Optimizing Waste Transportation for Miharja Shipping & Chempro Using Transshipment and Transportation Models

(University Project – Operational Research and Optimization @ UTM)
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Developed a mathematical model to optimize waste transportation routes for Miharia Shipping and Chempro using both transportation and transshipment models. The project involved formulating cost minimization strategies using operations research techniques.

Tools/Skills: Linear programming, Excel Solver, optimization modeling, logistics analysis

Sally is the manager of the North Atlantic office of the Miharja Shipping and Transport Company. She is in the process of negotiating a new shipping contract with ChemPro, a company that manufactures chemicals for industrial use. ChemPro want Miharja to pick up and transport waste products from its six plants to three waste disposal sites. Sally is very concerned about this proposed arrangement. The chemical wastes that will be hauled can be hazardous to humans and the environment if they leak. In addition, a number of towns and communities in the region where the plants are located prohibit hazardous materials from being shipped through their municipal limits. Thus, not only will the shipments have to be handled carefully and transported at reduced speeds, but they will also have to traverse circuitous routes in many cases. Sally has estimated the cost of shipping a barrel of waste from each of the six plants to each of the three waste disposal sites as shown in Table 1:

Table 1

	Waste Disposal Site		
Plant	Whitewater	Los Canos	Duras
Kingsport	12	15	17
Danville	14	9	10
Macon	13	20	11
Selma	17	16	19
Columbus	7	14	12
Allentown	22	16	18

The plants generate the mounts of waste products each week as in Table 2.

Table 2

Plant	**Waste per Week (bbl)**
Kingsport	35
Danville	26
Macon	42
Selma	53
Columbus	29
Allentown	38

The three waste disposal sites at Whitewater, Los Canos, and Duras can accommodate a maximum of 65, 80, and 105 barrels per week respectively. In addition to shipping directly from each of the six plants to one of the three waste disposal sites, Sally is also considering using each of the plants as intermediate shipping points. Trucks would be able to drop a load at a plant to be picked up and carried on to the final destination by another truck, and vice versa. Miharja would not incur any handling costs because ChemPro has agreed to take care of all local handling of the waste materials at the plants and the waste disposal sites. In other words, the only cost Miharja incurs is the actual transportation cost. So, Sally wants to be able to consider the possibility that it may be cheaper to drop and pick up loads at intermediate points rather than ship them directly. Sally estimates the shipping costs per barrel between each of the six plants to be as in Table 3.

Table 3

	Plant					
Plant	Kingsport	Danville	Macon	Selma	Columbus	Allentown
Kingsport	-	6	4	9	7	8
Danville	6	-	11	10	12	7
Macon	5	11	-	3	7	15
Selma	9	10	3	-	3	16
Columbus	7	12	7	3	-	14
Allentown	8	7	15	16	14	-

Sally wants to determine the shipping routes that will minimize Miharja's total cost in order to develop a contract proposal to submit to ChemPro for waste disposal. She particularly wants to know if it would be cheaper to ship directly from the plants to the waste sites or if she should drop and pick up some loads at the various plants. Develop a model to assist Sally and solve the model to determine the optimal routes.

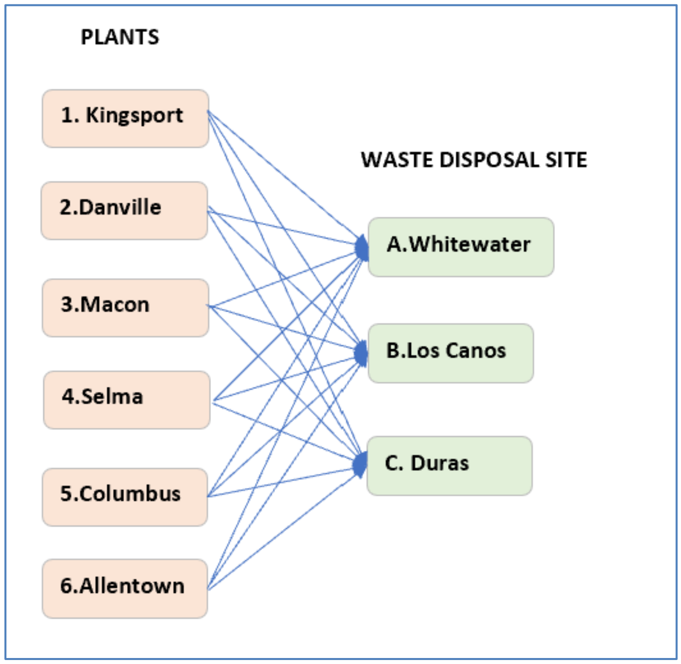
Analyze this case, as follows:

a) Develop a transportation model for shipping the waste directly from the 6 plants to the 3 waste disposal sites using Phyton. Solve the model and determine the optimal transportation cost.

b) Develop a transhipment model in which each of the plants can be used as intermediate points using Excel Solver and Python to determine the optimal cost.

c) Interpret the results and determine the best model for Sally to be implemented.

a) Develop a transportation model for shipping the waste directly from the 6 plants to the 3 waste disposal sites using Phyton. Solve the model and determine the optimal transportation cost.



Transportation Network Routes

Shipping cost from plants to waste disposal sites			
Waste Disposal Site			
Plant	A.Whitewater	B.Los Canos	C.Duras
1. Kingsport	12	15	17
2.Danville	14	9	10
3.Macon	13	20	11
4.Selma	17	16	19
5.Columbus	7	14	12
6.Allentown	22	16	18

Supply	
Plant	**Waste/Week (bbl)**
1. Kingsport	35
2.Danville	26
3.Macon	42
4.Selma	53
5.Columbus	29
6.Allentown	38
TOTAL	**223**

Demand	
Waste Disposal Site	**Maximum Demand**
A.Whitewater	65
B.Los Canos	80
C.Duras	105
TOTAL	**250**

Upon observation, this transportation model is unbalanced with demand exceeding the available supply. To rectify, a dummy supply is introduced to achieve even distribution.

Shipping cost from plants to waste disposal sites			
	Waste Disposal Site		
Plant	A.Whitewater	B.Los Canos	C.Duras
1. Kingsport	12	15	17
2.Danville	14	9	10
3.Macon	13	20	11
4.Selma	17	16	19
5.Columbus	7	14	12
6.Allentown	22	16	18
7. Dummy	0	0	0

Supply	
Plant	**Waste/Week (bbl)**
1. Kingsport	35
2.Danville	26
3.Macon	42
4.Selma	53
5.Columbus	29
6.Allentown	38
7. Dummy	27
TOTAL	**250**

Demand	
Waste Disposal Site	**Maximum Demand**
A.Whitewater	65
B.Los Canos	80
C.Duras	105
TOTAL	**250**

Model Formulation

Decision Variables:

- x_{ij}: Number of barrels of waste shipped from plant i to disposal site j.
- i=1,2,3,4,5,6,7 to j=A,B,C

Objective Function:

Minimize, $Z = 12x_{1A} + 15x_{1B} + 17x_{1C} + 14x_{2A} + 9x_{2B} + 10x_{2C} + 13x_{3A} + 20x_{3B} + 11x3_{C} + 17x_{4A} + 16x_{4B} + 19x_{4C} + 7x_{5A} + 14x_{5B} + 12x_{5C} + 22x_{6A} + 16x_{6B} + 18x_{6C} + 0x_{7A} + 0x_{7B} + 0x_{7C}$

Minimize the total transportation cost.

Subject to Constraints:

Supply Constraints: The amount of waste shipped from each plant should be equal its supply.
$x_{1A}+x_{1B}+x_{1C}=35$ (1.Kingsport)
$x_{2A}+x_{2B}+x_{2C}=26$ (2.Danville)
$x_{3A}+x_{3B}+x_{3C}=42$ (3.Macon)
$x_{4A}+x_{4B}+x_{4C}=53$ (4.Selma)
$x_{5A}+x_{5B}+x_{5C}=29$ (5.Columbus)
$x_{6A}+x_{6B}+x_{6C}=38$ (6.Allentown)
$x_{7A}+x_{7B}+x_{7C}=27$ (7.Dummy)

Demand Constraints: The amount of waste received at each disposal site should meet its demand.
$x_{1A}+x_{2A}+x_{3A}+x_{4A}+x_{5A}+x_{6A}=65$ (A.Whitewater)
$x_{1B}+x_{2B}+x_{3B}+x_{4B}+x_{5B}+x_{6B}=80$ (B.Los Canos)
$x_{1C}+x_{2C}+x_{3C}+x_{4C}+x_{5C}+x_{6C}=105$ (C.Duras)

Non-negativity Constraints: The number of barrels shipped must be non-negative.
$x_{ij}\geq 0$ for all i,j

Results:

```
Route_1_A = 35.0
Route_2_C = 26.0
Route_3_C = 42.0
Route_4_A = 1.0
Route_4_B = 52.0
Route_5_A = 29.0
Route_6_B = 28.0
Route_6_C = 10.0
Route_7_C = 27.0
Optimal Direct Transportation Cost: 2822.0
```

Optimal Solution using Python

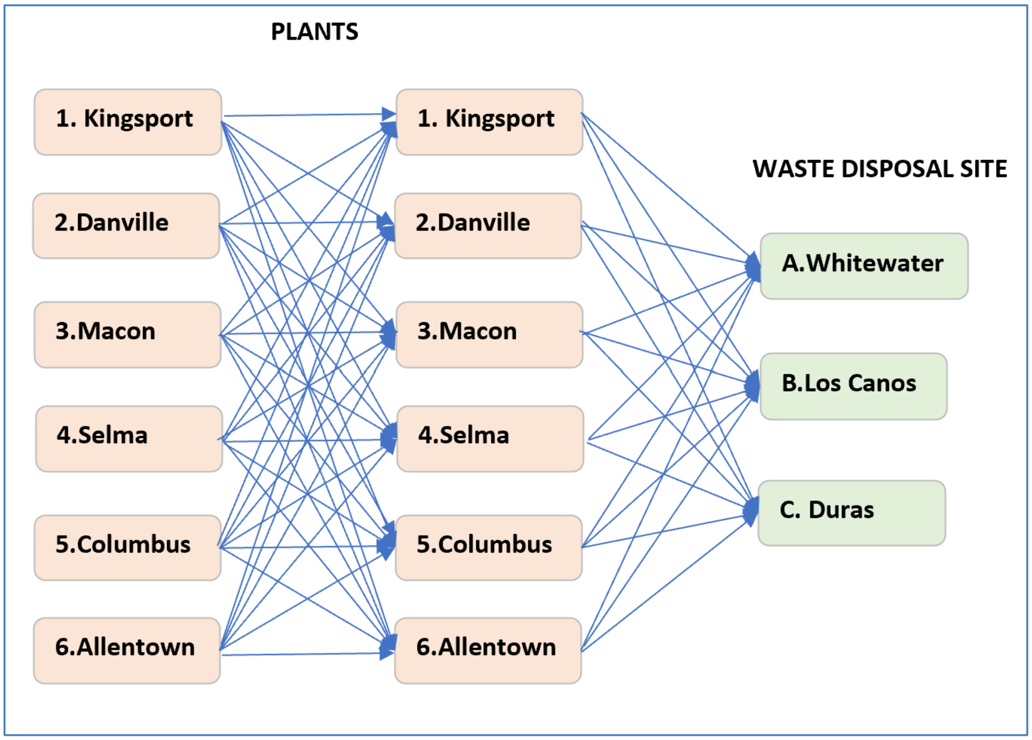
Thefore, the optimum solution of the transportation problem can be conclude as the table below.

From	To	Shipment unit	Cost per unit	Shipment cost
1.Kingsport	A.Whitewater	35	$12	$420
2.Danville	C.Duras	26	$10	$260
3.Macon	C.Duras	42	$11	$462
4.Selma	A.Whitewater	1	$17	$17
4.Selma	B.Los Canos	52	$16	$832
5.Columbus	A.Whitewater	29	$7	$203
6.Allentown	B.Los Canos	28	$16	$448
6.Allentown	C.Duras	10	$18	$180
Total Cost				**$2822**

When Miharja directly ships waste from plants to disposal sites, the total transportation cost for the optimal solution is $2822. This cost is calculated by considering the coefficients (shipping costs) and the optimal flow values for each route.

However, the rest of the demand at disposal site Duras (27bbl), cannot be fully met due to insufficient supply. To account for this shortfall, a dummy route was introduced in the optimization model. Overall, the solution optimally allocates the transportation of waste to meet demand at disposal sites while minimizing the total transportation cost.

b) <u>Develop a transshipment model in which each of the plants can be used as intermediate points using Excel Solver and Python to determine the optimal cost.</u>



Transshipment Network Routes

Since each plants used as intermediate shipping points, the transportation model becomes a transshipment model. The additional decision variables included in the new model formulation as below:

<u>Model Formulation</u>

Decision Variables:

- $x_{hi:}$ Number of barrels of waste shipped from plant h to intermediate plant h.
- x_{ij}: Number of barrels of waste shipped from plant i to disposal site j.

where

h=1,2,3,4,5,6

i=1,2,3,4,5,6,7 (the same 6 plants as h but additional of 1 dummy supply)

j=A,B,C

Objective Function:
Minimize, $Z = 6x_{12} + 4x_{13} + 9x_{14} + 7x_{15} + 8x_{16} + 6x_{21} + 11x_{23} + 10x_{24} + 12x_{25} + 7x_{26} + 5x_{31} + 11x_{32} + 3x_{34} + 7x_{35} + 15x_{36} + 9x_{41} + 10x_{42} + 3x_{43} + 3x_{45} + 16x_{46} + 7x_{51} + 12x_{52} + 7x_{53} + 3x_{54} + 14x_{56} + 8x_{61} + 7x_{62} + 15x_{63} + 16x_{64} + 14x_{65} + 12x_{1A} + 15x_{1B} + 17x_{1C} + 14x_{2A} + 9x_{2B} + 10x_{2C} + 13x_{3A} + 20x_{3B} + 11x3_{C} + 17x_{4A} + 16x_{4B} + 19x_{4C} + 7x_{5A} + 14x_{5B} + 12x_{5C} + 22x_{6A} + 16x_{6B} + 18x_{6C}$

Minimize the total transportation cost.

Subject to Constraints:

Supply Constraints 1: The amount of waste shipped from each plant should be equal its supply.
$x_{1A+}x_{1B}+x_{1C} = 35$ (1.Kingsport)
$x_{2A}+x_{2B}+x_{2C} = 26$ (2.Danville)
$x_{3A}+x_{3B}+x_{3C} = 42$ (3.Macon)
$x_{4A}+x_{4B}+x_{4C} = 53$ (4.Selma)
$x_{5A}+x_{5B}+x_{5C} = 29$ (5.Columbus)
$x_{6A}+x_{6B}+x_{6C} = 38$ (6.Allentown)

Supply Constraints 2: The amount of waste shipped from each plant should be equal its supply.
$x_{1A+}x_{1B}+x_{1C} = 35$ (1.Kingsport)
$x_{2A}+x_{2B}+x_{2C} = 26$ (2.Danville)
$x_{3A}+x_{3B}+x_{3C} = 42$ (3.Macon)
$x_{4A}+x_{4B}+x_{4C} = 53$ (4.Selma)
$x_{5A}+x_{5B}+x_{5C} = 29$ (5.Columbus)
$x_{6A}+x_{6B}+x_{6C} = 38$ (6.Allentown)
$x_{7A}+x_{7B}+x_{7C} = 27$ (7.Allentown)

Demand Constraints: The amount of waste received at each disposal site should meet its demand.
$x_{1A}+x_{2A}+x_{3A}+x_{4A}+x_{5A}+x_{6A}=65$ (A.Whitewater)
$x_{1B}+x_{2B}+x_{3B}+x_{4B}+x_{5B}+x_{6B}=80$ (B.Los Canos)
$x_{1C}+x_{2C}+x_{3C}+x_{4C}+x_{5C}+x_{6C}=105$ (C.Duras)

Transshipment Constraint: The amount of waste shipped from supply equal to amount received by demand
$\sum x_{hh} - \sum x_{ij} = 0$

Non-negativity Constraints: The number of barrels shipped must be non-negative.
x_{hi} and $x_{ij} \geq 0$ for all h, i, and j

Results:

The solution produced for this model formulation using Excel Solver and Python are as below;

```
Route_1_Allentown_Allentown = 38.0
Route_1_Columbus_Columbus = 29.0
Route_1_Danville_Danville = 26.0
Route_1_Kingsport_Kingsport = 16.0
Route_1_Kingsport_Macon = 19.0
Route_1_Macon_Macon = 42.0
Route_1_Selma_Columbus = 36.0
Route_1_Selma_Macon = 17.0
Route_2_Allentown_Los_Canos = 38.0
Route_2_Columbus_Whitewater = 65.0
Route_2_Danville_Los_Canos = 26.0
Route_2_Dummy_Duras = 27.0
Route_2_Kingsport_Los_Canos = 16.0
Route_2_Macon_Duras = 78.0
Optimal Transshipment Cost: 2630.0
```

Solution using Python

Plant	Plant							
	1. Kingsport	2.Danville	3.Macon	4.Selma	5.Columbus	6.Allentown	Shipped	Supply
1. Kingsport	0	16	19	0	0	0	35	35
2.Danville	0	26	0	0	0	0	26	26
3.Macon	0	0	42	0	0	0	42	42
4.Selma	0	0	17	0	36	0	53	53
5.Columbus	0	0	0	0	29	0	29	29
6.Allentown	0	38	0	0	0	0	38	38
Shipped	0	80	78	0	65	0		

Plant	Waste disposal site			
	A.Whitewater	B.Los Canos	C.Duras	Shipped
1. Kingsport	0	0	0	0
2.Danville	0	80	0	80
3.Macon	0	0	78	78
4.Selma	0	0	0	0
5.Columbus	65	0	0	65
6.Allentown	0	0	0	0
7. Dummy	0	0	27	27
Shipped	65	80	105	
Demand	65	80	105	

Total Cost	**2630**

Solution using Excel Solver

Thus, the optimum solution for this transshipment problem can be concluded in the following table.

From Plants	**To Intermediate Plants**	**Shipment unit**	**Cost per unit**	**Shipment cost**	**Total**
1.Kingsport	1.Danville	16	$6	$96	$597
2.Danville		26	$0	$0	
6.Allentown		38	$7	$266	
1.Kingsport	3.Macon	19	$4	$76	
3.Macon		42	$0	$0	
4.Selma		17	$3	$51	
4.Selma	5.Columbus	36	$3	$108	
5.Columbus		29	$0	$0	
From Intermediate Plants	**To Waste Disposal Sites**	**Shipment unit**	**Cost per unit**	**Shipment cost**	**Total**
1.Danville	B.Los Canos	80	$9	$720	$2033
3.Macon	C.Duras	78	$11	$858	
5.Columbus	A.Whitewater	65	$7	$455	
				TOTAL	**$2630**

When transshipping waste within Miharja's plants and subsequently to disposal sites, the total transshipment cost for the optimal solution amounts to $2630. This cost is computed by taking into consideration the coefficients (shipping costs) and the optimal flow values for each route.

Again, the remaining demand at the disposal site for Duras (27 barrels) cannot be entirely fulfilled due to inadequate supply. Hence, a dummy route was incorporated into the optimization model.

c) <u>Interpret the results and determine the best model for Sally to be implemented</u>

a) Direct Transportation Model

From Plants	To Disposal Sites	Shipment unit	Cost per unit	Shipment cost
1.Kingsport	A.Whitewater	35	$12	$420
2.Danville	C.Duras	26	$10	$260
3.Macon	C.Duras	42	$11	$462
4.Selma	A.Whitewater	1	$17	$17
4.Selma	B.Los Canos	52	$16	$832
5.Columbus	A.Whitewater	29	$7	$203
6.Allentown	B.Los Canos	28	$16	$448
6.Allentown	C.Duras	10	$18	$180
Total Cost				$2822

b) Transshipment Model

From Plants	To Intermediate Plants	Shipment unit	Cost per unit	Shipment cost	Total
1.Kingsport	1.Danville	16	$6	$96	$597
2.Danville		26	$0	$0	
6.Allentown		38	$7	$266	
1.Kingsport	3.Macon	19	$4	$76	
3.Macon		42	$0	$0	
4.Selma		17	$3	$51	
4.Selma	5.Columbus	36	$3	$108	
5.Columbus		29	$0	$0	
From Intermediate Plants	**To Waste Disposal Sites**	**Shipment unit**	**Cost per unit**	**Shipment cost**	**Total**
1.Danville	B.Los Canos	80	$9	$720	$2033
3.Macon	C.Duras	78	$11	$858	
5.Columbus	A.Whitewater	65	$7	$455	
				TOTAL	$2630

Upon analyzing the results, optimal cost for direct transportation from plants to disposal sites is $2882. Conversely, employing a transshipment strategy within each plants before onward transportation to disposal sites yields a total cost of $2630. From a cost perspective alone, the transshipment model appears as the more advantageous choice by presenting a solution with a lower optimal cost compared to the direct transportation model. This translates to a weekly cost savings of $192 for Miharja which is significant amount of cost that can be reduce.

In summary, the transshipment model stands as the best choice for Sally to implement in achieving cost efficiency with a noticeably reduced optimal cost.

Appendixes

1. Question a) using Python

a) Direct Transportation Model

```
# Import PuLP modeler functions
from pulp import *

# Creates a list of all the supply nodes
plants = ['1','2','3','4','5','6','7']

# Creates a dictionary for the number of units of supply (Weekly waste generation)
supply = {'1': 35,
          '2': 26,
          '3': 42,
          '4': 53,
          '5': 29,
          '6': 38,
          '7': 27}

# Creates a list of all the demand nodes
waste_disposal_sites = ['A', 'B', 'C']

# Creates a dictionary for the number of units of demand (Maximum capacity at disposal sites)
demand = {'A': 65,
          'B': 80,
          'C': 105}
```

```
# Cost matrix
costs = [[12, 15, 17],
         [14, 9, 10],
         [13, 20, 11],
         [17, 16, 19],
         [7, 14, 12],
         [22, 16, 18],
         [0, 0, 0]]

# The cost data is made into a dictionary
costs = makeDict([plants,waste_disposal_sites],costs,0)
```

```
# Define the problem
prob = LpProblem("Waste_DirectShipping_Problem", LpMinimize)
```

```
# Create routes
routes = [(i, j) for i in plants for j in waste_disposal_sites]
```

```
# Create dictionary
route_vars = LpVariable.dicts("Route", (plants, waste_disposal_sites), 0, None, LpInteger)
```

```
# Objective function
prob += lpSum([route_vars[i][j] * costs[i][j] for (i,j) in routes]), "Total Shipping Cost"
```

```
# Supply constraints
for i in plants:
    prob += lpSum([route_vars[i][j] for j in waste_disposal_sites]) == supply[i], f"Supply_{i}"
```

```
# Demand constraints
for j in waste_disposal_sites:
    prob += lpSum([route_vars[i][j] for i in plants]) == demand[j], f"Demand_{j}"
```

```
# The problem data is written to an .lp file
prob.writeLP("DirectShippingProblem.lp")
```

```
[Route_1_A,
 Route_1_B,
 Route_1_C,
 Route_2_A,
 Route_2_B,
 Route_2_C,
 Route_3_A,
 Route_3_B,
 Route_3_C,
 Route_4_A,
 Route_4_B,
 Route_4_C,
 Route_5_A,
 Route_5_B,
 Route_5_C,
 Route_6_A,
 Route_6_B,
 Route_6_C,
 Route_7_A,
 Route_7_B,
 Route_7_C]
```

```python
# Solve the problem
prob.solve()
```

```
1
```

```python
# The status of the solution is printed to the screen
print ("Status:", LpStatus[prob.status])
```

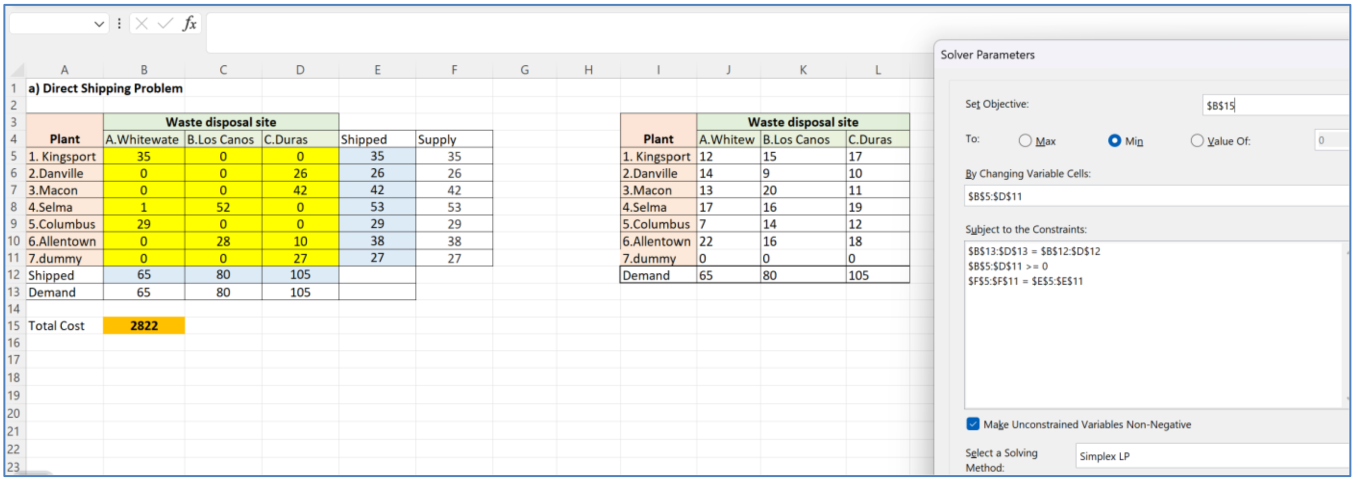
```
Status: Optimal
```

```python
# Print the results
for v in prob.variables():
    if v.varValue > 0:
        print(f"{v.name} = {v.varValue}")

print("Optimal Direct Transportation Cost:", value(prob.objective))
```

```
Route_1_A = 35.0
Route_2_C = 26.0
Route_3_C = 42.0
Route_4_A = 1.0
Route_4_B = 52.0
Route_5_A = 29.0
Route_6_B = 28.0
Route_6_C = 10.0
Route_7_C = 27.0
Optimal Direct Transportation Cost: 2822.0
```

2. Question a) using Excel Solver



a) Direct Shipping Problem

Plant	Waste disposal site			Shipped	Supply
	A.Whitewate	B.Los Canos	C.Duras		
1. Kingsport	35	0	0	35	35
2.Danville	0	0	26	26	26
3.Macon	0	0	42	42	42
4.Selma	1	52	0	53	53
5.Columbus	29	0	0	29	29
6.Allentown	0	28	10	38	38
7.dummy	0	0	27	27	27
Shipped	65	80	105		
Demand	65	80	105		

Total Cost	2822

Plant	Waste disposal site		
	A.Whitew	B.Los Canos	C.Duras
1. Kingsport	12	15	17
2.Danville	14	9	10
3.Macon	13	20	11
4.Selma	17	16	19
5.Columbus	7	14	12
6.Allentown	22	16	18
7.dummy	0	0	0
Demand	65	80	105

3. Question b) using Python

b) Transshipment Model- with dummy

```python
# Import PuLP modeler functions
from pulp import *

# Creates a list of all the supply nodes
plants = ['Kingsport', 'Danville', 'Macon', 'Selma', 'Columbus', 'Allentown']

# Creates a dictionary for the number of units of supply for each supply node
supply = {'Kingsport': 35,
          'Danville': 26,
          'Macon': 42,
          'Selma': 53,
          'Columbus': 29,
          'Allentown': 38}

# Creates a list of costs of each transportation path
costs_1 = [[0, 6, 4, 9, 7, 8],
           [6, 0, 11, 10, 12, 7],
           [5, 11, 0, 3, 7, 15],
           [9, 10, 3, 0, 3, 16],
           [7, 12, 7, 3, 0, 14],
           [8, 7, 15, 16, 14, 0]]

intermediate_plants = ['Kingsport', 'Danville', 'Macon', 'Selma', 'Columbus', 'Allentown','Dummy']
supply1 = {'Kingsport': 35,
          'Danville': 26,
          'Macon': 42,
          'Selma': 53,
          'Columbus': 29,
          'Allentown': 38,
          'Dummy': 27}

# Creates a list of all demand nodes
disposal_sites = ['Whitewater', 'Los Canos', 'Duras']
```

```python
# Creates a dictionary for the number of units of demand for each demand node
demand = {
    'Whitewater': 65,
    'Los Canos': 80,
    'Duras': 105}

costs_2 = [[12, 15, 17],
         [14, 9, 10],
         [13, 20, 11],
         [17, 16, 19],
         [7, 14, 12],
         [22, 16, 18],
          [0,0,0]]
```

```python
# The cost data is made into a dictionary
costs_1 = makeDict([plants, plants], costs_1, 0)

# The cost data is made into a dictionary
costs_2 = makeDict([intermediate_plants, disposal_sites], costs_2, 0)
```

```python
# Creates the 'prob' variable to contain the problem data
prob = LpProblem("Material_Supply_Problem", LpMinimize)
```

```python
# Creates a list of tuples containing all the possible routes for transport
Routes = [(i, j) for i in plants for j in plants]
```

```python
# A dictionary called 'Vars' is created to contain the referenced variables(the routes)
vars = LpVariable.dicts("Route_1", (plants, plants), 0, None, LpInteger)
```

```python
# Creates a list of tuples containing all the possible routes for transport
Routes_2 = [(w, b) for w in intermediate_plants for b in disposal_sites]
```

```python
# A dictionary called 'Vars_2' is created to contain the referenced variables(the routes)
vars_2 = LpVariable.dicts("Route_2", (intermediate_plants, disposal_sites), 0, None, LpInteger)
```

```python
# The supply maximum constraints are added to prob for each supply node (plants)
for s in plants:
    prob += (lpSum([vars[s][w] for w in plants]) == supply[s],
        "Sum_of_Products_out_of_plants%s" % s,)
```

```python
# The demand minimum constraints are added to prob for each demand node (project)
for b in disposal_sites:
    prob += (lpSum([vars_2[w][b] for w in intermediate_plants]) == demand[b],
        "Sum_of_Products_into_disposal_sites%s" % b,)
```

```python
# Transshipment constraints: What is shipped into a transshipment node must ne shipped out.
for w in intermediate_plants[:-1]:
    prob += (lpSum([vars[f][w] for f in plants]) - lpSum([vars_2[w][p] for p in disposal_sites]) == 0,
        "Sum_of_Products_out_of_intermediate_plants_%s" % w,)
```

```python
# The problem data is written to an .lp file
prob.writeLP("TransshipmentProblem1.lp")
```

```python
# Solve the problem
prob.solve()
```

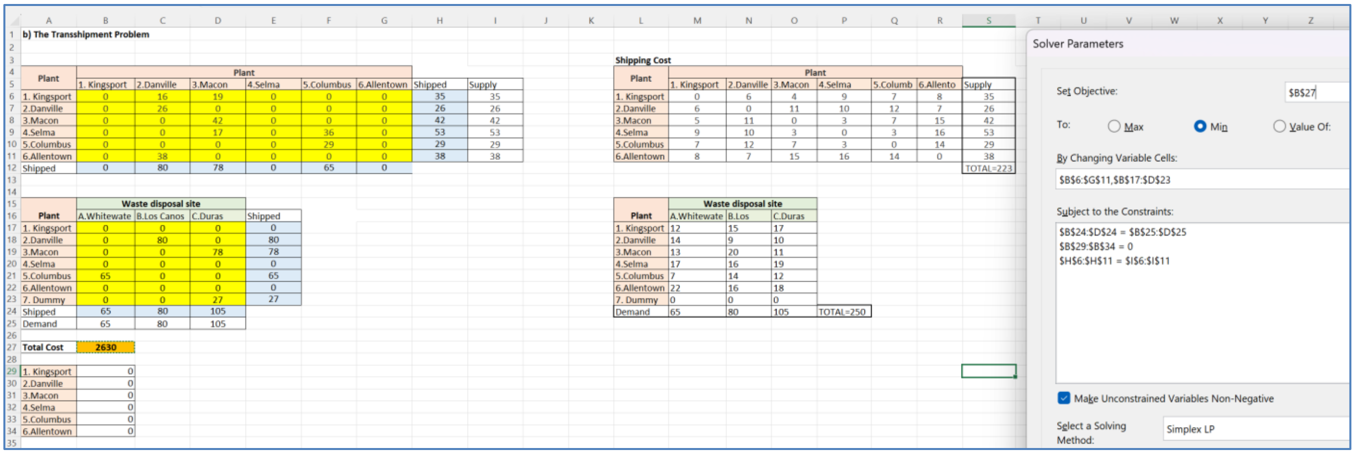
```
1
```

```python
# Print the results
for v in prob.variables():
    if v.varValue > 0:
        print(f"{v.name} = {v.varValue}")

print("Optimal Transshipment Cost:", value(prob.objective))
```

```
Route_1_Allentown_Allentown = 38.0
Route_1_Columbus_Columbus = 29.0
Route_1_Danville_Danville = 26.0
Route_1_Kingsport_Kingsport = 16.0
Route_1_Kingsport_Macon = 19.0
Route_1_Macon_Macon = 42.0
Route_1_Selma_Columbus = 36.0
Route_1_Selma_Macon = 17.0
Route_2_Allentown_Los_Canos = 38.0
Route_2_Columbus_Whitewater = 65.0
Route_2_Danville_Los_Canos = 26.0
Route_2_Dummy_Duras = 27.0
Route_2_Kingsport_Los_Canos = 16.0
Route_2_Macon_Duras = 78.0
Optimal Transshipment Cost: 2630.0
```

4. Question b) using Excel Solver



b) The Transshipment Problem

Plant	Plant							
	1. Kingsport	2.Danville	3.Macon	4.Selma	5.Columbus	6.Allentown	Shipped	Supply
1. Kingsport	0	16	19	0	0	0	35	35
2.Danville	0	26	0	0	0	0	26	26
3.Macon	0	0	42	0	0	0	42	42
4.Selma	0	0	17	0	36	0	53	53
5.Columbus	0	0	0	0	29	0	29	29
6.Allentown	0	38	0	0	0	0	38	38
Shipped	0	80	78	0	65	0		

Plant	Waste disposal site			
	A.Whitewate	B.Los Canos	C.Duras	Shipped
1. Kingsport	0	0	0	0
2.Danville	0	80	0	80
3.Macon	0	0	78	78
4.Selma	0	0	0	0
5.Columbus	65	0	0	65
6.Allentown	0	0	0	0
7. Dummy	0	0	27	27
Shipped	65	80	105	
Demand	65	80	105	

Total Cost	2630
1. Kingsport	0
2.Danville	0
3.Macon	0
4.Selma	0
5.Columbus	0
6.Allentown	0

Shipping Cost

Plant	Plant						
	1. Kingsport	2.Danville	3.Macon	4.Selma	5.Columb	6.Allento	Supply
1. Kingsport	0	6	4	9	7	8	35
2.Danville	6	0	11	10	12	7	26
3.Macon	5	11	0	3	7	15	42
4.Selma	9	10	3	0	3	16	53
5.Columbus	7	12	7	3	0	14	29
6.Allentown	8	7	15	16	14	0	38
							TOTAL=223

Plant	Waste disposal site			
	A.Whitewate	B.Los	C.Duras	
1. Kingsport	12	15	17	
2.Danville	14	9	10	
3.Macon	13	20	11	
4.Selma	17	16	19	
5.Columbus	7	14	12	
6.Allentown	22	16	18	
7. Dummy	0	0	0	
Demand	65	80	105	TOTAL=250

Solver Parameters

Set Objective: $B$27

To: Max Min Value Of:

By Changing Variable Cells: $B$6:$G$11,$B$17:$D$23

Subject to the Constraints:
$B$24:$D$24 = $B$25:$D$25
$B$29:$B$34 = 0
$H$6:$H$11 = $I$6:$I$11

Make Unconstrained Variables Non-Negative

Select a Solving Method: Simplex LP