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Частные производные

Функция n переменных:

$$F : \mathbb{R}^n \to \mathbb{R}$$

$$F(x_1, \ldots, x_n)$$

Частная производная по i-й переменной:

$$\frac{\partial F}{\partial x_i}(x_1, \ldots, x_n) =$$

$$= \lim_{\varepsilon \to 0} \frac{F(x_1, x_2, \ldots, x_i + \varepsilon, \ldots, x_n) - F(x_1, x_2, \ldots, x_i, \ldots, x_n)}{\varepsilon}$$

$$\frac{\partial F}{\partial x_i} : \mathbb{R}^n \to \mathbb{R}$$

Частные производные

$$F(x, y, z, u) = x^3 + y^u + \sin z^2 u^3$$

$$\frac{\partial F}{\partial x} =$$

$$\frac{\partial F}{\partial y} =$$

$$\frac{\partial F}{\partial z} =$$

Частные производные

$$F(x, y, z, u) = x^3 + y^u + \sin z^2 u^3$$

$$\begin{aligned} \frac{\partial F}{\partial x} &= 3x^2 \\ \frac{\partial F}{\partial y} &= \\ \frac{\partial F}{\partial z} &= \end{aligned}$$

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$$\begin{aligned} \frac{\partial F}{\partial x} &= 3x^2 \\ \frac{\partial F}{\partial y} &= uy^u - 1 \\ \frac{\partial F}{\partial z} &= (-\cos z^2 u^3)(u^3 2z) \end{aligned}$$

Производная сложной функции

$$F = F(x_1, \ldots, x_n)$$

$$G_i = G_i(y_1, \ldots, y_m)$$

$$H(y_1, \ldots, y_n) = F(G_1(y_1, \ldots, y_m), \ldots, G_n(y_1, \ldots, y_n))$$

$$\frac{\partial H}{\partial y_i} = \sum_{j=1}^{n} \frac{\partial H}{\partial G_j} \frac{\partial G_j}{\partial y_i}$$

Производная сложной функции

$$F(x_1, \ldots, x_n) = \sum_{k=1} n a_i x_i$$

$$G_i(y_1, \ldots, y_m) = \sum_{k=1} m y_k^i$$

$$H(y_1, \ldots, y_n) = F(G_1(y_1, \ldots, y_m), \ldots, G_n(y_1, \ldots, y_n))$$

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$$\frac{\partial H}{\partial y_j} = \sum_{i=1}^{n} \frac{\partial H}{\partial G_i} \frac{\partial G_i}{\partial y_j} = \sum_{i=1}^{n} a_i i y_j^{i-1}$$

Градиент

$$\nabla F = \left(\frac{\partial F}{\partial x_1}, \frac{\partial F}{\partial x_2}, \ldots, \frac{\partial F}{\partial x_n} \right)$$

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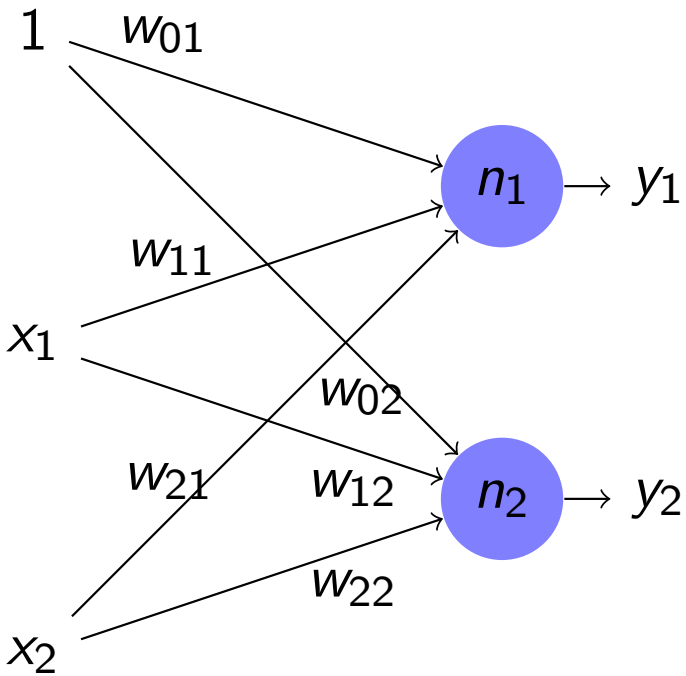
Градиент

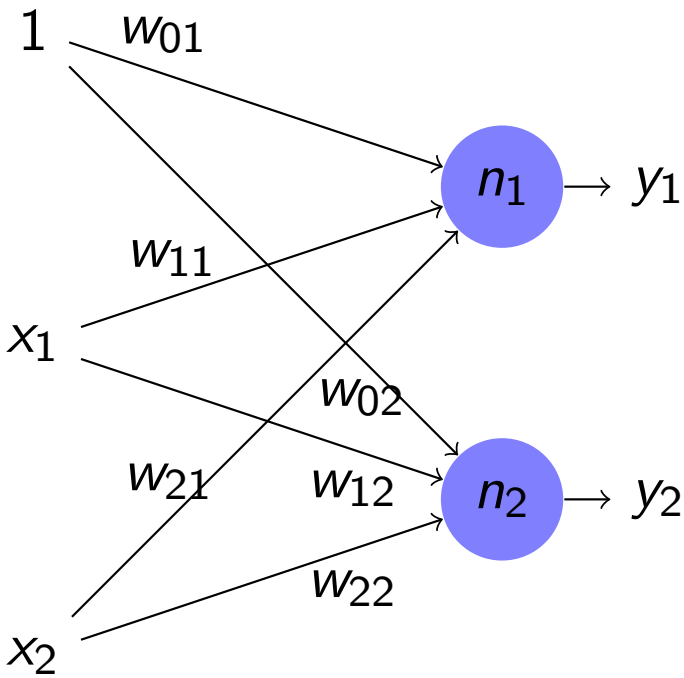
$$\nabla F = \left(\frac{\partial F}{\partial x_1}, \frac{\partial F}{\partial x_2}, \ldots, \frac{\partial F}{\partial x_n} \right)$$

$$\nabla F : \mathbb{R}^n \to \mathbb{R}^n$$

Обратное распространение ошибки



Обратное распространение ошибки



$$E(y_1, y_2) = (y_1 - a_1)^2 + (y_2 - a_2)^2$$

$$\frac{\partial E}{\partial y_1} = 2(y_1 - a_1)$$

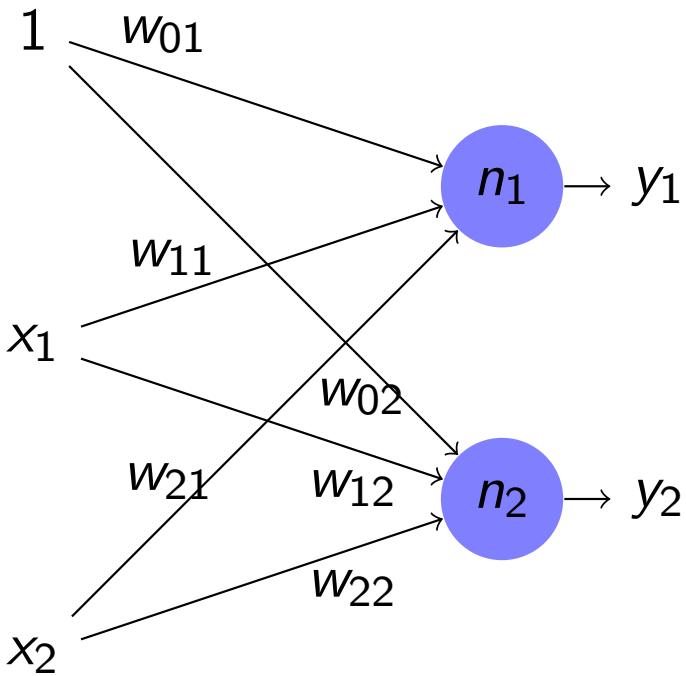
$$y_1 = f\underbrace{(w_{01} + x_1 w_{11} + x_2 w_{21})}_{S_1}$$

$$\frac{\partial y_1}{\partial w_{21}} = f'(S_1)x_2$$

$$\frac{\partial E}{\partial w_{21}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial w_{21}}$$

$$= 2(y_1 - a_1)f'(S_1)x_2$$

Обратное распространение ошибки



$$E(y_1, ..., y_n) = (y_i - a_i)^2 + ... + (y_n - a_n)^2$$

$$\frac{\partial E}{\partial y_i} = 2(y_i - a_i)$$

$$y_i = f\underbrace{(w_{0i} + x_1 w_{1i} + ... + x_n w_{ni})}_{S_i}$$

$$\frac{\partial y_i}{\partial w_{ji}} = f'(S_i)x_j$$

$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial E}{\partial y_i}\frac{\partial y_i}{\partial w_{ji}}$$

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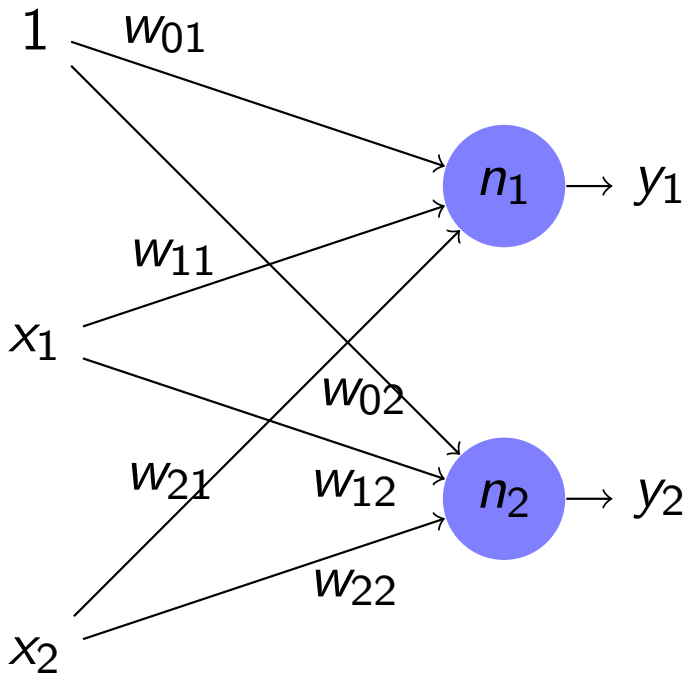


$$E(y_1, y_2) = (y_1 - a_1)^2 + (y_2 - a_2)^2$$

$$\frac{\partial E}{\partial y_1} = 2(y_1 - a_1)$$

$$y_1 = f\underbrace{(w_{01} + x_1 w_{11} + x_2 w_{21})}_{S_1}$$

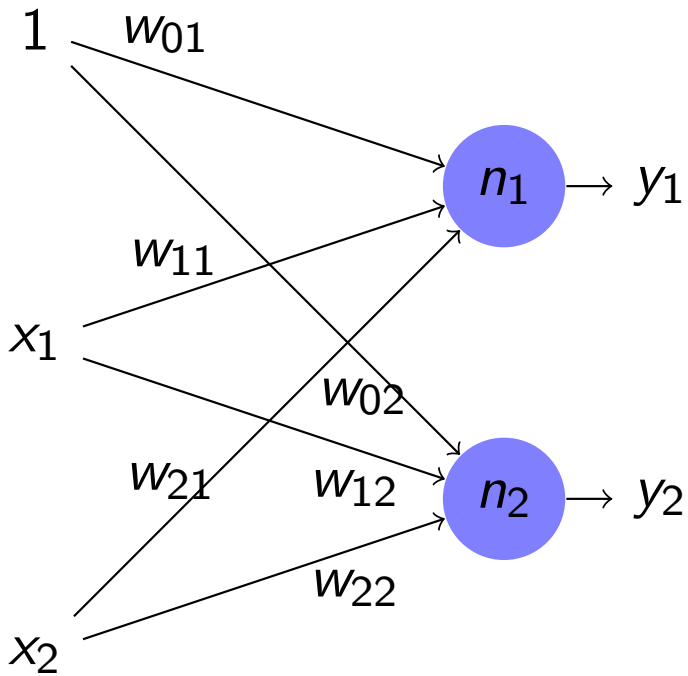
$$\frac{\partial y_1}{\partial x_2} = f'(S_1) w_{12}$$

$$\frac{\partial y_2}{\partial x_2} = f'(S_2) w_{22}$$

$$\frac{\partial E}{\partial x_2} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial x_2} + \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial x_2} =$$

$$= 2(y_1 - a_1) f'(S_1) w_{12} + 2(y_2 - a_2) f'(S_2) w_{22}$$

Обратное распространение ошибки



$$E(y_1, ..., y_n) = (y_i - a_i)^2 + ... + (y_n - a_n)^2$$

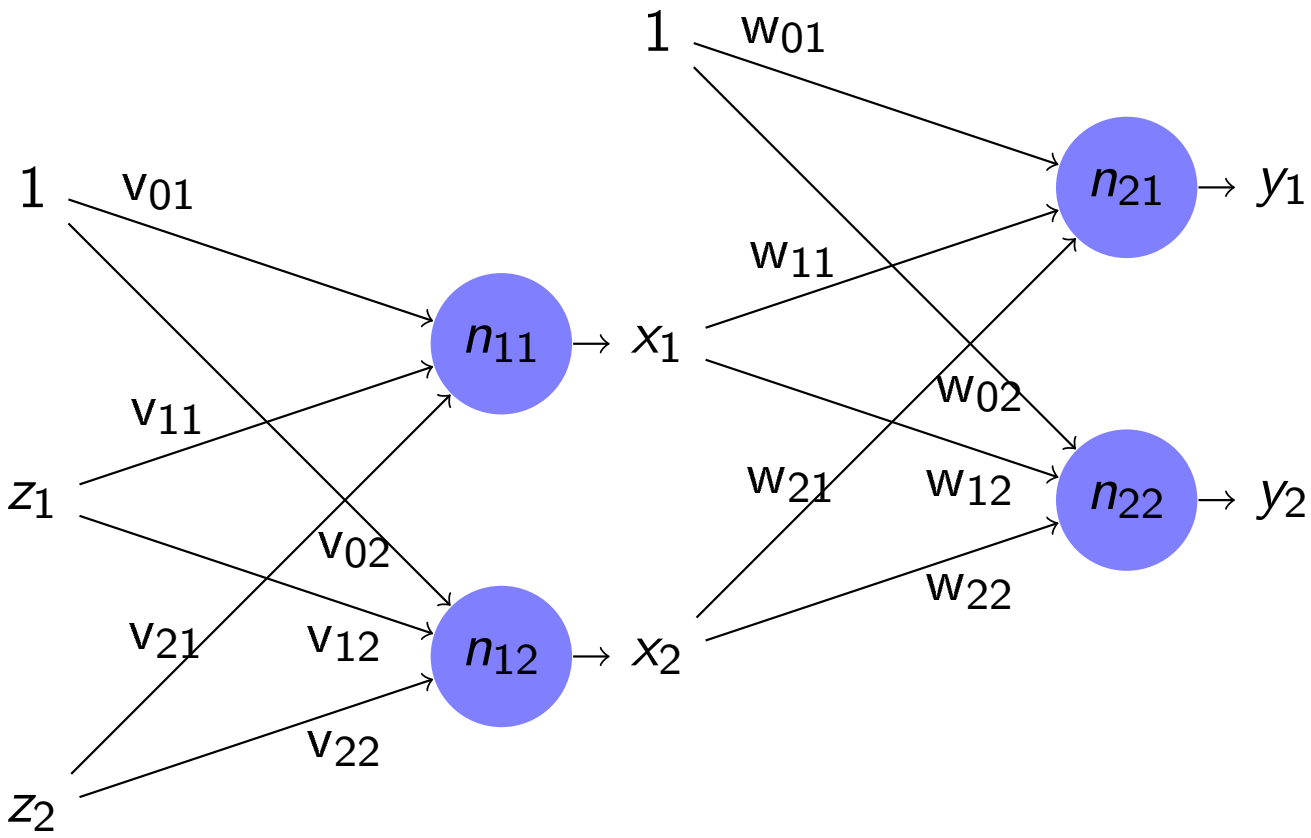
$$\frac{\partial E}{\partial y_i} = 2(y_i - a_i)$$

$$y_i = f\underbrace{\left(w_{0i} + x_1 w_{1i} + ... + x_n w_{ni}\right)}_{S_i}$$

$$\frac{\partial y_i}{\partial x_j} = f'(S_i) w_{ji}$$

$$\frac{\partial E}{\partial x_j} = \sum_{i=1}^{n} \frac{\partial E}{\partial y_i} \frac{\partial y_i}{\partial x_j} = \sum_{i=1}^{n} 2(y_i - a_i) f'(S_i) w_{ji}$$

Обратное распространение ошибки



$$\frac{\partial E}{\partial v_{ij}} = \frac{\partial E}{\partial x_j} \frac{\partial x_j}{\partial v_{ij}}$$

$$\frac{\partial E}{\partial z_i} = \sum_{j=1}^{m} \frac{\partial E}{\partial x_j} \frac{\partial x_j}{\partial z_i}$$