Technical Report:
Double-Sided Auction based Data-Energy Trading Architecture in Internet of Vehicles

Honggang He, Yang Xu, Jia Liu

May, 2022

In this technical report, we provide the detailed proofs for Proposition 1 and Proposition 2 in the manuscript *Double-Sided Auction based Data-Energy Trading Architecture in Internet of Vehicles*.

1 Proof of Proposition 1

Proof. According to the update functions (25) and (26), the dynamics of dual variables λ_i and $\mu_{j,i}$, defined as $\hat{\lambda}_i(t) = \lambda_i^{(t+1)} - \lambda_i^{(t)}$ and $\hat{\mu}_{j,i}(t) = \mu_{j,i}^{(t+1)} - \mu_{j,i}^{(t)}$, are given by

$$\hat{\lambda}_i(t) = \begin{cases} \sum_{j=1}^{M} y_{i,j} - 1, & \lambda_i > 0, \\ 0, & \lambda_i = 0, \end{cases} \quad \forall i \in \mathcal{N}, \tag{P1}$$

$$\hat{\mu}_{j,i}(t) = \begin{cases} x_{j,i} - y_{i,j}, & \mu_{j,i} > 0, \\ 0, & \mu_{j,i} = 0, \end{cases} \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{M}. \tag{P2}$$

We define the Lyapunov function as

$$V(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \frac{1}{2}\sum_{i=1}^{N}(\lambda_i - \lambda_i^{\dagger})^2 + \frac{1}{2}\sum_{j=1}^{M}\sum_{i=1}^{N}(\mu_{j,i} - \mu_{j,i}^{\dagger})^2. \tag{P3}$$

Taking the first derivative of Lyapunov function $V(\boldsymbol{\lambda}, \boldsymbol{\mu})$ with respect to t yields

$$\begin{aligned}
\frac{\mathrm{d}V(\boldsymbol{\lambda}, \boldsymbol{\mu})}{\mathrm{d}t} &= \sum_{i=1}^{N}(\lambda_i - \lambda_i^{\dagger})\hat{\lambda}_i(t) + \sum_{j=1}^{M}\sum_{i=1}^{N}(\mu_{j,i} - \mu_{j,i}^{\dagger})\hat{\mu}_{j,i}(t) \\
&\leq \sum_{i=1}^{N}(\lambda_i - \lambda_i^{\dagger})(\sum_{j=1}^{M} y_{i,j} - 1) + \sum_{j=1}^{M}\sum_{i=1}^{N}(\mu_{j,i} - \mu_{j,i}^{\dagger})(x_{j,i} - y_{i,j}) \\
&= \sum_{i=1}^{N}(\lambda_i - \lambda_i^{\dagger})(\sum_{j=1}^{M} y_{i,j} - \sum_{j=1}^{M} y_{i,j}^{\dagger}) + \sum_{i=1}^{N}(\lambda_i - \lambda_i^{\dagger})(\sum_{j=1}^{M} y_{i,j}^{\dagger} - 1) \\
&+ \sum_{j=1}^{M}\sum_{i=1}^{N}(\mu_{j,i} - \mu_{j,i}^{\dagger})(x_{j,i}^{\dagger} - y_{i,j}^{\dagger}) + \sum_{j=1}^{M}\sum_{i=1}^{N}(\mu_{j,i} - \mu_{j,i}^{\dagger})[(x_{j,i} - x_{j,i}^{\dagger}) - (y_{i,j} - y_{i,j}^{\dagger})].
\end{aligned} \tag{P4}$$

From the KKT conditions we have $\frac{\partial U_{RSU_j}(\boldsymbol{x}_j^{\dagger})}{\partial x_{j,i}^{\dagger}} = \mu_{j,i}^{\dagger}$ and $\frac{\partial U_{EV_i}(\boldsymbol{y}_i^{\dagger})}{\partial y_{i,j}^{\dagger}} = \mu_{j,i}^{\dagger} - \lambda_i^{\dagger}$. Then, inequality (P4) can be transformed as

$$\begin{aligned}
\frac{\mathrm{d}V(\boldsymbol{\lambda}, \boldsymbol{\mu})}{\mathrm{d}t} \leq & \sum_{j=1}^{M}\sum_{i=1}^{N}\left(\frac{\partial U_{RSU_j}(\boldsymbol{x}_j)}{\partial \boldsymbol{x}_j} - \frac{\partial U_{RSU_j}(\boldsymbol{x}_j^{\dagger})}{\partial \boldsymbol{x}_j^{\dagger}}\right)(x_{j,i} - x_{j,i}^{\dagger}) \\
& + \sum_{j=1}^{M}\sum_{i=1}^{N}\left(\frac{\partial U_{EV_i}(\boldsymbol{y}_i)}{\partial \boldsymbol{y}_i} - \frac{\partial U_{EV_i}(\boldsymbol{y}_i^{\dagger})}{\partial \boldsymbol{y}_i^{\dagger}}\right)(y_{i,j} - y_{i,j}^{\dagger}).
\end{aligned} \tag{P5}$$

Due to the strictly concave property of RSUs' payoff functions and the strictly convex property of EVs' cost functions, we have

$$\begin{aligned}
&\left(\frac{\partial U_{RSU_j}(\boldsymbol{x}_j)}{\partial \boldsymbol{x}_j} - \frac{\partial U_{RSU_j}(\boldsymbol{x}_j^{\dagger})}{\partial \boldsymbol{x}_j^{\dagger}}\right)(x_{j,i} - x_{j,i}^{\dagger}) \leq 0, \\
&\left(\frac{\partial U_{EV_i}(\boldsymbol{y}_i)}{\partial \boldsymbol{y}_i} - \frac{\partial U_{EV_i}(\boldsymbol{y}_i^{\dagger})}{\partial \boldsymbol{y}_i^{\dagger}}\right)(y_{i,j} - y_{i,j}^{\dagger}) \leq 0.
\end{aligned} \tag{P6}$$

We can conclude that the first derivative of the Lyapunov function is less than 0, i.e., $\mathrm{d}V(\boldsymbol{\lambda}, \boldsymbol{\mu})/\mathrm{d}t < 0$, and thus the IDA algorithm converges within t iterations. □

2 Proof of Proposition 2

We will prove the individual rationality, incentive compatibility, economic efficiency, and budget balance of the IDA-based data-energy trading algorithm in the following subsections, respectively.

2.1 Individual Rationality (IR)

Proof. Since $U_{RSU_j}(\boldsymbol{x}_j)$ is a strictly concave function of $\boldsymbol{x}_j$ and $U_{RSU_j}(\mathbf{0}) = 0$, according to the Lagrange's Mean Value Theorem, we have

$$U_{RSU_j}(\boldsymbol{x}_j^\dagger) \geq U_{RSU_j}(\mathbf{0}) + x_{j,i}^\dagger \frac{\partial U_{RSU_j}(\boldsymbol{x}_j^\dagger)}{\partial x_{j,i}^\dagger} = x_{j,i}^\dagger \frac{\partial U_{RSU_j}(\boldsymbol{x}_j^\dagger)}{\partial x_{j,i}^\dagger}. \tag{P7}$$

Considering the optimal bid and payment of RSU, there is

$$x_{j,i}^\dagger \frac{\partial U_{RSU_j}(\boldsymbol{x}_j^\dagger)}{\partial x_{j,i}^\dagger} = x_{j,i}^\dagger \frac{u_{j,i}}{x_{j,i}^\dagger} = P_{RSU_j}(\boldsymbol{u}_j^\dagger), \tag{P8}$$

which implies the following inequalities always hold.

$$U_{RSU_j}(\boldsymbol{x}_j^\dagger) \geq P_{RSU_j}(\boldsymbol{u}_j^\dagger), \quad \forall j \in \mathcal{M}. \tag{P9}$$

Similarly, considering the strictly convex property of EVs' cost function $U_{EV_i}(\boldsymbol{y}_i)$ and the optimal bid and reward of EV, we can deduce that the following inequalities always hold.

$$Q_{EV_i}(\boldsymbol{v}_i^\dagger) - U_{EV_i}(\boldsymbol{y}_i^\dagger) \geq 0, \quad \forall i \in \mathcal{N}. \tag{P10}$$

We have the following conclusions: on one hand, RSUs and EVs can always gain a positive utility by participating in the double-sided auction-based data-energy trading process; on the other hand, they have no gain if they do not join the trading. Therefore, the designed IDA algorithm satisfies the IR property. □

2.2 Incentive Compatibility (IC)

Proof. From our derivation, it is clear that EVs and RSUs do not need to report their private information to the market operator, but submit their current optimal bids to the operator by locally solving the payoff maximization problem. In addition, these iteratively updated optimal bids can gradually reveal RSUs' hidden payoff and EVs' hidden cost. In other words, EVs and RSUs do not share information with the operator, but the optimal bids that they submit will gradually eliminate confidential information and ultimately maximize social welfare. Therefore, the designed IDA algorithm satisfies the IC property. □

2.3 Economic Efficiency

Proof. According to Proposition 1, we know that the IDA algorithm converges to the optimal solution when the KKT conditions of the ORA problem are satisfied. In addition, when EVs and RSUs submit their optimal bids according to our derived payment and reward functions, the IDA algorithm finally enables social welfare maximization to be achieved. This completes the proof. □

2.4 Budget Balance

Proof. According to the local payoff maximization problem of each RSU and EV, the budget of the market operator, denoted as $\Theta(\boldsymbol{U}, \boldsymbol{V})$, is given by

$$\begin{aligned}\Theta(\boldsymbol{U}, \boldsymbol{V}) &= \sum_{j=1}^{M} P_{RSU_j}(\boldsymbol{u}_j) - \sum_{i=1}^{N} Q_{EV_i}(\boldsymbol{v}_i), \\ &= \sum_{j=1}^{M}\sum_{i=1}^{N}\left[u_{j,i} - \frac{(\mu_{j,i}-\lambda_i)^2}{v_{i,j}}\right], \\ &= \sum_{j=1}^{M}\sum_{i=1}^{N}\left[\mu_{j,i}x_{j,i} - y_{i,j}(\mu_{j,i}-\lambda_i)\right].\end{aligned} \tag{P11}$$

When all participants submit their optimal bids, there is

$$\Theta(\boldsymbol{U}^\dagger, \boldsymbol{V}^\dagger) = \sum_{j=1}^{M}\sum_{i=1}^{N}\mu_{j,i}^\dagger(x_{j,i}^\dagger - y_{i,j}^\dagger) + \sum_{i=1}^{N}\lambda_i^\dagger\sum_{j=1}^{M}\sum_{i=1}^{N}(y_{i,j}^\dagger - 1) + \sum_{i=1}^{N}\lambda_i^\dagger \geq 0. \tag{P12}$$

It indicates that when the market operator has the optimal bids $\boldsymbol{u}_j$ and $\boldsymbol{v}_i$ from both sides, due to the KKT constraints, it will always gain a non-negative budget. This completes the proof. □