Verbose proof for `vc`.

`vc`:

$$
\begin{array}{|ll}
& \\
\hline
\{1\} & \forall\ (\text{value}\colon\ \mathbb{Z},\ \text{length}\colon\ \mathbb{N},\ x\colon\ [\mathbb{N} \to \mathbb{Z}],\ y\colon\ [\mathbb{N} \to \mathbb{Z}])\colon \\
& \quad (0 < \text{length}) \supset \text{rep}(\text{length},\ 0,\ x,\ y) = \text{DP\_RESULT}(0,\ \text{length},\ x,\ y)
\end{array}
$$

Inducting on length on formula 1,
we get 2 subgoals:

`vc.1`:

$$
\begin{array}{|ll}
& \\
\hline
\{1\} & \forall\ (\text{value}\colon\ \mathbb{Z},\ x\colon\ [\mathbb{N} \to \mathbb{Z}],\ y\colon\ [\mathbb{N} \to \mathbb{Z}])\colon \\
& \quad (0 < 0) \supset \text{rep}(0,\ 0,\ x,\ y) = \text{DP\_RESULT}(0,\ 0,\ x,\ y)
\end{array}
$$

Simplifying, rewriting, and recording with decision procedures,
This completes the proof of `vc.1`.

`vc.2`:

$$
\begin{array}{|ll}
& \\
\hline
\{1\} & \forall\ j\colon \\
& \quad (\forall\ (\text{value}\colon\ \mathbb{Z},\ x\colon\ [\mathbb{N} \to \mathbb{Z}],\ y\colon\ [\mathbb{N} \to \mathbb{Z}])\colon \\
& \qquad (0 < j) \supset \text{rep}(j,\ 0,\ x,\ y) = \text{DP\_RESULT}(0,\ j,\ x,\ y)) \\
& \quad\ \supset \\
& \quad\ (\forall\ (\text{value}\colon\ \mathbb{Z},\ x\colon\ [\mathbb{N} \to \mathbb{Z}],\ y\colon\ [\mathbb{N} \to \mathbb{Z}])\colon \\
& \qquad (0 < j+1) \supset \\
& \qquad\ \text{rep}(j+1,\ 0,\ x,\ y) = \text{DP\_RESULT}(0,\ j+1,\ x,\ y))
\end{array}
$$

Skolemizing,

`vc.2`:

$$
\begin{array}{|ll}
& \\
\hline
\{1\} & (\forall\ (\text{value}\colon\ \mathbb{Z},\ x\colon\ [\mathbb{N} \to \mathbb{Z}],\ y\colon\ [\mathbb{N} \to \mathbb{Z}])\colon \\
& \qquad (0 < j') \supset \text{rep}(j',\ 0,\ x,\ y) = \text{DP\_RESULT}(0,\ j',\ x,\ y)) \\
& \supset \\
& (\forall\ (\text{value}\colon\ \mathbb{Z},\ x\colon\ [\mathbb{N} \to \mathbb{Z}],\ y\colon\ [\mathbb{N} \to \mathbb{Z}])\colon \\
& \qquad (0 < j'+1) \supset \\
& \qquad\ \text{rep}(j'+1,\ 0,\ x,\ y) = \text{DP\_RESULT}(0,\ j'+1,\ x,\ y))
\end{array}
$$

Applying disjunctive simplification to flatten sequent,

`vc.2`:

$$
\begin{array}{|ll}
\{\text{-}1\} & \forall\ (\text{value}\colon\ \mathbb{Z},\ x\colon\ [\mathbb{N} \to \mathbb{Z}],\ y\colon\ [\mathbb{N} \to \mathbb{Z}])\colon \\
& (0 < j') \supset \text{rep}(j',\ 0,\ x,\ y) = \text{DP\_RESULT}(0,\ j',\ x,\ y) \\
\hline
\{1\} & \forall\ (\text{value}\colon\ \mathbb{Z},\ x\colon\ [\mathbb{N} \to \mathbb{Z}],\ y\colon\ [\mathbb{N} \to \mathbb{Z}])\colon \\
& (0 < j'+1) \supset \\
& \ \text{rep}(j'+1,\ 0,\ x,\ y) = \text{DP\_RESULT}(0,\ j'+1,\ x,\ y)
\end{array}
$$

Skolemizing,

`vc.2`:

$$\begin{array}{l|ll} & \{\text{-}1\} & \forall\ (\text{value}:\ \mathbb{Z},\ x:\ [\mathbb{N} \to \mathbb{Z}],\ y:\ [\mathbb{N} \to \mathbb{Z}]): \\ & & \quad (0 < j') \supset \text{rep}(j',\ 0,\ x,\ y) = \text{DP\_RESULT}(0,\ j',\ x,\ y) \\ \hline & \{1\} & (0 < j'+1) \supset \\ & & \quad \text{rep}(j'+1,\ 0,\ x',\ y') = \text{DP\_RESULT}(0,\ j'+1,\ x',\ y') \end{array}$$

Applying disjunctive simplification to flatten sequent,

`vc.2`:

$$\begin{array}{l|ll} & \{\text{-}1\} & \forall\ (\text{value}:\ \mathbb{Z},\ x:\ [\mathbb{N} \to \mathbb{Z}],\ y:\ [\mathbb{N} \to \mathbb{Z}]): \\ & & \quad (0 < j') \supset \text{rep}(j',\ 0,\ x,\ y) = \text{DP\_RESULT}(0,\ j',\ x,\ y) \\ & \{\text{-}2\} & (0 < j'+1) \\ \hline & \{1\} & \text{rep}(j'+1,\ 0,\ x',\ y') = \text{DP\_RESULT}(0,\ j'+1,\ x',\ y') \end{array}$$

Instantiating the top quantifier in -1 with the terms: 0, x', y',

`vc.2`:

$$\begin{array}{l|ll} & \{\text{-}1\} & (0 < j') \supset \text{rep}(j',\ 0,\ x',\ y') = \text{DP\_RESULT}(0,\ j',\ x',\ y') \\ & \{\text{-}2\} & (0 < j'+1) \\ \hline & \{1\} & \text{rep}(j'+1,\ 0,\ x',\ y') = \text{DP\_RESULT}(0,\ j'+1,\ x',\ y') \end{array}$$

Splitting conjunctions,
we get 2 subgoals:

`vc.2.1`:

$$\begin{array}{l|ll} & \{\text{-}1\} & \text{rep}(j',\ 0,\ x',\ y') = \text{DP\_RESULT}(0,\ j',\ x',\ y') \\ & \{\text{-}2\} & (0 < j'+1) \\ \hline & \{1\} & \text{rep}(j'+1,\ 0,\ x',\ y') = \text{DP\_RESULT}(0,\ j'+1,\ x',\ y') \end{array}$$

Expanding the definition of rep,

`vc.2.1`:

$$\begin{array}{l|ll} & \{\text{-}1\} & \text{rep}(j',\ 0,\ x',\ y') = \text{DP\_RESULT}(0,\ j',\ x',\ y') \\ & \{\text{-}2\} & (0 < j'+1) \\ \hline & \{1\} & \text{rep}(j',\ 0,\ x',\ y') + x'(j') \times y'(j') = \\ & & \quad \text{DP\_RESULT}(0,\ 1+j',\ x',\ y') \end{array}$$

Expanding the definition of DP_RESULT,

`vc.2.1`:

$$\begin{array}{l|ll} & \{\text{-}1\} & \text{rep}(j',\ 0,\ x',\ y') = \text{DP\_RESULT}(0,\ j',\ x',\ y') \\ & \{\text{-}2\} & (0 < j'+1) \\ \hline & \{1\} & \text{rep}(j',\ 0,\ x',\ y') = \text{DP\_RESULT}(0,\ j',\ x',\ y') \end{array}$$

which is trivially true.
This completes the proof of `vc.2.1`.

vc.2.2:

$$\begin{array}{l|l}
\{\text{-1}\} & (0 < j' + 1) \\ \hline
\{1\} & (0 < j') \\
\{2\} & \text{rep}(j' + 1,\ 0,\ x',\ y') = \text{DP_RESULT}(0,\ j' + 1,\ x',\ y')
\end{array}$$

Simplifying, rewriting, and recording with decision procedures,
vc.2.2:

$$\begin{array}{l|l}
\{\text{-1}\} & (0 < 1 + j') \\ \hline
\{1\} & (0 < j') \\
\{2\} & \text{rep}(1 + j',\ 0,\ x',\ y') = \text{DP_RESULT}(0,\ 1 + j',\ x',\ y')
\end{array}$$

Simplifying with decision procedures,
vc.2.2:

$$\begin{array}{l|l}
\{\text{-1}\} & (0 < 1 + j') \\ \hline
\{1\} & (0 < j') \\
\{2\} & \text{rep}(1 + j',\ 0,\ x',\ y') = \text{DP_RESULT}(0,\ 1 + j',\ x',\ y')
\end{array}$$

Case splitting on 0 ¡ j!1,
we get 2 subgoals:
vc.2.2.1:

$$\begin{array}{l|l}
\{\text{-1}\} & 0 < j' \\
\{\text{-2}\} & (0 < 1 + j') \\ \hline
\{1\} & (0 < j') \\
\{2\} & \text{rep}(1 + j',\ 0,\ x',\ y') = \text{DP_RESULT}(0,\ 1 + j',\ x',\ y')
\end{array}$$

which is trivially true.
This completes the proof of vc.2.2.1.
vc.2.2.2:

$$\begin{array}{l|l}
\{\text{-1}\} & (0 < 1 + j') \\ \hline
\{1\} & 0 < j' \\
\{2\} & (0 < j') \\
\{3\} & \text{rep}(1 + j',\ 0,\ x',\ y') = \text{DP_RESULT}(0,\ 1 + j',\ x',\ y')
\end{array}$$

Case splitting on j!1 ¡ 0,
we get 2 subgoals:
vc.2.2.2.1:

$$\begin{array}{l|l}
\{\text{-1}\} & j' < 0 \\
\{\text{-2}\} & (0 < 1 + j') \\ \hline
\{1\} & 0 < j' \\
\{2\} & (0 < j') \\
\{3\} & \text{rep}(1 + j',\ 0,\ x',\ y') = \text{DP_RESULT}(0,\ 1 + j',\ x',\ y')
\end{array}$$

Simplifying, rewriting, and recording with decision procedures,
This completes the proof of vc.2.2.2.1.

`vc.2.2.2.2:`

$$
\begin{array}{|ll}
\{\text{-1}\} & (0 \ < \ 1 + j') \\
\hline
\{1\} & j' \ < \ 0 \\
\{2\} & 0 \ < \ j' \\
\{3\} & (0 \ < \ j') \\
\{4\} & \mathrm{rep}(1 + j',\ 0,\ x',\ y') \ = \ \mathrm{DP\_RESULT}(0,\ 1 + j',\ x',\ y')
\end{array}
$$

Case splitting on NOT j!1 = 0,
we get 2 subgoals:
`vc.2.2.2.2.1:`

$$
\begin{array}{|ll}
\{\text{-1}\} & (0 \ < \ 1 + j') \\
\hline
\{1\} & j' \ = \ 0 \\
\{2\} & j' \ < \ 0 \\
\{3\} & 0 \ < \ j' \\
\{4\} & (0 \ < \ j') \\
\{5\} & \mathrm{rep}(1 + j',\ 0,\ x',\ y') \ = \ \mathrm{DP\_RESULT}(0,\ 1 + j',\ x',\ y')
\end{array}
$$

Simplifying, rewriting, and recording with decision procedures,
This completes the proof of `vc.2.2.2.2.1`.
`vc.2.2.2.2.2:`

$$
\begin{array}{|ll}
\{\text{-1}\} & j' \ = \ 0 \\
\{\text{-2}\} & (0 \ < \ 1 + j') \\
\hline
\{1\} & j' \ < \ 0 \\
\{2\} & 0 \ < \ j' \\
\{3\} & (0 \ < \ j') \\
\{4\} & \mathrm{rep}(1 + j',\ 0,\ x',\ y') \ = \ \mathrm{DP\_RESULT}(0,\ 1 + j',\ x',\ y')
\end{array}
$$

Expanding the definition of rep,
`vc.2.2.2.2.2:`

$$
\begin{array}{|ll}
\{\text{-1}\} & j' \ = \ 0 \\
\{\text{-2}\} & (0 \ < \ 1 + j') \\
\hline
\{1\} & j' \ < \ 0 \\
\{2\} & 0 \ < \ j' \\
\{3\} & (0 \ < \ j') \\
\{4\} & \mathrm{rep}(j',\ 0,\ x',\ y') + x'(j') \times y'(j') \ = \\
 & \quad \mathrm{DP\_RESULT}(0,\ 1 + j',\ x',\ y')
\end{array}
$$

Expanding the definition of DP_RESULT,

`vc.2.2.2.2.2`:

$$
\begin{array}{|ll}
\{\text{-}1\} & j' = 0 \\
\{\text{-}2\} & (0 < 1 + j') \\
\hline
\{1\} & j' < 0 \\
\{2\} & 0 < j' \\
\{3\} & (0 < j') \\
\{4\} & \mathrm{rep}(j',\ 0,\ x',\ y') = \mathrm{DP\_RESULT}(0,\ j',\ x',\ y')
\end{array}
$$

Expanding the definition of rep,
`vc.2.2.2.2.2`:

$$
\begin{array}{|ll}
\{\text{-}1\} & j' = 0 \\
\{\text{-}2\} & (0 < 1 + j') \\
\hline
\{1\} & j' < 0 \\
\{2\} & 0 < j' \\
\{3\} & (0 < j') \\
\{4\} & 0 = \mathrm{DP\_RESULT}(0,\ j',\ x',\ y')
\end{array}
$$

Expanding the definition of DP_RESULT,
`vc.2.2.2.2.2`:

$$
\begin{array}{|ll}
\{\text{-}1\} & j' = 0 \\
\{\text{-}2\} & (0 < 1 + j') \\
\hline
\{1\} & j' < 0 \\
\{2\} & 0 < j' \\
\{3\} & (0 < j') \\
\{4\} & \text{TRUE}
\end{array}
$$

which is trivially true.
This completes the proof of `vc.2.2.2.2.2`.
Q.E.D.