Sino Bright School Chongqing

Monthly Exam, 2024-2025 T2

Subject:	Further Math
Date:	May 2025
Time:	60 Minutes
Teacher:	Shi Feng

IMPORTANT: Read together before beginning the exam.

Students must complete the assessments to the best of their ability. Any students found cheating during an exam will be given a zero and referred to the Principal for further disciplinary measures. Cheating is a serious offense.

"Cheating" may include any of the following:

- Having course papers, notes, unapproved data sheets or other course-related materials in the exam room without the express permission of the subject class teacher in charge of the exam.
- Using or even having a cellphone or other electronic device in the exam room without the express permission of the subject class teacher in charge of the exam.
- Attempting to communicate, either verbally or non-verbally with other students during the exam.
- Having exam-related materials prior to the writing of the exam.

Mark Breakdown:	Student Name:
Part I: Multiple Choice Questions $5pts \times 10$ Part II: Short Answer Questions $5pts \times 4$ Part III: Long Answer Questions $30pts$	
Total Score: 100	Score: _____

Special Instructions/Materials Allowed:

- Pen
- Calculator

Part I: Multiple Choice Questions ($5pts \times 10$)

1. If a root of $f(x) = 0$ lies in $[3, 4]$, what is the midpoint after the first iteration of interval bisection?
   A) 3.25
   B) 3.5
   C) 3.75
   D) 3.0

2. The formula for the first approximation x_1 using linear interpolation is:
   A) $x_1 = a - \frac{f(a)(b-a)}{f(b)-f(a)}$
   B) $x_1 = \frac{a+b}{2}$
   C) $x_1 = a - \frac{f(a)}{f'(a)}$
   D) $x_1 = \frac{f(b)-f(a)}{b-a}$

3. The directrix of the parabola $y^2 = 24x$ is:
   A) $x = -6$
   B) $x = 6$
   C) $y = -6$
   D) $y = 6$

4. The modulus of $z = 3 + 4i$ is:
   A) 5
   B) 7
   C) 12
   D) 25

5. The iteration formula is $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$. What condition is critical for convergence?
   A) $f'(x_n) \neq 0$
   B) $f(x_n) > 0$
   C) $f(x_n) < 0$
   D) $f'(x_n) = 0$

6. If $z = 2 - 5i$, then $z + z^*$ equals:
   A) 4
   B) $-10i$
   C) $4 - 10i$
   D) 0

7. A root of a continuous function $f(x)$ exists in $[a, b]$ if:
   A) $f(a)$ and $f(b)$ are positive
   B) $f(a) \cdot f(b) < 0$
   C) $f(a) = f(b)$
   D) $f(a) \cdot f(b) > 0$

8. The Cartesian equation for $x = 2t$, $y = 4t^2$ is:
   A) $y = x^2$
   B) $y = 2x^2$
   C) $y = \frac{x^2}{4}$
   D) $y = 4x^2$

9. The asymptotes of $xy = 9$ are:
   A) $x = 0$ and $y = 0$
   B) $x = 3$ and $y = 3$
   C) $x = 9$ and $y = 9$
   D) None

10. If a quadratic equation has roots α and β, then $\alpha + \beta =$
    A) $\frac{b}{a}$
    B) $-\frac{b}{a}$
    C) $\frac{c}{a}$
    D) $-\frac{c}{a}$

Part II: Short Answer Questions ($5pts \times 4$)

1. After 3 iterations of interval bisection on $[3, 4]$, the interval length is _____.
2. For $f(2) = -3$ and $f(4) = 5$, the next approximation using linear interpolation is _____.
3. The focus of the parabola $y^2 = 28x$ is at _____.
4. The argument of $z = -1 - i$ (in radians) is _____.

Part III: Long Answer Questions ($30pts$)

1. Let $z^6 = 1$.

   (a) Find all the solutions to the equation.

   (b) Show each solution on an Argand diagram.

(c) Show that each solution lies on a circle with center $(0, 0)$ and radius 1.

2. Given the quadratic equation $x^2 - 5x + 6 = 0$ with roots α and β:

   (a) Use Vieta's formulas to find $\alpha + \beta$ and $\alpha\beta$.

   (b) Construct a new quadratic equation with roots 2α and 2β.

   (c) Use the Newton-Raphson method with $x_0 = 3$ to approximate a root of the new equation. Perform 2 iterations and round your answers to 4 decimal places.