Spaces

$$X = \{x_1, \dots, x_n\} \qquad Y = \{y_1, \dots, y_m\}$$

Functions

$$a : X \to \mathbb{R} \qquad b : Y \to \mathbb{R}$$

$$a(x_i) = a_i \qquad b(y_i) = b_i$$

$$\Rightarrow a \equiv [a_1, \dots, a_n] \qquad b \equiv [b_1, \dots, b_m]$$

$$K : X \times Y \to \mathbb{R} \qquad K(x_i, y_j) = K_{ij}$$

$$\Rightarrow K \equiv \begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1m} \\ \vdots & & & \\ K_{n1} & \cdots & \cdots & K_{nm} \end{bmatrix}$$

What should $K \times a$ or $K \times b$ mean?

~~If we can~~

Note: $a : X \to \mathbb{R}$ can be considered as function on $X \times Y$ as well:

$$a(x_i, y_j) = a_i \quad \left(\text{for all } y_j \;\; a(x_i, y_j) = a(x_i) = a_i \right.$$

$$\text{So} \quad a \equiv \begin{bmatrix} a_1 & a_1 & \cdots & a_1 \\ a_2 & a_2 & \cdots & a_2 \\ \vdots & & & \\ a_n & a_n & \cdots & a_n \end{bmatrix}$$

Same for $b: Y \to \mathbb{R}$ can be considered as function on $X \times Y$ as follows

$$b(x_i, y_j) = b(y_j) = b_j \quad (\text{for all } x_i\text{'s})$$

Thus
$$b \equiv \begin{bmatrix} b_1 & b_2 & \cdots & b_m \\ b_1 & b_2 & \cdots & b_m \\ \vdots & & & \\ b_1 & b_2 & \cdots & b_m \end{bmatrix}$$

Moreover, what does it mean if we have two functions

$C: X \times Y \to \mathbb{R}$ & $D: X \times Y \to \mathbb{R}$

& we want to compute $C \times D$?

Since ~~$C \times D$ is~~ $C \equiv [C_{ij}]_{n \times m}$ & $D = [D_{ij}]_{n \times m}$

& Since $C \times D(x_i, y_j) = C(x_i, y_j) D(x_i, y_j)$

$$= C_{ij} D_{ij}$$

Thus
$$C \times D = \begin{bmatrix} C_{11} D_{11} & C_{12} D_{12} & \cdots & C_{1m} D_{1m} \\ \vdots & & & \\ C_{n1} D_{n1} & C_{n2} D_{n2} & \cdots & C_{nm} D_{nm} \end{bmatrix}$$

it means product component by component

So, In the case

$$(*) \quad \boxed{a \equiv \frac{1}{\int K b \, dP_2}} \quad \text{we have}$$

$$K * b = \begin{bmatrix} K_{ij} \end{bmatrix} \begin{bmatrix} b_1 & b_2 & \cdots & b_m \\ \vdots & & & \\ b_1 & b_2 & \cdots & b_m \end{bmatrix}$$

same as $C * D$ $\overset{\curvearrowleft}{=}$

$$\begin{bmatrix} K_{11}b_1 & K_{12}b_2 & \cdots & K_{1m}b_m \\ K_{21}b_1 & K_{22}b_2 & \cdots & K_{2m}b_m \\ \vdots & & & \\ K_{n1}b_1 & K_{n2}b_2 & \cdots & K_{nm}b_m \end{bmatrix}$$

(*) In discrete case

$$\int K b \, dP_2 = \underbrace{\begin{bmatrix} K * b \end{bmatrix}}_{\text{on } X \times Y} \begin{bmatrix} ? \end{bmatrix}$$

In fact, Let us assume that $C = [c_{ij}]$ &

P_2 is prob. meas. on Y & let us compute

$$\int c \, dP_2 = ?$$

Note $\int c(x,y) \, dP_2(y) =$ is a function of x

Thus $\quad \|$

$$\begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1m} \\ c_{21} & c_{22} & \cdots & c_{2m} \\ \vdots & & & \\ c_{n1} & c_{n2} & \cdots & c_{nm} \end{bmatrix} \underbrace{\begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_m \end{bmatrix}}_{\text{This is } P_2} = \text{is a vector on } X = \{x_1, \dots, x_n\}$$

So
$$\int K \times b \, dP_2 = \begin{bmatrix} K_{11}b_1 & K_{12}b_2 & \cdots & K_{1m}b_m \\ K_{21}b_1 & K_{22}b_2 & \cdots & K_{2m}b_m \\ \vdots & & & \\ K_{n1}b_1 & K_{n2}b_2 & \cdots & K_{nm}b_m \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_m \end{bmatrix}$$

For 3-Marg. We can do exactly the same but by sum Modification