------------
##  General model description
## ----------------------------------------
## Model fit with 113 observations.
##
## Number of covariates 3 (including intercept if specified).
##
## Using the exponential spatial correlation model.
##
## Number of MCMC samples 1000.
##
```

```
## Priors and hyperpriors:
##  beta flat.
##  sigma.sq IG hyperpriors shape=0.10000 and scale=0.10000
##  tau.sq IG hyperpriors shape=0.10000 and scale=0.10000
##  phi Unif hyperpriors a=0.00200 and b=0.06000
## -------------------------------------------------
##      Sampling
## -------------------------------------------------
## Sampled: 100 of 1000, 10.00%
## Report interval Metrop. Acceptance rate: 61.00%
## Overall Metrop. Acceptance rate: 61.00%
## -------------------------------------------------
## Sampled: 200 of 1000, 20.00%
## Report interval Metrop. Acceptance rate: 58.00%
## Overall Metrop. Acceptance rate: 59.50%
## -------------------------------------------------
## Sampled: 300 of 1000, 30.00%
## Report interval Metrop. Acceptance rate: 56.00%
## Overall Metrop. Acceptance rate: 58.33%
## -------------------------------------------------
## Sampled: 400 of 1000, 40.00%
## Report interval Metrop. Acceptance rate: 60.00%
## Overall Metrop. Acceptance rate: 58.75%
## -------------------------------------------------
## Sampled: 500 of 1000, 50.00%
## Report interval Metrop. Acceptance rate: 55.00%
## Overall Metrop. Acceptance rate: 58.00%
## -------------------------------------------------
## Sampled: 600 of 1000, 60.00%
## Report interval Metrop. Acceptance rate: 56.00%
## Overall Metrop. Acceptance rate: 57.67%
## -------------------------------------------------
## Sampled: 700 of 1000, 70.00%
## Report interval Metrop. Acceptance rate: 58.00%
## Overall Metrop. Acceptance rate: 57.71%
## -------------------------------------------------
## Sampled: 800 of 1000, 80.00%
## Report interval Metrop. Acceptance rate: 59.00%
## Overall Metrop. Acceptance rate: 57.88%
## -------------------------------------------------
## Sampled: 900 of 1000, 90.00%
## Report interval Metrop. Acceptance rate: 64.00%
## Overall Metrop. Acceptance rate: 58.56%
## -------------------------------------------------
## Sampled: 1000 of 1000, 100.00%
## Report interval Metrop. Acceptance rate: 59.00%
## Overall Metrop. Acceptance rate: 58.60%
## -------------------------------------------------
```

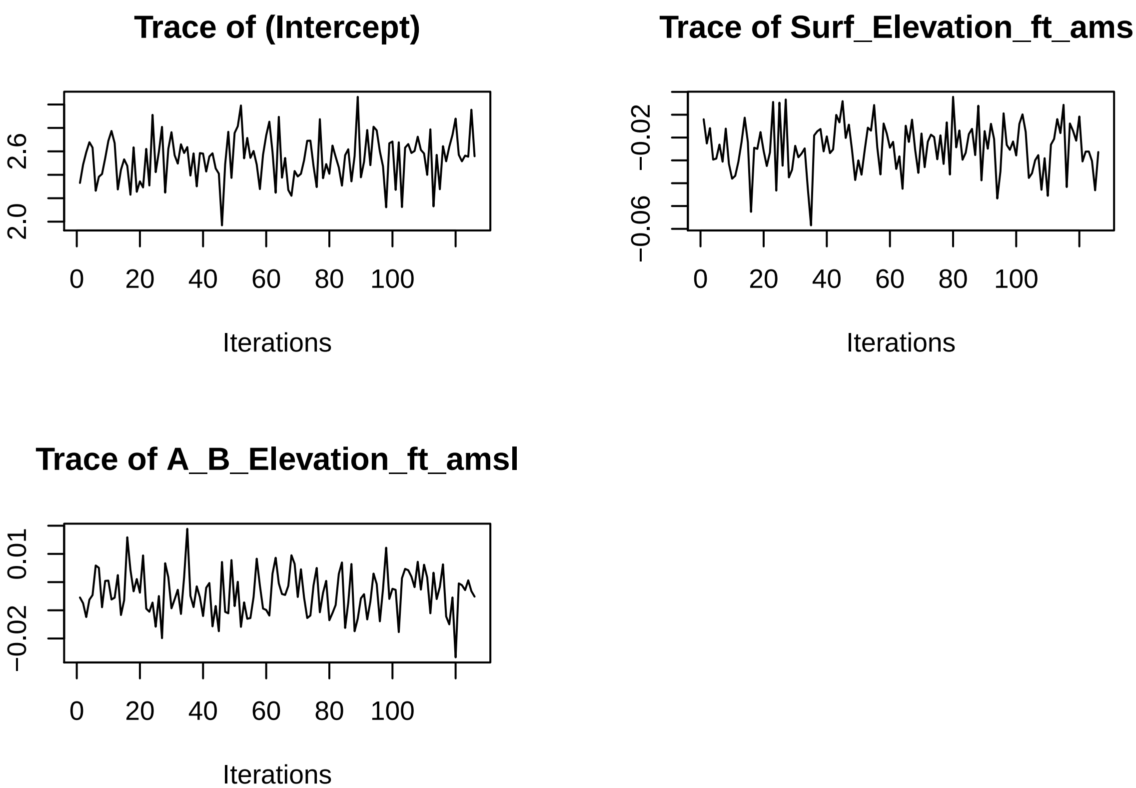
```r
round(summary(mcmc(thickness.sp$p.theta.samples))$quantiles,3)
```

```
##           2.5%   25%   50%   75% 97.5%
## sigma.sq 0.206 0.332 0.394 0.462 0.640
## tau.sq   0.029 0.054 0.081 0.121 0.256
## phi      0.002 0.003 0.005 0.006 0.012
```

```
## Recover spatial residuals using spRecover
burn.in <- floor(0.75*n.samples)
thickness.sp <- spRecover(thickness.sp, start=burn.in, thin=2)
```

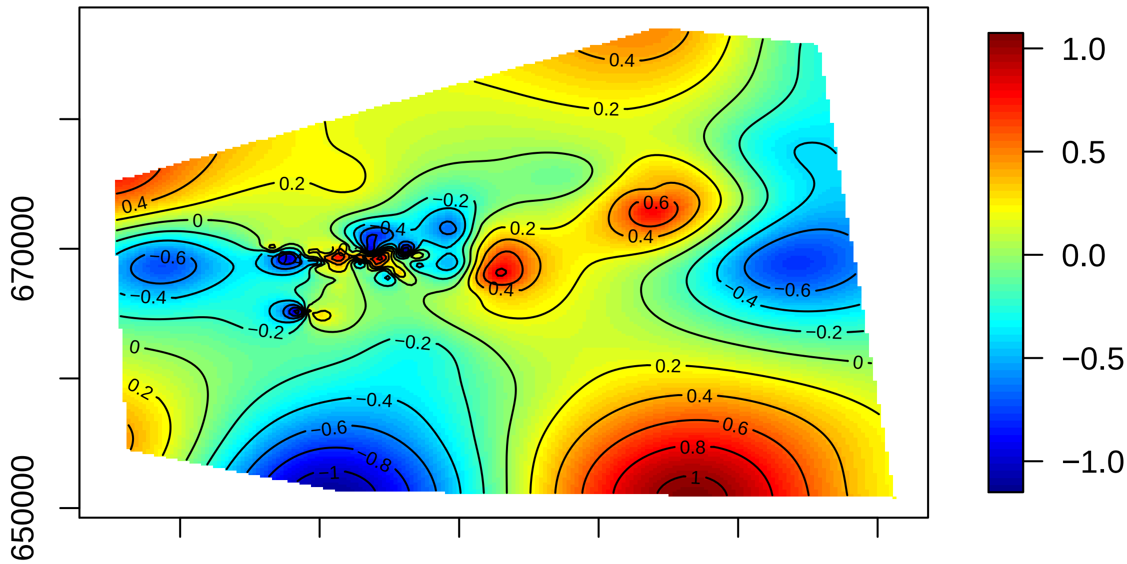
```
## -------------------------------------------------
##      Recovering beta and w
## -------------------------------------------------
## Sampled: 99 of 126, 78.57%
```

```
## The posterior samples of the regression coefficients and the spatial effects can then be obtained as
beta.samples = thickness.sp$p.beta.recover.samples
w.samples = thickness.sp$p.w.recover.samples

## Obtain trace plots for regression coefficients
par(mfrow=c(2,2))
plot(beta.samples, auto.layout=TRUE, density=FALSE)
```



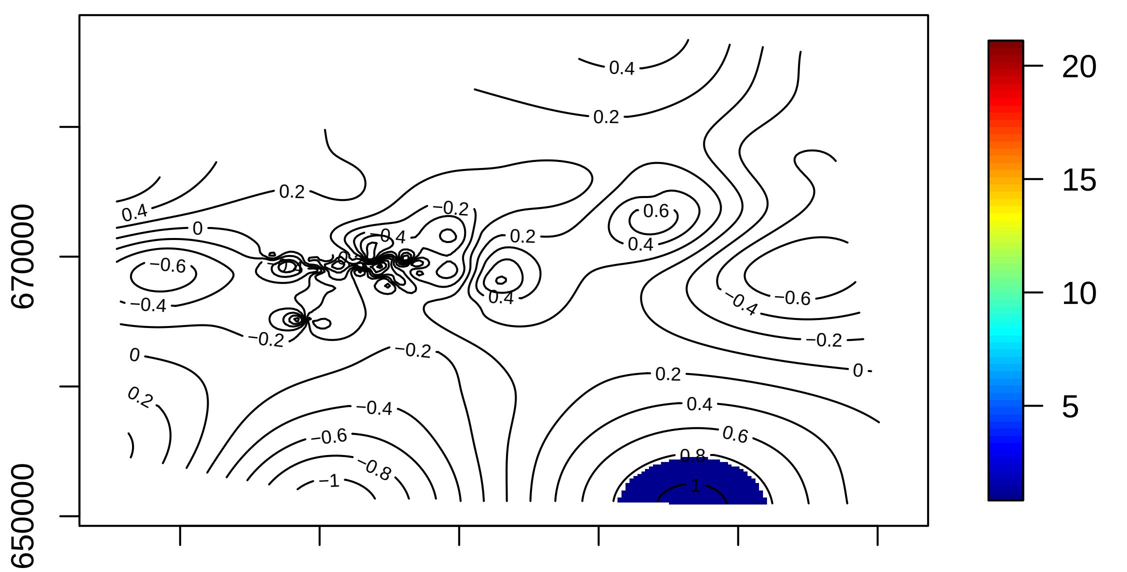

```
par(mfrow=c(1,1))
## Obtain posterior means and sd's of spatial residuals for each location
w.hat.mu <- apply(w.samples,1,mean)
w.hat.sd <- apply(w.samples,1,sd)


surf <- mba.surf(cbind(coords, w.hat.mu),
                 no.X=x.res, no.Y=y.res, extend=FALSE)$xyz.est
image.plot(surf, xaxs = "r", yaxs = "r",
           main="Mean Spatial Effects")
contour(surf,add = T)
```

Mean Spatial Effects



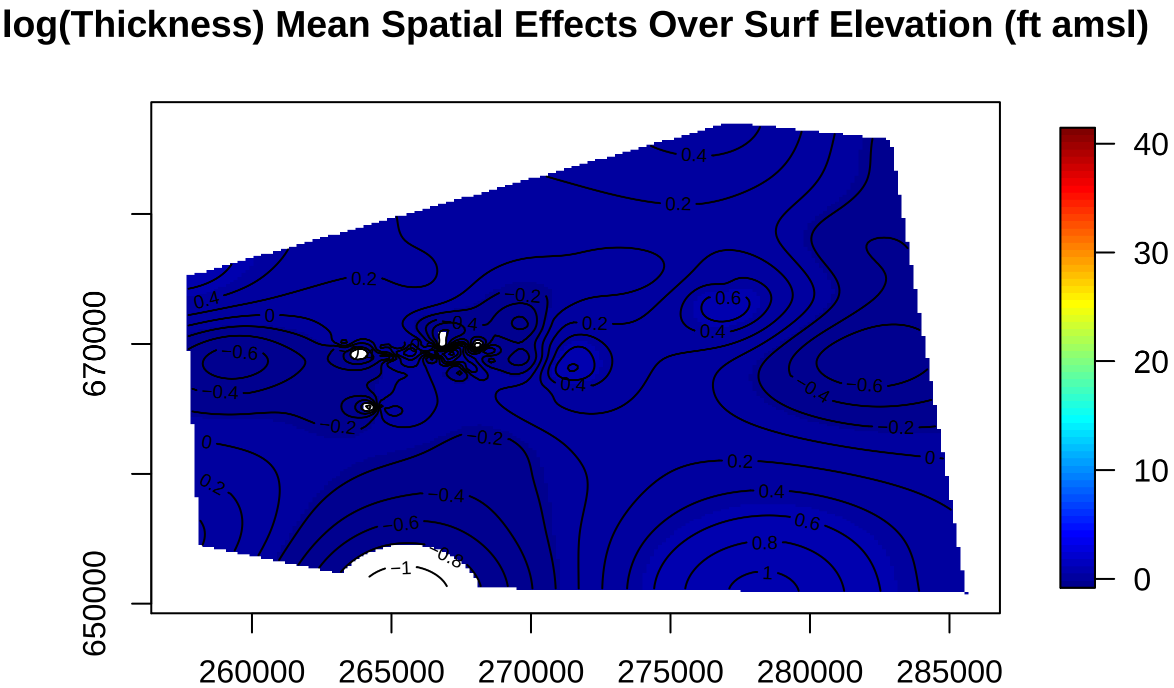

```
surf <- mba.surf(cbind(coords, w.hat.mu),
                 no.X=x.res, no.Y=y.res, extend=FALSE)$xyz.est
image.plot(surf, xaxs = "r", yaxs = "r",
           zlim = zlim.Thickness_ft,
           main="log(Thickness) Mean Spatial Effects Over Thickness(ft)")
contour(surf,add = T)
```

log(Thickness) Mean Spatial Effects Over Thickness(ft)

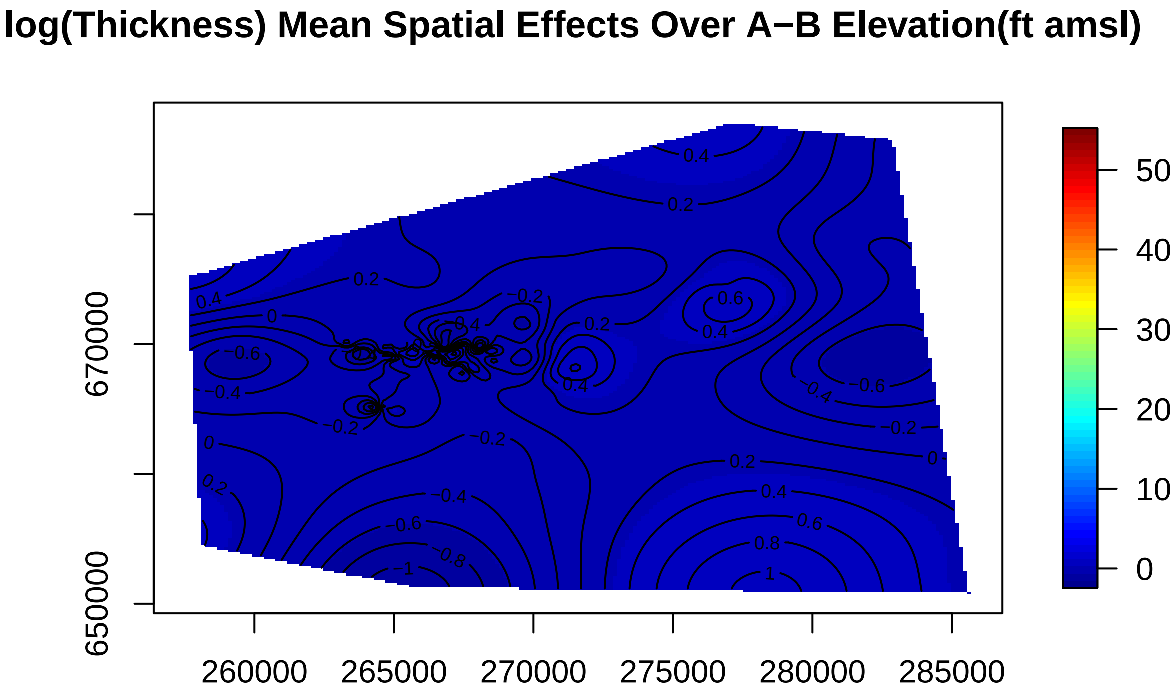



```
surf <- mba.surf(cbind(coords, w.hat.mu),
                 no.X=x.res, no.Y=y.res, extend=FALSE)$xyz.est
image.plot(surf, xaxs = "r", yaxs = "r",
           zlim = zlim.Surf_Elevation_ft_amsl,
```

```
          main="log(Thickness) Mean Spatial Effects Over Surf Elevation (ft amsl)")
contour(surf,add = T)
```




```
surf <- mba.surf(cbind(coords, w.hat.mu),
                no.X=x.res, no.Y=y.res, extend=FALSE)$xyz.est
image.plot(surf, xaxs = "r", yaxs = "r",
          zlim = zlim.A_B_Elevation_ft_amsl,
          main="log(Thickness) Mean Spatial Effects Over A-B Elevation(ft amsl) ")
contour(surf,add = T)
```



d)

```r
# run model with purely spatial
# note: set tau.sq=0 will work on spRecover and spDiag, therefore we set it a value close to 0
thickness.sp2 <- spLM(log.thickness ~ Surf_Elevation_ft_amsl + A_B_Elevation_ft_amsl,
               data=lithology_df, coords=coords,
               starting=list("phi"=3/1100,"sigma.sq"=0.08,"tau.sq"=1E-10),
               tuning=list("phi"=0.1, "sigma.sq"=0.05, "tau.sq"=1E-10),
               priors=list("phi.Unif"=c(3/1500, 3/50),
                           "sigma.sq.IG"=c(0.1,0.1),
                           "tau.sq.IG"=c(1E-10, 1E-10)),
               cov.model="exponential",n.samples=n.samples)
```

```
## ----------------------------------------
##  General model description
## ----------------------------------------
## Model fit with 113 observations.
##
## Number of covariates 3 (including intercept if specified).
##
## Using the exponential spatial correlation model.
##
## Number of MCMC samples 1000.
##
## Priors and hyperpriors:
##  beta flat.
##  sigma.sq IG hyperpriors shape=0.10000 and scale=0.10000
##  tau.sq IG hyperpriors shape=0.00000 and scale=0.00000
##  phi Unif hyperpriors a=0.00200 and b=0.06000
## -------------------------------------------------
##      Sampling
## -------------------------------------------------
## Sampled: 100 of 1000, 10.00%
## Report interval Metrop. Acceptance rate: 53.00%
## Overall Metrop. Acceptance rate: 53.00%
## -------------------------------------------------
## Sampled: 200 of 1000, 20.00%
## Report interval Metrop. Acceptance rate: 49.00%
## Overall Metrop. Acceptance rate: 51.00%
## -------------------------------------------------
## Sampled: 300 of 1000, 30.00%
## Report interval Metrop. Acceptance rate: 42.00%
## Overall Metrop. Acceptance rate: 48.00%
## -------------------------------------------------
## Sampled: 400 of 1000, 40.00%
## Report interval Metrop. Acceptance rate: 54.00%
## Overall Metrop. Acceptance rate: 49.50%
## -------------------------------------------------
## Sampled: 500 of 1000, 50.00%
## Report interval Metrop. Acceptance rate: 43.00%
## Overall Metrop. Acceptance rate: 48.20%
## -------------------------------------------------
## Sampled: 600 of 1000, 60.00%
## Report interval Metrop. Acceptance rate: 40.00%
```

```
## Overall Metrop. Acceptance rate: 46.83%
## -------------------------------------------------
## Sampled: 700 of 1000, 70.00%
## Report interval Metrop. Acceptance rate: 55.00%
## Overall Metrop. Acceptance rate: 48.00%
## -------------------------------------------------
## Sampled: 800 of 1000, 80.00%
## Report interval Metrop. Acceptance rate: 48.00%
## Overall Metrop. Acceptance rate: 48.00%
## -------------------------------------------------
## Sampled: 900 of 1000, 90.00%
## Report interval Metrop. Acceptance rate: 55.00%
## Overall Metrop. Acceptance rate: 48.78%
## -------------------------------------------------
## Sampled: 1000 of 1000, 100.00%
## Report interval Metrop. Acceptance rate: 54.00%
## Overall Metrop. Acceptance rate: 49.30%
## -------------------------------------------------
```

```
round(summary(mcmc(thickness.sp2$p.theta.samples))$quantiles,3)
```

```
##           2.5%   25%   50%   75% 97.5%
## sigma.sq 0.350 0.444 0.493 0.556 0.685
## tau.sq   0.000 0.000 0.000 0.000 0.000
## phi      0.003 0.007 0.010 0.012 0.020
```

```
thickness.sp2 <- spRecover(thickness.sp2, start=burn.in, thin=2)
```

```
## -------------------------------------------------
##      Recovering beta and w
## -------------------------------------------------
## Sampled: 99 of 126, 78.57%
```

```
Dic1 = spDiag(thickness.sp,start=burn.in,verbose=FALSE)
Dic1
```

```
## $DIC
##                  value
## bar.D       -154.55216
## D.bar.Omega -215.61318
## pD            61.06103
## DIC          -93.49113
##
## $GP
##       value
## G  3.496594
## P 20.734411
## D 24.231005
##
## $GRS
## [1] 174.5656
```

```
Dic2 = spDiag(thickness.sp2,start=burn.in,verbose=FALSE)
Dic2
```

```
## $DIC
##                  value
```

```
## bar.D       -2487.2611
## D.bar.Omega -2601.0949
## pD            113.8338
## DIC         -2373.4273
##
## $GP
##          value
## G 1.511482e-10
## P 2.278086e-08
## D 2.293201e-08
##
## $GRS
## [1] 2522.933
```

Compare the two models with above DIC values, the second model with much nugget is better than the first one with nugget.