

HIR-Diff: Unsupervised Hyperspectral Image Restoration Via Improved Diffusion Models

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Abstract

Hyperspectral image (HSI) restoration aims at recovering clean images from degraded observations and plays a vital role in downstream tasks. Existing model-based methods have limitations in accurately modeling the complex image characteristics with handcraft priors, and deep learning-based methods suffer from poor generalization ability. To alleviate these issues, this paper proposes an unsupervised HSI restoration framework with pre-trained diffusion model (HIR-Diff), which restores the clean HSIs from the product of two low-rank components, i.e., the reduced image and the coefficient matrix. Specifically, the reduced image, which has a low spectral dimension, lies in the image field and can be inferred from our improved diffusion model where a new guidance function with total variation (TV) prior is designed to ensure that the reduced image can be well sampled. The coefficient matrix can be effectively pre-estimated based on singular value decomposition (SVD) and rank-revealing QR (RRQR) factorization. Furthermore, a novel exponential noise schedule is proposed to accelerate the restoration process (about 5× acceleration for denoising) with little performance decrease. Extensive experimental results validate the superiority of our method in both performance and speed on a variety of HSI restoration tasks, including HSI denoising, noisy HSI super-resolution, and noisy HSI inpainting. The code is available at <https://github.com/LiPang/HIRDifff>.

1. Introduction

Hyperspectral imaging is an advanced imaging technique that captures and processes a large number of narrow, contiguous spectral bands across the electromagnetic spectrum. Thus Hyperspectral images (HSIs) can provide more abundant spatial and spectral information than natural images and have extensive applications in various fields, e.g., agriculture [7, 19], mineral exploration [26, 41] and environmental monitoring [22]. However, due to sensor limitations and atmospheric interference, HSIs suffer various degeneration during the acquisition process, significantly impairing the performance of downstream tasks. Therefore, HSI restoration is very important for HSI applications and numerous methods have emerged in the past decades.

Existing HSI restoration methods can be mainly categorized into two classes, i.e. model-based approaches [50, 52, 53] and deep learning (DL)-based approaches [2, 11, 34, 37]. Model-based methods reformulate the image inverse problem as an optimization problem which contains data fidelity term and regularization term. The data fidelity term ensures that the restored image is close to the observed image and the regularization term constrains the solving space by exploiting the prior knowledge of the clean images, e.g. low-rank and total variation property. This type of method obtains good generalization ability but the handcraft priors are always subjective and thus cannot fully reveal image characteristics, leaving a large room for improvement. Additionally, the optimization problem could be very complex, resulting in high time costs and suboptimal solutions.

In the past decade, extensive DL-based approaches have been proposed for HSI restoration and these methods can learn the image structure and details from a substantial amount of clean and degraded image pairs. Despite the promising performance, the DL-based methods suffer from a poor generalization problem. Besides, training deep neural networks (DNNs) requires a lot of image pairs, but HSIs are very precious and only limited data are available in most cases. Recently, extensive diffusion-based models have emerged for image restoration tasks such as denoising [18], super-resolution [5, 18, 40, 49] and inpainting [18, 28, 39, 45, 49]. Among these methods, DDRM [18] solves linear inverse problems by performing singular value

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decomposition (SVD) of the linear degradation matrix and performing diffusion in the spectral space with a pre-trained diffusion model. However, the method cannot be applied directly for HSI restoration since HSIs have more bands than natural images, and the number of bands varies in different datasets due to different sensors. After that, DDS2M [32] presents a self-supervised diffusion model for HSI restoration, which restores HSI only using the degraded HSI and can be adaptive to different HSI datasets. Although demonstrating desirable HSI restoration performance and superior generalization ability, DDS2M fails to utilize prior knowledge of available datasets and takes a lot of time to complete the self-supervised training process for each dataset.

To alleviate these issues, inspired by [38] which employs pre-trained diffusion models for pansharpening, we propose an unsupervised HSI restoration framework with pre-trained diffusion model (HIR-Diff). We start from the point that HSIs can be restored from the product of two low-rank components, i.e. the reduced image and the coefficient matrix, which can be estimated separately. Concretely, the reduced image which has a low spectral dimension is defined as several linearly independent bands from the HSI and can be restored using our improved diffusion model, where we propose a new guidance function and a novel exponential noise schedule. The guidance function consisting of a data fidelity term and a total variation (TV) regularization ensures that the reduced image can be better estimated during the diffusion sampling process. The exponential noise schedule can accelerate the diffusion process, that is, it can enable the reduced image to be restored within 20 sampling steps ($5\times$ acceleration for denoising) with little performance decrease. The coefficient matrix, which is needed for the restoration of the reduced image, can be pre-estimated from the degraded image and a predefined band index of the reduced image without any additional information. In this work, we resort to singular value decomposition (SVD) to estimate the coefficient matrix since SVD is noise-robust. Besides, the predefined band index of the reduced image needs to be carefully designed so that each band in the reduced image contains different contents and the estimated coefficient matrix is robust to perturbations. To this end, the ranking-revealing QR (RRQR) factorization [12] which can identify the numerical rank of a matrix, is adopted to determine the band selection index.

In summary, our contributions are as follows.

- We propose an unsupervised HSI restoration framework with an improved diffusion model (HIR-Diff), which recovers the clean HSIs from the product of the reduced image and the coefficient matrix. Additionally, a new guidance function with TV prior is designed in the reverse sampling process of the diffusion model to ensure the reduced image can be well sampled.
- We propose an efficient and noise-robust method to es-

timate the coefficient matrix utilizing SVD and RRQR factorization. Specifically, the RRQR is adopted to pre-define the band index of the reduced image so that the estimated coefficient matrix is robust to perturbations.

- We propose a novel exponential noise schedule, which can significantly accelerate the diffusion process compared with existing noise schedules (e.g., $5\times$ acceleration for denoising) with little performance decrease.

2. Preliminaries

2.1. Diffusion Models

Diffusion models are probabilistic generative models that capture the underlying dynamics of data evolution over time [14, 44]. Owing to the powerful generation ability, diffusion models have gained considerable attention in various tasks, e.g., audio and text generation [16, 20]. A typical diffusion process contains a T -step forward process and a T -step reverse process. The forward process starts with a clean image and progressively adds noise over T steps, while the reverse process operates in the opposite direction. Given a noise scheduler $\{\bar{\alpha}_t\}_{t=1}^T$ and a clean data x_0 , adding Gaussian noise to x_0 iteratively over t iterations yields

$$\mathbf{x}_t = \sqrt{\bar{\alpha}_t} \mathbf{x}_0 + (\sqrt{1 - \bar{\alpha}_t}) \epsilon, \epsilon \sim \mathcal{N}(0, \mathbf{I}). \quad (1)$$

A diffusion model ϵ_θ is trained to predict the noise as

$$\epsilon_\theta(\mathbf{x}_t, t) \approx \epsilon = \frac{\mathbf{x}_t - \sqrt{\bar{\alpha}_t} \mathbf{x}_0}{\sqrt{1 - \bar{\alpha}_t}}. \quad (2)$$

The reverse process starts with random noise and progressively refines the sample over T steps. Recently numerous approaches are proposed to accelerate the sampling process of diffusion models. A typical method is the denoising diffusion implicit model (DDIM) [43], where the forward process is modeled as non-Markovian. The model predicts the starting point as

$$\hat{\mathbf{x}}_0 = \frac{\mathbf{x}_t - (\sqrt{1 - \bar{\alpha}_t}) \epsilon_\theta(\mathbf{x}_t, t)}{\sqrt{\bar{\alpha}_t}}, \quad (3)$$

and then \mathbf{x}_{t-1} is sampled as

$$\mathbf{x}_{t-1} = \sqrt{\bar{\alpha}_{t-1}} \hat{\mathbf{x}}_0 + (\sqrt{1 - \bar{\alpha}_{t-1}}) \epsilon_\theta. \quad (4)$$

We adopt DDIM as the sampling method in this work since this model can accelerate sampling without significant performance degradation.

2.2. Hyperspectral Image Restoration

HSI restoration task aims to recover clean images from degraded observations and its degradation process is

$$\mathcal{Y} = \mathbf{H}(\mathcal{X}) + \mathcal{Z}, \quad (5)$$

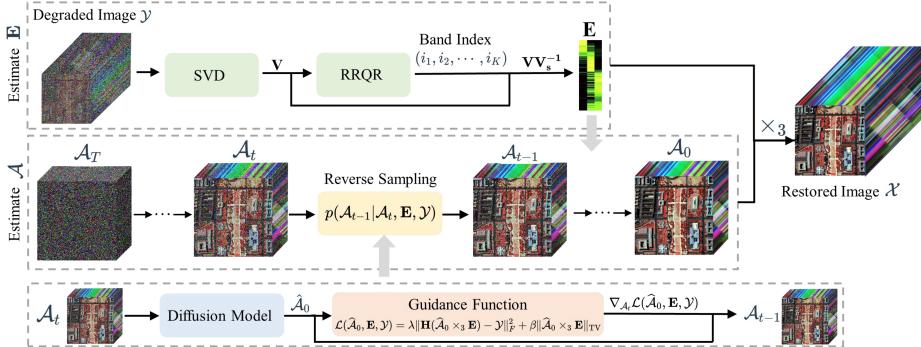


Figure 1. The overall framework of the proposed HIR-Diff. First, the coefficient matrix \mathbf{E} is estimated from the degraded image using SVD and RRQR. Then, taking the degraded image and the estimated matrix \mathbf{E} as conditions, the reduced image \mathcal{A} is reconstructed with an improved pre-trained diffusion model that contains a newly designed guidance function. Finally, the clean image is restored from the product of the estimated \mathbf{E} and \mathcal{A} .

where \mathcal{Y} is the degraded image, \mathcal{X} is the clean HSI, \mathbf{H} is a known degradation operation, and \mathcal{Z} denotes an additive Gaussian noise. For the denoising task, \mathbf{H} is an identity operation. For the super-resolution task, \mathbf{H} contains a blur operation and a down-sampling operation. For the inpainting task, \mathbf{H} denotes a random mask.

3. Proposed Model

3.1. Notations

The scalar, vector, matrix and tensor are defined as $x, \mathbf{x}, \mathbf{X}$ and \mathcal{X} , respectively. Given a third-order tensor $\mathcal{X} \in \mathbb{R}^{H \times W \times K}$ and a matrix $\mathbf{E} \in \mathbb{R}^{B \times K}$, mode-3 multiplication can be defined as $\mathcal{Y} = \mathcal{X} \times_3 \mathbf{E}$, resulting in a new tensor $\mathcal{Y} \in \mathbb{R}^{H \times W \times B}$. $\mathbf{X}_{(3)} \in \mathbb{R}^{B \times HW}$ denotes the mode-3 unfolding of \mathcal{X} and is obtained by flattening the first two dimensions of \mathcal{X} . And $\text{fold}_3(\mathbf{X}_{(3)})$ means reshaping $\mathbf{X}_{(3)}$ back to \mathcal{X} .

3.2. Overall Framework

Due to strong spectral correlation, HSIs thus exhibit low-rank property, which has been widely used for HSI restoration [3, 13, 21]. Following this research line, we assume that the clean HSI $\mathcal{X} \in \mathbb{R}^{H \times W \times B}$ can be recovered as

$$\mathcal{X} = \mathcal{A} \times_3 \mathbf{E}, \quad (6)$$

where $\mathcal{A} \in \mathbb{R}^{H \times W \times K}$ denotes the reduced image, $\mathbf{E} \in \mathbb{R}^{B \times K}$ denotes the coefficient matrix and $K \ll B$. Therefore, to restore the clean HSI, we first need to estimate the reduced image \mathcal{A} and the coefficient matrix \mathbf{E} . In this work, we propose a method to separately estimate \mathcal{A} and \mathbf{E} . The overall framework is illustrated in Fig. 1.

Overall, given a degraded HSI \mathcal{Y} , we first estimate the coefficient matrix \mathbf{E} , which is a necessary condition for the restoration of the reduced image \mathcal{A} . Then, we estimate the

reduced image \mathcal{A} using our improved diffusion model. \mathcal{A} is supposed to be a sequence of bands selected from the HSI so that the distribution is consistent with the diffusion priors. In other words, defining (i_1, i_2, \dots, i_K) as the band index, \mathcal{A} satisfies $\mathbf{A}_{(3)} = [\mathbf{x}_{i_1}^T, \mathbf{x}_{i_2}^T, \dots, \mathbf{x}_{i_K}^T]^T$, where \mathbf{x}_{i_n} denotes the i_n th row of $\mathbf{X}_{(3)}$. Given a predefined band index (i_1, i_2, \dots, i_K) of the reduced image and a degraded image \mathcal{Y} , the coefficient matrix \mathbf{E} can be estimated by singular value decomposition (SVD) without any additional information. Once \mathbf{E} is well estimated, \mathcal{A} could be inferred by applying our improved diffusion model with our newly designed guidance function. Moreover, an exponential noise schedule is also proposed to accelerate the sampling.

Although a restored image can be obtained for each possible predefined band selection index, the band index needs to be carefully designed to ensure that each band in \mathcal{A} can encode different contents, which helps to generate a more robust estimation of \mathbf{E} and thus obtain better restoration results. To this end, we resort to a classical rank revealing QR (RRQR) decomposition [12] to determine the band index. Next, we provide a more detailed description for estimating the coefficient matrix \mathbf{E} and the reduced image \mathcal{A} .

3.3. Coefficient Matrix Estimation

3.3.1 Coefficient Matrix Estimation Using SVD

As discussed before, the coefficient matrix \mathbf{E} needs to be pre-estimated from an observed degraded image \mathcal{Y} and a predefined band index (i_1, i_2, \dots, i_K) without any additional information about the reduced image \mathcal{A} and the clean image \mathcal{X} . To this end, inspired by [13], we resort to SVD to estimate the coefficient matrix \mathbf{E} . First, the matrix version of the HSI \mathcal{Y} is decomposed using the rank- K SVD as

$$\mathbf{Y}_{(3)}^T = (\mathbf{U}\mathbf{S})\mathbf{V}^T, \quad (7)$$

where $\mathbf{U} \in \mathbb{R}^{HW \times K}$, $\mathbf{S} \in \mathbb{R}^{K \times K}$ and $\mathbf{V} \in \mathbb{R}^{B \times K}$. We define $\mathbf{V}_s = [\mathbf{v}_{i_1}^T, \mathbf{v}_{i_2}^T, \dots, \mathbf{v}_{i_K}^T]^T$, where \mathbf{v}_{i_n} denotes the i_n th row of \mathbf{V} . Then we can obtain

$$\mathbf{A}_{\mathbf{Y}(3)}^T = (\mathbf{US})\mathbf{V}_s^T, \quad (8)$$

Combining Eq. (7) and Eq. (8), we have

$$\mathbf{Y}_{(3)}^T = \mathbf{A}_{\mathbf{Y}(3)}^T (\mathbf{V}_s^T)^{-1} \mathbf{V}^T, \quad (9)$$

which is equivalent to

$$\mathcal{Y} = \mathcal{A}_{\mathbf{Y}} \times_3 (\mathbf{V}\mathbf{V}_s^{-1}). \quad (10)$$

Then we have the following remark

Remark 3.1 Let $\mathbf{Y}_{(3)}^T = (\mathbf{US})\mathbf{V}^T$ be the rank- K SVD of the observed HSI \mathcal{Y} , and define \mathbf{V}_s as the (i_1, i_2, \dots, i_K) th rows of \mathbf{V} . Then $\mathbf{V}\mathbf{V}_s^{-1}$ is equivalent to the expected \mathbf{E} in Eq. (6) when rank K is small.

Here we provide a rough proof for this remark. Defining $\bar{\mathcal{Y}} = \mathbf{H}(\mathcal{X})$, the degraded image \mathcal{Y} could be regarded as $\bar{\mathcal{Y}}$ with *i.i.d.* Gaussian noise. $\bar{\mathcal{Y}}$ can be decomposed similarly to Eq. (10), namely

$$\bar{\mathcal{Y}} = \bar{\mathcal{A}}_{\mathbf{Y}} \times_3 (\bar{\mathbf{V}}\bar{\mathbf{V}}_s^{-1}), \quad (11)$$

where $\bar{\mathcal{A}}_{\mathbf{Y}}$ is the (i_1, i_2, \dots, i_K) th bands of $\bar{\mathcal{Y}}$, $\bar{\mathbf{V}}$ is the right singular vectors obtained from the SVD of $\bar{\mathcal{Y}}$ and $\bar{\mathbf{V}}_s$ is the (i_1, i_2, \dots, i_K) th rows of $\bar{\mathbf{V}}$. Due to the favourable mathematical property of the SVD, \mathbf{V} is an orthonormal matrix, i.e. $\mathbf{V}^T \mathbf{V} = \mathbf{I}$. The orthogonal property encourages the representations of \mathbf{V} to be more distinguishable from each other, helping to keep the noise of \mathbf{V} [13]. Additionally, when K is small, there is no significant difference between \mathbf{V} and $\bar{\mathbf{V}}$ as the columns of \mathbf{V} indicate the directions that capture the most significant variations in the data. Therefore, the decomposition of $\bar{\mathcal{Y}}$ can be rewritten as

$$\bar{\mathcal{Y}} = \bar{\mathcal{A}}_{\mathbf{Y}} \times_3 (\mathbf{V}\mathbf{V}_s^{-1}), \quad (12)$$

As the degradation operation \mathbf{H} is linear and is performed in the spatial dimension, then we have

$$\bar{\mathcal{Y}} = \mathbf{H}(\mathcal{X}) = \mathbf{H}(\mathcal{A} \times_3 \mathbf{E}) = \mathbf{H}(\mathcal{A}) \times_3 \mathbf{E} = \bar{\mathcal{A}}_{\mathbf{Y}} \times_3 \mathbf{E}. \quad (13)$$

By comparing Eq. (12) and Eq. (13), it can be easily seen that the coefficient matrix can be defined as

$$\mathbf{E} = \mathbf{V}\mathbf{V}_s^{-1}. \quad (14)$$

3.3.2 Band Index Selection Using RRQR

While the coefficient matrix could be estimated from Eq. (14), the predefined index (i_1, i_2, \dots, i_K) of the reduced image needs to be carefully selected so that $|\det(\mathbf{V}_s)| > 0$ and \mathbf{V}_s^{-1} exists. Actually, $|\det(\mathbf{V}_s)|$ is expected to be large owing to the following two reasons: A larger $|\det(\mathbf{V}_s)|$ indicates that each band in the reduced image \mathcal{A} encodes different image information, improving the HSI restoration performance. In addition, if $|\det(\mathbf{V}_s)|$ is small, the value in the coefficient matrix \mathbf{E} could be very large, which could lead to numerical instability during the estimation process of the reduced image \mathcal{A} . To this end, a rank-revealing QR (RRQR) algorithm proposed in [12] is employed to determine the band selection index. Using the RRQR factorization on the matrix \mathbf{V}^T can be decomposed as

$$\mathbf{V}^T \mathbf{\Pi} = \mathbf{Q} \mathbf{R} \equiv [\mathbf{Q} \mathbf{R}_1 \mathbf{Q} \mathbf{R}_2], \quad (15)$$

where $\mathbf{\Pi} \in \mathbb{R}^{B \times B}$ is a permutation matrix, $\mathbf{Q} \in \mathbb{R}^{K \times K}$ is an orthogonal matrix and $\mathbf{R} \in \mathbb{R}^{K \times B}$ is an upper triangular matrix. The RRQR factorization algorithm works by interchanging any pair of columns that increases sufficiently $|\det(\mathbf{R}_1)|$. We define \mathbf{V}_s^T as $\mathbf{Q} \mathbf{R}_1$ and then we could obtain $|\det(\mathbf{V}_s)| = |\det(\mathbf{R}_1)|$. Therefore, when the algorithm terminates, $|\det(\mathbf{V}_s)|$ is maximized and the indices corresponding to the first K columns of the permuted matrix $\mathbf{V}^T \mathbf{\Pi}$ can be defined as the band selection index. The choice of selection index could not cause $|\det(\mathbf{V}_s)|$ to be too large, which could also result in numerical instability. Concretely, from Hadamard's inequality [9], we have

$$|\det(\mathbf{V}_s)| \leq \prod_{i=1}^K \|\mathbf{v}\mathbf{s}_i\|_2, \quad (16)$$

where $\mathbf{v}\mathbf{s}_i$ is the i th column of \mathbf{V}_s . Since each column \mathbf{v} in matrix \mathbf{V} satisfies $\|\mathbf{v}\|_2 = 1$, we have $\|\mathbf{v}\mathbf{s}_i\|_2 \leq \|\mathbf{v}_i\|_2 = 1$ and $|\det(\mathbf{V}_s)| \leq 1$.

3.4. Conditional Denoising Diffusion Model

With the pre-estimated coefficient matrix \mathbf{E} and a predefined band index, we employ a pre-trained diffusion model to estimate the reduced image \mathcal{A} . Recently, emerging approaches [1, 6, 8] are proposed for image generation with various conditions utilizing a pre-trained diffusion model. In general, given a condition or a guidance \mathbf{y} , these approaches model $p_\theta(\mathbf{x}_{t-1} | \mathbf{x}_t, \mathbf{y})$ during the reverse sampling process by introducing gradient on ϵ_θ as

$$\hat{\epsilon}_\theta(\mathbf{x}_t, t) = \epsilon_\theta(\mathbf{x}_t, t) + s(t) \cdot \nabla_{\mathbf{x}_t} \mathcal{L}(\hat{\mathbf{x}}_0, \mathbf{y}), \quad (17)$$

where $s(t)$ denotes the guidance strength in the t th step and \mathcal{L} denotes a loss function which measures the distance between the current predicted $\hat{\mathbf{x}}_0$ and the expected image. In our case, taking the degraded image \mathcal{Y} and the pre-estimated

Algorithm 1: HIR-Diff Method

Input: \mathcal{A}_T sampled from $\mathcal{N}(0, I)$, diffusion model ϵ_θ , noise scales $\{\bar{\alpha}_t\}_{t=1}^T$, guidance strength $s(t)$, loss function \mathcal{L} , degraded HSI \mathcal{Y} , estimated coefficient \mathbf{E}

Output: Clean HSI \mathcal{X}_0

for $t = T, T-1, \dots, 1$ **do**

- step 1:** estimate $\hat{\mathcal{A}}_0$ by Eq. (19)
- step 2:** calculate $\mathcal{L}(\hat{\mathcal{A}}_0, \mathbf{E}, \mathcal{Y})$ by Eq. (21)
- step 3:** estimate $\hat{\epsilon}_\theta(\mathcal{A}_t, t)$ by Eq. (18)
- step 4:** sample \mathcal{A}_{t-1} by Eq. (20)

end

$\mathcal{X}_0 = \mathcal{A}_0 \times_3 \mathbf{E}$

return \mathcal{X}_0

coefficient matrix \mathbf{E} as conditions, we employ diffusion models to estimate the reduced image \mathcal{A} as shown in Fig. 1. Specifically, Eq. (17) can be rewritten as

$$\hat{\epsilon}_\theta(\mathcal{A}_t, t) = \epsilon_\theta(\mathcal{A}_t, t) + s(t) \cdot \nabla_{\mathcal{A}_t} \mathcal{L}(\hat{\mathcal{A}}_0, \mathbf{E}, \mathcal{Y}), \quad (18)$$

where $\hat{\mathcal{A}}_0$ is estimated by Eq.(3) as

$$\hat{\mathcal{A}}_0 = \frac{\mathcal{A}_t - (\sqrt{1 - \bar{\alpha}_t}) \epsilon_\theta(\mathcal{A}_t, t)}{\sqrt{\bar{\alpha}_t}}. \quad (19)$$

Then \mathcal{A}_{t-1} is sampled from Eq. (1) as

$$\mathcal{A}_{t-1} = \sqrt{\bar{\alpha}_{t-1}} \hat{\mathcal{A}}_0 + (\sqrt{1 - \bar{\alpha}_{t-1}}) \hat{\epsilon}_\theta(\mathcal{A}_t, t). \quad (20)$$

The loss function $\mathcal{L}(\hat{\mathcal{A}}_0, \mathbf{E}, \mathcal{Y})$ proposed in our work contains a data fidelity term and a TV regularization. The data fidelity term ensures that the predicted image closely matches the observed data. The TV term that has been widely adopted in HSI restoration [35, 36, 48] contributes to denoising and edge preservation. Specifically, the guidance function $\mathcal{L}(\hat{\mathcal{A}}_0, \mathbf{E}, \mathcal{Y})$ is defined as

$$\mathcal{L}(\hat{\mathcal{A}}_0, \mathbf{E}, \mathcal{Y}) = \lambda \|\mathbf{H}(\hat{\mathcal{A}}_0 \times_3 \mathbf{E}) - \mathcal{Y}\|_F^2 + \beta \|\hat{\mathcal{A}}_0 \times_3 \mathbf{E}\|_{TV}, \quad (21)$$

where λ and β are hyperparameters. The proposed method is summarized in Algorithm 1. When diffusion sampling finishes, the restored HSI \mathcal{X}_0 is represented as $\mathcal{A}_0 \times_3 \mathbf{E}$.

3.5. Exponential Noise Schedule

We found that when the widely used linear noise schedule [14] and the cosine noise schedule [33] are used, the Peak Signal-to-Noise Ratio (PSNR) value of the restored image fluctuates drastically at the beginning and does not converge at the end of the sampling process, as shown in Fig. 2. Increasing the sampling steps or improving the strength of the guide does not alleviate the problem. One

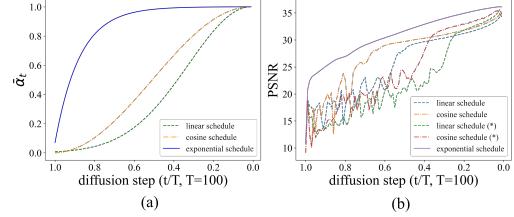


Figure 2. (a) The $\bar{\alpha}_t$ in the linear schedule, cosine schedule and our proposed exponential schedule. (b) The PSNR values throughout the diffusion process with different noise schedules. Linear schedule (*) and cosine schedule (*) denote the results when the guidance strength is enhanced.

possible reason is that with the guidance information as illustrated in Eq. (17), the noise rapidly decays at the beginning of the sampling process since the diffusion process starts with random noise while the conditional image contains abundant image details. It can be observed from Eq. (1) that the noise schedule $\bar{\alpha}_t$ reflects how much information is contained in the current sample. Therefore, the $\bar{\alpha}_t$ is supposed to increase rapidly at the beginning, which is inconsistent with the linear schedule and cosine schedule. Besides, the PSNR value increases rapidly at the end of the sampling process, indicating that a larger $\bar{\alpha}_t$ helps to refine the image details and achieve the local optimum at the end. During this stage, the diffusion prior progressively works to generate a desirable image. Therefore, we design a different noise schedule in terms of $\bar{\alpha}_t$ as

$$\bar{\alpha}_t = e^{-kt/T}, \quad t = 1, 2, \dots, T, \quad (22)$$

$$\bar{\alpha}_t = \frac{\bar{\alpha}_t - \min\{\bar{\alpha}_t\}_{t=1}^T}{\max\{\bar{\alpha}_t\}_{t=1}^T - \min\{\bar{\alpha}_t\}_{t=1}^T} \times (1 - \epsilon) + \epsilon, \quad (23)$$

$$t = 1, 2, \dots, T. \quad (24)$$

where k is a hyperparameter and ϵ is a small value to prevent $\hat{\alpha}_0$ from being zero. Our exponential noise schedule increases rapidly at the beginning and changes slowly at the end so that the random noise can converge to the observed image rapidly and the image details can be well refined.

Remark: It should be noted that our improved diffusion model modifies DDIM in two aspects, i.e. the proposed exponential noise schedule and the designed guidance function introduced in Sec. 3.4, and thus providing better sample quality and faster sampling speed. The effectiveness of the two modifications will be discussed in Sec. 4.5.

4. Experiment

4.1. Datasets and Evaluation Metrics

We evaluate the HSI restoration performance on three publicly available datasets, namely Washington DC (WDC)

Mall¹ whose size is $1208 \times 307 \times 191$, Houston² whose size is $349 \times 1905 \times 144$, and Salinas³ whose size is $512 \times 217 \times 224$. For each dataset, we crop the center area and remove some noisy bands, deriving three HSIs with size $256 \times 256 \times 191$, $256 \times 256 \times 124$ and $128 \times 128 \times 190$, respectively. Two commonly used evaluation metrics, i.e. peak signal-to-noise ratio (PSNR) and structure similarity (SSIM), are adopted to evaluate the performance.

4.2. Competing Methods

HSI Denoising: The HSI denoising aims at recovering the clean HSI from its noisy observation. We mainly consider Gaussian noise, and the standard deviation of Gaussian noise σ in the range of $[0, 255]$ is set as 30, 50, and 70, respectively. Three model based methods including BM4D [31], NGMeet [13], ETPTV [4], four deep learning based methods including T3SC [2], MACNet [51], SST [23], SERT [24] and an unsupervised deep learning based methods (i.e. DDS2M [32]) are adopted for comparison. For the DL-based methods, we use available models⁴ trained on ICVL⁵ with Gaussian noise for comparison.

Noisy HSI Super-Resolution: The noisy HSI super-resolution aims at recovering the clean HSI from a noisy low-resolution observation. Specifically, the low-resolution image is obtained by first spatially blurring the clean image using a Gaussian-shape filter, then downsampling the blurred image and finally adding the image with Gaussian noise with $\sigma = 30$. Seven methods, including two unsupervised deep learning based methods, i.e. DIP2d [42] and DIP3d [42], and five supervised deep learning based methods, i.e. SFCCSR [46], SSPSR [17], MCNet [25], RFSR [47], PDENet [15] are adopted for comparison. Since the pre-trained model is not available, we train all the supervised models on CAVE⁶ with the setting proposed in [46].

Noisy HSI Inpainting: The noisy HSI inpainting aims at recovering the clean HSI from noisy and incomplete data. Concretely, we randomly mask a portion of pixels and add the Gaussian noise ($\sigma = 30$) to the observed area. The mask rate is set as 0.7, 0.8 and 0.9, respectively. Four model-based methods (i.e. TRPCA [27], TRLRF [52], S2NTNN [30] and HLRTF [29]) and three unsupervised DL-based methods (i.e. DIP2d [42], DIP3d [42] and DDS2M [32]) are adopted for comparison.

¹<http://lesun.weebly.com/hyperspectral-data-set.html>

²https://hyperspectral.ee.uh.edu/?page_id=459

³https://www.ehu.eus/ccwintco/index.php/Hyperspectral_Remote_Sensing_Scenes

⁴<https://github.com/MyuLi/SERT>

⁵<https://icvl.cs.bgu.ac.il/hyperspectral/>

⁶<https://www.cs.columbia.edu/CAVE/databases/multispectral/>

4.3. Implementation Details

A pre-trained diffusion model⁷ trained on a large amount of 3-channel (i.e. RGB) remote sensing images [10] is used to generate the reduced image \mathcal{A} . The diffusion sampling step T is set as 20 for all the HSI restoration tasks. All DL-based models are trained on an NVIDIA Geforce RTX 3090 GPU.

Table 1. The average quantitative results for HSI denoising. The **best** and **second-best** values are highlighted.

Dataset	Method	standard deviation		30		50		70		Time (s)
		PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	
WDC mall	BM4D	37.09	0.90	34.48	0.83	32.81	0.77	286		
	NGMeet	43.77	0.98	41.07	0.96	39.82	0.94	65		
	ETPTV	41.08	0.95	37.12	0.91	35.15	0.87	629		
	T3SC	38.70	0.92	37.04	0.88	35.94	0.86	3		
	SST	39.09	0.93	37.24	0.90	36.19	0.87	5		
	SERT	38.98	0.93	37.08	0.89	35.90	0.86	5		
	DDS2M	41.58	0.95	39.13	0.93	38.83	0.92	3132		
Houston	Ours	<u>42.85</u>	<u>0.97</u>	<u>40.77</u>	<u>0.94</u>	<u>39.33</u>	<u>0.92</u>	17		
	BM4D	33.89	0.84	31.83	0.77	30.57	0.72	184		
	NGMeet	38.50	0.94	35.77	0.89	34.36	0.85	62		
	ETPTV	35.78	0.90	33.75	0.84	32.47	0.80	322		
	T3SC	35.74	0.90	33.91	0.85	32.75	0.81	3		
	SST	36.26	0.92	34.19	0.87	32.87	0.83	6		
	SERT	36.11	0.91	33.96	0.86	32.58	0.82	4		
Salinas	DDS2M	35.64	0.91	33.44	0.85	31.96	0.79	1813		
	Ours	<u>38.13</u>	<u>0.94</u>	36.01	0.90	34.56	0.86	25		
	BM4D	38.62	0.91	35.96	0.86	34.23	0.82	68		
	NGMeet	44.76	0.98	42.23	0.96	40.96	0.95	16		
	ETPTV	42.52	0.95	40.44	0.93	38.03	0.91	91		
	T3SC	41.06	0.95	39.47	0.94	38.36	0.92	3		
	SST	38.93	0.94	37.51	0.92	36.39	0.91	4		
CAVE	SERT	38.80	0.94	37.26	0.92	36.07	0.90	3		
	DDS2M	43.29	0.96	40.05	0.93	38.10	0.90	846		
	Ours	<u>43.79</u>	<u>0.96</u>	<u>41.95</u>	<u>0.95</u>	<u>39.48</u>	<u>0.93</u>	13		

4.4. Results for HSI Restoration

The results of different methods on HSI denoising, noisy HSI super-resolution and noisy HSI inpainting are demonstrated in Table 1, Table 2 and Table 3, respectively. It can be seen that our proposed method demonstrates superior HSI restoration performance compared to other competitive models. Compared with model-based approaches (e.g., BM4D and ETPTV) which highly rely on handcraft priors, the pre-trained diffusion model employed in our method is able to capture complex intrinsic characteristics and image details from abundant data. The supervised DL-based methods suffer limited generalization ability and struggle with the restoration task of unseen data. In contrast, our model demonstrates desirable generalization ability as the employed diffusion model learns the image structure and details in a self-supervised manner, which enables the model to be competitive for various tasks and datasets. In addition, prior knowledge including low rank and total variation is introduced in our model, which helps to regularize the restoration process and thus obtain better restoration

⁷<https://github.com/wgcbn/ddpm-cd>

results. Moreover, other unsupervised DL-based methods, e.g., DIP2d, DIP3d and DDS2M, take lots of time to exploit the image inherent structure with an untrained neural network for each individual dataset, while our method benefitting from self-supervised training is considerably more efficient and fast. Some visual results for different tasks are shown in Fig. 3, from which it can be observed that our method can restore more accurate and reliable visual results.

Table 2. The average quantitative results for noisy HSI super-resolution. The **best** and second-best values are highlighted.

Dataset	Scale	$\times 2$				$\times 4$				$\times 8$				Time (s)
		PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	
WDC mall	DIP2d	32.18	0.58	31.62	0.57	29.92	0.53	206						
	DIP3d	31.81	0.57	32.05	0.58	30.10	0.51	16644						
	SFCSR	33.74	0.71	32.92	0.66	31.85	0.58	6						
	SSPSR	33.32	0.71	32.38	0.66	30.63	0.55	6						
	MCNet	34.55	0.74	33.55	0.69	31.76	0.59	22						
	RFSR	33.70	0.72	32.73	0.66	31.06	0.56	15						
	PDENet	29.79	0.64	28.78	0.57	29.28	0.55	16						
	Ours	36.67	0.81	34.68	0.74	32.20	0.60	22						
Houston	DIP2d	27.67	0.61	27.61	0.61	27.15	0.60	178						
	DIP3d	27.72	0.61	27.67	0.61	27.29	0.60	4841						
	SFCSR	29.90	0.69	29.00	0.65	27.35	0.60	5						
	SSPSR	29.99	0.72	29.12	0.67	27.49	0.61	5						
	MCNet	30.52	0.72	29.60	0.68	28.03	0.62	16						
	RFSR	30.27	0.71	29.22	0.66	27.64	0.62	10						
	PDENet	28.83	0.60	28.06	0.56	27.71	0.58	11						
	Ours	31.83	0.78	30.68	0.72	29.10	0.65	28						
Salinas	DIP2d	36.29	0.88	34.35	0.84	30.72	0.75	85						
	DIP3d	35.24	0.86	34.60	0.85	32.34	0.82	1866						
	SFCSR	35.63	0.87	34.42	0.86	32.15	0.82	4						
	SSPSR	33.71	0.86	32.40	0.84	30.08	0.79	5						
	MCNet	36.01	0.89	35.13	0.87	32.47	0.82	10						
	RFSR	35.26	0.87	34.02	0.85	31.59	0.81	12						
	PDENet	31.45	0.70	30.52	0.67	31.14	0.75	8						
	Ours	39.44	0.91	37.53	0.88	34.48	0.82	14						

4.5. Ablation Study

Reduced Image Estimation In our work, an HSI is restored from a reduced image \mathcal{A} which is supposed to be several bands, and a corresponding coefficient matrix \mathbf{E} . Nevertheless, there exists other alternative tensor decomposition methods to restore an HSI. One of them is directly defining the \mathbf{V} matrix obtained from SVD as the coefficient matrix \mathbf{E} , and employing the diffusion model to restore the "pseudo" image $\mathcal{A} = \text{fold}_3(\mathbf{US})$. Another solution [38] is that the coefficient matrix \mathbf{E} is estimated by solving a least square problem with several observed noisy bands, and then \mathcal{A} is estimated from the diffusion model. The definition of these decomposition strategies is inconsistent with the diffusion prior or is susceptible to noise, leading to poor restoration results. The HSI restoration results of different decomposition strategies on WDC dataset are illustrated in Table 4 and Fig. 4. Our method demonstrates superior HSI restoration performance as the SVD operation effectively suppresses noise and the definition of the tensor \mathcal{A} lies in the image field, which is consistent with the diffusion prior.

Table 3. The average quantitative results for noisy HSI inpainting. The **best** and second-best values are highlighted.

Dataset	Masking Rate	0.7		0.8		0.9		Time (s)
		PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	
WDC mall	TRPCA	22.29	0.23	22.72	0.22	23.30	0.22	346
	TRLRF	22.03	0.27	20.61	0.19	11.79	0.03	1000
	S2NTNN	35.64	0.80	33.70	0.73	31.03	0.62	32
	WDC mall	HLRTF	37.03	0.85	36.07	0.82	<u>34.08</u>	<u>0.72</u>
	DIP2d	31.55	0.57	31.54	0.57	30.71	0.56	50
	DIP3d	31.47	0.57	31.90	0.57	31.69	0.57	12876
	DDS2M	38.32	0.91	<u>36.18</u>	0.85	32.35	0.71	4920
	Ours	37.90	0.87	36.82	<u>0.83</u>	35.08	0.76	17
Houston	TRPCA	22.13	0.21	22.46	0.21	22.84	0.22	175
	TRLRF	21.16	0.20	19.11	0.13	13.32	0.04	768
	S2NTNN	31.01	0.76	28.08	0.67	24.48	0.46	16
	Houston	HLRTF	30.82	0.77	29.92	0.72	28.53	0.65
	DIP2d	26.45	0.59	26.35	0.58	26.19	0.58	48
	DIP3d	26.51	0.59	26.28	0.58	26.12	0.58	3037
	DDS2M	<u>31.91</u>	<u>0.81</u>	<u>30.05</u>	<u>0.73</u>	<u>29.13</u>	<u>0.67</u>	2638
	Ours	32.42	0.82	31.19	0.78	29.67	0.72	20
Salinas	TRPCA	22.67	0.18	23.30	0.20	23.96	0.23	79
	TRLRF	22.75	0.19	20.42	0.13	13.09	0.03	324
	S2NTNN	37.74	0.88	34.14	0.75	26.79	0.61	11
	Salinas	HLRTF	33.97	0.79	33.40	0.78	32.56	0.77
	DIP2d	34.71	0.86	33.65	0.84	32.15	0.82	41
	DIP3d	33.76	0.85	33.04	0.83	31.77	0.81	1274
	DDS2M	40.49	0.94	<u>38.23</u>	<u>0.91</u>	<u>36.80</u>	<u>0.89</u>	1354
	Ours	40.70	0.94	39.40	0.92	37.61	0.90	16

Table 4. The HSI restoration results of different tensor decomposition methods. "SVD Only" denotes that the tensor \mathcal{A} is defined as the pseudo image of SVD and "Least Square" denotes that the matrix \mathbf{E} is estimated by solving the least square problem.

Task	Denoising		Super-Resolution		Inpainting		PSNR
	Methods	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM
SVD Only	28.88	0.55	27.86	0.43	28.16	0.44	
Least Square	37.53	0.89	34.37	0.73	35.55	0.79	
Ours	40.73	0.94	34.79	0.74	37.02	0.85	

Table 5. The HSI restoration results of different band selection indexes. $(\cdot, \cdot, \cdot)^*$ denotes the bands selected using our method.

Task	Bands	$ \det(\mathbf{V}_s) $	The Maximum Value in \mathbf{E}		PSNR
			Value	Index	
Denoising	(1, 48, 96)	0.000681	7.91		27.85
	(37, 84, 132)	0.001427	5.86		28.00
	(73, 120, 168)	0.000068	11.57		18.40
	(21, 31, 60)*	0.011500	1.01		44.27
Super-Resolution	(1, 48, 96)	0.000195	24.86		11.02
	(37, 84, 132)	0.000566	10.28		20.41
	(73, 120, 168)	0.000193	12.58		9.96
	(22, 33, 58)*	0.012835	1.00		39.68
Inpainting	(1, 48, 96)	0.001308	3.11		34.54
	(37, 84, 132)	0.000617	16.07		9.93
	(73, 120, 168)	0.000007	47.34		6.51
	(21, 34, 58)*	0.014779	1.00		41.02

Coefficient Matrix Estimation As introduced in Sec. 3.3, we employ RRQR to determine the band selection index (i_1, i_2, \dots, i_K) . The HSI restoration results on Salinas dataset of several indexes selected at equal intervals are

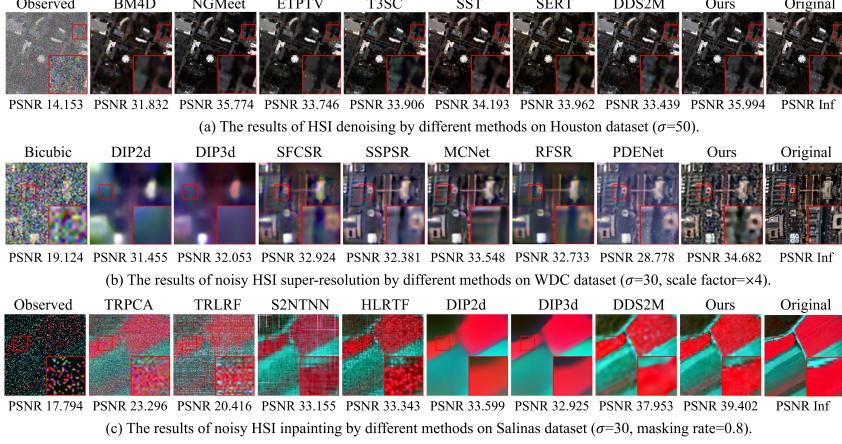


Figure 3. The visual result comparison of all the competing methods on the HSI restoration task.

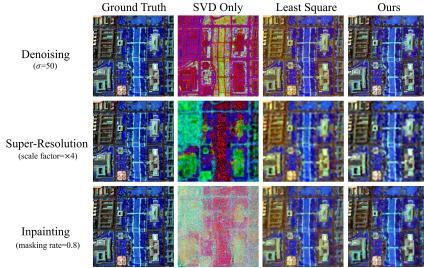


Figure 4. The reduced image \mathcal{A} estimated by different methods. Ground Truth denotes the bands selected from the clean HSI.

Table 6. Ablation study of the proposed guidance function.

Data Fidelity Term	TV Regularization Term	Denoising		Super-Resolution		Inpainting	
		PSNR	SSIM	PSNR	SSIM	PSNR	SSIM
✗	✗	10.12	0.07	11.49	0.16	11.13	0.08
✓	✗	34.99	0.87	27.10	0.45	29.90	0.67
✓	✓	36.14	0.90	30.70	0.72	31.71	0.80

provided in Table 5 to prove the necessity of RRQR. As can be observed, due to the low-rank property of HSIs, the determinant of \mathbf{V}_s could be extremely small, indicating a high similarity between the selected bands and thus resulting in large values in \mathbf{E} and numerical instability. In contrast, the bands selected using our method encode more abundant information and provide desirable results.

Guidance Function The results of different guidance functions are provided in Table 6. As can be seen, the absence of any condition leads to the worst performance since the model becomes a pure generative model. In addition, the TV regularization adopted in our work contributes to the improvement of restoration performance, verifying the effectiveness of our proposed guidance function.

Noise Schedule The results of different noise schedules are shown in Table 7. The restoration performance is improved with our exponential schedule and declines much slower when the number of sampling step decreases since there are more effective steps as introduced in Sec. 3.5.

Table 7. The HSI restoration results of different noise schedules.

Task	Step $\times T$ Schedule	20		50		100	
		PSNR	SSIM	PSNR	SSIM	PSNR	SSIM
Denoising	Linear	34.34	0.86	35.18	0.88	35.70	0.89
	Cosine	34.61	0.86	35.46	0.88	35.86	0.89
	Exponential	36.01	0.90	36.10	0.90	36.14	0.90
Super-Resolution	Linear	29.47	0.66	30.01	0.68	30.39	0.70
	Cosine	30.25	0.70	30.45	0.70	30.56	0.71
	Exponential	30.68	0.72	30.69	0.72	30.70	0.72
Inpainting	Linear	29.96	0.70	30.86	0.74	31.44	0.77
	Cosine	30.17	0.71	31.07	0.76	31.46	0.77
	Exponential	31.19	0.78	31.65	0.79	31.71	0.80

5. Conclusion

In this paper, we propose an unsupervised HSI restoration approach. Using the low-rank property of HSIs, the clean HSI can be restored by the product of a reduced image and a coefficient matrix. Specifically, the coefficient matrix can be pre-estimated from the observed image using SVD and Rank-Revealing QR (RRQR), and the reduced image can be estimated from the pre-trained diffusion model with a newly designed guidance function. Additionally, the proposed exponential noise schedule has been proven to be more reasonable for our conditional diffusion model and can significantly accelerate the sampling process. Our proposed method is a universal HSI restoration framework and can perform better than other state-of-the-art methods on various HSI restoration tasks.

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