Numerical analysis laboratory work №6

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Варіант завдання (4)

Рівняння $y' = \left(1 - x^2\right) y + F(x)$

$h = 0.1$

Точний розв'язок $y = x \cos x$

Визначаємо $F(x)$:

$$\cos x - x \sin x = \left(1 - x^2\right) x \cos x + F(x)$$
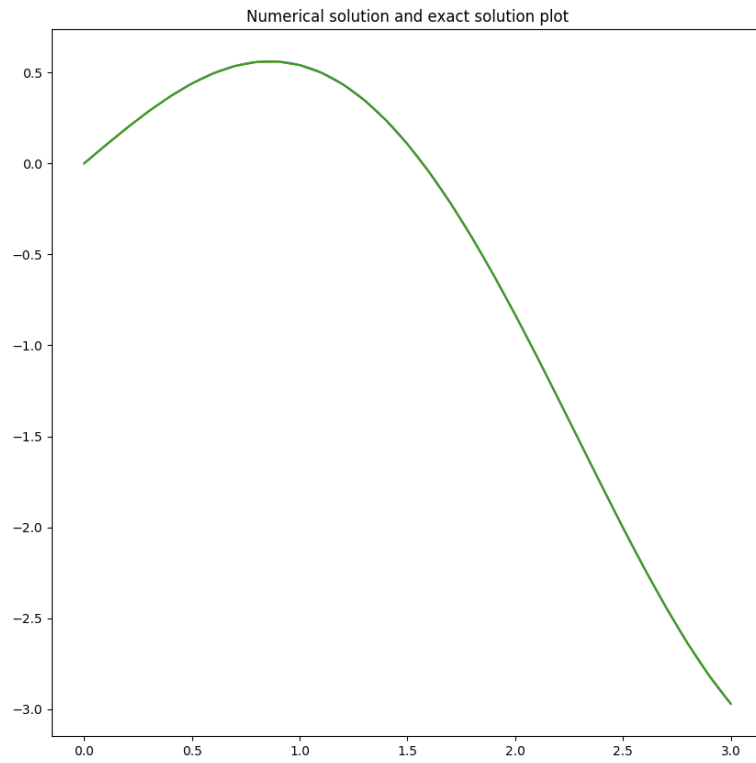$$F(x) = \left(x^3 - x + 1\right) \cos x - x \sin x$$

Умови:

$$\begin{cases} \dfrac{\partial y}{\partial x} = (1 - x^2)y + (x^3 - x + 1)\cos x - x \sin x \\ y(0) = 0 \end{cases} \tag{1}$$

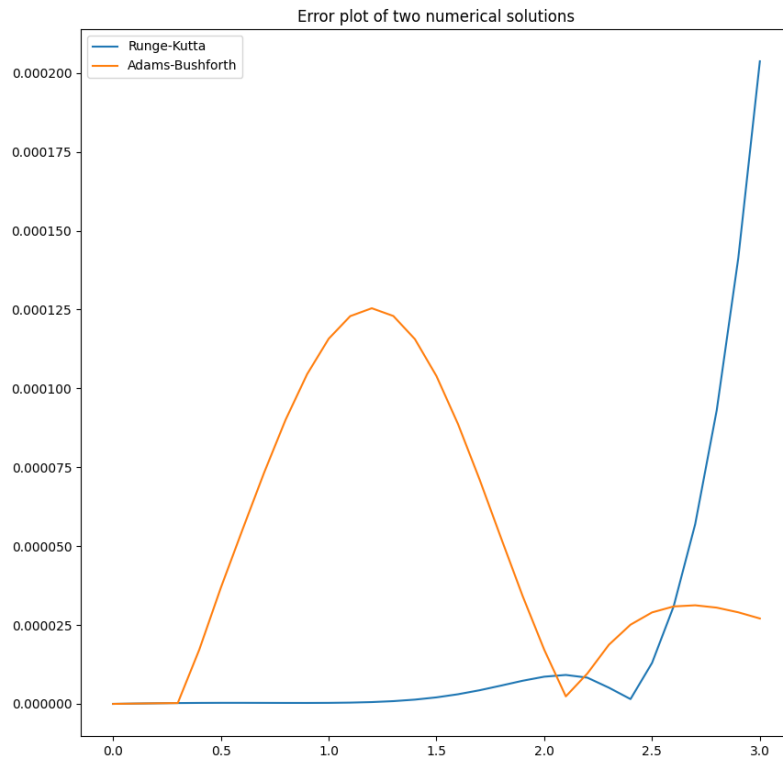## Program output

```
Function values
      x       f(x)   Runge-Kutta   Adams-Bushforth
0   0.0  0.000000      0.000000          0.000000
1   0.1  0.099500      0.099500          0.099500
2   0.2  0.196013      0.196013          0.196013
3   0.3  0.286601      0.286601          0.286601
4   0.4  0.368424      0.368424          0.368407
5   0.5  0.438791      0.438791          0.438754
6   0.6  0.495201      0.495201          0.495146
7   0.7  0.535390      0.535389          0.535316
8   0.8  0.557365      0.557365          0.557275
9   0.9  0.559449      0.559449          0.559344
10  1.0  0.540302      0.540302          0.540187
11  1.1  0.498956      0.498955          0.498833
```

```
12  1.2  0.434829     0.434829      0.434704
13  1.3  0.347748     0.347748      0.347626
14  1.4  0.237954     0.237953      0.237838
15  1.5  0.106106     0.106104      0.106002
16  1.6 -0.046719    -0.046722     -0.046808
17  1.7 -0.219036    -0.219040     -0.219107
18  1.8 -0.408964    -0.408970     -0.409016
19  1.9 -0.614250    -0.614258     -0.614284
20  2.0 -0.832294    -0.832302     -0.832311
21  2.1 -1.060177    -1.060186     -1.060179
22  2.2 -1.294702    -1.294711     -1.294693
23  2.3 -1.532435    -1.532440     -1.532416
24  2.4 -1.769745    -1.769743     -1.769720
25  2.5 -2.002859    -2.002846     -2.002830
26  2.6 -2.227911    -2.227880     -2.227880
27  2.7 -2.440995    -2.440938     -2.440964
28  2.8 -2.638223    -2.638129     -2.638192
29  2.9 -2.815779    -2.815637     -2.815750
```

Numerical solution and exact solution plot

Error plot of two numerical solutions

Code

```python
import numpy as np
import pandas
import matplotlib.pyplot as plt


def solve_runge_kutta(f, x, y, h, n):
    y_arr = [y]
    x_arr = [x]
    for _ in range(0, n):
        k_1 = h*f(x, y)
        k_2 = h*f(x + 1/2*h, y + 1/2*k_1)
        k_3 = h*f(x + 1/2*h, y + 1/2*k_2)
        k_4 = h*f(x+h, y + k_3)
```

```python
            y = y + 1/6*(k_1 + 2*k_2 + 2*k_3 + k_4)
            x = x + h
            y_arr.append(y)
            x_arr.append(x)
        return x_arr, y_arr


def solve_adams_bushforth(f, x_0, y_0, h, n):
    x, y = solve_runge_kutta(f, x_0, y_0, h, 3)
    for i in range(3, n):
        y.append(
            y[i] + h*(55*f(x[i], y[i]) - 59*f(x[i-1], y[i-1]) +
                37*f(x[i-2], y[i-2]) - 9*f(x[i-3], y[i-3]))/24
        )
        x.append(x[i] + h)
    return x, y


def main():
    def f(x, y):
        return (1 - x**2) * y + (x**3 - x + 1)*np.cos(x) - x * np.sin(x)
    h = 0.1
    n = 30
    exact = lambda x: x * np.cos(x)

    x, y1 = solve_runge_kutta(f, 0, 0, h, n)
    x, y2 = solve_adams_bushforth(f, 0, 0, h, n)
    f_x = list(map(exact, x))

    print("Function values")
    table = [
        [x[i], f_x[i], y1[i], y2[i]] for i in range(0, n)]
    print(pandas.DataFrame(table, columns=['x', 'f(x)', 'Runge-Kutta',
                          'Adams-Bushforth']), '\n')

    err1 = [np.absolute(y1[i] - f_x[i]) for i in range(0, n+1)]
    err2 = [np.absolute(y2[i] - f_x[i]) for i in range(0, n+1)]

    fig, ax = plt.subplots(figsize=(12, 12))
    xx = [0.01 * k for k in range(1, 10*n)]
    ax.plot(xx, list(map(exact, xx)))
    ax.plot(x, y1)
    ax.plot(x, y2)
    ax.set_title("Numerical solution and exact solution plot")
    fig.savefig('solution.png')
```

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```python
    fig, ax = plt.subplots(figsize=(12, 12))
    ax.plot(x, err1, label="Runge-Kutta")
    ax.plot(x, err2, label="Adams-Bushforth")
    ax.set_title("Error plot of two numerical solutions")
    ax.legend()
    fig.savefig('errors.png')


if __name__ == "__main__":
    main()
```