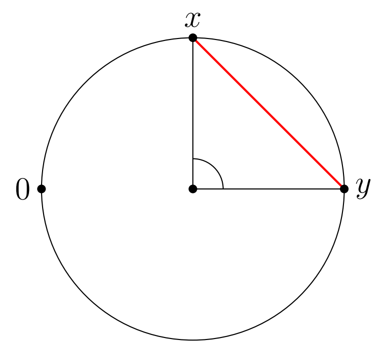
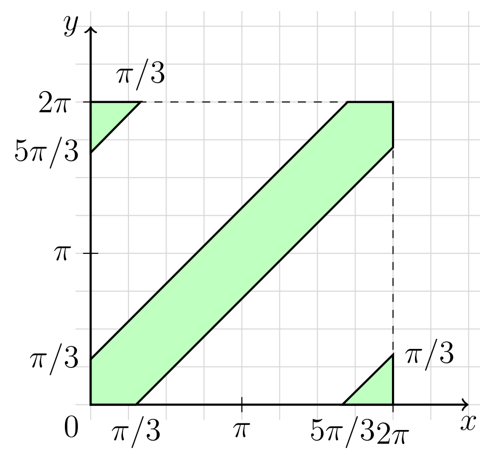
Problem 4.26



$$\Omega = \{(x,y) : x, y \in (0, 2\pi)\}$$



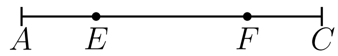
$$A = \{(x,y) \in \Omega : |x-y| < \frac{\pi}{3} \text{ or } (2\pi - y) + x < \frac{\pi}{3} \text{ or } (2\pi - x) + y < \frac{\pi}{3}\}$$

$$\left[\begin{array}{l} \begin{cases} x - y < \frac{\pi}{3} \\ y - x < \frac{\pi}{3} \end{cases} \\ 2\pi - y + x < \frac{\pi}{3} \\ 2\pi - x + y < \frac{\pi}{3} \end{array}\right.$$

$$\mu(A) = 4\pi^2 - \left(\frac{\pi}{3}\right)^2 + \left(\frac{\pi}{3}\right)^2 = 4\pi^2 - \frac{25}{9}\pi^2 + \frac{1}{9}\pi^2 = \frac{4}{3}\pi^2$$

$$P(A) = \frac{1}{3}$$

Problem 4.27

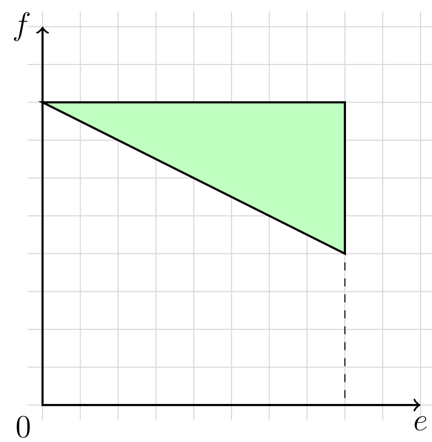


$$\Omega = \left\{(e, f) : e, f \in (0, \sqrt{13})\right\}$$

$$A = \left\{(e, f) : \frac{e^2}{(\sqrt{13} - f)^2} \geq 3\right\}$$

$$f \geq -\frac{1}{\sqrt{3}}e + \sqrt{13}$$

$$e = \sqrt{13} : f = \sqrt{13} - \frac{\sqrt{13}}{\sqrt{3}} = \sqrt{13}\left(1 - \frac{1}{\sqrt{3}}\right)$$



$$\mu(A) = \frac{1}{2}\left(\sqrt{13} \cdot (\sqrt{13} - \sqrt{13}(1 - \frac{1}{\sqrt{3}}))\right) = \frac{13}{2}\left(1 - 1 + \frac{1}{\sqrt{3}}\right) = \frac{13}{2\sqrt{3}}$$

$$\mu(\Omega) = 13$$

$$P(A) = \frac{1}{2\sqrt{3}}$$

Further assume the Ω is set using the following declaration:

$$\Omega = \{(\varepsilon_1, \dots, \varepsilon_j) : \varepsilon_i = \pm 1, \sum \varepsilon_i = k\}$$

Where k is y coordinate of destination point and j is x coordinate of one.

Problem 4.28

(a) (1)

$$(0,0) \to (2n,0)$$

$$S_1 > 0, \dots, S_{2n-1} > 0$$

$$\begin{aligned}
L^{+}_{(1,1)}(2n-1,1) &= L_{(1,1)}(2n-1,1) - L_{(1,-1)}(2n-1,1) = \\
&= L_{(0,0)}(2n-2,0) - L_{(0,0)}(2n-2,2) = C^{n-1}_{2n-2} - C^{n}_{2n-2} = \\
&= \frac{(2n-2)!}{(n-1)!(n-1)!} - \frac{(2n-2)!}{n!(n-2)!} = (2n-2)!\left(\frac{1}{((n-1)!)^2} - \frac{1}{n!(n-2)!}\right) = \\
&= \frac{(2n-2)!}{(n-1)!}\left(\frac{1}{(n-1)!} - \frac{n-1}{n!}\right) = \frac{(2n-2)!}{(n-1)!}\left(\frac{n-n+1}{n!}\right) = \frac{(2n-2)!}{(n-1)!n!}
\end{aligned}$$

(2)

$$(0,0) \to (2n,0)$$

$$S_1 \geq 0, \dots, S_{2n-1} \geq 0$$

$$\begin{aligned}
L^{\geq 0}_{(1,1)}(2n,0) = L_{(1,1)}(2n,0) - L_{(1,-3)}(2n,0) = L_{(0,0)}(2n-1,-1) - L_{(0,0)}(2n-1,3) = \\
C^{n}_{2n-1} - C^{n-2}_{2n-1} = (2n-1)!\left(\frac{1}{n!(n-1)!} - \frac{1}{(n-2)!(n+1)!}\right) = \\
= (2n-1)!\frac{n+1-n+1}{(n+1)!(n-1)!} = \frac{2(2n-1)!}{(n+1)!(n-1)!}
\end{aligned}$$