Routing-Based Methodology for Evaluation of Electric Vehicle Supply Equipment (EVSE) Networks

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Contents

1 Introduction

Road vehicles require a network of roads and a network of whose nodes are coincident with a subset of the nodes of the road network. Allocation of to road network nodes is a central concern in transportation planning. To a great extent, the topology of the network will be determined by the topology of the road network, itself determined by and determinative of population and economic topology. In other words, roads connect places, serve vehicles on roads. It is unlikely that, for the sake of the network, new roads will be built or population concentrations will shift. Thus, the problem of optimal resource allocation, with respect to should aim to propose locations for such that the performance of the road network is maximized with respect to the demand generated by population and economic topology.

2 EVSE Network

For EVSE networks, the nodes are charging stations. The focus of this document is long distance travel and , thus, only DC Fast Charging (DCFC) nodes will be considered. Because interoperability between vehicles and chargers is not universal, the network will look different for different types of vehicle. An EVSE network's links are ephemeral end vehicle-dependent. Links exist between all chargers and all other chargers within range. This creates a highly connected network with significant locality. A key feature of any EVSE network is the low probability of availability at all nodes. Chargers are unreliable and may be unavailable for reasons including hardware and software faults, physical blockage, plug damage, payment system failures, and occupation. Because Plug-in Electric Vehicle (PEV) fast charging events take a relatively long amount of time compared to Internal Combustion Engine Vehicle (ICEV) fueling events and the relatively low number of plugs per station and stations per mile of road network, the probability of plug occupation is high, especially for central nodes. This can lead to a frustrating driving experience as limited prior knowledge of charger availability is the norm.

Constructing an EVSE network requires the locations of usable chargers, vehicle usable range, and a road network which has nodes coincident with the chargers, or nearly so. In this document, the road network will be referred to as the atlas. The links of the EVSE network are constructed by finding the lowest cost (however defined) route for all node pairs which do not violate cost limits. Routing along the network involves the additional step of adding origins and destinations to the network in the same manner as adding chargers.

Random Graph

Shown in Figure 1 is a randomly generated graph ($G = \{V, E\}$) with a cardinality of $N = 100$ on a 100 km by 100 km area with a link probability determined by $P(E|L_E) = e^{-L/S}$ where E is the link from V_i to V_j, L_E is the Pythagorean distance between V_i and V_j, and S is a characteristic distance, (10 km in this case). Among the nodes in the graph, one has been designated the origin, five have been assigned as destinations, and ten have been assigned as chargers.

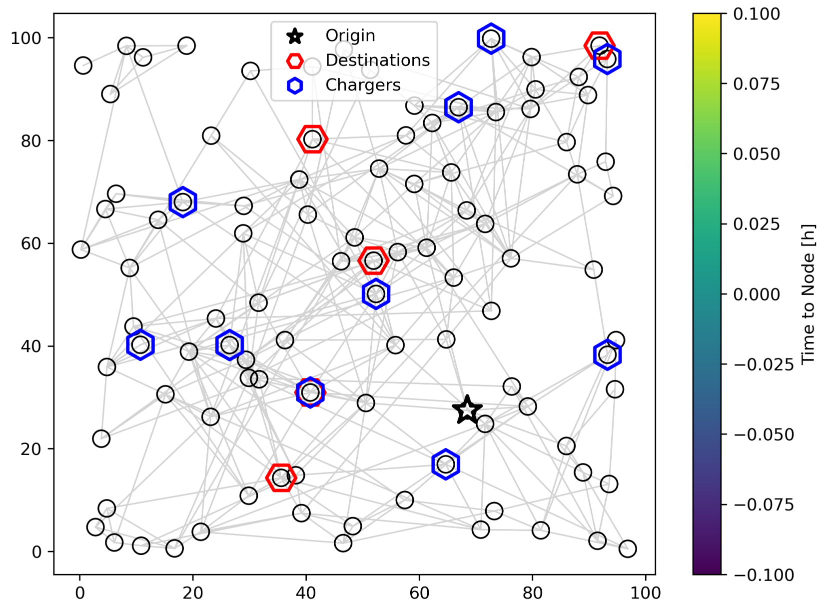


Figure 1: Randomly generated graph with cardinality of 100 with 1 origin, 5 destinations, and 10 chargers.

The links are traversed at speeds randomly selected from $\{35, 55, 90, 105\}$ kph. Like a real road network, all nodes are connected to the Giant Connected Component (GCC) but not directly to all others. Because the random graph is based on nodes drawn from a uniform random distribution, the distances between nodes should be normally distributed. Because the probability of a given link is a function of distance, one would also expect the nodes in the middle to have the highest valency and for valency to be Poisson distributed and this is, indeed, the case.

California Graph

The state of California contains 1,618 towns and cities, collectively referred to as places, per the US Census Bureau and 1,906 public DC charging stations per Alternative Fuels Data Center (AFDC). Itineraries entirely contained in California will originate and terminate at California places. If additional range is needed beyond the remaining range at the start of the trip, Electric Vehicles (EVs) will utilize one of the DC charging stations. The places and stations are completely connected by roads. Thus, an entity atlas can be computed by routing from each entity to each entity. For the purposes of reducing computation time and memory requirements, a limit on link ranges can be implemented which may result in a non-completely-connected entity atlas which should, nevertheless, be entirely contained within the GCC. The components of the California entity atlas are shown in Figure 2.

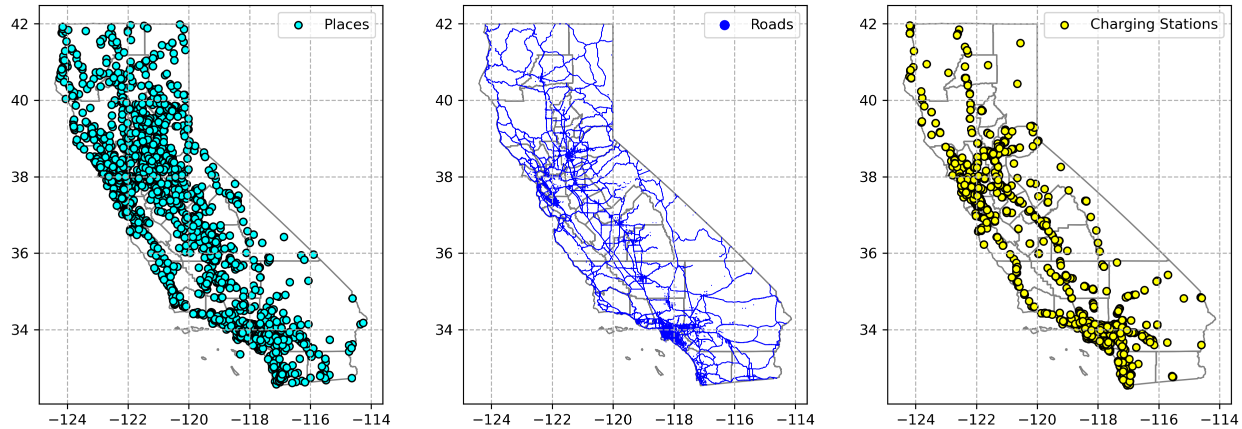


Figure 2: Components of California entity atlas

3 Routing

3.1 Background

For a graph $G = \{V, E\}$ where $O \in V$ is an origin and $D \in V$ is a destination, there is an optimal route $R^* = \{O, \Phi_0, \dots, \Phi_N, D\} | R^* \subset V$ such that $J^* = F(G, R^*)$ is the global minimum value of J for routing cost function $J = F(G, R)$. There are, in essence, two approaches to finding R^*, Dijkstra's method and Bellman's method. Both methods are founded on the Hamilton-Jacobi-Bellman (HJB) equation

$$J_k = F(U_k, \dots) + F(U^*_{k+1}, \dots) \tag{1}$$

which states that the cost of a given control at the current step is the sum of the cost of the control and the cost of the optimal control at the next step subject to the control for a time-varying Optimal Control Problem (OCP). A corollary is that the optimal control at a given step is influenced by the optimal controls at all following steps. Both Dijkstra's and Bellman's methods work by starting at a given point on a graph with a cost of zero and assigning infinite cost to all other points. The edges of the graph are then explored and for each edge (u, v) if the cost $c(v)$ is greater than the path cost $c(u) + c(u, v)$ then the cost at v is updated to the path cost. This process is called relaxation. The fundamental difference between the methods is that Bellman's method calls for evaluating every edge to check if a relaxation is possible for $|V|$ iterations where $|V|$ is the cardinality of the set of vertices belonging to the graph where Dijkstra's method calls for evaluating only those edges belonging to the "closest" unseen node at each iteration. Dijkstra's method uses a heap queue (priority queue) to keep track of nodes where every node that is reached is added and, at the beginning of each iteration, the edges of node with the smallest cost are evaluated. Essentially, the benefit of Bellman's method over Dijkstra's is that it allows for "upstream" changes to be evaluated constantly where Dijkstra's method only focuses downstream. However Dijkstra's is more often used because it is far more efficient than Bellman's method for the same problem. Generic algorithms can be found in the Appendix. Both methods will usually produce the same globally optimal route. The cases were Bellman's method is needed generally have to do with negative edge costs which are not common in transportation and many other fields. Both methods may be deployed with forward integration (sources to targets) or reverse integration (targets to sources) as diagrammed in Figure 3. It is worth noting that **forward integration creates a tree from a single origin to all destinations** where **reverse integration creates a tree from all origins to a single destination**.

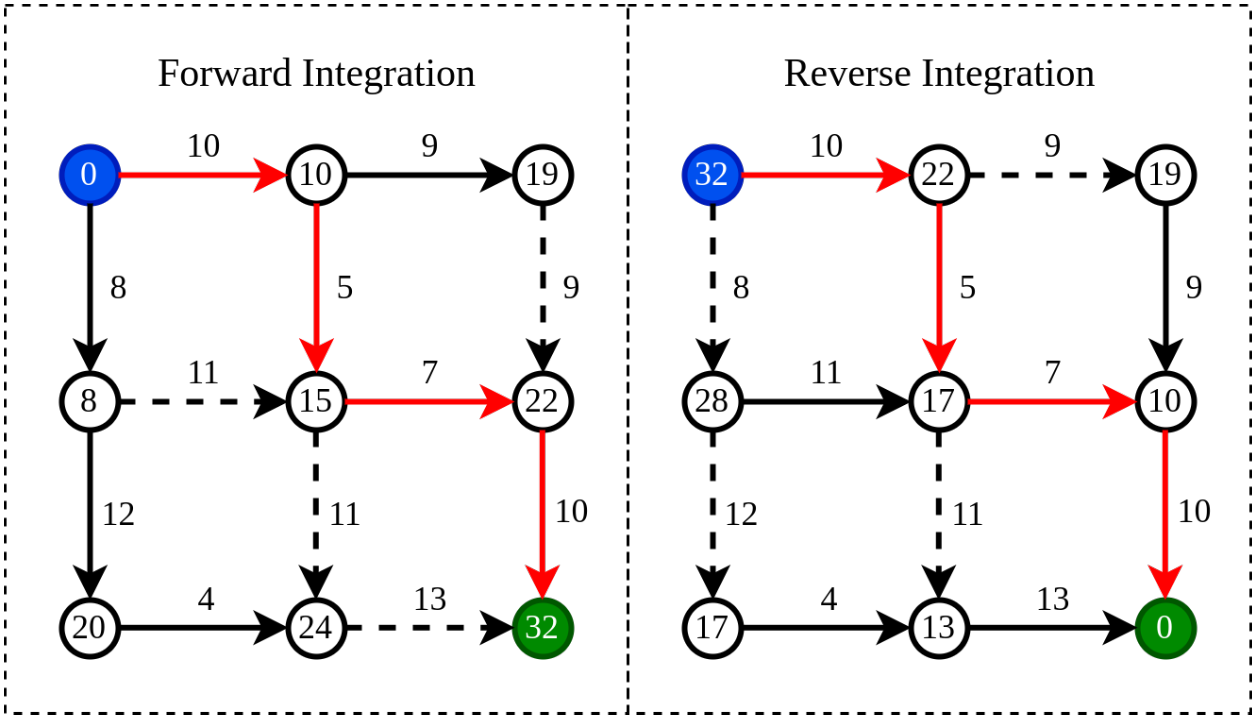


Figure 3: Comparison between Forward and Reverse Integration for Dijkstra's

3.2 Route Constraints

In the basic formulation presented in the Appendix, minimum cost routes will be found for all destinations. However, in reality, routes can be infeasible for many reasons. Herein two reasons will be focused on these being excessive route cost and insufficient route probability. The cost of a route (R) is

$$J_R = \sum_{v \in V_R} \sum_{f \in F_V} f(v) + \sum_{e \in E_R} \sum_{f \in F_E} f(e) \tag{2}$$

where V_R is the set of nodes in R, E_R is the set of links connecting V_R in sequence, F_V is the set of cost functions for nodes and F_E is the set of cost functions for links. The probability of route R is

$$P(R) = \prod_{v \in V_R} P(v) * \prod_{e \in E_R} P(e) \tag{3}$$

In general, costs can be either positive or negative and limits on cost can have upper and lower limits. Probabilities for nodes and links will never be less than 0 or more than 1 meaning that increasing route cardinality can only lead to sustained or lowered route probability.

For vehicles, the primary routing constraint will be range. Each vehicle has a limited amount of range and must utilize energizing stations to add range in order to complete certain trips. Graph nodes may be designated as energizing stations. In routing, when an energizing node is reached the vehicle's range is restored and time is added to the route. In this manner, more trips are possible as seen in Figure 4.

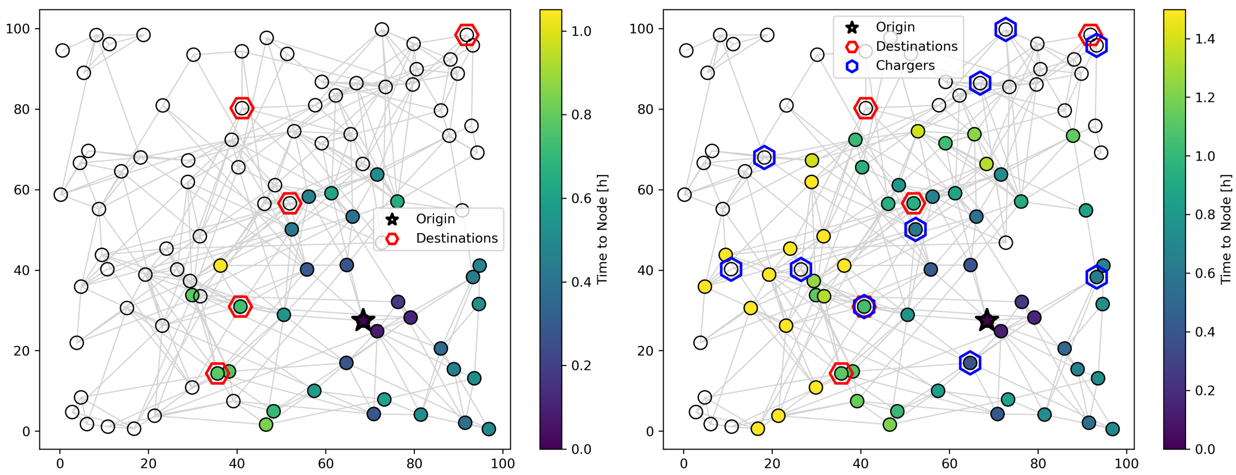


Figure 4: Routing with (right) and without (left) chargers

4 Stochastic Routing Methodology

The principle of stochastic optimization is to find the set of controls which returns the best expected outcome for an uncertain situation modeled as a set of discrete situations of known likelihood. The goal of stochastic optimization is

$$\min_{\overline{U}} \quad J(U) = \sum_{S \in \overline{S}} P(S)F(U,S) \tag{4}$$

where $\overline{S}$ is the set of possible scenarios, $P(S)$ is the probability of S, $F(U,S)$ is the cost of control vector U for scenario S, and $\overline{U}$ is the set of possible controls. A control vector $U = \{U_g, U_p\}$ contains general and specific controls (sometimes called first and second stage controls). General controls are shared among all scenarios and specific controls apply only ot one scenario. An example of stochastic optimization is provided in the Appendix.

For a Battery Electric Vehicle (BEV) driving between an origin and a destination the controls relate to where to charge. If there is less than 100% certainty of a charger being usable (functional, accessible, and available) at the time of arrival to the charger, the optimization becomes stochastic. The technical non-functionality of a given charger may be known to the driver before the trip but accessibility and availability are not likely to be known prior to arrival. When a driver arrives at a charger and becomes aware that the charger is not usable the driver has the following options: (1) Wait at the charger until it becomes available, (2) re-route from current location to destination. If the charger is found to be non-functional or inaccessible then only option (2) is possible. Less than perfect reliability may result in an optimal route which is, otherwise, higher cost than that for perfect reliability. Less than perfect certainty may result in a route which is, otherwise, higher cost than that produced with perfect information. Sufficiently poor reliability/certainty may render a trip infeasible from the start or after several failed charge events.

4.1 Algorithms

Where deterministic routing occurs on a graph G, stochastic routing occurs on the spanning set of possible graphs $\overline{G}$ which has infinite cardinality in which all graph are isomorphic to all others ($G_i \cong G_j \ \forall \ i, j \in \hat{G}$). A corollary is that producing a route for $\overline{G}$ requires that there is some flat bijection $f : V_G \to V_H$ such that $(u,v) \in E_G \iff (f(u), f(v)) \in E_H \ \forall u,v \in V_G$ common to all graphs in $\overline{G}$ which contains the primary structure of the graph (which nodes and links are present). The flat isomorphic graph is $G' = \{V', E'\}$. Variability in route cost is caused by the differences in non-primary node and edge properties between graphs. An example would be a road network where the streets are always present but have variable speeds, some of which may be zero, at a given time.

In reality, optimization must occur over a finite set. Thus stochastic optimization occurs on $\hat{G} \subset \overline{G}$ of cardinality N. Higher values of N better approximate $\overline{G}$ at the cost of increased computational load. Dijkstra's method for stochastic routing results in a single general optimal route R^*. R^* is found using a Monte-Carlo style solution wherein N integrations are simultaneously computed along the same paths experiencing different parameters along the way. The goal of the optimization is to produce the route with the lowest expected cost $\mathbb{J}^* = F(R^*, \hat{G})$ where R^* is the expected optimal route.

R^* is subject to charger usability for O/D pairs of a greater distance than the vehicle's remaining range. The net result of charging (or fueling etc.) is that the vehicle's range is reset, time is added to the route, and money is added to the route price. Time additions are due to charge time and a possible delay due to queuing. Price additions are due to charge pricing. As such, the chargers in $\hat{G}$ (present at a subset of nodes V_C') are defined by function Θ whose parameters are $\hat{\Psi} = \{\Psi_G, \quad \forall G \in \hat{G}\}$ where $\Psi_G = \{\psi_v^r, \psi_v^p, \psi_v^d, \psi_v^c \ \ \forall v \in V_C'\}$. The parameters of Θ are the range completion of the charge event (the reset range), the charge power, the wait time before charging, and the charging cost respectively. If the vehicle's remaining range is greater than ψ_v^r no charge event will occur. For stochastic optimization, all parameters are sampled from defined distributions each time the charger is visited.

Depending on the configuration of G, certain routes may be feasible or infeasible (because they cause the vehicle range to go below a minimum value) potentially creating a discontinuous cost space where the costs of the feasible routes must be considered alongside those of infeasible routes. A probabilistic risk measure, the superquantile provides an elegant solution. The superquantile function for a given distribution is

$$S_p(D) = \frac{1}{1-p} \int_p^1 Q_\alpha(D) d\alpha \tag{5}$$

where D is a given Probability Distribution Function (PDF), $p \in [0,1]$ is a probability threshold, and Q_α is the quantile function of a given PDF for probability α. In effect, the superquantile is the mean of the PDF after p. For route R on graphs $\hat{G}$ of cardinality N, there will be a revealed distribution D_r of remaining range at all points. The remaining range constraint can thus be written as

$$S_p(-D_r) \geq -r^{min} \tag{6}$$

where r^{min} is the lowest allowable remaining range and p is set by the user. In effect, (6) guarantees that the expected value of the worst $1-p$ portion of outcomes is within the feasible range. General values of other route costs are computed similarly. The routing framework described above will be referred to as Stochastic Cost with Risk Allowance Minimization (SCRAM).

4.1.1 SCRAM-D

The SCRAM-D algorithm is outlined below.

Algorithm 1 SCRAM-D Routing Algorithm

$G' = f(G \in \hat{G}) = \{V', E'\}$ ▷ Flat isomorph common to all graphs in $\hat{G}$
$\hat{C} = \{\hat{C_V}, \hat{C_E}\}$ ▷ Costs for graph elements for each graph in $\hat{G}$
$\hat{\Psi} = \{\Psi_v, \quad \forall v \in V'_C \subset V'\}$ ▷ Charger parameters for the subset of nodes with chargers in G'
$O \in V'$ ▷ Single origin node
$D \in V'$ ▷ Set of destination nodes

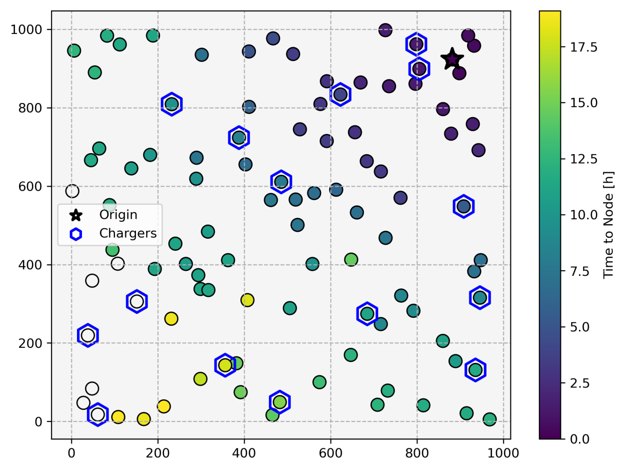
$S = \{\}$ ▷ Set of visited nodes
$W = \{\{\infty\}_{G \in \hat{G}}\}_{v \in V'}$ ▷ All node weights initialized to infinity for all graphs
$R = \{\{\}\}_{d \in D}$ ▷ Initializing empty path for each destination
$F = \{(w_{o_0}, o_0)\}$
▷ Heap queue containing nodes to be visited. Elements are tuples (weight, node) and the heap is ordered by weight. The heap is initialized with the origin node.

while $F \neq \emptyset$ **do** ▷ Iterate while there are reachable nodes which have not been visited
  $w_v, v = f_0$
  $F = F \not\cup f_0$ ▷ Remove current node from heap queue
  if $v \in V'_C$ **then** ▷ If current node is a charger ...
    $l_G, w_{v,G} = \theta_G(l_G, w_{v,G}, \psi_G)$ ▷ Update remaining range and cost after charging
  end if
  $E'_v = \{(v_s, v_t) \in E' | v_s = v\}$
  for all $(v, v_t) \in E'_v$ **do** ▷ Iterate through links of v
    $\mathbb{E}[w_{v_t}] = \mathbb{E}[\{w_v + c_{v,(v,v_t)} + c_{v_t}\}_{G \in \hat{G}}]$ ▷ Compute an expectation for cost of current path
    if $\mathbb{E}[w_{v_t}] < w_{v_t}$ **then** ▷ If current path represents savings ...
      $r_{v_t} = r_v \cup \{v_t\}$ ▷ Update path to v_t
      $w_{v_t} = \mathbb{E}[w_{v_t}]$ ▷ Update cost at v_t
      if $v_t \notin S$ **then**
        $F = F \cup \{(w_t, v_t)\}$ ▷ If v_t has not been visited add to heap queue
      end if
    end if
  end for
  $S = S \cup \{v\}$ ▷ Add current node to set of visited nodes
end while

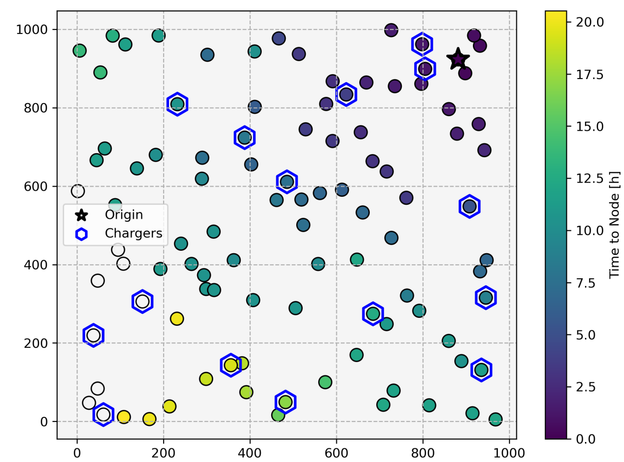
4.2 Routing-Based Metrology

SCRAM is valid for any N. The added value of SCRAM is that is that it allows for risk-tolerance-sensitive routing. This is of particular value for PEVs because charger usability is not guaranteed. When a PEV driver arrives at a charger it may be non-functional or inaccessible in which case it cannot be used or unavailable in which case the driver must wait to use it. Non-functionality and inaccessibility are complete failures where unavailability is a partial failure. Complete failures are modeled using ψ_v^r. If the vehicle's remaining range is greater than ψ_v^r then the vehicle does not charge, if $\psi_v^r = 0$ then no vehicle can charge which is equivalent to a complete failure. Thus ψ_v^r can be sampled from $\{0, r\}$ with defined probabilities for each. Partial failures are modeled with the ψ_v^d term which is sampled from $[0,\infty)$ and added to the time required for route R which includes v. Risk tolerance is modeled using the superquantile function parameter $p \in [0,1]$ where higher values indicate a tighter focus on worse outcomes. **Thus R^* is a function of both graph parameter distributions and driver risk tolerance**.

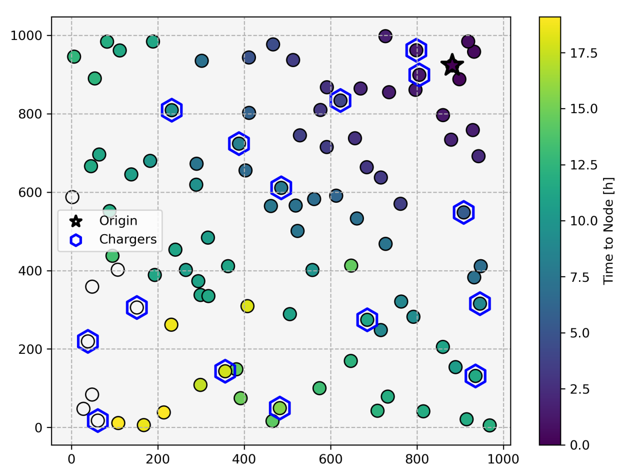
Consider a randomly generated graph of 100 nodes on a 1,000 by 1,000 km grid where chargers are located at 15 nodes. Link probabilities are proportional to distance by the formula $P((v,u)) = \exp(\frac{-\|(v,u)\|}{s})$ where the characteristic distance s is 200 km. A driver is currently at a single origin node and wants to find R^* for every other node. The driver's car has a range of 400 km. The driver has risk tolerance p. Charger parameters are randomly sample as above where the probability of charger complete failure is η. Risk tolerance applies both the the probability of route completion and the metric of optimization which is travel time in this case. A factorial of cases for $p \in \{.5,.9\}$ and $\eta \in \{.1,.5\}$ are shown in Figure 5.



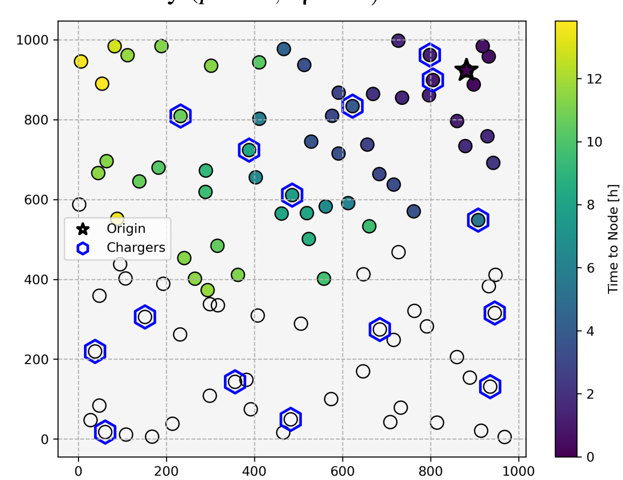
(a) High risk tolerance and high charger reliability ($p = .5,\ \eta = .1$)



(b) High risk tolerance and low charger reliability ($p = .5,\ \eta = .5$)



(c) Low risk tolerance and high charger reliability ($p = .9,\ \eta = .1$)



(d) Low risk tolerance and low charger reliability ($p = .9,\ \eta = .5$)

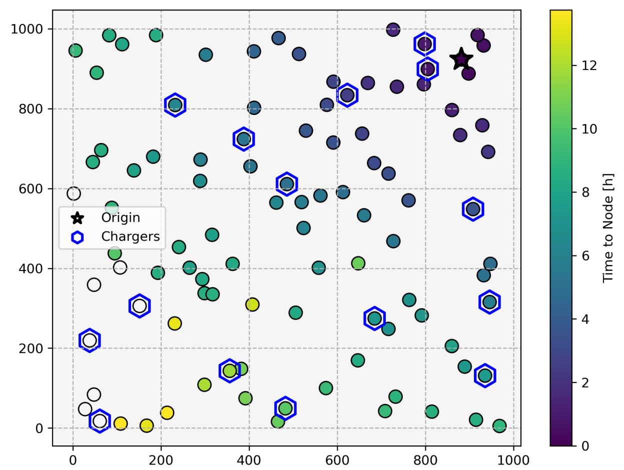
Figure 5: Expected travel times for optimal routes generated by SCRAM-D for varying charger reliability and driver risk tolerance

As seen in Figure 5 SCRAM-D optimal routes are effected by both reliability and risk tolerance in an additive manner. The high risk tolerance scenarios in the top row show a small effect of charger reliability which leads to slightly longer travel times. The high reliability scenarios in the first column shows a small effect of risk tolerance, even smaller than the previously mentioned effect. However, the low risk tolerance scenarios in the second row show a huge effect of charger reliability and the low reliability scenarios in the second column show a huge effect of risk tolerance. This result reinforces the need for SCRAM - **the underlying parameters of PEV charging networks go hand-in-hand with user risk tolerance to effect PEV dis-utility**.

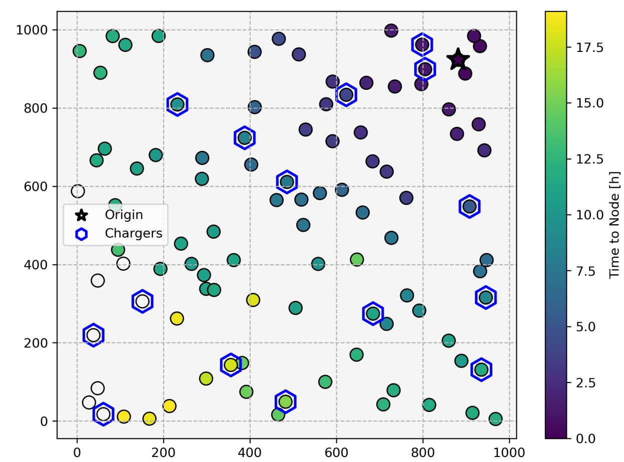
4.3 Cost Differential

If the user experience for PEVs is different than that for ICEVs the difference is due to charging vs. fueling. Charging a PEV at a public charging station is, overall, a worse experience than ICEV refueling or low-rate private PEV charging. The worse experience is due to the longer time required compared to fueling and charger sparsity and unreliability. Although DC charging is, on a cost-per-unit-range-added basis often cheaper than fueling, the pricing structure associated with DC charging is variable and, often, opaque. Gasoline prices, in the US, are determined by individual stations but are, in general, set on the basis of weeks-ahead futures contracts. The price of a gallon of gasoline will be fairly predictable on a several-day timescale and within a region. In effect, most of the time ICEV drivers will have a reasonable expectation that they will be able to find a working fueling station with predictably priced gasoline before running out of fuel when their refueling warning light comes on. It is, however, common for ICEV drivers who are operating in areas which are sparsely populated and/or unfamiliar to them to maintain a higher fuel state by stopping to fuel more often. Comparing the regularity and reliability of the fueling infrastructure present on interstate highways which traverse the emptiest expanses of the American west to the DC charging infrastructure in the most populated portions of the country is a sobering exercise.

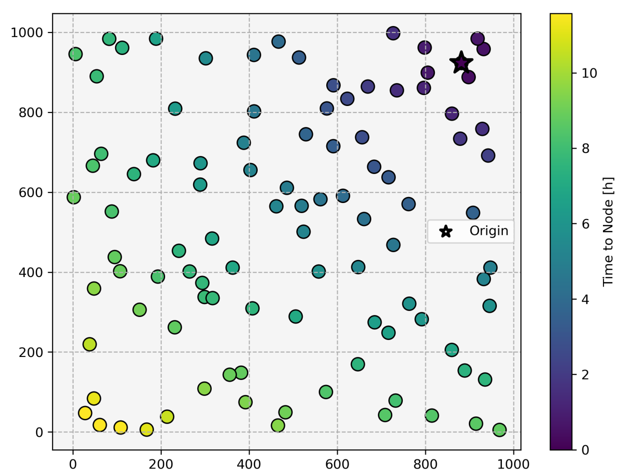
The cost-of-travel for a given O/D pair for a given mode is effected by the network structure as well as underlying mode characteristics. If the driving distance for an O/D is considerably less than half of a given vehicle's range then it might be assumed that traversal will not be effected by charging/fueling. For longer trips, wherein charging/fueling is required itinerary cost (time, distance, price) may be directly added by a chartering/fueling event or may be increased due to the need to take a sub-optimal route to accommodate the event. Figure **??** shows this effect by comparing BEV travel times with and without charging time included with ICEV travel times for the same graph while assuming that all chargers are always usable and available.



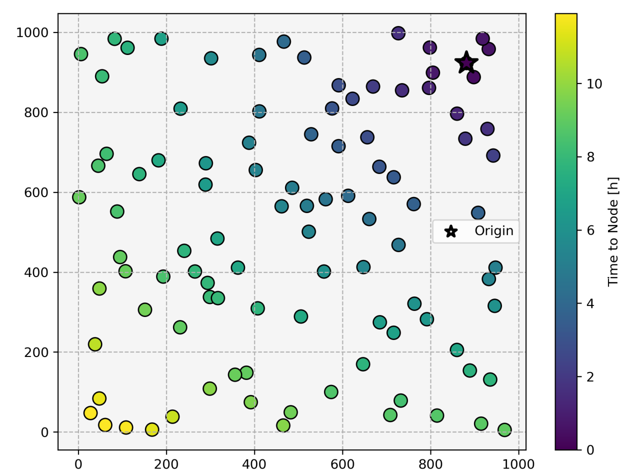
(a) BEV travel times without charging time



(b) BEV travel times with charging time



(c) ICEV travel times without fueling time



(d) ICEV travel times with fueling time

Figure 6: Expected travel times with and without charging/fueling time included for BEV with 400 km of range and ICEV with 600 km of range.

In this case the ICEV is assumed to not be effected by limited fueling opportunities and may purchase fuel while pursuing the optimal route. ICEV fueling time and price were modeled on the basis of 30 seconds per gallon (to account for setup time) and $3.50 per gallon where fueling requirements were assessed. ICEV fueling was assumed to be available at all nodes. The added fueling time for long ICEV trips is not substantial compared to overall trip duration. This is in contrast to the substantial contribution of charging time to overall trip time for long trips taken with BEVs. However, the limited charger availability compared to fueling stations dictates that equivalent BEV trips would take longer even if charging was instantaneous. Constrained BEV routing generally also means longer distance trips following less direct routes which eats into the price advantage provided by BEVs. Optimal route cost quantiles are shown in Figure 7.

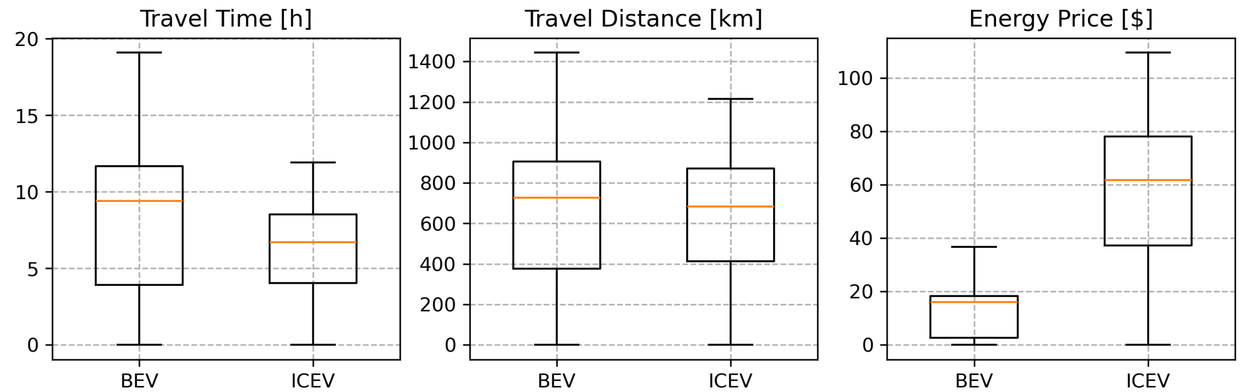


Figure 7: Quantiles of costs for optimal routes for BEVs and ICEVs

While the differential in range-addition rates between charging and fueling is unlikely to be massively reduced in the near future, the influence of charging station location on optimal routing can be reduced by more and better-placed charging stations.

5 Station Centrality

Network centrality is a measure of the importance of a given node to a network. Central nodes need not be at the geometric center of a network map, indeed certain networks wont have a geometric center. The centrality of a given node can be computed relative to position, relative to adjacency, or relative to flow. A computationally simple way to compute adjacency-focused centrality is degree centrality. Degree centrality is defined as

$$\Omega_d(v) = \frac{\eta(v)}{n-1} \tag{7}$$

where $\eta(v)$ is a function returning a value computed from the adjacency of node v. Degree is the number of adjacent nodes adjacent to a given node. Nodes of high degree will often be important to a network but are not necessarily so. As an example, several major US freeways meet in the greater Los Angeles area and service a huge amount of traffic to and from the area. Los Angeles thus has high valency with respect to the US freeway network. However, it is intuitive to note that Los Angeles is not central to the US freeway network being located at a near fringe. Were the freeways in Los Angeles to be shut down performance across the network would be less effected than it would be for the loss of Denver, Kansas City, Oklahoma City, or St. Louis even though Los Angeles is more populated and produces more traffic than any of those mentioned cities. Because of Los Angeles's geographic location, relatively little traffic will flow through it from other origins to other destinations. Thus Los Angeles has relatively low flow centrality. Fundamentally, the reason that degree centrality is misleading for the US freeway network is that its nodes are spatially defined. Indeed, the US transportation network would be more efficient if high-traffic-generation nodes like Los Angeles were located in its geometric center. For networks wherein nodes can be moved or node capacity can be changed (such as logistics networks) over time degree centrality will come to approximate flow centrality.

Flow centrality requires an estimation of network traffic at a node level. To compute traffic, one generally needs to compute lowest-cost paths. Suppose that for a graph G, the sets of shortest paths from each node v to all other nodes $R_{v,U}$ and vice versa $R_{U,v}$ are known. From the optimal routes betweenness centrality and closeness centrality can be computed.

Betweenness centrality is defined as

$$\Omega_b(v) = \sum_{o,d \in V} \frac{\sigma(o,d \mid v)}{\sigma(o,d)} \tag{8}$$

where $\sigma(o,d \mid v)$ is the weighted sum of the set of shortest paths between o and d which contain v and $\sigma(o,d)$ is the weighted sum of the set of shortest paths between o and d. Betweenness centrality reflects the relative volume of use for a node in a graph (and may also be computed for edges). Returning to the US freeway system, since very few O/D pairs would feature shortest paths through Los Angeles, the betweenness centrality of Los Angeles would be appropriately low. However, if weighted by O/D traffic volume, Los Angeles's centrality may be inflated. Another way to look at centrality is closeness centrality defined as

$$\Omega_c(v) = \frac{n-1}{\sum_{u \in U} \eta(v,u)} \tag{9}$$

where n is the cardinality of V and $\eta(v,u)$ is the cost of the shortest path from v to u. For the US freeway network cities like Denver and St. Louis will have high closeness centrality where Los Angeles will have low closeness centrality. The flow based measures of centrality model centrality on the basis of network behavior as observed. Both flow based measures could be applied to networks on the basis of recorded traffic rather than computed shortest paths. In other words, flow-based centrality measures only relate to network structure to the extent that observed traffic is determined by network structure. Flow-based centrality measures are perfect information metrics.

In reality, particularly with respect to transportation, information is imperfect. What this means in practice is that individual drivers have many options for each O/D pair and will not necessarily pick the shortest path. Furthermore, many paths may share edges or sequences of edged and, for much of the route, the remaining route will be uncertain even if the O/D pair is known. As one progresses along a route information will increase and entropy will decrease but this will not be a linear relationship. Certain nodes will be much more determinative than others. A practical example is the decision to merge onto a freeway. Once a driver merges onto a freeway it is very likely that the driver will stay on freeways until close until the destination. On the other hand, when traversing a street grid diagonally no action is particularly determinative over future actions. Information centrality measures the inverse of the information gained upon reaching a given node on the theory that the most central nodes provide the most options. Information centrality for the nodes in $G = \{V, E\}$ is based on the inverse incidence matrix B whose elements are

$$b_{ij} = \begin{cases} 1 & (i,j) \notin E \\ 1 - \eta(i,j) & (i,j) \in E \end{cases} \quad i \neq j \tag{10}$$

$$b_{ii} = 1 + \sum_{(i,j) \in E_i} \eta(i,j) \tag{11}$$

where $\eta(i,j)$ is a function returning edge costs. The diagonal elements of B are the degrees of V plus self-connection and the non-diagonal elements are 1 if no direct connection exists and less than 1 where one does exist. Rows and columns of B all have the same sum if G is reciprocal. The information for each node can be computed form the incidence matrix $C = D^{-1}$ as

$$\Omega_i(v) = \frac{n}{nc_{vv} + \sum_{u=1}^{n} c_{uu} - 2\sum_{u=1}^{n} c_{vu}} \tag{12}$$

The methods of computing centrality are compared for the simple graph shown in Figure 8.

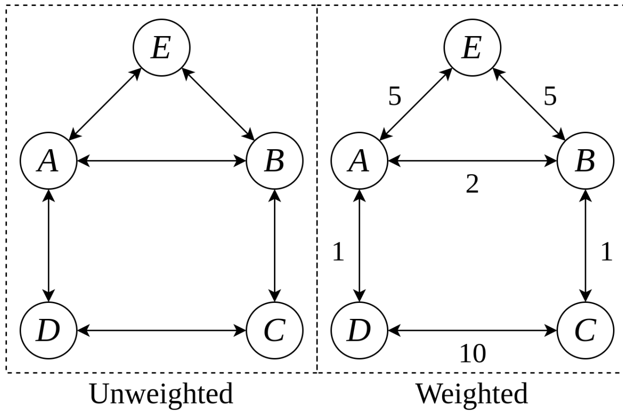


Figure 8: Simple example undirected graph unweighted and weighted isomorphs

Table 1: Centrality for simple undirected and unweighted example graph

Node	Degree Centrality	Betweenness Centrality	Closeness Centrality	Information Centrality
A	0.75	0.25	0.8	0.355
B	0.75	0.25	0.8	0.355
C	0.5	0.083	0.667	0.282
D	0.5	0.083	0.667	0.282
E	0.5	0	0.667	0.275

Table 2: Centrality for simple undirected and weighted example graph

Node	Degree Centrality	Betweenness Centrality	Closeness Centrality	Information Centrality
A	0.75	0.5	0.364	0.667
B	0.75	0.5	0.364	0.667
C	0.5	0	0.286	0.535
D	0.5	0	0.286	0.535
E	0.5	0	0.182	0.646

Concerning the isomorphs in Figure 8, intuition would say that nodes A and B are most central and this is supported by all centrality measures presented. The principle effects of considering edge weights are that the shortest paths between C and D no longer utilize (C,D), all shortest paths include A or B, and 3 include (A,B). The differences are not attested to by degree centrality which is identical among isomorphs. The increased reliance of the network on A and B is reflected in the remaining centrality metrics. It is worth noting that, although the relative closeness centrality of A and B increased to reflect their greater importance, the gross values for A and B decreased due to the greater-than-one edge weights. The lower diversity of paths for the weighted graph is also reflected in the higher information centrality values for the nodes.

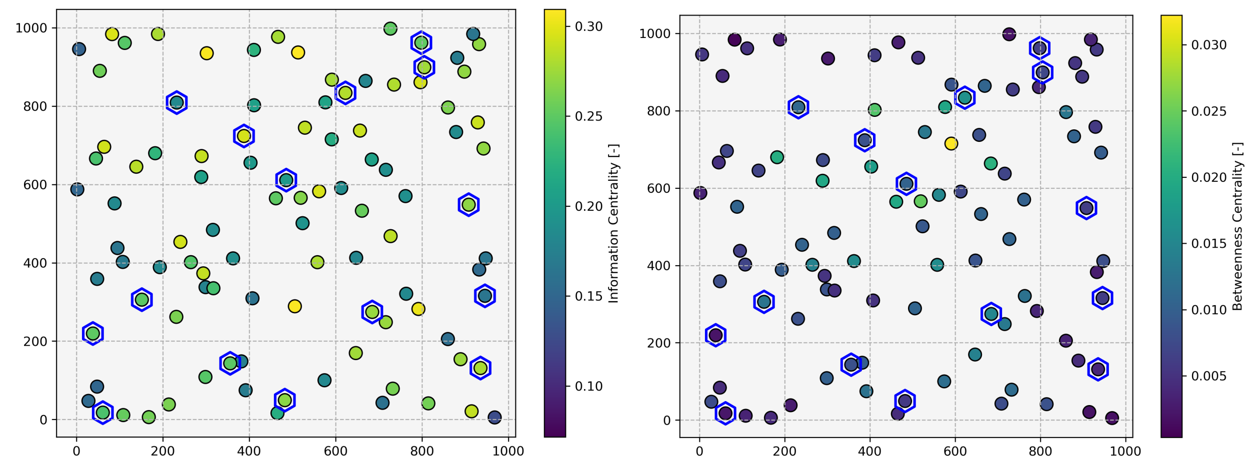
Ultimately, for a given application, some definition of centrality must be chosen. Ultimately, some metric must be computed for whole networks or subnetworks of interest. For transportation network resilience betweenness and information centrality are particularly relevant. It is straightforward that removing a node which is highly central by either definition will cause a shift in traffic which results in worse overall performance. The difference between the measures is exemplified by node E. E has the same degree as C and D but is directly connected to the high degree nodes A and B where C and D are connected to a high degree node and each-other. For the weighted graph, nodes C, D, and E occur on no shortest paths and are, thus, equal from a betweenness perspective. Information centrality, however, captures the subtle difference that after leaving E it will be known if A or B is next. Thus the information gained through visiting E is higher than that gained by visiting C or D because someone *could* traverse (C,D). In other words, **betweenness centrality is descriptive while information centrality is fundamental**. Philosophically a network with generally low and evenly distributed information centrality at all nodes will be quite redundant increasing its resilience while its efficiency and vice versa. A corollary is that network resilience can be optimized for via entropy maximization. Information centrality has the additional advantage of being less costly to compute.

In order to apply information centrality to BEV transportation a slight reformulation is required and this measure is herein called Range-Sensitive Information Centrality (RSIC). The elements of the B matrix for RSIC are

$$b_{ij} = \begin{cases} 1 & (i,j) \notin E \\ 1 & \eta(i,j) > l \\ 1 - \eta(i,j) & otherwise \end{cases} \tag{13}$$

$$b_{ii} = 1 + \sum_{(i,j)\in E_i} \eta(i,j) \tag{14}$$

where l is vehicle range. This modification reflects the inadmissibility of edges greater than vehicle range. RSIC for node i is calculated as in (12). Range-Sensitive Betweenness Centrality (RSBC) is the equivalent of betweenness centrality for vehicles and is computed as in (8) also neglecting those edges which exceed vehicle range. Figure 9 shows RSIC and RSBC for the randomly generated transportation system example.



(a) Information Centrality

(b) Betweenness Centrality

Figure 9: Centrality measures for example transportation network.

Information RSIC is likely the more useful metric for the above scenario. RSBC is more heavily influenced by geometric proximity and thus, minimizing RSBC will be less useful for increasing resilience. RSIC minimization could be done on the basis of gross or average RSIC or on the basis of inequality (Gini coefficient).

More to come ...

A Appendix: Stochastic Optimization Example - Crop Allocation

Problem Description:

A farmer has 100 total acres of fields to plant and may choose from among three crops. Each crop has a different cost per acre for seeds. Depending on meteorological conditions, each crop will have a different revenue per acre in low, medium, and high yield scenarios. Seeds must be purchased in the fall but the farmer has until the spring to actually decide what to plant and at that point he will have more information about the weather for the coming year. Any excess seeds may be re-sold for a 20% loss. What quantities of seed should the farmer purchase in order to maximize profit?

Discussion:

In the example the farmer has to make one set of decisions (what quantities of seeds to buy for the three crops) which holds for all yield scenarios. These are general controls. The farmer then has a set of decisions (what quantities of seed to plant and to resell for each crop) which may vary in different yield scenarios. These are specific controls. The problem contains 3 general controls and 9 specific controls (3 per scenario) for a total of 12 controls. The expected profit for the farmer at the time of seed purchase is the total cost from (4) over the number of scenarios.

If the farmer had complete information about the following year's weather at the time of seed purchase he could optimize both seed purchase and seed planting for specific scenarios. In this case the farmer could compute optimal controls for each scenario individually (6 specific controls per scenario). The difference between the expected profit from the stochastic optimization and the mean value of the profits from the deterministic optimizations is called the "cost-of-uncertainty" or the "value-of-information".

A few things are worth mentioning specifically. first, the cost-of-uncertainty should be positive (expected profit less than mean profit). In the example, the farmer may have to buy more seeds than he can possibly plant due to uncertainty and have to sell the excess at a loss. In the deterministic scenarios the optimal solution will never call for purchasing more seeds than needed for planting if the resell value is less than the purchase value. It is intuitive to see that the farmer is paying a price for his decision to purchase seeds before it is possible to predict the weather with sufficient accuracy. Second, stochastic optimization reduces to deterministic optimization with the removal of general controls. In other words, a stochastic optimization problem is an optimization problem with at least one general control and need not have any specific controls.

B Appendix: Generic Routing Algorithm Formulations

A generic formulation of Dijkstra's algorithm is as follows:

Algorithm 2 Dijkstra Routing Algorithm

$G = \{V, E\}$ ▷ Graph consisting of nodes and links
$C = \{C_V, C_E\}$ ▷ Traversal costs corresponding to each node and link
$O \in V$ ▷ Single origin node
$D \in V$ ▷ Set of destination nodes

$S = \{\}$ ▷ Set of visited nodes
$W = \{\infty\}_{v \in V}$ ▷ All node weights initialized to infinity
$P = \{\{\}\}_{d \in D}$ ▷ Initializing empty path for each destination
$F = \{(w_{o_0}, o_0)\}$
▷ Heap queue containing nodes to be visited. Elements are tuples (weight, node) and the heap is ordered by weight. The heap is initialized with the origin node.

while $F \neq \emptyset$ **do** ▷ Iterate while there are reachable nodes which have not been visited
  $w_v, v = f_0$
  $F = F \not\cup f_0$ ▷ Remove current node from heap queue
  $E_v = \{(v_s, v_t) \in E | v_s = v\}$
  for all $(v, v_t) \in E_v$ **do** ▷ Iterate through links of v
    if $w_v + c_{v,(v,v_t)} < w_{v_t}$ **then** ▷ If current path represents savings ...
      $p_{v_t} = p_v \cup \{v_t\}$ ▷ Update path to v_t
      $w_{v_t} = w_v + c_{v,(v,v_t)}$ ▷ Update cost at v_t
      if $v_t \notin S$ **then**
        $F = F \cup \{(w_t, v_t)\}$ ▷ If v_t has not been visited add to heap queue
      end if
    end if
  end for
  $S = S \cup \{v\}$ ▷ Add current node to set of visited nodes
end while