Technical Report:
Incentive Routing Design for Covert Communication in Multi-hop Decentralized Wireless Networks

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In this technical report, we provide the detailed proof for Proposition 1 in the manuscript *Incentive Routing Design for Covert Communication in Multi-hop Decentralized Wireless Networks*. Let $G_{\mathrm{JC}} = (\mathcal{J}, \{\mathbf{P}_J\}, \{U_{J_i}\})$ denote the jammer-competition (JC) game, where $\mathcal{J}$, $\{\mathbf{P}_J\}$ and $\{U_{J_i}\}$ are the sets of jammers, strategy profiles and utility functions, respectively. In the following, we will prove the existence, uniqueness, and calculation of the Nash Equilibrium (NE) in the JC game, sequentially.

1 Existence of Nash Equilibrium

Note that the strategy of any player J_i satisfies $P_{J_i} \geq 0$, the strategy space of the JC game $\{\mathbf{P}_J\}$ is a nonempty, compact, and convex subset of the M-dimensional Euclidean space $\mathbf{R}^M$. Taking the first- and second-order derivatives of U_{J_i} with respect to P_{J_i} yields

$$\frac{\partial U_{J_i}}{\partial P_{J_i}} = \frac{Rr_i}{\sum_{J_j \in \mathcal{J}} P_{J_j} r_j} - \frac{RP_{J_i} r_i^2}{\left(\sum_{J_j \in \mathcal{J}} P_{J_j} r_j\right)^2} - c \tag{A1}$$

$$\frac{\partial^2 U_{J_i}}{\partial P_{J_i}^2} = -2\frac{Rr_i^2 \sum_{J_j \in \mathcal{J} \setminus \{J_i\}} P_{J_j} r_j}{\left(\sum_{J_j \in \mathcal{J}} P_{J_j} r_j\right)^3} < 0. \tag{A2}$$

We can obtain that U_{J_i} is continuous and differentiable on P_{J_i} and its second-order derivative is negative, and thus U_{J_i} is a concave function of P_{J_i}. According to the Game Theory [1, Theorem 1], we can conclude that there exists at least an NE in the JC game $G_{\mathrm{JC}} = (\mathcal{J}, \{\mathbf{P}_J\}, \{U_{J_i}\})$.

2 Uniqueness of Nash Equilibrium

In the JC game (non-cooperative game), the objective of each player is to play a strategy to achieve its maximum utility. Such a strategy is termed the best response strategy defined as follows.

Definition 1. *(Best Response Strategy): Given $\mathbf{P}_{-J_i}$, a strategy is the best response strategy of jammer J_i, denoted as $b_i\left(\mathbf{P}_{-J_i}\right)$, if it satisfies $U_{J_i}\left(b_i\left(\mathbf{P}_{-J_i}\right), \ \mathbf{P}_{-J_i}\right) \geq U_{J_i}\left(P_{J_i}, \mathbf{P}_{-J_i}\right)$ for all $P_{J_i} \geq 0$.*

Let $\mathbf{b}\left(\mathbf{P}_J\right) \triangleq \left(b_1\left(\mathbf{P}_{-J_1}\right), b_2\left(\mathbf{P}_{-J_2}\right), \ldots, \ \ b_M\left(\mathbf{P}_{-J_M}\right)\right)$, and we term $\mathbf{b}\left(\mathbf{P}_J\right)$ the best response correspondence of the JC game. According to the definition of NE, we know that an NE is actually a fixed point of the best response correspondence $\mathbf{b}\left(\mathbf{P}_J\right)$, i.e., $\mathbf{P}_J^{\mathrm{ne}} = \mathbf{b}\left(\mathbf{P}_J^{\mathrm{ne}}\right)$. Therefore, the uniqueness of NE is equivalent to that the function $\mathbf{b}\left(\mathbf{P}_J\right)$ has a unique fixed point.

Next, we check the properties of $\mathbf{b}\left(\mathbf{P}_J\right)$. Note that U_{J_i} is a concave function on P_{J_i}, so we can derive the best response strategy $b_i\left(\mathbf{P}_{-J_i}\right)$ by solving the zero of the first-order derivative of U_{J_i} with respect to P_{J_i}, which yields

$$P_{J_i} = \sqrt{\frac{R\sum_{J_j \in \mathcal{J}\setminus\{J_i\}} P_{J_j} r_j}{c r_i}} - \frac{1}{r_i} \sum_{J_j \in \mathcal{J}\setminus\{J_i\}} P_{J_j} r_j. \tag{A3}$$

If the right-hand side of (A3) is positive, it is the best response strategy of jammer J_i. Otherwise, jammer J_i will not participate in the JC game by setting $P_{J_i} = 0$. Therefore, the best response strategy of J_i can be expressed as

$$b_i\left(\mathbf{P}_{-J_i}\right) = \begin{cases} 0, & R \leq \frac{c}{r_i} \sum_{J_j \in \mathcal{J}\setminus\{J_i\}} P_{J_j} r_j \\ \sqrt{\frac{R\sum_{J_j \in \mathcal{J}\setminus\{J_i\}} P_{J_j} r_j}{c r_i}} - \frac{1}{r_i} \sum_{J_j \in \mathcal{J}\setminus\{J_i\}} P_{J_j} r_j. & \text{otherwise} \end{cases} \tag{A4}$$

Obviously, for every jammer J_i which joins the JC game, $b_i\left(\mathbf{P}_{-J_i}\right)$ is always positive and monotonic. In addition, for any $\alpha > 1$, we have

$$\alpha \cdot b_i\left(\mathbf{P}_{-J_i}\right) - b_i\left(\alpha \cdot \mathbf{P}_{-J_i}\right) = \left(\alpha - \sqrt{\alpha}\right)\sqrt{\frac{R\sum_{J_j \in \mathcal{J}\setminus\{J_i\}} P_{J_j} r_j}{c r_i}} > 0. \tag{A5}$$

Therefore, $\mathbf{b}\left(\mathbf{P}_J\right)$ satisfies the properties of positivity, monotonicity, and scalability, and thus it is a *standard* function. According to [2, Theorem 1], the fixed point of a standard function is unique. Equivalently, the uniqueness of NE in the JC game has been proved.

3 Calculation of Nash Equilibrium

Let $A_i = \sum_{J_j \in \mathcal{J}\setminus\{J_i\}} P_{J_j}^{ne} r_j$. From (A1) we have

$$\begin{aligned} r_i A_i &= \frac{c}{R}\left(A_i + P_{J_i}^{ne} r_i\right)^2 = \frac{c}{R}\left(\sum_{J_k \in \mathcal{J}} P_{J_k}^{ne} r_k\right)^2 \\ &= \frac{c}{R}\left(A_j + P_{J_j}^{ne} r_j\right)^2 = r_j A_j \quad \forall J_i, J_j \in \mathcal{J}. \end{aligned} \tag{A6}$$

Then, the following system of equations can be obtained:

$$\begin{cases} A_1 = \frac{r_i}{r_1} A_i \\ A_2 = \frac{r_i}{r_2} A_i \\ \vdots \\ A_M = \frac{r_i}{r_M} A_i. \end{cases} \tag{A7}$$

Performing some basic algebraic transformations yields

$$A_1 + A_2 + \cdots + A_M = \frac{r_i}{r_1} A_i + \frac{r_i}{r_2} A_i + \cdots + \frac{r_i}{r_M} A_i = \sum_{j=1}^{M} \frac{r_i}{r_j} A_i. \tag{A8}$$

$$A_j = \sum_{J_k \in \mathcal{J} \setminus \{J_j\}} P_{J_k}^{ne} r_k = A_i + P_{J_i}^{ne} r_i - P_{J_j}^{ne} r_j. \tag{A9}$$

Then, we have

$$\begin{aligned} \sum_{i=1}^{M} A_i &= M \left(A_i + P_{J_i}^{ne} r_i \right) - \sum_{j=1}^{M} P_{J_j}^{ne} r_j \\ &= M \left(A_i + P_{J_i}^{ne} r_i \right) - \left(A_i + P_{J_i}^{ne} r_i \right) \\ &= (M-1) \left(A_i + P_{J_i}^{ne} r_i \right) = \sum_{j=1}^{M} \frac{r_i}{r_j} A_i, \end{aligned} \tag{A10}$$

and thus

$$A_i = \frac{(M-1) r_i P_{J_i}^{ne}}{\sum_{j=1}^{M} \frac{r_i}{r_j} - M + 1}. \tag{A11}$$

Substituting (A11) into (A3), there is

$$P_{J_i}^{ne} = \frac{R(M-1)\left(\sum_{j=1}^{M} \frac{D_{J_i}}{D_{J_j}} - M + 1 \right)}{c \cdot \left(\sum_{j=1}^{M} \frac{D_{J_i}}{D_{J_j}} \right)^2} = R \cdot \kappa_i. \tag{A12}$$

This completes the calculation of NE in the JC game.

References

[1] J. B. Rosen, "Existence and uniqueness of equilibrium points for concave n-person games," *Econometrica*, vol. 33, no. 3, pp. 520–534, 1965.

[2] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Sel. Areas Commun.*, vol. 13, no. 7, pp. 1341–1347, 1995.