Homework4

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Note: This file is produced by RMarkdown , and the lines start with ## are the outputs of R codes.

```
library(spdep)
library(maps)
library(maptools)
library(classInt)
library(RColorBrewer)
```

Excercise 11

(a)

```
state.sat.scores = read.table("state-sat.dat", header=F)
colnames(state.sat.scores) <- c("STATE","VERBAL","MATH","ELIGIBLE")
#state.sat.scores$ELIGIBLE = state.sat.scores$ELIGIBLE/100
head(state.sat.scores)
```

```
##    STATE VERBAL MATH ELIGIBLE
## 1    ala    561  555        9
## 2 alaska    516  514       50
## 3   ariz    524  525       34
## 4    ark    563  556        6
## 5  calif    497  514       49
## 6   colo    536  540       32
```

```
# create listw
usa.state <- map(database="state", fill=TRUE, plot=FALSE)
state.ID <- sapply(strsplit(usa.state$names, ":"),
                   function(x) x[1])

usa.poly <- map2SpatialPolygons(usa.state,
                                IDs=state.ID)
usa.nb <- poly2nb(usa.poly)
usa.listb <- nb2listw(usa.nb, style="B")
usa.listw <- nb2listw(usa.nb, style="W")

# train SAR model

x = ((state.sat.scores$STATE=="alaska") |
       (state.sat.scores$STATE=="hawaii") |
       (state.sat.scores$STATE=="us"))

index = c(1:nrow(state.sat.scores))[x]
state.sat.scores = state.sat.scores[-index,]

# binnary weights
stat.sat.sar.b <- spautolm(ELIGIBLE~ VERBAL,
```

```r
                         data=state.sat.scores,
                         family="SAR",
                         listw=usa.listb,
                         zero.policy=TRUE)
summary(stat.sat.sar.b)
```

```
##
## Call: spautolm(formula = ELIGIBLE ~ VERBAL, data = state.sat.scores,
##     listw = usa.listb, family = "SAR", zero.policy = TRUE)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -24.88699  -7.47460   0.97745   6.14293  16.45480
##
## Coefficients:
##               Estimate Std. Error z value  Pr(>|z|)
## (Intercept) 390.605790  30.166352  12.948 < 2.2e-16
## VERBAL       -0.653476   0.058125 -11.243 < 2.2e-16
##
## Lambda: 0.15957 LR test value: 26.465 p-value: 2.6836e-07
## Numerical Hessian standard error of lambda: 0.016179
##
## Log likelihood: -179.7985
## ML residual variance (sigma squared): 75.436, (sigma: 8.6854)
## Number of observations: 49
## Number of parameters estimated: 4
## AIC: 367.6
```

```r
# row-normalized weights
stat.sat.sar.w <- spautolm(ELIGIBLE~ VERBAL,
                         data=state.sat.scores,
                         family="SAR",
                         listw=usa.listw,
                         zero.policy=TRUE)
summary(stat.sat.sar.w)
```

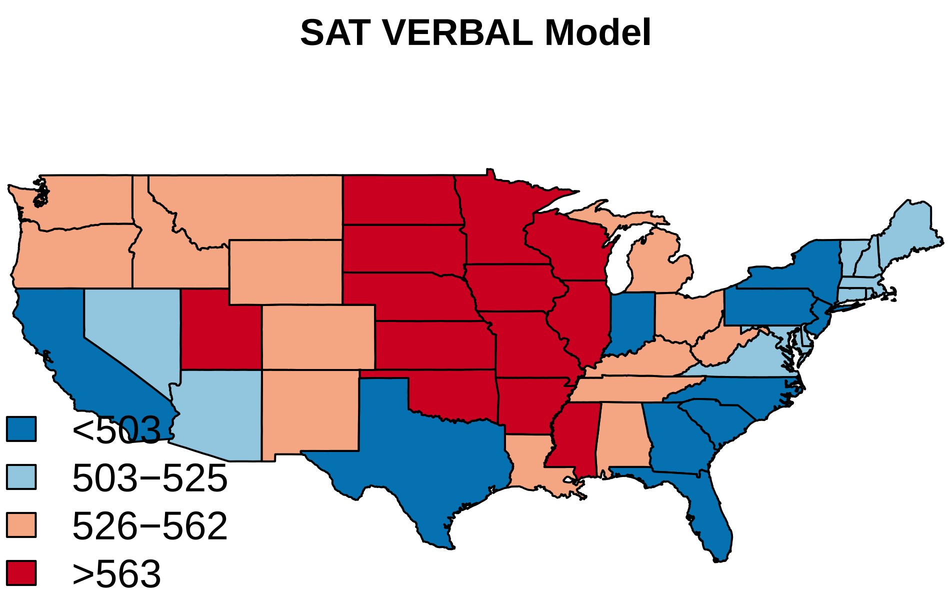
```
##
## Call: spautolm(formula = ELIGIBLE ~ VERBAL, data = state.sat.scores,
##     listw = usa.listw, family = "SAR", zero.policy = TRUE)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -23.69268  -5.31293  -0.21455   5.86328  17.06011
##
## Coefficients:
##               Estimate Std. Error z value  Pr(>|z|)
## (Intercept) 387.625170  30.011970  12.916 < 2.2e-16
## VERBAL       -0.658120   0.055979 -11.757 < 2.2e-16
##
## Lambda: 0.74267 LR test value: 31.772 p-value: 1.7338e-08
## Numerical Hessian standard error of lambda: 0.088661
##
## Log likelihood: -177.145
## ML residual variance (sigma squared): 67.444, (sigma: 8.2124)
## Number of observations: 49
```

```
## Number of parameters estimated: 4
## AIC: 362.29
```

As we can see from the above model that the VERBAL is significant since their P-Value (lower than 0.05) is small enough.The binnary weights model is $ELIGIBLE = 390.605790 - 0.653476 * VERBAL$,and The row-normalized weights model is $ELIGIBLE = 387.625170 - 0.658120 * VERBAL$. The ELIGIBLE has a negative relation with VERBAL. A possible reason VERBAL score has a negative relative with ELIGIBLE is that the erea higher verbal score the higher the more competitive in terms of admission for the area. The spatial autocorrelation parameter Lambda is significant based on the corresponded P-value. The Y is not highly sensitive the of weights.

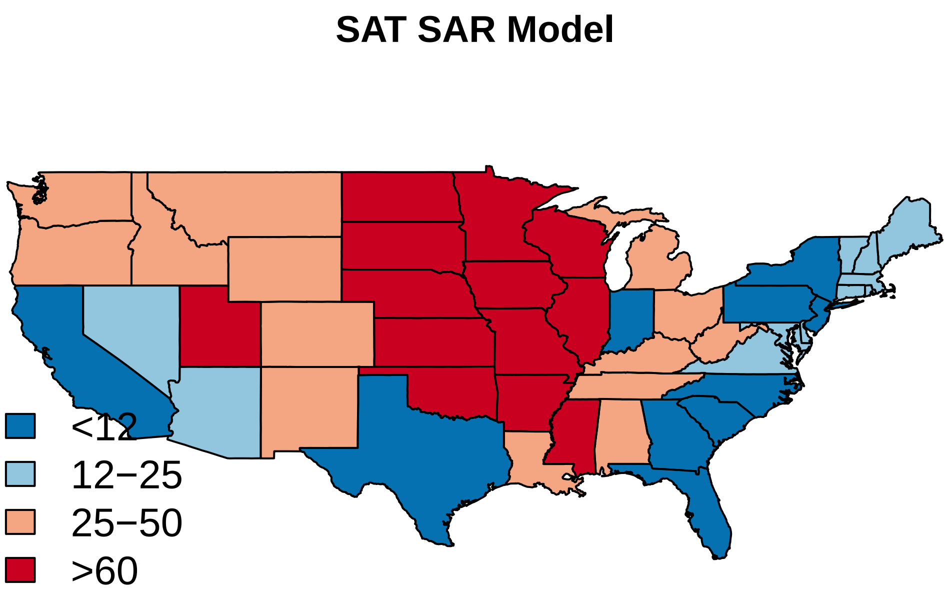
(b)

```
raw_brks = c(0,503,525,562,563)
color.pallete = rev(brewer.pal(4,"RdBu"))
state.sat.scores.fitted.sar = fitted(stat.sat.sar.w)


library(classInt)
usa.poly$VERBAL <- state.sat.scores$VERBAL
class.sar= classIntervals(var=usa.poly$VERBAL, n=4, style="fixed",
                          fixedBreaks=raw_brks, dataPrecision=4)
color.code.sar = findColours(class.sar, color.pallete)

raw.leg.txt = c("<503", "503-525", "526-562",">563")

plot(usa.poly, col=color.code.sar)
title("SAT VERBAL Model" )
legend("bottomleft", legend=raw.leg.txt, cex=1.25, bty="n", horiz = FALSE, fill = color.pallete)
```




```
sar_brks = c(0, 12, 25, 50, 60)
usa.poly$fitted.sar <- state.sat.scores.fitted.sar
class.car= classIntervals(var=usa.poly$fitted.sar, n=4, style="fixed",
                          fixedBreaks=sar_brks, dataPrecision=4)
color.code.car = findColours(class.sar, color.pallete)
sar.leg.txt = c("<12", "12-25", "25-50",">60")
```

```r
plot(usa.poly, col=color.code.sar)
title("SAT SAR Model" )
legend("bottomleft", legend=sar.leg.txt, cex=1.25, bty="n", horiz = FALSE, fill = color.pallete)
```



Given the map, the SAT scores are spatially correlated, and the Lambda value and its P-value also support this result.

(c)

```r
# train CAR model

# binnary weights
stat.sat.sar.b = spautolm(ELIGIBLE~ VERBAL,
                          data=state.sat.scores,
                          family="CAR",
                          listw=usa.listb,
                          zero.policy=TRUE)
summary(stat.sat.sar.b)
```

```
##
## Call: spautolm(formula = ELIGIBLE ~ VERBAL, data = state.sat.scores,
##     listw = usa.listb, family = "CAR", zero.policy = TRUE)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -24.64722  -6.35266   0.71498   6.07228  16.50271
##
## Coefficients:
##               Estimate Std. Error z value  Pr(>|z|)
## (Intercept) 387.833260  30.498183  12.717 < 2.2e-16
## VERBAL       -0.644767   0.059126 -10.905 < 2.2e-16
##
## Lambda: 0.18197 LR test value: 22.766 p-value: 1.8298e-06
## Numerical Hessian standard error of lambda: 0.0040577
##
```

```
## Log likelihood: -181.648
## ML residual variance (sigma squared): 82.672, (sigma: 9.0924)
## Number of observations: 49
## Number of parameters estimated: 4
## AIC: 371.3
```

```
# row-normalized weights
stat.sat.sar.w = spautolm(ELIGIBLE~ VERBAL,
                          data=state.sat.scores,
                          family="CAR",
                          listw=usa.listw,
                          zero.policy=TRUE)
summary(stat.sat.sar.w)
```

```
##
## Call: spautolm(formula = ELIGIBLE ~ VERBAL, data = state.sat.scores,
##     listw = usa.listw, family = "CAR", zero.policy = TRUE)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -25.18521  -5.83751  -0.67678   5.11028  16.94816
##
## Coefficients:
##               Estimate Std. Error z value  Pr(>|z|)
## (Intercept) 342.236050  30.175560 11.3415 < 2.2e-16
## VERBAL       -0.562578   0.056616 -9.9367 < 2.2e-16
##
## Lambda: 0.8659 LR test value: 25.452 p-value: 4.5356e-07
## Numerical Hessian standard error of lambda: NaN
##
## Log likelihood: -180.3051
## ML residual variance (sigma squared): 79.67, (sigma: 8.9258)
## Number of observations: 49
## Number of parameters estimated: 4
## AIC: 368.61
```

As we can see from the above model that the VERBAL is significant since their P-Value is small enough. The binnary weights model is $ELIGIBLE = 387.83326 - 0.644767 * VERBAL$,and The row-normalized weights model is $ELIGIBLE = 342.236050 - 0.562578 * VERBAL$. The ELIGIBLE has a negative relation with VERBAL. A possible reason VERBAL score has a negative relative with ELIGIBLE is that the erea higher verbal score the higher the more competitive in terms of admission for the area. The spatial autocorrelation parameter Lambda is significant based on the corresponded P-value.

(d)

```
## add recipical colunm
state.sat.scores[,"RC_ELIGIBLE"] <- 1.0/state.sat.scores$ELIGIBLE

# row-normalized weights
stat.sat.sar.w = spautolm(RC_ELIGIBLE~ VERBAL,
                          data=state.sat.scores,
                          family="SAR",
                          listw=usa.listw,
                          zero.policy=TRUE)
summary(stat.sat.sar.w)
```

```
##
## Call: spautolm(formula = RC_ELIGIBLE ~ VERBAL, data = state.sat.scores,
##     listw = usa.listw, family = "SAR", zero.policy = TRUE)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0514200 -0.0197687 -0.0048896  0.0140497  0.1290361
##
## Coefficients:
##               Estimate Std. Error z value  Pr(>|z|)
## (Intercept) -0.8718330  0.0800584 -10.890 < 2.2e-16
## VERBAL       0.0017632  0.0001497  11.779 < 2.2e-16
##
## Lambda: 0.035559 LR test value: 0.030181 p-value: 0.86208
## Numerical Hessian standard error of lambda: 0.20246
##
## Log likelihood: 96.5972
## ML residual variance (sigma squared): 0.0011352, (sigma: 0.033693)
## Number of observations: 49
## Number of parameters estimated: 4
## AIC: -185.19
```

The model of recipical of ELIGIBLE is $RC\_ELIGIBLE = 0.0017632 * VERBAL - 0.871833$. The spatial autocorrelation parameter Lambda is not significant based on the corresponded P-value in this case; threfore, it is not spatial correlated, and it is not really make sense.

Excercise 12

(a)

```
## Constructing neighbors using distances: Columbus example
columbus.poly <- readShapePoly(system.file("etc/shapes/columbus.shp", package="spdep")[1])

##Distance based neighbors in spdep
columbus.coords <- coordinates(columbus.poly)
columbus.knn <- knearneigh(columbus.coords)
columbus.knn2nb <- knn2nb(columbus.knn)
columbus.dist.list <- nbdists(columbus.knn2nb, columbus.coords)
columbus.dist.vec <- unlist(columbus.dist.list)
columbus.dist.max <- max(columbus.dist.vec)
columbus.dnn.nb <- dnearneigh(columbus.coords, 0, columbus.dist.max)
# 25% of the maximum intercentroid distances.
columbus.dnn.nb_25 <- dnearneigh(columbus.coords, 0, 0.25*columbus.dist.max)
columbus.dnn.nb_25
```

```
## Neighbour list object:
## Number of regions: 49
## Number of nonzero links: 4
## Percentage nonzero weights: 0.1665973
## Average number of links: 0.08163265
## 45 regions with no links:
## 1 2 3 4 5 6 7 8 9 10 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 4
```

```
# 50% of the maximum intercentroid distances.
columbus.dnn.nb_50 <- dnearneigh(columbus.coords, 0, 0.5*columbus.dist.max)
```

```
columbus.dnn.nb_50
```

```
## Neighbour list object:
## Number of regions: 49
## Number of nonzero links: 34
## Percentage nonzero weights: 1.416077
## Average number of links: 0.6938776
## 28 regions with no links:
## 1 2 3 4 5 6 7 8 9 10 15 17 20 21 22 23 25 26 28 31 32 34 36 40 41 42 45 47
```

```
# 75% of the maximum intercentroid distances.
columbus.dnn.nb_75 <- dnearneigh(columbus.coords, 0, 0.75*columbus.dist.max)
columbus.dnn.nb_75
```

```
## Neighbour list object:
## Number of regions: 49
## Number of nonzero links: 136
## Percentage nonzero weights: 5.664307
## Average number of links: 2.77551
## 7 regions with no links:
## 1 3 6 9 20 21 47
```

(b)

```
# SAR model with 25% of the maximum intercentroid distances.
columbus.dnn.listw25 = nb2listw(columbus.dnn.nb_25, style="B", zero.policy=TRUE)
columbus.dnn.sar.out_25 = spautolm(CRIME~HOVAL+INC+OPEN+PLUMB+DISCBD,
                                data=columbus.poly, family="SAR",
                                listw=columbus.dnn.listw25,
                                zero.policy=TRUE)
summary(columbus.dnn.sar.out_25)
```

```
##
## Call: spautolm(formula = CRIME ~ HOVAL + INC + OPEN + PLUMB + DISCBD,
##     data = columbus.poly, listw = columbus.dnn.listw25, family = "SAR",
##     zero.policy = TRUE)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -31.81329  -6.51404   0.64317   6.87722  18.52797
##
## Regions with no neighbours included:
##  1 2 3 4 5 6 7 8 9 10 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40
##
## Coefficients:
##              Estimate Std. Error z value  Pr(>|z|)
## (Intercept) 65.614250   4.534242 14.4708 < 2.2e-16
## HOVAL       -0.220072   0.092197 -2.3870  0.016988
## INC         -0.909138   0.304376 -2.9869  0.002818
## OPEN         0.132942   0.298311  0.4456  0.655850
## PLUMB        0.568859   0.445951  1.2756  0.202094
## DISCBD      -3.946217   1.467254 -2.6895  0.007155
##
## Lambda: 0.42853 LR test value: 1.7161 p-value: 0.19019
## Numerical Hessian standard error of lambda: 0.28721
```

```
##
## Log likelihood: -177.8164
## ML residual variance (sigma squared): 81.726, (sigma: 9.0402)
## Number of observations: 49
## Number of parameters estimated: 8
## AIC: 371.63
```

```r
# SAR model with 50% of the maximum intercentroid distances.
columbus.dnn.listw50 = nb2listw(columbus.dnn.nb_50, style="B", zero.policy=TRUE)
columbus.dnn.sar.out_50 = spautolm(CRIME~HOVAL+INC+OPEN+PLUMB+DISCBD,
                                   data=columbus.poly, family="SAR",
                                   listw=columbus.dnn.listw50,
                                   zero.policy=TRUE)
summary(columbus.dnn.sar.out_50)
```

```
##
## Call: spautolm(formula = CRIME ~ HOVAL + INC + OPEN + PLUMB + DISCBD,
##     data = columbus.poly, listw = columbus.dnn.listw50, family = "SAR",
##     zero.policy = TRUE)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -30.4210  -5.3446   1.1742   7.6049  19.2439
##
## Regions with no neighbours included:
##  1 2 3 4 5 6 7 8 9 10 15 17 20 21 22 23 25 26 28 31 32 34 36 40 41 42 45 47
##
## Coefficients:
##              Estimate Std. Error z value  Pr(>|z|)
## (Intercept) 64.354954   4.603757 13.9788 < 2.2e-16
## HOVAL       -0.238251   0.083726 -2.8456  0.004432
## INC         -0.908360   0.283752 -3.2012  0.001368
## OPEN         0.175997   0.239703  0.7342  0.462808
## PLUMB        0.481019   0.445153  1.0806  0.279889
## DISCBD      -3.422517   1.464512 -2.3370  0.019441
##
## Lambda: 0.27073 LR test value: 5.4933 p-value: 0.01909
## Numerical Hessian standard error of lambda: 0.090858
##
## Log likelihood: -175.9278
## ML residual variance (sigma squared): 72.445, (sigma: 8.5115)
## Number of observations: 49
## Number of parameters estimated: 8
## AIC: 367.86
```

```r
# SAR model with 75% of the maximum intercentroid distances.
columbus.dnn.listw75 = nb2listw(columbus.dnn.nb_75, style="B", zero.policy=TRUE)
columbus.dnn.sar.out_75 = spautolm(CRIME~HOVAL+INC+OPEN+PLUMB+DISCBD,
                                   data=columbus.poly, family="SAR",
                                   listw=columbus.dnn.listw75,
                                   zero.policy=TRUE)
summary(columbus.dnn.sar.out_75)
```

```
##
## Call: spautolm(formula = CRIME ~ HOVAL + INC + OPEN + PLUMB + DISCBD,
```

```
##     data = columbus.poly, listw = columbus.dnn.listw75, family = "SAR",
##     zero.policy = TRUE)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -27.82384  -4.60966   0.24482   6.49501  20.32682
##
## Regions with no neighbours included:
##  1 3 6 9 20 21 47
##
## Coefficients:
##              Estimate Std. Error z value  Pr(>|z|)
## (Intercept) 56.214935   6.191956  9.0787 < 2.2e-16
## HOVAL       -0.263497   0.080785 -3.2617  0.001108
## INC         -0.677682   0.285948 -2.3699  0.017791
## OPEN         0.178092   0.247245  0.7203  0.471338
## PLUMB        0.599812   0.396126  1.5142  0.129977
## DISCBD      -2.222274   1.610364 -1.3800  0.167592
##
## Lambda: 0.15152 LR test value: 10.334 p-value: 0.0013063
## Numerical Hessian standard error of lambda: 0.022277
##
## Log likelihood: -173.5076
## ML residual variance (sigma squared): 63.104, (sigma: 7.9438)
## Number of observations: 49
## Number of parameters estimated: 8
## AIC: 363.02
```

As we can see from above results, the models are:

SAR model with 25% of the maximum intercentroid:

$CRIME = 65.614250 - 0.220072 * HOVAL - 0.909138 * INC + 0.132942 * OPEN + 0.568859 * PLUMB - 3.946217 * DISCBD$

SAR model with 50% of the maximum intercentroid:

$CRIME = 64.354954 - 0.238251 * HOVAL - 0.908360 * INC + 0.175997 * OPEN + 0.481019 * PLUMB - 3.422517 * DISCBD$

SAR model with 75% of the maximum intercentroid: $CRIME = 56.214935 - 0.263497 * HOVAL - 0.677682 * INC + 0.178092 * OPEN + 0.599812 * PLUMB - 3.946217 * DISCBD$

As we can see from the above models the CRIME has a negative relation with HOVAL, INC, and DISCBD; and positive relation with OPEN and PLUMB. The significant corresponded with models are below. As we can see from the table below, the DISCBD become insignificant with the distance increased.

Table 1: Significant table for SAR models

Variables	25 precent SAR model	50 precent SAR model	75 precent SAR model
HOVAL	YES	YES	YES
INC	YES	YES	YES
OPEN	NO	NO	NO
PLUMB	NO	NO	NO
DISCBD	YES	YES	NO
Lambda	NO	YES	YES

(d)

```r
# SAR model with 25% of the maximum intercentroid distances.
columbus.dnn.listw25 = nb2listw(columbus.dnn.nb_25, style="B", zero.policy=TRUE)
columbus.dnn.car.out_25 = spautolm(CRIME~HOVAL+INC+OPEN+PLUMB+DISCBD,
                                data=columbus.poly, family="CAR",
                                listw=columbus.dnn.listw25,
                                zero.policy=TRUE)
summary(columbus.dnn.car.out_25)
```

```
##
## Call: spautolm(formula = CRIME ~ HOVAL + INC + OPEN + PLUMB + DISCBD,
##     data = columbus.poly, listw = columbus.dnn.listw25, family = "CAR",
##     zero.policy = TRUE)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -31.76585  -6.50955   0.51605   6.86591  18.57521
##
## Regions with no neighbours included:
##  1 2 3 4 5 6 7 8 9 10 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 4
##
## Coefficients:
##              Estimate Std. Error z value  Pr(>|z|)
## (Intercept) 65.557404   4.522815 14.4948 < 2.2e-16
## HOVAL       -0.221292   0.091966 -2.4062  0.016118
## INC         -0.909192   0.303664 -2.9941  0.002753
## OPEN         0.133654   0.297657  0.4490  0.653417
## PLUMB        0.579156   0.445048  1.3013  0.193144
## DISCBD      -3.918245   1.464026 -2.6763  0.007443
##
## Lambda: 0.67159 LR test value: 1.613 p-value: 0.20408
## Numerical Hessian standard error of lambda: 0.34472
##
## Log likelihood: -177.868
## ML residual variance (sigma squared): 81.253, (sigma: 9.014)
## Number of observations: 49
## Number of parameters estimated: 8
## AIC: 371.74
```

```r
# SAR model with 50% of the maximum intercentroid distances.
columbus.dnn.listw50 = nb2listw(columbus.dnn.nb_50, style="B", zero.policy=TRUE)
columbus.dnn.car.out_50 = spautolm(CRIME~HOVAL+INC+OPEN+PLUMB+DISCBD,
                                data=columbus.poly, family="CAR",
                                listw=columbus.dnn.listw50,
                                zero.policy=TRUE)
summary(columbus.dnn.car.out_50)
```

```
##
## Call: spautolm(formula = CRIME ~ HOVAL + INC + OPEN + PLUMB + DISCBD,
##     data = columbus.poly, listw = columbus.dnn.listw50, family = "CAR",
##     zero.policy = TRUE)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
```

```
## -30.5492  -5.4425  -0.5656   5.9001  19.2863
##
## Regions with no neighbours included:
##  1 2 3 4 5 6 7 8 9 10 15 17 20 21 22 23 25 26 28 31 32 34 36 40 41 42 45 47
##
## Coefficients:
##              Estimate Std. Error z value  Pr(>|z|)
## (Intercept) 64.712769   4.611392 14.0332 < 2.2e-16
## HOVAL       -0.237664   0.085611 -2.7761  0.005502
## INC         -0.902190   0.288791 -3.1240  0.001784
## OPEN         0.162894   0.261103  0.6239  0.532712
## PLUMB        0.482755   0.453080  1.0655  0.286651
## DISCBD      -3.542936   1.472339 -2.4063  0.016114
##
## Lambda: 0.361 LR test value: 4.4253 p-value: 0.03541
## Numerical Hessian standard error of lambda: 0.086496
##
## Log likelihood: -176.4618
## ML residual variance (sigma squared): 73.748, (sigma: 8.5877)
## Number of observations: 49
## Number of parameters estimated: 8
## AIC: 368.92
```

```r
# SAR model with 75% of the maximum intercentroid distances.
columbus.dnn.listw75 = nb2listw(columbus.dnn.nb_75, style="B", zero.policy=TRUE)
columbus.dnn.car.out_75 = spautolm(CRIME~HOVAL+INC+OPEN+PLUMB+DISCBD,
                                data=columbus.poly, family="CAR",
                                listw=columbus.dnn.listw75,
                                zero.policy=TRUE)
summary(columbus.dnn.car.out_75)
```

```
##
## Call: spautolm(formula = CRIME ~ HOVAL + INC + OPEN + PLUMB + DISCBD,
##     data = columbus.poly, listw = columbus.dnn.listw75, family = "CAR",
##     zero.policy = TRUE)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -29.61587  -4.76948  -0.20693   6.01413  19.35546
##
## Regions with no neighbours included:
##  1 3 6 9 20 21 47
##
## Coefficients:
##              Estimate Std. Error z value  Pr(>|z|)
## (Intercept) 60.020801   5.629926 10.6610 < 2.2e-16
## HOVAL       -0.250413   0.084817 -2.9524  0.003153
## INC         -0.768608   0.297212 -2.5861  0.009708
## OPEN         0.149635   0.267933  0.5585  0.576517
## PLUMB        0.511153   0.420342  1.2160  0.223969
## DISCBD      -2.871965   1.551668 -1.8509  0.064186
##
## Lambda: 0.16746 LR test value: 7.0791 p-value: 0.0077989
## Numerical Hessian standard error of lambda: 0.015017
##
```

```
## Log likelihood: -175.1349
## ML residual variance (sigma squared): 69.116, (sigma: 8.3136)
## Number of observations: 49
## Number of parameters estimated: 8
## AIC: 366.27
```

As we can see from above results, the models are:

CAR model with 25% of the maximum intercentroid:

$CRIME = 65.557404 - 0.221292 * HOVAL - 0.909192 * INC + 0.133654 * OPEN + 0.579156 * PLUMB - 3.918245 * DISCBD$

CAR model with 50% of the maximum intercentroid:

$CRIME = 64.712769 - 0.237664 * HOVAL - 0.902190 * INC + 0.162894 * OPEN + 0.482755 * PLUMB - 3.542936 * DISCBD$

CAR model with 75% of the maximum intercentroid:

$CRIME = 60.020801 - 0.250413 * HOVAL - 0.768608 * INC + 0.149635 * OPEN + 0.511153 * PLUMB - 2.871965 * DISCBD$

As we can see from the above models the CRIME has a negative relation with HOVAL, INC, and DISCBD; and positive relation with OPEN and PLUMB. The significant corresponded with models are below. As we can see from the table below, the DISCBD become insignificant with the distance increased. Overall the SAR models and CAR models show similar results. Based on the above models and results.

Table 2: Significant table for CAR models

Variables	25 precent CAR model	50 precent CAR model	75 precent CAR model
HOVAL	YES	YES	YES
INC	YES	YES	YES
OPEN	NO	NO	NO
PLUMB	NO	NO	NO
DISCBD	YES	YES	NO
Lambda	NO	YES	YES