Assessing bank's default probability using the ASRF model

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Abstract

In this paper it is shown how a Vasicek-model approach and the assumptions in Basel 2 regulatory framework can be used to develop measures of the probability of banks' failure. The Basel 2 framework is based on a Vasicek-model approach. The estimation of the propose measure of bank probability of default could be made over the capital ratio from supervisory authorities (non-public information) or over the capital ratio from balance sheet data (public available information).

Key words: Vasicek model, ASRF model, Basel 2, banks' probability of default

Mathematics Subject Classification 2000: 60J05; 60J10; 60J20; 91B24.

1   Introduction

The Vasicek-model approaches and the assumptions in Basel 2 regulatory framework could be used to develop measures of the probability of banks' failure. Capital requirements' set by Basel 2 are based on standardize approach or advanced internal risk based (IRB) approach. The banks' probability of default measure could be extracted by the capital ratio from supervisory authorities (non-public information). The same ratio could be derived

from balance sheet data (public available information).When we know the probability of default of loans portfolio according to the Vasicek-model approach (see Vasicek 2002) we could calculate the requirement bank capital equal to 99.9The modified model could be used to estimate the banks' probability of default where the IRB approach set by Basel 2 is not accepted from regulatory institutions or the banks do not use it.

The remainder of this paper is organized as follows. The second section presents a brief review .. In the third section we describe our model.

2   Background

Usually in practice there are two alternative methodologies for estimating the probability of default. The first one is based on the historical data analysis. The second is based on the conception of imply probability measure from the option pricing theory, developed by Merton (1974) and then continuing by Vasicek (1987). The last method is extended by Basel 2 as a regulatory framework to valuate the loss on loan portfolio and to calculate requirement capitals. This paper is closely related to the second concept for quantifying the banks probability of default.

3   Methodology

Basel 2 takes the Vasicek model which is used to determine the regulatory capital in the Basel 2 framework. Under the assumptions of the model could be estimate the expected loss (EL) and unexpected loss (UL) of the banks. The expected loss is determined by three main ingredients: Probability of default (PD), Exposure at default (EAD) and Loss given default (LGD). The EAD means the amount of money owed at the moment counterparty goes into default. The LGD is the percentage of the EAD lost during a default. The expected loss is given by the following product

$$(1) \qquad\qquad EL = PD * EAD * LGD$$

and determines the amount of provisions which should be taken by the banks as a reserves. The UL is the additional loss in nonregular circumstances, for which tier capital should be retained. In the Vasicek model approach the UL is defined by the PD in a downturn situation. The model assumes that the EAD and LGD are not affected by dire circumstances. The both parameters are considered constant for a company.

VaR is defined as a threshold value x such that the probability of exceedance x is equal to $\alpha < 1$. α is called a confidential level. Basel 2 uses VaR coefficient as a measure of risk. The VaR is obtained by algebraically inverting the distribution function $F(x)$

$$(2) \qquad\qquad P(L \leq x) = F(x),$$

where x is set to be equal to the maximum losses (Value at Risk). Basel 2 sets the probability $1 - \alpha$ equal to 0.999 (BIS 2005) and $VaR_9 9.9$ is equal to $x = F^{-1}(0.999)$.

The unexpected loss is calculated as following

$$(3) \qquad\qquad UL = VaR_{99.9} - PD * LGD$$

More specifically, the new capital accord uses the Vasicek result to compute the Value at Risk (VaR) for each obligor, which is the maximum loss amount that is not expected to be exceeded with a given level and within a defined time horizon. In general the banks are expected to cover their expected loss on an ongoing basis by **provisions and write-offs** while Basel 2 requires banks to hold capital only to protect against unexpected losses.

Our task is to assess banks' default probability from the banks' required capital or capital from the balance sheets. That is an inverting problem of Vasicek model. VaR is computed for each obligor and found out the total required capital when the probability of default

of each obligor is known. The estimation of the proposed measure of bank probability of default could be made over the capital ratio from supervisory authorities (non-public information) or over the capital ratio from balance sheet data (public available information). The assumptions under which we calculate the implied probability of default are the following:

- Banks portfolios are exposed only to the credit risk, Banks' credit portfolio is homogeneous and the portfolio is infinitely divisible,

- The maximum loss (VaR) is set by Basel 2 to be equal to 99.9

- Required capital determinate by standardized approach or by internal rating based approach IRB (foundation and advanced approaches) both of them base on Basel 2 is a sufficient to cover the unexpected loss in 0.999.

The basis of the Basel 2 IRB approach is the Vasicek portfolio credit loss model (Vasicek, 1987, Vasicek, 2002). Vasicek offers one factor model which was firstly introduce by Merton in 1974 . Consider a portfolio consisting of n loans. Dynamic of the value of the assets of a borrower i for $i = 1, 2, ...n$ can be described by the geometrical Brownian motion:

$$(4) \qquad\qquad dA_i = \mu A_i(t)dt + \sigma A_i(t)dW_i(t)$$

where $W(t)$ is a Wiener process and $W(t)$ $N(0, t)$.

Thus the value of assets at time T can be expressed as:

$$(5) \qquad\qquad A_i(T) = A_i(0)e^{(\mu - \frac{\sigma^2}{2})T + \sigma\sqrt{T}W(T)}$$

where W(T) is a normal distributed random variable.

Basel's 2 formula uses Merton's model interpretation of default probability of the borrower i, so an obligor will default if the value of his assets falls below the value of his debt:

$$(6) \qquad p = P(A_i(T) < B_i) = P(W_i < c_i) = N(c_i) = N(-d_i)$$

$$(7) \qquad c_i = \frac{\ln(B_i/A_i) - (\mu + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

where $N(-d_2)$ is the standard normal distribution function giving the default probability which is equal to p. Then $p = N(c_i)$ and therefore $c_i = N^{-1}(p)$. For fixed t we assume that W_i has the following expression:

$$(8) \qquad W_i = by + a\varepsilon_i$$

where y represents systemic risk affecting the entire portfolio and is the specific risk for borrower i. It is assumed that both of them follow standard normal distributions. The values of b and a are given by:

$$(9) \qquad b = \sqrt{\rho} \qquad a = \sqrt{1 - \rho}$$

where ρ measures the asset correlation between borrowers' assets. It could be described as the dependence of the assets of a borrower on the general state of the economy; so all borrowers are linked to each other by the single risk factor, etc the common pairwise correlation between obligors (see Vasicek 1991).

Under those conditions, default probability of any loan, conditional on the single risk factor y, can be written as:

$$P(x|y) = P\left(by + a\varepsilon_i < c_i\right) = P\left(\varepsilon_i < \frac{c_i - by}{a}\right)$$

$$(10) \qquad P(x|y) = P\left(\varepsilon_i < \frac{c_i - by}{a}\right) = P\left(\varepsilon_i < \frac{N^{-1}(p) - \sqrt{\rho}y}{\sqrt{1 - \rho}}\right)$$

5

We can establish that the percentage of default on total portfolio equals the number of individuals that default divided by total number of obligators. If n is large enough, by the law of the large numbers it can be stated that the fraction of clients x that default is equal to the default probability conditional on y (4).

Under certain homogeneity assumptions, the Vasicek model leads us to very simple analytic asymptotic approximation of loss distribution due to each obligor (see Vasicek 2002). The cumulative distribution function of loan losses L on a very large portfolio is given by the limit:

$$(11) \qquad P(L \leq x) = F(x) = N\left[\frac{\sqrt{1-\rho}N^{-1}(x) - N^{-1}(PD)}{\sqrt{\rho}}\right]$$

where N is the standard normal distribution function. We use the notation PD for the probability of default, previously noted by p.

According to the Basel 2, if the LGD is equal to 1, 0.999 VaR (BIS 2005) has the following representation:

$$(12) \qquad VaR_{99.9} = N\left[\sqrt{\frac{1}{1-\rho}}N^{-1}(PD) + \sqrt{\frac{\rho}{1-\rho}}N^{-1}(0.999)\right]$$

the loss could be recovered, i.e. the LGD is not equal to 1 then

$$(13) \qquad VaR_{99.9} = LGD * N\left[\sqrt{\frac{1}{1-\rho}}N^{-1}(PD) + \sqrt{\frac{\rho}{1-\rho}}N^{-1}(0.999)\right]$$

The VaR quantity represents the basis for the computation of the bank capital requirement.

Then from (3) and (6) we get the formula for calculating capital requirements, known as a capital allocation parameter:

$$(14) \qquad C = \left[LGD * N\left[\sqrt{\frac{1}{1-\rho}}N^{-1}(PD) + \sqrt{\frac{\rho}{1-\rho}}N^{-1}(0.999)\right] - PD * LGD\right]$$

According to the Basel 2 the formula (7) is adjusted to the following result

$$C = \left[LGD * N\left[\sqrt{\frac{1}{1-\rho}}N^{-1}(PD) + \sqrt{\frac{\rho}{1-\rho}}N^{-1}(0.999)\right] - PD * LGD\right]$$

$$(1 - 1.5B)^{-1}[1 + (M - 2.5)B]1.06,$$

where M is the time to maturity and B is a function of PD computed as

$$B = [0.11852 - 0.05478\ln(PD)]^2$$

The equation (8) represents the adjusted capital in order to take into account the expected loss and the maturity adjustment factors.

According to the Basel 2 the correlation coefficient ρ is given by:

$$\rho = 0.12\frac{1-e^{-50PD}}{1-e^{-50}} + 0.24\left[1 - \frac{1-e^{-50PD}}{1-e^{-50}}\right] - 0.04\left[1 - \frac{S-5}{45}\right],$$

where S is the size of the firm.

Given the assumption of the infinite divisibility, the overall capital requirement for a bank is computed as a sum of the capital requirements of each loan exposure:

$$(15) \qquad\qquad K = \sum_i C_i(PD_i)A_i$$

that is the sum of the capital allocation parameter C_i of each exposure i multiplied by its amount A_i. If the credit portfolio is homogenous then there is one implied probability of default which could be extracted from capital adequacy ratio:

$$(16) \qquad\qquad \frac{K}{\sum_i A_i} = C(PD^*)$$

We can derive the average PD^* of a banks' credit portfolio as a function of capital requirement K for the bank and the sum of risk weight assets:

$$(17) \qquad\qquad PD^* = f\left(\sum A_i, K\right)$$

estimate the implied probability of default by equation (13) under the previous assumptions. The capital ratio could be taken from supervisory authorities as a non-public information (capital adequate ratio) or from balance sheet data as a public available information.

4 Numerical examples

Coming after the described method for assessing the implied probability, we implement it in MatLab. We find out the implied probability of default as a function of capital ratio, given in Figure 1. The function of implied probability of default vs. capital ratio is an increasing function which describes the fact that more risk taken from the bank costs more capital to it.The loss distribution as a function of default probability is given in Figure 2. The loss distribution has been found with 1 million Monte Carlo simulations.

5 Conclusion

This paper describes the derivation of implied default probabilities from the Vasicek model and Basel 2 framework. Our method for measuring probability of default is very simple and easy to implement in the both cases when we use the capital ratio from balance sheet data (public available information) and the capital ratio from supervisory authorities (non-public information).

Once we know the implied probability of default we could get the following: 1. Find out the loss distribution via Monte Carlo simulations (Figure2); 2. Find out the expected loss which should be coved by provisions and write-offs. 3. Make simulations and stress tests what will happen if the probability of default rose sharply for one bank or for the total banking system.
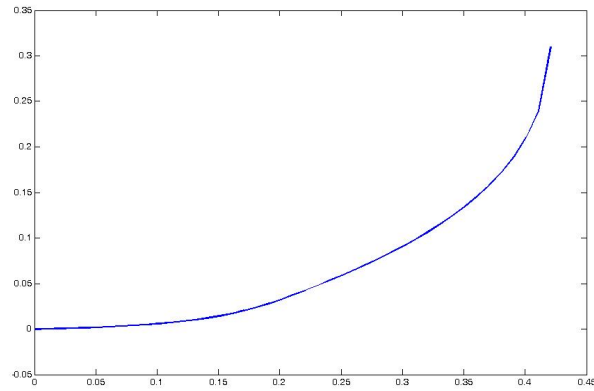
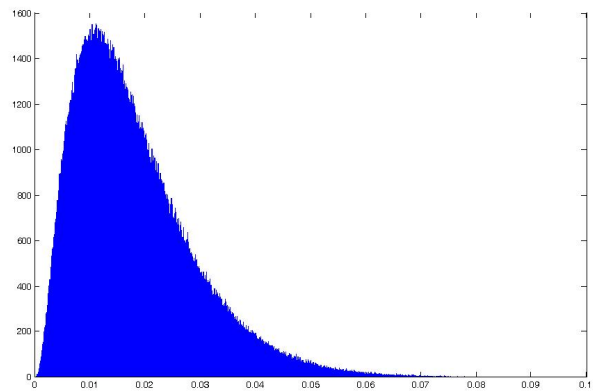Figure 1: The relation between implied probability of default as a function of capital ratio

Figure 2: The Loss distribution as a function of default probability found with 1 million Monte Carlo simulations.

Acknowledgments

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