Linear Algebra

Vectors

By Roghaye Faraji

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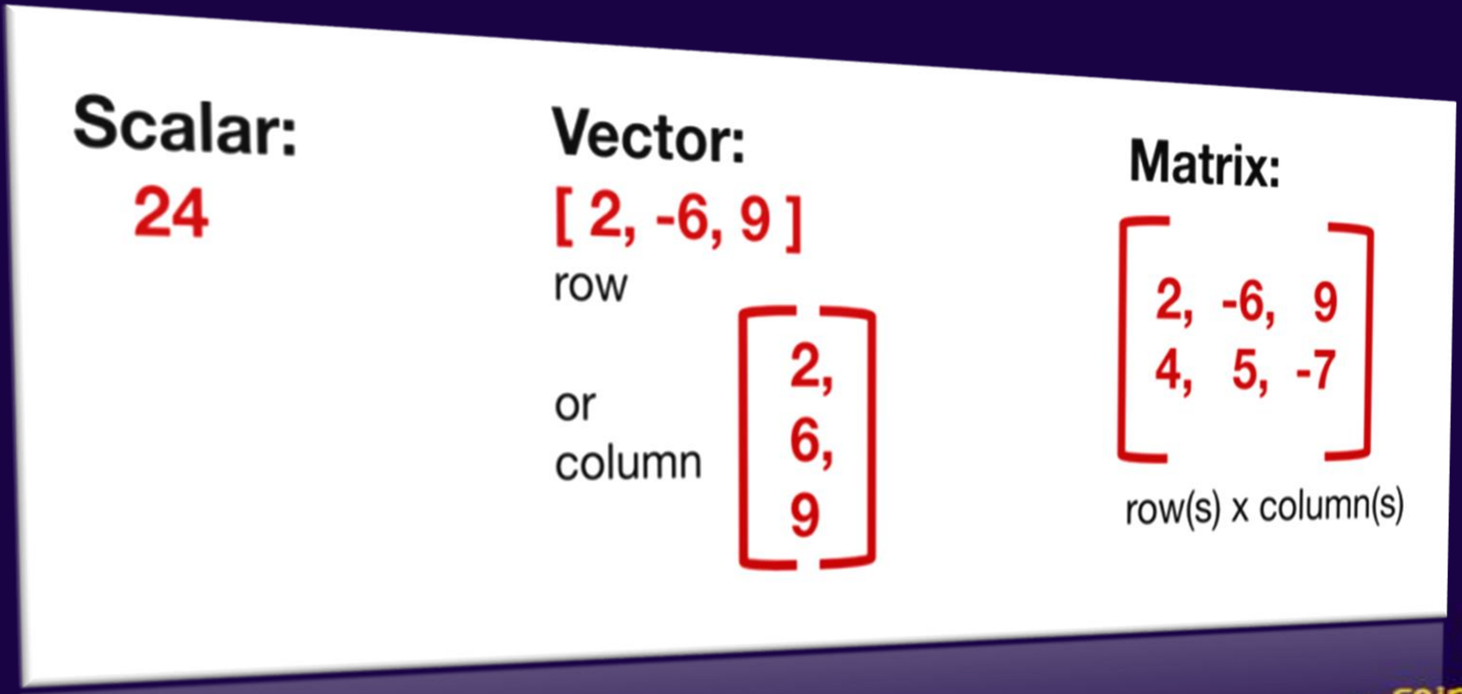
- Definition of Vectors
- Sum/Subtraction of Vectors
- Magnitude of a Vector
- Multiply and Dot product
- Distance Between Vectors

What is Linear Algebra

Linear algebra is the branch of mathematics that deals with vector spaces.



Scalar:

24

Vector:

[2, -6, 9]

row

or

column $\begin{bmatrix} 2, \\ 6, \\ 9 \end{bmatrix}$

Matrix:

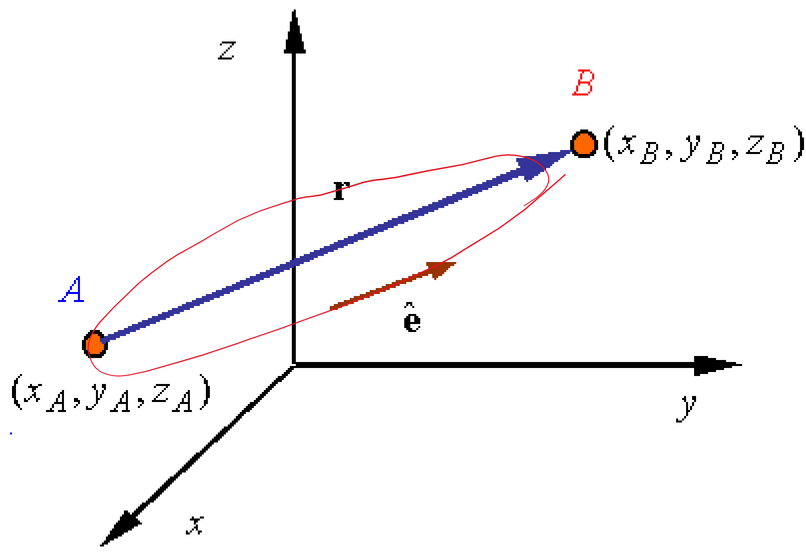
$\begin{bmatrix} 2, & -6, & 9 \\ 4, & 5, & -7 \end{bmatrix}$

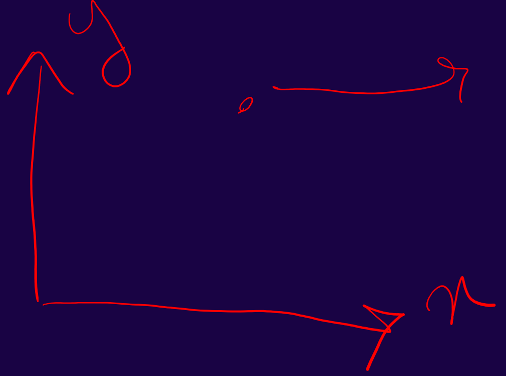
row(s) x column(s)

Definition of Vectors

- A vector is an ordered array of numbers and can be in a row or a column.

- For example, if you have the heights, weights, and ages of large number of people, you can treat your data as three-dimensional vectors [height, weight, age]





$$\longrightarrow \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\to$ weight

$\to$ height

$\to$ age

$$\longrightarrow \underbrace{[\text{weight}, \text{height}, \text{age}]}$$

Componentwise Addition

$$\mathbf{A} = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$$

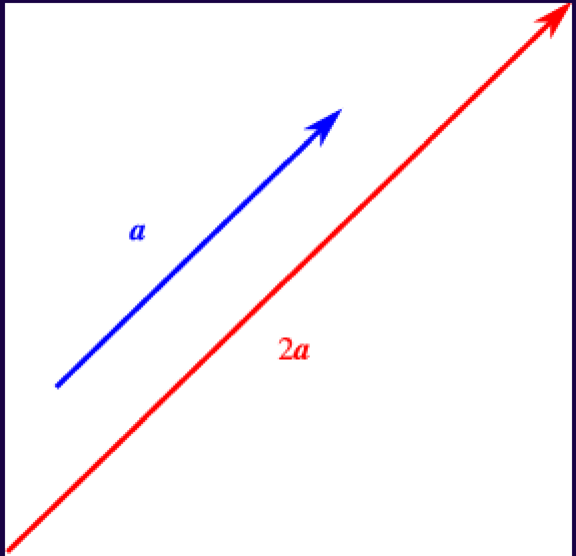
$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} A_x + B_x \\ A_y + B_y \\ A_z + B_z \end{bmatrix}$$

Subtract Two Vectors

$$\begin{bmatrix}5\\7\\9\end{bmatrix} - \begin{bmatrix}4\\5\\6\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$a\begin{bmatrix}x\\y\\z\end{bmatrix} \qquad \begin{bmatrix}a\times x\\a\times y\\a\times z\end{bmatrix}$$

Multiply a Vector by a Scalar



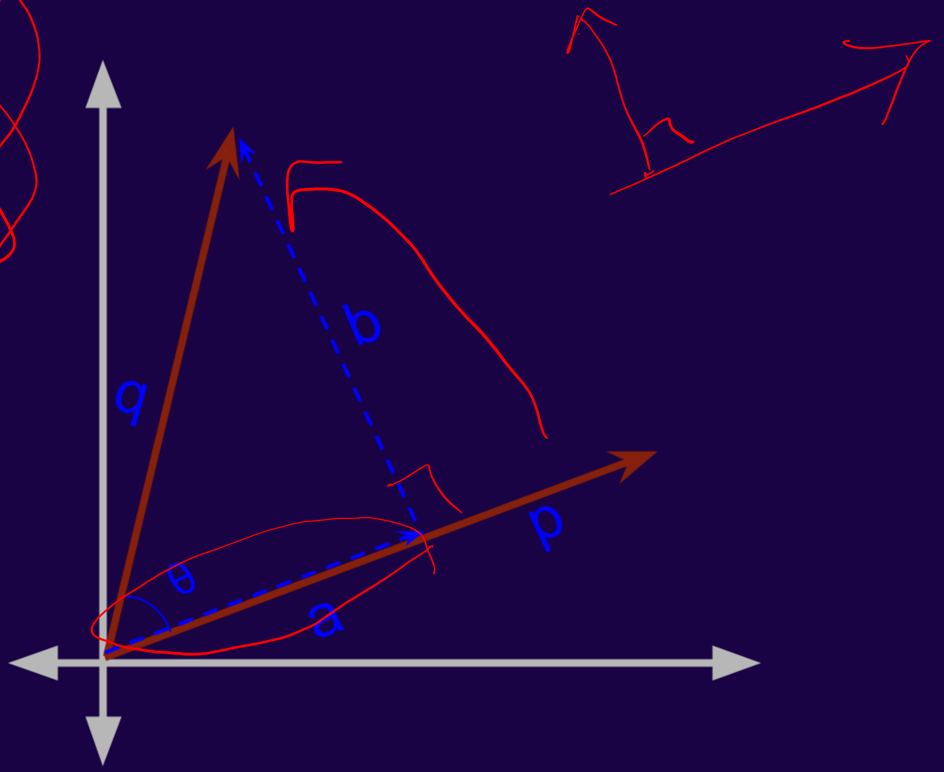
Dot Product

a . b = ۲ × ۱ + ۰ × ۰ + ۵ × ۹ = ۴

a = [۲ ۰ ۵]

۳ ۵

b = [۱ ۹]



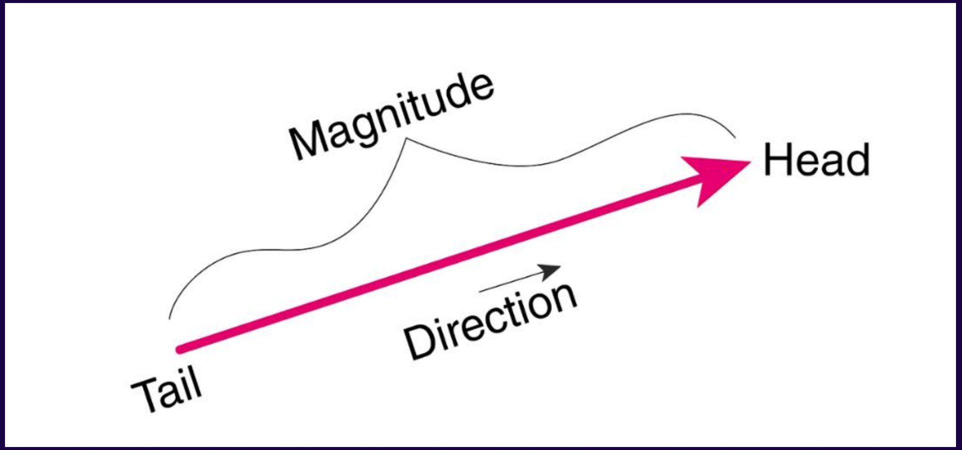
$x = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$\|x\| = \sqrt{a^2+b^2+c^2}$

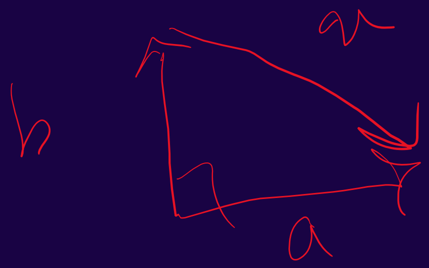
$x = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$\sqrt{1^2+1^2+0^2} = \sqrt{2}$

Magnitude



$x = \begin{bmatrix} a \\ b \end{bmatrix}$



$x^2 = a^2 + b^2$

$\rightarrow x = \sqrt{a^2+b^2}$

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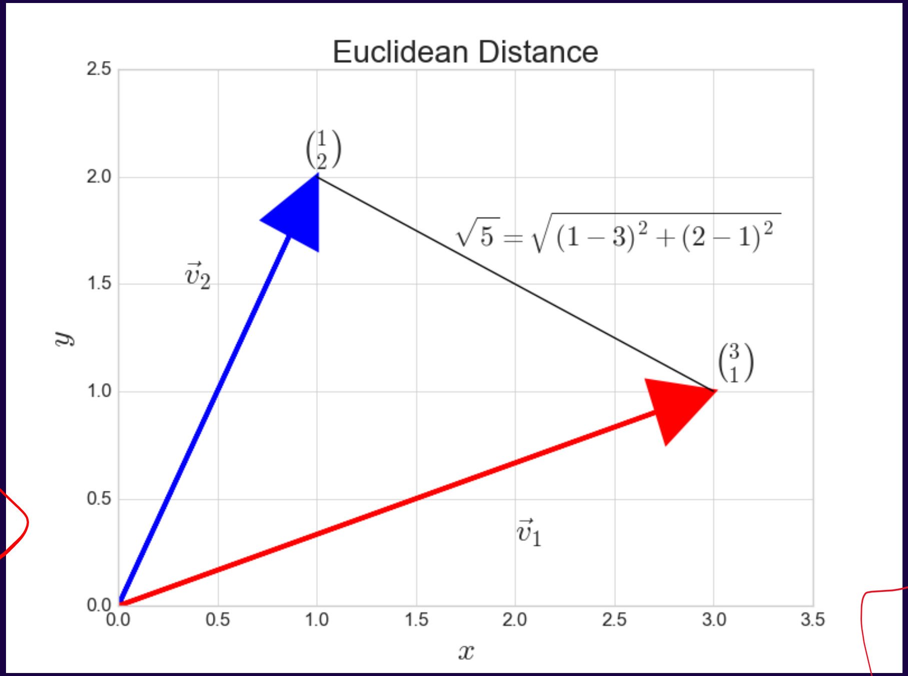
P = [۰/۸ , ۵ , ۰/۹]

γ = [۴ , ۴ , ۱]

S = [۰/۱ , ۵ , ۰/۱]

Distance Between Two Vectors

$$\sqrt{(v_1 - w_1)^2 + \ldots + (v_n - w_n)^2}$$



$S^۲ = [۰/۱^۲ , ۵^۲ , (-۰/۱)^۲]$

X S+S

۰/۱ + 0 + ۰/۱ = ۰/۲ → $\sqrt{۰/۲}$

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Exercise 1:

$w \cdot x$

Suppose the input vector to your linear model is x=(1,2) and the weight vector is w=(0.5,−0.3) Calculate the model's prediction.

Exercise 2:

Consider two vectors u=(1,2,8,-6) and v=(3,4,5,9). Find the sum of these two vectors.

Exercise 3:

Suppose you have two document vectors d1=(1,0,2) and d2=(0,1,2). Calculate the cosine similarity between these two document vectors.