That time I did some “math research”

Quang Ong: A glimpse into the world of “research”

Disclaimer: I only talk about Unimelb in particular because… I don’t know about the process for other unis

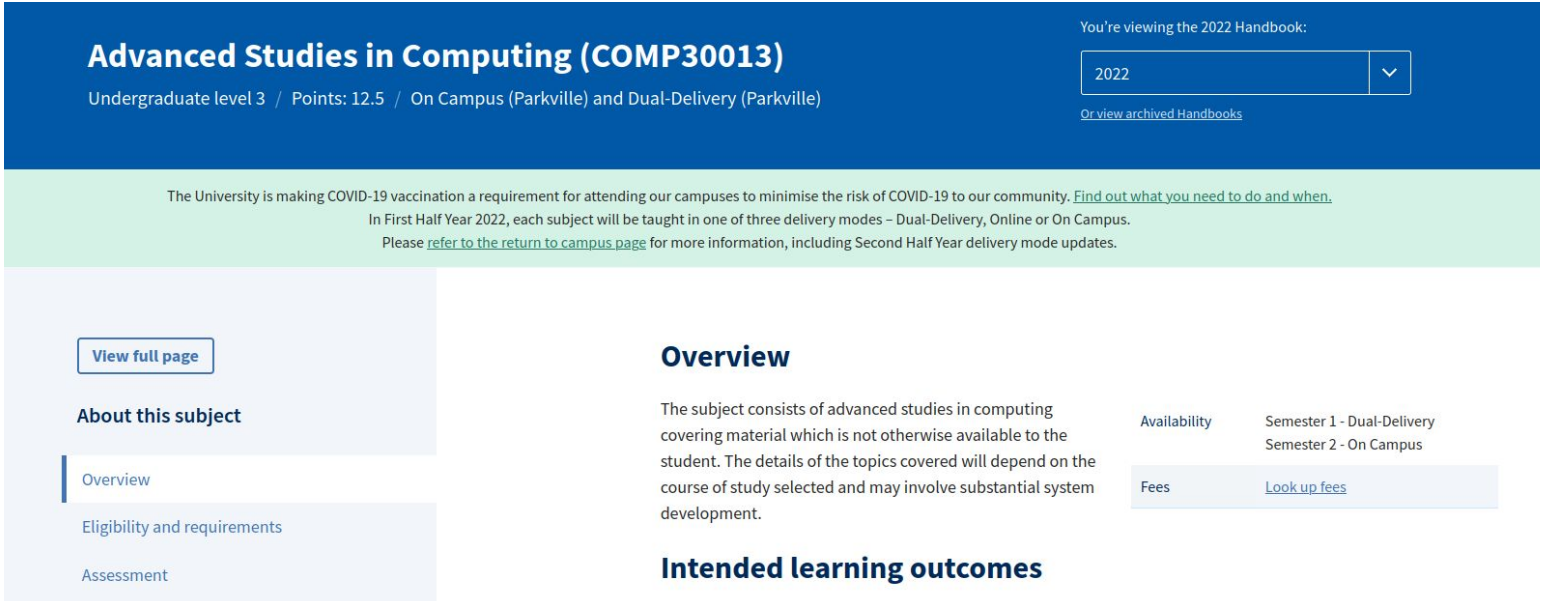
Step 1: Figure out how to get paid for this

I mean, uhhh, which undergrad programs are available at your university

Your university's undergraduate research program

Sometimes they can be hard to find E.G. Unimelb compsci research

https://handbook.unimelb.edu.au/2022/subjects/comp30013



Your university’s undergraduate research program

Unimelb Compsci research:

Pay $1000 (if you’re domestic, otherwise it’s $6000) to do research as a university subject. Doesn’t sound like a good deal but…

- You need to do a subject anyways
- You get a supervisor!!!!!!!! (more on this later)
- It’s a good resume thing,
- It’s a unique experience if you don’t plan on going into research

Your university's undergraduate research program

Unimelb Math Research:

https://ms.unimelb.edu.au/engage/vacation-scholarships

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The problem: You have to actually get really good scores in second year subjects to get into the program (like 85+)

Step 2: Find your supervisor

It’s apparently “just like dating”

You’ll have a list of potential supervisors

https://ms.unimelb.edu.au/engage/vacation-scholarships/vacation-scholarships-projects

https://cis.unimelb.edu.au/people#academic

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And now you’ll go through them one by one!!!

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Talk slides are actually a pretty good place to start (tend to be more accessible)

https://blogs.unimelb.edu.au/paul-zinn-justin/#tab12 (pure mathematicians are scary)

https://matthewktam.github.io/pages/research.html (the person that ended up being my supervisor)

You’ll have a list of potential supervisors
And now you’ll go through them one by one!!!

Email them, schedule a Zoom ~~date~~ appointment

Quang Ong <quango@student.unimelb.edu.au> Aug 22, 2021, 9:59 PM

to matthew.tam

Dear Matthew Tam

I saw that you had posted a project on vacation scholars, and as someone very interested in algorithms and computing, I am also interested. I looked through some of your talks and although the literature/background knowledge is a little bit over my head, your talk on applying Douglas-Rachford (not that I know what that is) on sudoku reminded me of the times I have applied simulated annealing and beam search to solve similar combinatorial problems for fun/competitive programming. Hence I am really keen to do a vacation scholars under you, and study the required literature to get up to speed.

Is it possible to schedule a zoom call soon to talk about our experiences in more detail, and see where our interests line up? I have attached my uni schedule with the majority of my commitments to aid this process.

23/08/2021 - 29/08/2021 (next week)

	23/8 (Mon)	24/8 (Tue)	25/8 (Wed)	26/8 (Thu)	27/8 (Fri)	28/8 (Sat)
8:00 AM						
9:00 AM				MAST20022_U Lecture2	MAST20030_U Lecture3	

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What do they get out of this?

What do supervisors get out of this?

- It’s literally in their job description
- They just need to tell their research to someone
- Having lots of successful students makes them look good, helps with promotion.
- Students, young talent, can be quite inspiring

What do you get out of a supervisor

- You get a mind-boggling smart, experienced, and interesting person as your personal expert.
- It’s like having a private tutor, except extremely specialised, and you have to take the initiative.

Other ways to meet supervisors.

- I chatted with my lecturers, chatted with friends for recommendations…
- You can apparently get bounced around a lot, i.e. “Our interests don’t exactly align, but I know that my colleague [other potential supervisor] is really interested in that stuff!”

Step 3: Actually do the research

Actually research is 99% just reading about what other people have done.

What did I research?

I investigated projection algorithms for non-convex combinatorial feasibility problems

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Yeah it’s a long story…

What did I research?

I investigated projection algorithms for non-convex combinatorial feasibility problems

Yeah it’s a long story…

But here’s a sneak peak!

https://www.youtube.com/watch?v=5vmsg40DOX4

Here’s how it started

https://arxiv.org/pdf/1904.09148.pdf

Do some reading/research to get yourself up to speed.

I was quite fortunate, my topic only required about two days to get up to speed to some frontier of human knowledge.

Disclaimer

I will tell you when I actually do new original stuff.

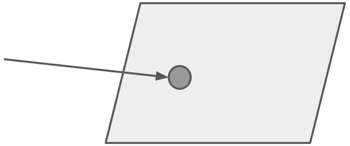
These concepts have been studied before!

What is $\mathbb{R}^n$?

$\mathbb{R}^1 = \mathbb{R} =$ Real number line

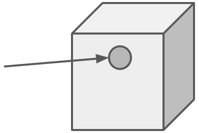
$\mathbb{R}^2 =$ 2d space

(x,y)



$\mathbb{R}^3 =$ 3d space

(x,y,z)



$\mathbb{R}^n =$ n-dimensional space

$(x_1, x_2, x_3 \ldots x_n)$, where $x_i \in \mathbb{R}$

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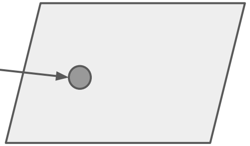

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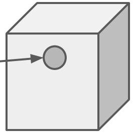

$\mathbb{R}^n$ = n-dimensional space

$(x_1, x_2, x_3 \ldots x_n)$, where $x_i \in \mathbb{R}$

By the way, $\in$ means “in”, or “is an element of”

Feasibility problems

Suppose you had some space $E = \mathbb{R}^n$, and n constraint sets $C_1, C_2, \ldots C_n \subseteq E$ A feasibility problem asks to find an $x = (x_1, x_2, \ldots x_n) \in E$ such that

$$x \in C_1 \cap C_2 \cdots \cap C_n$$

or determine that no such x exists.

Notes: E is basically a shorthand for R^n, and actually a lot of theorems here work in a much more general space, but I don't understand them

A ⊆ B means "A is a subset of B"

Feasibility problem examples

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Examples:

Finding an intersection of some sets in R^n (like maybe you need to find the intersection of some number of balls)

You can convert problems like Sudoku, N-Queens, 3-SAT, Graph Coloring, Protein Folding to feasibility problems.

Projection operator.

The projection operator (if you can read math notation) is defined as follows:

Given a set S, define the projection operator P_S as

$$P_S(x) = \{c : \mathrm{dist}(c, x) \text{ is minimal for } c \in S\}$$

In other words, $P_S(x)$ shoves x to the closest point in S.

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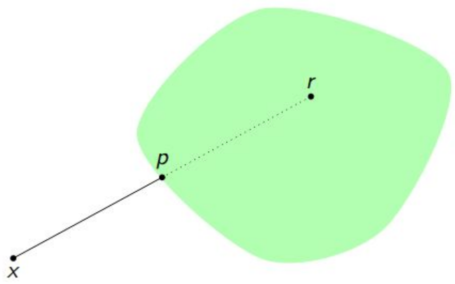
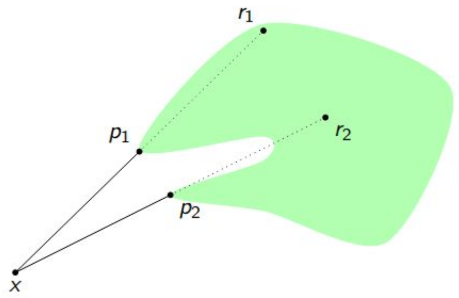
We also define the reflection operator $R_S(x)$ as $R_S(x) = 2P_S(x) - x$
It literally “reflects” point x into the set S.





Method of cyclic projections

Mathematical description of the algorithm:

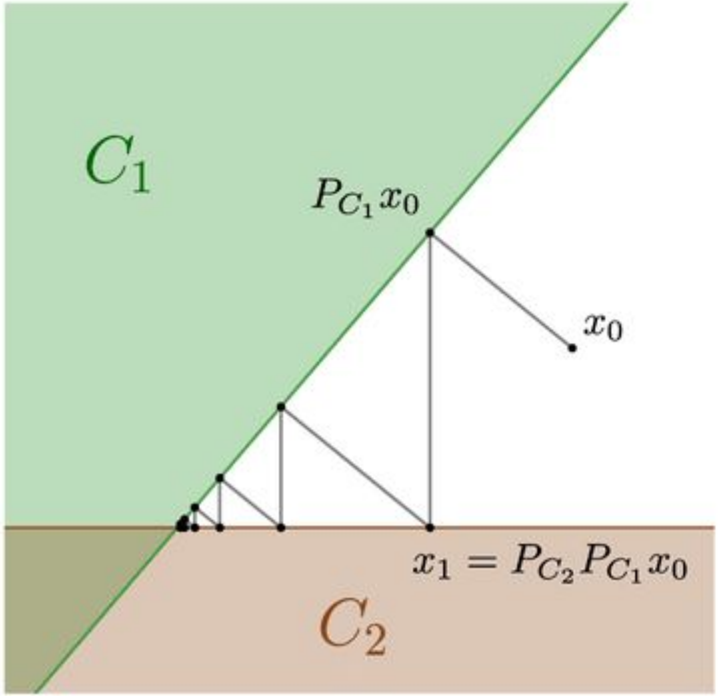
Given n constraint sets $C_1, \ldots, C_n$, choose an arbitrary $x_0 \in E$, then define the following sequence

$$x_{k+1} = (P_{C_n} P_{C_{n-1}} \ldots P_{C_1})(x_k)$$

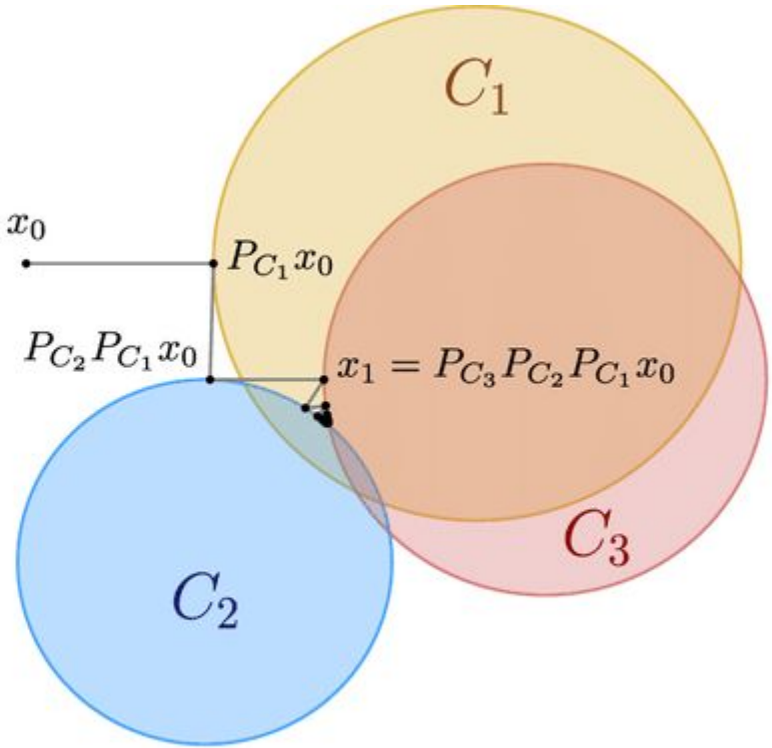
Translation into pseudocode

```
Let x be some random element in E

While x is not in the intersection of C_1, C_2 … C_n:

    for i in [1,n]:

        x = Projection of x onto the set C_i

return x
```

Method of cyclic projections

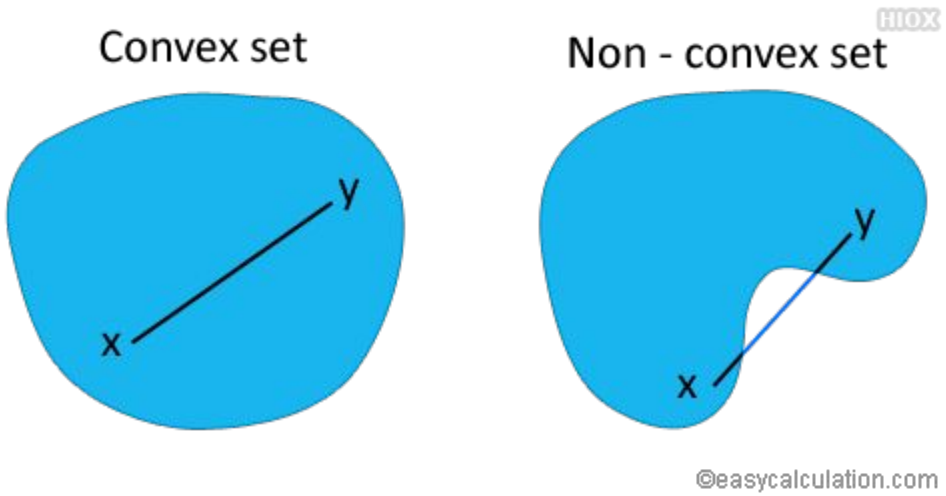


(a) The method of cyclic projections for two halfspaces



(b) The method of cyclic projections for three balls

What does convex mean (in $\mathbb{R}^n$)?



A convex set S is such that for any two points x,y $\in$ S, the line connecting x and y is fully contained within the set.

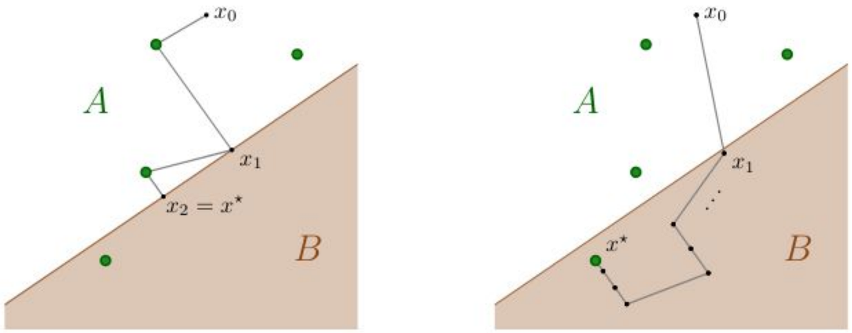
In general, projection algorithms are very well behaved when they ask for the intersection of convex sets. However, I was looking to find out more about non-convex sets!

Douglas Rachford Algorithm

This is really weird… but it works. Let our two constraint sets be A,B.

First, initialise x_0 to some arbitrary point in E. Then calculate the sequence using the following iteration step:

$$R_S(x) = 2P_S(x) - x \qquad\qquad x_{k+1} = \frac{1}{2}x_k + \frac{1}{2}R_B R_A x_k$$



(a) Cyclic projections (b) Douglas–Rachford

Douglas Rachford Algorithm

It can only work with 2 sets… right?

Wrong!

Firstly, define the cartesian product of sets as follows. Here, A,B, C_1, C_2 … C_n are sets.

$$A \times B = \{(x, y) : x \in A \text{ and } y \in B\}$$

$$C_1 \times C_2 \times ... \times C_n = \{(x_1, x_2 ... x_n) : x_i \in C_i \text{ for all } i\}$$

Douglas Rachford Algorithm

Suppose we had n sets $C_1, \ldots, C_n$. Define new constraint sets $\mathbf{C}, \mathbf{D} \subseteq E^n$ so that

$$\mathbf{C} = C_1 \times \cdots \times C_n \text{ and } \mathbf{D} = \{(x, x, \ldots, x) \mid x \in E\}$$

$$C_1 \times C_2 \times ... \times C_n = \{(x_1, x_2 ... x_n) : x_i \in C_i \text{ for all } i\}$$

Douglas Rachford Algorithm

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$$\mathbf{C} = C_1 \times \cdots \times C_n \text{ and } \mathbf{D} = \{(x, x, \ldots, x) \mid x \in E\}$$

Consider what happens when $\mathbf{x} = (x_1, x_2, \ldots x_n) \in \mathbf{C} \cap \mathbf{D}$

$$C_1 \times C_2 \times \ldots \times C_n = \{(x_1, x_2 \ldots x_n) : x_i \in C_i \text{ for all } i\}$$

Douglas Rachford Algorithm

Summary:
Douglas Rachford can run on any number of constraint sets, and is weird but powerful.

Tam-Malitsky algorithm

A work of magic by my supervisor, it was apparently used to prove a theoretical bound

Given n closed sets $C_1 \times \cdots \times C_n \subseteq E$, initialise $\mathbf{z}^0 = (z_1^0, \ldots, z_{n-1}^0) \in E^{n-1}$ and $\gamma \in (0,1)$

Then compute $\mathbf{z}^{k+1} = (z_1^{k+1}, \ldots, z_{n-1}^{k+1}) \in E^{n-1}$ according to

$$\mathbf{z}^{k+1} = T_A(\mathbf{z}^k) = \mathbf{z}^k + \gamma \begin{pmatrix} x_2^k - x_1^k \\ x_3^k - x_2^k \\ \vdots \\ x_n^k - x_{n-1}^k \end{pmatrix}$$

where $\mathbf{x}^k = (x_1^k, \ldots, x_n^k) \in E^n$ is given by

$$\begin{cases} x_1^k = P_{C_1}(z_1^k), \\ x_i^k = P_{C_i}(z_i^k - z_{i-1}^k + x_{i-1}^k) \qquad \forall i \in [[2, n-1]] \\ x_n^k = P_{C_n}(x_1^k + x_{n-1}^k - z_{n-1}^k). \end{cases}$$

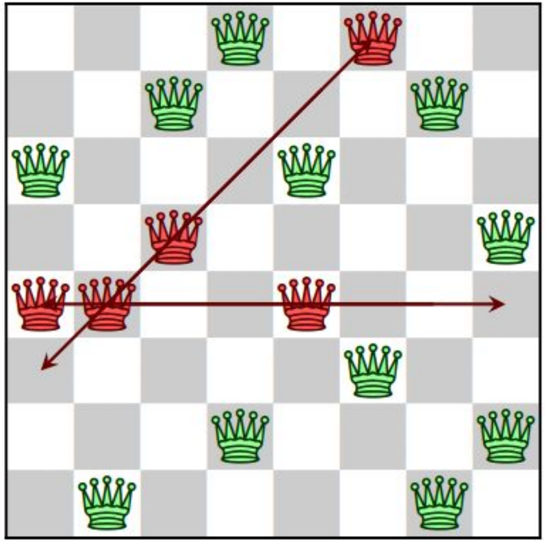
where the notation $[[a, b]]$ denotes "The set of integers between a and b inclusive."

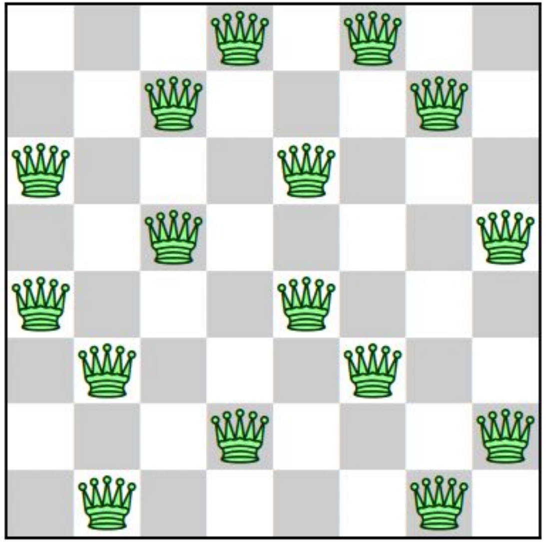
Finished reading… now what?

Kinda lost, so I guess a good place to start was to reproduce some results, and do some visualisation work!

N-M Queens

-You are given an N x N board. Fill the board with NM queens such there are exactly m queens on each row and column, and at most m queens on each diagonal.





Pictured is an invalid solution and a valid solution to the (8-2) queens problem.

N-M Queens

Choose E to be $\mathbb{R}^{n \times n}$

For any $x \in E$,

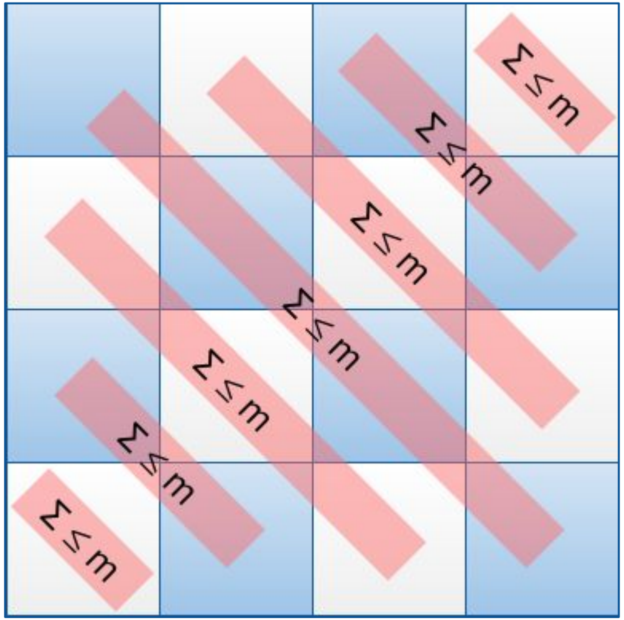
$$x_{ij} = 1 \text{ if there is a queen at position } (i, j)$$
$$x_{ij} = 0 \text{ if it is empty at position } (i, j)$$

However, x_{ij} can be something other than 0 or 1, and in those cases it represents the "confidence" that the position (i, j) is a queen

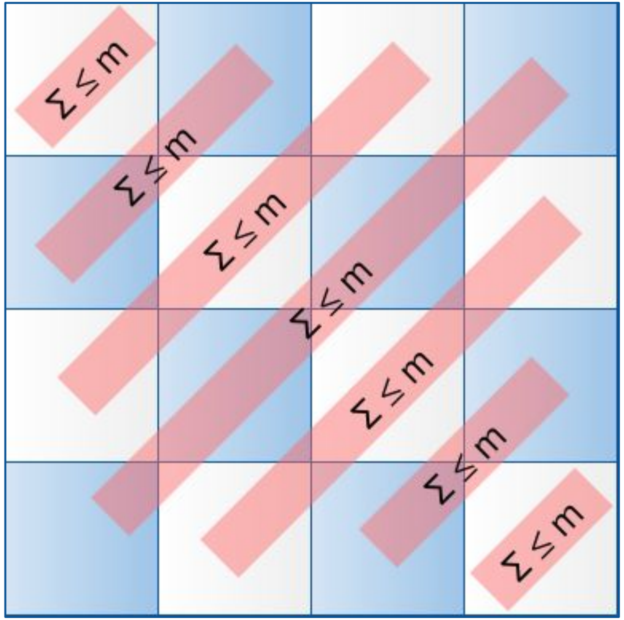
N-M Queens

There are 5 constraint sets (link to my blog explaining them, and the formulation)

C_3

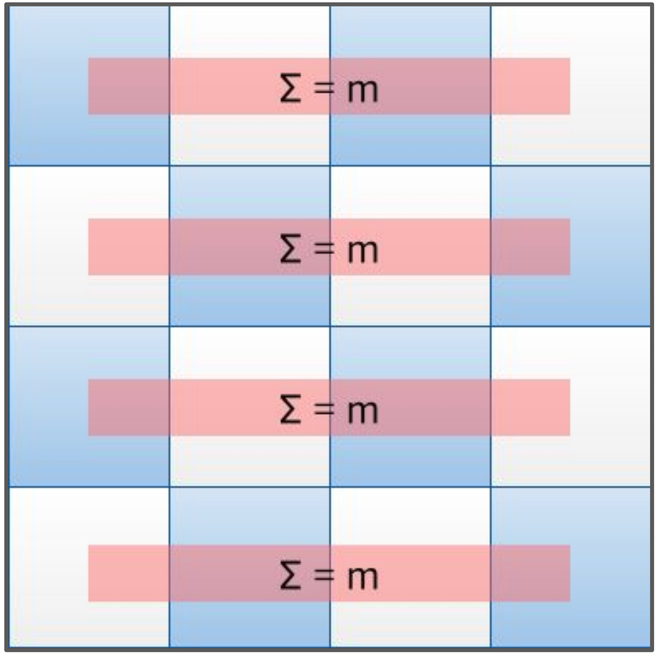


C_4



N-M Queens

Let's suppose we want to project to $\hat{C}_1$ (this is the set such that each row sums to m, and all entries are 0 or 1)



We can project each row separately!

Remember: projecting x to a set S means we want to find the closest point in S to x. Another way of thinking about it is finding a way to move x to a point in S with minimum distance/effort

N-M Queens

Let's suppose we want to project to $\hat{C}_1$ (this is the set such that each row sums to m, and all entries are 0 or 1)

Let's say m = 2, and we want to change the row y = (0.4,0.2,0.8,1.1) as little as possible such that it had exactly two 1's and two 0's.

What do we change it to?

What about y = (-0.1,0.6,0.2,0.9)?

N-M Queens

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It's a greedy algorithm! Pick the largest m elements to be 1, and then the rest 0.

N-M Queens

See the algorithms in action! Zoom doesn’t share screen at a very high framerate!

https://www.youtube.com/watch?v=HgHH8aXgjZU&feature=youtu.be

https://www.youtube.com/watch?v=xiA94KMdB80

https://www.youtube.com/watch?v=AF3S30rRdN4

We will move over to my blog where I will discuss some of the actual insights.

https://theepiccowoflife.github.io/2022/02/05/projalgo.html

I tried submitting to FARIO Secret Chamber of the Giza Pyramid in the middle of my research.

(I will screenshare how that went)

Sudoku

Here is a cool visualisation. Time permitting, I'll explain this in lecture.

https://www.youtube.com/watch?v=J_UXjyfXUu8