

# BAYESIAN EVIDENCE SYNTHESIS: OPIOID CRISIS

HYEONGCHEOL PARK\* & PAUL GUSTAFSON\* & MICHEAL A IRVINE\*<sup>1</sup>

## CONTENTS

1	Introduction	2
2	methods	2
3	template	4
3.1	Paragraphs . . . . .	4
3.2	Math . . . . .	4
4	Results and Discussion	4
4.1	Subsection . . . . .	4
4.2	Figure Composed of Subfigures . . . . .	5

## LIST OF FIGURES

Figure 1	Boxplot of posterior 2000 samples of $O_t$ with actual data as red dot. . . . .	3
Figure 2	An example of a floating figure . . . . .	4
Figure 3	A number of pictures. . . . .	5

## LIST OF TABLES

Table 1	Table of Grades . . . . .	5
---------	---------------------------	---

## ABSTRACT

---

\* Department of Statistics, University of British Columbia, Vancouver, Canada

<sup>1</sup> Department of Mathematics, University of British Columbia, Vancouver, Canada

## 1 INTRODUCTION

Opioid crisis is one of major issues in North America continents including Canada. There were 1,490 deaths and 15,598 paramedic- attended overdose events during 2017 alone. [1] (need to know about bib in latex, change statistics to 2018 later) The goal of this project is to apply Bayesian evidence synthesis to help reduce the effect of opoid crisis in Vancouver, Canada.

All examples here were performed in Python 3.7 using the library pyMC (reference) and JAGS (reference). Training was performed using No U-Turn Sampling (NUTS) over two chains with 1000 iterations (is it sample size?). Fitting was performed on a GHz Intel Core i5 with 8GM of LPDD3 RAM and typically had wall times under ten minutes. Data processing was carried out using the Pandas and SciPy library [reference]. Data visualization was performed using the libraries Seaborn and Matplotlib [ref]. Code for all examples in this study are provided.

## 2 METHODS

### Process

The number of overdoses is our ultimate interest of estimation. Let  $O_t$  the number of overdose in a given month  $t$ . Suppose there was a survey conducted to estimate the proportion of ambulance call  $p_A$  among the subjects of overdoses. Let  $n_A$  the sample size of the survey and  $x_A$  to be the total number who confirmed they did call ambulance. It is assumed that  $x_A$  follows Binomial distribution.

$$x_A \sim \text{Bin}(n_A, p_A) \text{ ambulance call-outs model} \quad (1)$$

The total overdoses need to be modeled. The simplest conceptual model is to take an underlying log-rate  $z_t$  that is independent and identically distributed according to a normal distribution with mean  $\mu$  and variance  $\sigma^2$ . [1] Denote  $\lambda_t$  the rate of overdose at time  $t$ . It is assumed that the total overdose  $O_t$  follows Poission distribution where the population of the region of interest is  $N$ .

$$\left. \begin{aligned} z_t &\sim N(\mu, \sigma^2) \\ \lambda_t^{\text{OD}} &= \exp(z_t) \\ O_t &\sim \text{Poi}(\lambda_t^{\text{OD}} N) \end{aligned} \right\} \text{ overdose model} \quad (2)$$

Estimation of  $O_t$  is not straightforward since none of the variables ( $\mu$ ,  $\sigma$ ,  $N$ ) determining  $O_t$  is known. Hence  $O_t$  should be inferred from using  $U_t$  and  $p_A$ , where  $p_A$  is the ambulance call out rate and  $U_t$  is the number of ambulance-attended overdoses at a time point  $t$ . In general, the data of ambulance-attended overdoses  $U_t$  can be obtained. It is assumed that  $U_t$  follows Binomial distribution:

$$U_t \sim \text{Bin}(O_t, p_A) \quad (3)$$

Now  $O_t$  can be estimated as  $p_A$  can be infered by survey data and the data regarding  $U_t$  is given. We suggest a simple model as a start where the model only combines Ambulance Call-outs Model (1) and Overdose Model (2).

The next step is to run some simulations to figure out how different types of inputs lead some changes of output. To do so, the simple model illustrated below.

### Simulation

The first simulation simplifies the assumptions of variables as much as possible; We assumed  $N = 10000, n_A = 1000$ . The assumptions will change later to see the impact of the likelihood over the posterior distributions of variables of interest; The total number of population for a region  $N$  could vary over time or it can be stratified for a better realization of the real world.  $n_A$  can be vary as  $n_A = 100$  or  $n_A = 10000$ .

## Likelihood

There exist two data sets; survey data  $(n_A, x_A)$ , and ambulance attended overdose data  $(U_t)$ . The two data set is simulated as follows. The true value of  $p_A$  was set  $p_A = 0.8$  for the survey data. It is assumed that the data was collected for a year ( $t=1,2,3, \dots, 12$ ) and  $x_t$  values were independently generated from the Binomial distribution (1). It is assumed that the true values of parameters for overdose model were  $\mu = \log 0.05, \sigma = 1$ . The vector of  $O_t$  was generated following the overdose model (2). The vector of  $U_t$  was generated from the Binomial relation of the two variables (3). The two generated vectors have the same length with the survey data ( $t=1,2,3, \dots, 12$ ).

Note that only  $U_t$  and  $x_t$  are known as the likelihood and  $p_A$  needs to be estimated first so as to estimate  $O_t$  which is the ultimate interest of the research.

## Prior Distributions

Noninformative prior distributions are presumed as a start for simplicity.

$$p(p_A) \sim \text{Beta}(1, 1) \text{ noninformative prior of ambulance model} \quad (4)$$

$$\left. \begin{array}{l} \mu \sim U(-10, 0) \\ \sigma \sim U(0, 5) \end{array} \right\} \text{noninformative prior of overdose model} \quad (5)$$

This leads the posterior distribution of variables of interest to heavily depend on the likelihood. Later, the noninformative priors will be changed and the impact of the changes over posteriors will be investigated.

## Early Result

Figure 1 is the boxplot of posterior samples of  $O_t$ . It is shown that

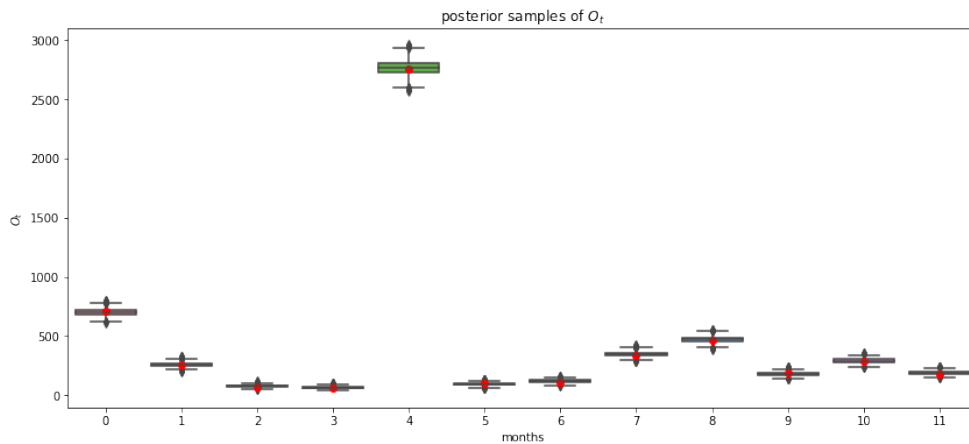


Figure 1: Boxplot of posterior 2000 samples of  $O_t$  with actual data as red dot.

### 3 TEMPLATE

We first focus on the simplest situation that can describe the data sets and the general idea.

1. First item in a list
2. Second item in a list
3. Third item in a list

#### 3.1 Paragraphs

PARAGRAPH DESCRIPTION

DIFFERENT PARAGRAPH DESCRIPTION

#### 3.2 Math

$$\cos^3 \theta = \frac{1}{4} \cos \theta + \frac{3}{4} \cos 3\theta \quad (6)$$

**Definition 1** (Gauss). To a mathematician it is obvious that  $\int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}$ .

**Theorem 1** (Pythagoras). *The square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.*

*Proof.* We have that  $\log(1)^2 = 2 \log(1)$ . But we also have that  $\log(-1)^2 = \log(1) = 0$ . Then  $2 \log(-1) = 0$ , from which the proof.  $\square$

## 4 RESULTS AND DISCUSSION

Reference to Figure 2.

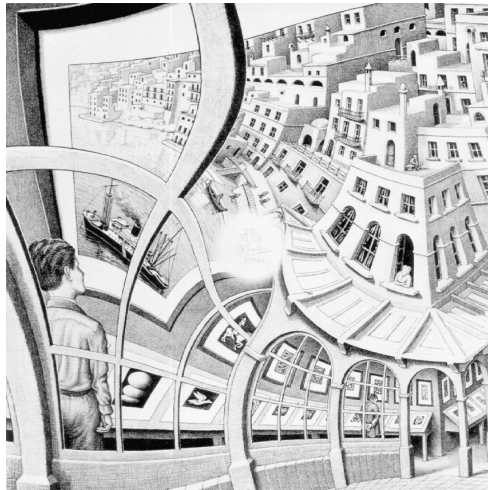


Figure 2: An example of a floating figure (a reproduction from the *Gallery of prints*, M. Escher, from <http://www.mcescher.com/>).

#### 4.1 Subsection

##### 4.1.1 Subsubsection

**WORD** Definition



**Figure 3:** A number of pictures with no common theme.

#### CONCEPT Explanation

##### IDEA Text

- First item in a list
- Second item in a list
- Third item in a list

#### 4.1.2 Table

**Table 1:** Table of Grades

Name		
First name	Last Name	Grade
John	Doe	7.5
Richard	Miles	2

Reference to [Table 1](#).

#### 4.2 Figure Composed of Subfigures

Reference the figure composed of multiple subfigures as [Figure 3](#). Reference one of the subfigures as [Figure 3b](#).

## REFERENCES

- [1] Buxton J Balshaw R Otterstatter M Macdougall L et al. Irvine MA, Kuo M. Modelling the combined impact of interventions in averting deaths during a

synthetic-opioid overdose epidemic. *Addiction*, 2019.

- [2] A. J. Figueredo and P. S. A. Wolf. Assortative pairing and life history strategy - a cross-cultural study. *Human Nature*, 20:317–330, 2009.