BAYESIAN EVIDENCE SYNTHESIS:OPIOID CRISIS

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ABSTRACT

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1 INTRODUCTION

Opioid crisis is one of major issues in North America continents including Canada. There were 1,490 deaths and 15,598 paramedic- attended overdose events during 2017 alone. [1] (need to know about bib in latex, change statistics to 2018 later) The goal of this project is to apply Bayesian evidence synthesis to help reduce the effect of opoid crisis in Vancouver, Canada.

All examples here were performed in Python 3.7 using the library pyMC (reference) and JAGS (reference). Training was performed using No U-Turn Sampling (NUTS) over two chains with 1000 iterations (is it sample size?). Fitting was performed on a GHz Intel Core i5 with 8GM of LPDD3 RAM and typically had wall times under ten minutes. Data processing was carried out using the Pandas and SciPy library [reference]. Data visualization was performed using the libraries Seaborn and Matplotlib [ref]. Code for all examples in this study are provided.

2 METHODS

Process

The number of overdoses is our ultimate interest of estimation. Let O_t the number of overdose in a given month t. Suppose there was a survey conducted to estimate the proportion of ambulance call p_A among the subjects of overdoses. Let n_A the sample size of the survey and x_A to be the total number who confirmed they did call ambulance. It is assumed that x_A follows Binomial distribution.

$$x_A \sim \text{Bin}(n_A, p_A) \quad \text{ambulance call-outs model} \tag{1}$$

The total overdoses need to be modeled. The simplest conceptual model is to take an underlying log-rate z_t that is independent and identically distributed according to a normal distribution with mean μ and variance σ^2. [1] Denote λ_t the rate of overdose at time t. It is assumed that the total overdose O_t follows Poission distribution where the population of the region of interest is N.

$$\left.\begin{aligned} z_t &\sim N(\mu, \sigma^2) \\ \lambda_t^{OD} &= \exp(z_t) \\ O_t &\sim \text{Poi}(\lambda_t^{OD} N) \end{aligned}\right\} \text{ overdose model} \tag{2}$$

Estimation of O_t is not straightforward since none of the variables (μ, σ, N) determining O_t is known. Hence O_t should be inferred from using U_t and p_A, where p_A is the ambulance call out rate and U_t is the number of ambulance-attended overdoses at a time point t. In general, the data of ambulance-attended overdoses U_t can be obtained. It is assumed that U_t follows Binomial distribution:

$$U_t \sim \text{Bin}(O_t, p_A) \tag{3}$$

Now O_t can be estimated as p_A can be infered by survey data and the data regarding U_t is given. We suggest a simple model as a start where the model only combines Ambulance Call-outs Model (1) and Overdose Model (2).

The next step is to run some simulations to figure out how different types of inputs lead some changes of output. To do so, the simple model illustrated below.

Simulation

The first simulation simplifies the assumptions of variables as much as possible; We assumed $N = 10000, n_A = 1000$. The assumptions will change later to see the impact of the likelihood over the posterior distributions of variables of interest; The total number of population for a region N could vary over time or it can be staritified for a better realization of the real world. n_A can be vary as $n_A = 100$ or $n_A = 10000$.

Likelihood

There exist two data sets; survey data (n_A, x_A), and ambulance attended overdose data (U_t). The two data set is simulated as follows. The true value of p_A was set $p_A = 0.8$ for the survey data. It is assumed that the data was collected for a year (t=1,2,3, ..., 12) and x_t values were independentally generated from the Binomial distribution (1). It is assumed that the true values of parameters for overdose model were $\mu = \log 0.05, \sigma = 1$. The vector of O_t was generated following the overdose model (2). The vector of U_t was gerated from the Binomial relation of the two variables (3). The two generated vectors have the same length with the survey data (t=1,2,3, ..., 12).

Note that only U_t and x_t are known as the likelihood and p_A needs to be estimated first so as to estimate O_t which is the ultimate interest of the research.

Prior Distributions

Noninformative prior distributions are presumed as a start for simplicity.

$$p(p_A) \sim Beta(1,1) \text{ noninformative prior of ambulance model} \tag{4}$$

$$\left.\begin{aligned} \mu &\sim U(-10,0) \\ \sigma &\sim U(0,5) \end{aligned}\right\} \text{ noninformative prior of overdose model} \tag{5}$$

This leads the posterior distribution of variables of interest to heavily depend on the likelihood. Later, the noninformative priors will be changed and the impact of the changes over posteriors will be investigated.

Early Result

The result from the simple case scenario is illustrated below.

Posterior Distribution

Figure 1 is the boxplot of posterior samples of O_t. It is shown that our posterior estimates of O_t is fairly accurate since (1) the boxplots contain actual values of O_t within their interquartile range (IQR) and (2) the ranges of IQR and 95% range seem narrow covering the actual values of O_t. Notice that the range of the boxplot from a higher O_t values (t=4) is wider than the other ranges of the boxplots from smaller estimates of O_t (all t values but 4)

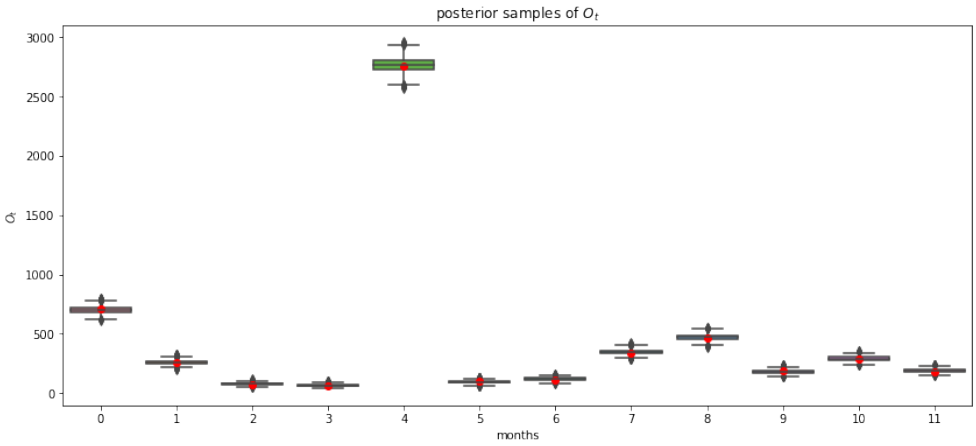


Figure 1: Boxplot of posterior samples of O_t (2000 samples for each month) with actual data points of simulated O_t values. The simulated values are shown as red dots.

Posterior Predictive Check

Figure2 is the boxplot of posterior predictive samples of U_t. It is shown that the posterior predictive estimates of U_t is failry accurate with the same two reasons regarding the accuracy of the posterior distribution of O_t It is more obvious here that the range of the boxplots from higher O_t values (t=1, 7, 11) is wider than the other ranges of the boxplots from smaller estimates of O_t (all t values but 1,7, 11)
.

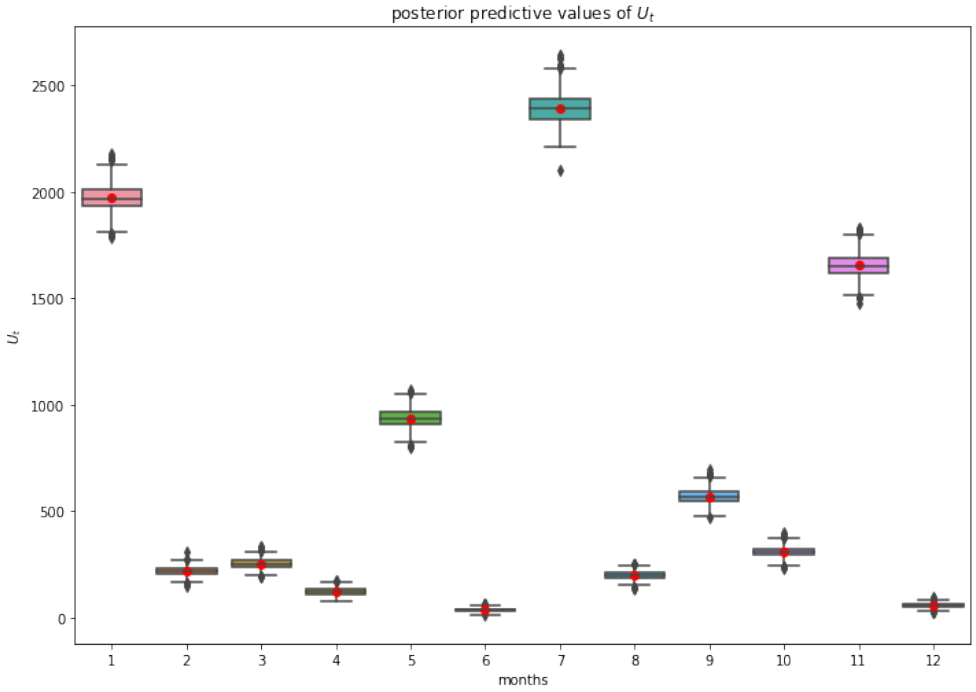


Figure 2: Boxplot of posterior predictive samples of U_t (2000 samples for each month) with actual data points of simulated U_t values. The simulated values are shown as red dots.

Figure 4 is the boxplot of posterior predictive samples of x_A. It is shown that the posterior predictive estimates of x_A is failry accurate with the same two reasons regarding the accuracy of the posterior distribution of x_A.

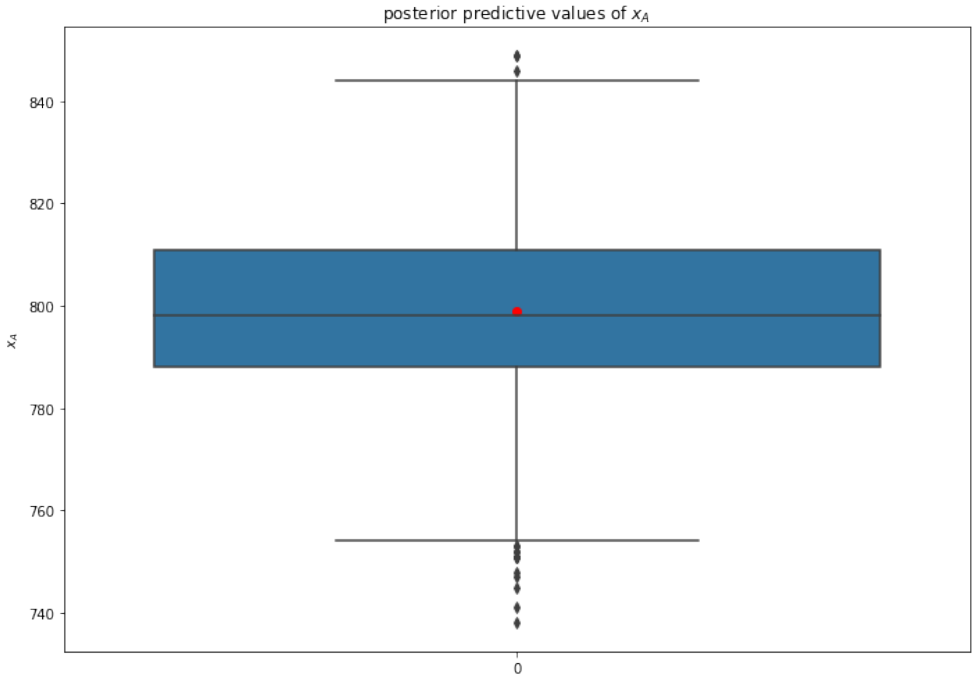


Figure 3: Boxplot of posterior predictive samples of x_A (1000 samples) with the actual data point of simulated x_A value. The simulated value is shown as a red dot.

Early Result: Contamination of p_A

One of the attention of this research project is to investigate how robust the model is from a contamination of a data set. The first inspection is to check an impact of a contamination of p_A; what would happen if the estimation of p_A is biased? It is assumed that the survey data gives us a wrong estimate of p_A and it would be underestimated or overestimated. We then want to see how the biased estimation of p_A affects the estimate of O_t, the total overdose.

Both of underestimation and overestimation were conducted for the analysis. In terms of underestimation, the simulated survey data (n_A, x_A) was generated with $p_A = 0.6$ while the true value of p_A is 0.8, and all the other assumptions hold the same. That is, x_A is generated from $x_A \sim \text{Bin}(n_A, 0.6)$, while U_t is generated from $U_t \sim \text{Bin}(O_t, 0.8)$ for every t. For overestimation, the simulated survey data was generated with $p_A = 0.9$ while the true value of p_A is 0.8, and all the other assumptions hold the same.

3 TEMPLATE

We first focus on the simplest sitaution that can describe the data sets and the general idea.

1. First item in a list
2. Second item in a list
3. Third item in a list

3.1 Paragraphs

PARAGRAPH DESCRIPTION

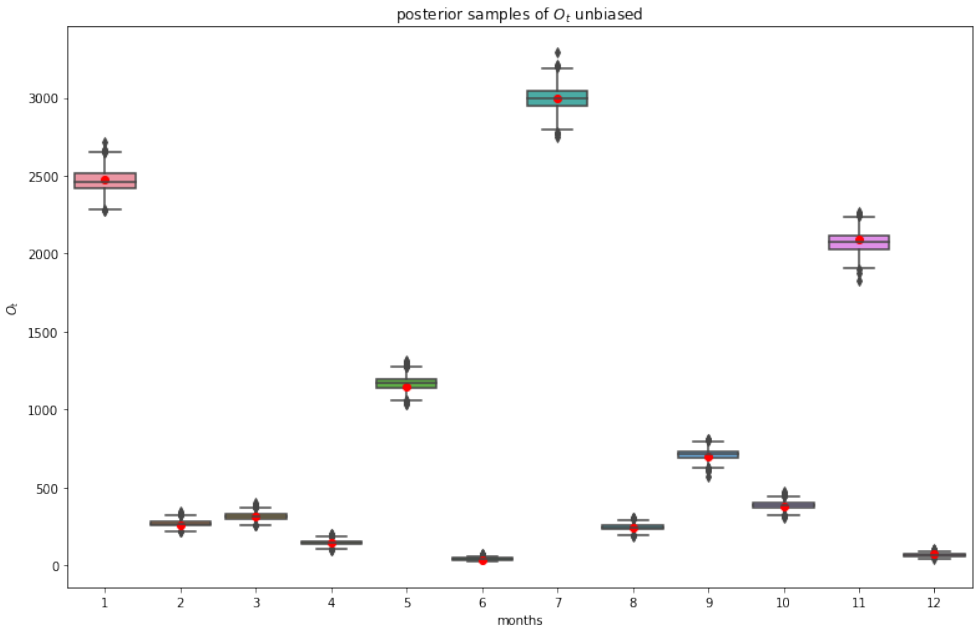


Figure 4: Boxplot of posterior predictive samples of x_A (1000 samples) with the actual data point of simulated x_A value. The simulated value is shown as a red dot.

DIFFERENT PARAGRAPH DESCRIPTION

3.2 Math

$$\cos^3\theta = \frac{1}{4}\cos\theta + \frac{3}{4}\cos 3\theta \tag{6}$$

Definition 1 (Gauss)**.** To a mathematician it is obvious that $\int_{-\infty}^{+\infty} e^{-x^2}\,dx = \sqrt{\pi}$.

Theorem 1 (Pythagoras)**.** *The square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.*

Proof. We have that $\log(1)^2 = 2\log(1)$. But we also have that $\log(-1)^2 = \log(1) = 0$. Then $2\log(-1) = 0$, from which the proof. □

4 RESULTS AND DISCUSSION

Reference to Figure 5 on the following page.

4.1 Subsection

4.1.1 *Subsubsection*

WORD Definition

CONCEPT Explanation

IDEA Text

- First item in a list
- Second item in a list
- Third item in a list

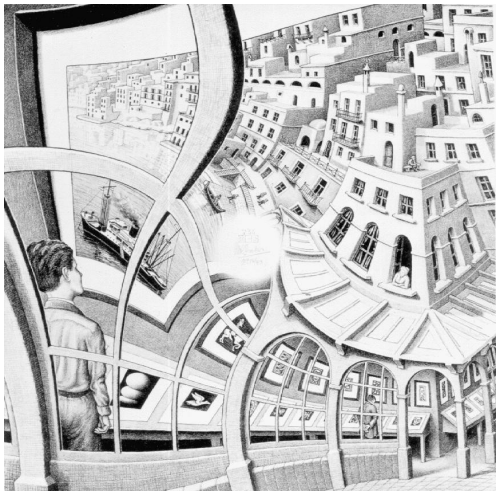


Figure 5: An example of a floating figure (a reproduction from the *Gallery of prints*, M. Escher, from http://www.mcescher.com/).

Table 1: Table of Grades

Name		
First name	Last Name	Grade
John	Doe	7.5
Richard	Miles	2

4.1.2 *Table*

Reference to Table 1.

4.2 Figure Composed of Subfigures

Reference the figure composed of multiple subfigures as Figure 6 on the following page. Reference one of the subfigures as Figure 6b on the next page.

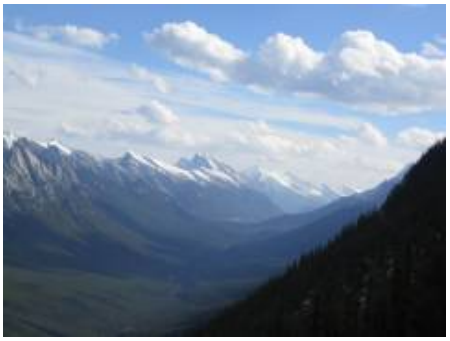
REFERENCES

[1] Buxton J Balshaw R Otterstatter M Macdougall L et al. Irvine MA, Kuo M. Modelling the combined impact of interventions in averting deaths during a synthetic-opioid overdose epidemic. *Addiction*, 2019.

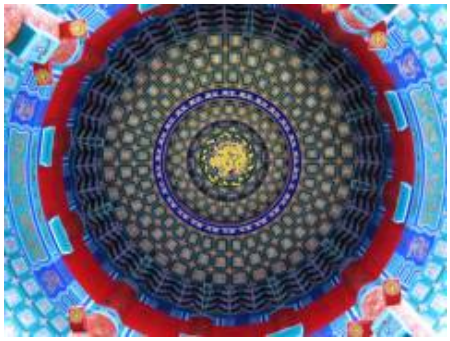
[2] A. J. Figueredo and P. S. A. Wolf. Assortative pairing and life history strategy - a cross-cultural study. *Human Nature*, 20:317–330, 2009.

(a) A city market.

(b) Forest landscape.



(c) Mountain landscape.



(d) A tile decoration.

Figure 6: A number of pictures with no common theme.