

An optimization model for electric vehicle charging infrastructure planning considering queuing behavior with finite queue length

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ABSTRACT

As clean energy vehicles, electric vehicles (EVs) have been paid unprecedented attention in dealing with serious energy crises and heavy tailpipe emission in recent years. Due to its limited battery range and long charging time, it's significant to reasonably determine the locations and capacities of EV charging infrastructure. There are two research gaps in existing researches: unrealistically assuming the infinite queuing length based on the $M/M/1$ or $M/M/S$ queuing model and lacking the research on variable quantities of chargers allocated at different charging stations. To fill up these gaps, we propose an optimal location model to determine the optimal locations and capacities of EV charging infrastructure to minimize the comprehensive total cost, which considers the charging queuing behavior with finite queue length and various siting constraints. And the results show that (1) the proposed model has a good performance in determining the optimal locations and capacities of EV charging infrastructure (i.e. the optimal locations of charging stations, the optimal quantities of chargers installed at each charging station, the optimal allowable maximum queue length and maximum capacity of each charging station); (2) the quantity of chargers and allowable maximum queue length at each charging station are consistent with the distribution densities of existing charging stations at these locations; (3) the two parameters of unit value of time and unit distance cost have a more significant impact on the total cost. Therefore, the total cost can be effectively reduced by appropriately increasing the quantity of chargers at each charging station and the distribution density of charging stations.

1. Introduction

1.1. Background

With the increasing quantities of oil-fuel vehicles, the environmental pollution, greenhouse emission and fossil energy shortage caused by road traffic are growing worse [1]. In 2015, about half of the air pollution in China was caused by tailpipe emissions from private vehicles [2]. Meanwhile, private vehicles was also a major contributor to about 70% of petroleum consumption and 27% of carbon emission in the USA [3]. Compared to traditional internal combustion engine (ICE) vehicles, electric vehicles (EVs) have great advantages of low gasoline consumption, high energy conversion rate, and low/zero tailpipe emissions [4]. This means that EV is an appropriate response to the challenges of energy shortage and environment pollution related to the traditional vehicles [5]. Therefore, many countries are actively promoting and popularizing EVs. However, on the one hand, although

there are large quantity of public charging stations worldwide, their quantity and distribution density still do not match the number of EVs. On the other hand, restricted by existing battery technology, the battery range of EVs is still limited. Although some studies claimed that the maximum range can reach to 300 miles [4], many existing studies still generally assumed that the default battery range of EVs was 80–100 miles [6–11]. Due to these reasons, people are prone to have range anxiety when they choose EVs, namely concern about insufficient range to reach their destinations, which is the steepest barrier to widespread adoption of EVs. These could further lead to the lower customer acceptance of EVs. The survey indicated that only about 18.1% of the consumers would be willing to replace their oil-fuel vehicles with EVs [12]. To improve the situation, an effective way is to optimize the deployment of EVs charging infrastructure (that is, the locations of charging stations and the quantity of chargers installed at each charging station). Therefore, the optimal deployment of EV charging infrastructure has attracted unprecedented attention, and this is also what

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Nomenclature

EV	electric vehicle
ICE	internal combustion engine
FLP	facility location problem
PEV	plug-in electric vehicle
FRLM	flow-refueling location model
BEV	battery electric vehicle
UE	user equilibrium
GPS	global positioning system
AC	alternative current
DC	direct current
FCFS	first-come and first-served
LCFS	last-come and first-served
kW	kilowatt
kWh	kilowatt hours

Decision variables

X_j	binary variable, which equals one if a charging station is built at location j and zero otherwise
s_j	the number of chargers installed at charging station j

Parameters

I	set of candidate locations
$C_{station}$	the charging stations construction cost
$C_{charger}$	the chargers cost (i.e., purchase cost and installation cost)
$C_{infrastructure}$	the comprehensive infrastructure cost

$C_{waiting}$	the queueing time cost
$C_{distance}$	the distance cost or the charging station access cost
$c_j^{station}$	the fixed construction cost of a charging station built at station j
$c_j^{charger}$	the cost of installing a charger
$c_j^{waiting}$	the value of unit time
$c_j^{distance}$	the unit distance cost
N_j	the maximum capacity of charging station j
$p_j(0)$	the probability of there has no EV at charging location j
L_{jq}	the effective queue length at charging location j
W_j	the expected queueing time of each EV at charging station j
Y_{ij}	binary variable, which equals one if the users drive to charge their EVs from location i to charging location j and zero otherwise
m_i	the number of EVs at a candidate location i
d_{ij}	the distance between location i and j
λ	EV mean arrival rate
μ	mean charging service rate
ρ	the service density
δ	the ratio of the chargers' quantity to the maximum queue length
A	a small positive number
B	a large positive number
D	the users' tolerable maximum range
T	the lifespan of the charging infrastructure
r	the discount rate
n	the number of EVs

we will focus on in this paper.

To make the research problem of this paper more clearly, we describe it with the following scenario. Daily, people usually travel from their trip origins (e.g., residences) to their destinations (e.g. workplaces, shopping malls, parks, schools, etc.) to conduct various social activities. EVs can be charged using household chargers when parked at home and using public chargers when outside. So we assume that EVs have enough power to arrive at the destinations without recharging halfway and charging demands are only generated on the return trips. If there are charging stations built at destinations, people can charge their EVs there directly. If not, they need to drive to a nearby charging station for charging based on the premise that the distance between destination and nearby charging station is within their tolerable range. EV is left to recharge at this station, and then they will return to their destinations on foot or by other transportations. Assuming that there are more than one charging station within the tolerable range, the nearest one is the choice. Actually, even though there is a charging station at the destination, when without available chargers or even already queueing to wait for charging, people may leave for another nearby charging station to avoid waiting too long. Obviously, the more charging stations and chargers mean the larger charging service capacity and the lower probability of queueing waiting for charging, but the higher construction cost. Therefore, taken service satisfaction and cost budget into consideration, where to properly build the public charging stations and how many chargers installed at each charging station are the most urgent problems we need to solve.

1.2. Literature review

Due to the limited battery range and long charging time of EVs, it's of great significance to reasonably deploy the EV charging infrastructure to achieve a win-win situation of meeting charging demand and minimizing cost. Quite a few scholars used "point demand approach" to site the charging infrastructure, which is to place these

stations near the trip generation and attraction points, such as residences, hospitals, schools, work places, and shopping malls [13–18]. The essence of this approach is a p-median or set-covering facility location problem (FLP). Some studies also investigated the maximum capture of traffic flow to minimize the number of facilities [8,9,17,19–21]. The method is generally based on the user equilibrium (UE) principle to connect flow distribution and the deployment of charging infrastructure. Moreover, data-driven approaches were also applied to deploy the charging infrastructure locations. They were developed based on a large number of available trajectory data and simulation data [10,22–27].

Furthermore, considering the limited charging capacity and the increasing quantity of EVs in recent years, it is more practical to consider the queueing charging when charging demand exceeds a station's charging capacity. Considering the charging station capacity, charging time, and waiting time, some researchers [28–30] combined queueing theory to build the EV charging station planning models, and determined the optimal locations of charging stations and the optimal scale (the quantity of chargers and waiting spaces). To ensure the service quality, a data-driven plug-in electric vehicle (PEV) charging station planning model was formulated to analyze the impact of charging queueing on the whole operating system [31]. An extended flow-refueling location model (FRLM) was proposed to design an optimal EV charging spots network that satisfied the maximum charging demand without exceed the given users' longest tolerable queueing time [32]. The charging strategy put users' cost on the top priority in two different situations (regular routes and irregular routes), and the irregular routes situation was combined queueing time into the optimal model [33]. In this model, the queues of multiple charging stations in a certain area were integrated into a single queue in advance, and EVs of the queue were centrally dispatched to the available charging stations. The simulation results showed that it can reduce over 60% queueing time compared to queueing at each charging station separately. Similarly, the study proposed a comprehensive model, which first regarded the

upcoming EVs as one queue and then scheduled them according to the current queuing charging situation to realize the minimum waiting time of users [34]. With the similar objective function, a bi-level model and simulation-optimization framework were proposed to determine the optimal locations of electric taxi charging stations and optimal allocation of chargers, which considered the queuing delay and stochastic dynamic demand to minimize the average delay [24]. Applied the queuing theory, an optimal EV charging station system was established with the minimum infrastructure cost, and simulation results showed that the distance between charging stations and queue length of each station both had impacts on queuing time [35]. Combined the $M/M/x/s$ queuing model with the location problem, these researchers [36] formulated an optimal model to allocate charging stations and chargers for battery electric vehicles (BEV) taxis using large-scale GPS trajectories of over 7,910 taxis to minimize the total cost.

From the above discussions, we can see that the existing literature has two research gaps. First, considering the queuing charging behavior, there are a handful of studies developing the optimal location models of EVs charging infrastructure with the minimum total cost of the operators and users as the objective function. And among them, the majority adopted the $M/M/1$ or the $M/M/S$ queuing model, assuming that the queue length is allowed to be infinite. Studies by behavioral scientists indicates that, generally, when persons wait for more than 10, 20 and 40min, they start feeling irritable, become bored, and leave with anger, respectively [35]. So obviously, assuming infinite queue length was out of line with reality. Second, while a few of studies considered finite queue length that adopted the $M/M/S/N$ queuing model, they assumed that the quantity of chargers installed at each charging station S were all the same among different charging stations. Actually, for each charging station, the value of S should be diverse from each other based on the different charging demand.

1.3. Contributions and organization of the paper

In this paper, we aim to fill these gaps by developing an optimization model to deploy the optimal locations and capacities of charging stations with the minimum total cost as the objective function. Specially, there are two main contributions: (1) we adopt the $M/M/s_j/N$ queuing model, which the queue length is finite and the maximum

queue length is allowed to be N ; (2) we assume that the quantity of chargers installed at each charging station are diverse. The s_j in the $M/M/s_j/N$ queuing model represents the quantity of chargers installed at charging station j . It means that the quantity of chargers at each station varies from the respective different charging demand. Then we apply the model to determine the optimal locations and capacities of charging infrastructure in a certain area of Manhattan, New York, USA as a case study. Finally, the sensitivity analyses of some key parameters are carried out, and the results and conclusions are summarized.

The rest of this paper is organized as follows. Section 2 is the optimization model which illustrates the cost structure of model, and then formulates the model. The results analyses and discussions are given in the Section 3 that describes the study area, lists the model parameters and illustrates how the various cost components and optimization results vary with several key parameters. The conclusions are summarized by Section 4 that also discusses the directions of future research.

2. Optimization model

2.1. Cost structure of model

The total cost of operating an EV charging station system can be decomposed into two parts: (1) the first part is the cost of construction stage which is called upper level cost: for the decision makers, it consists of initial infrastructure installation cost (i.e. costs of charging station construction and chargers installation) and equipment depreciation cost; and for the users, it includes the distance cost between destinations and charging stations and queuing waiting cost; (2) the second part is the cost of operation stage which is summarized as lower level cost: for the decision makers, it consists of equipment maintenance cost, staff salary cost and schedule cost; and for the users, it includes charging cost and valet parking cost, respectively. The total cost structure is shown in Fig. 1.

This research does not involve the charging process and schedule process, so the upper level cost minimization is the objective function of the optimal model. Due to no research about the charging process, the different charging modes (e.g., slow charging (alternative current (AC) level-1 charging and AC level-2 charging) and fast charging (direct current (DC) charging)) are not studied separately. That is, we assume

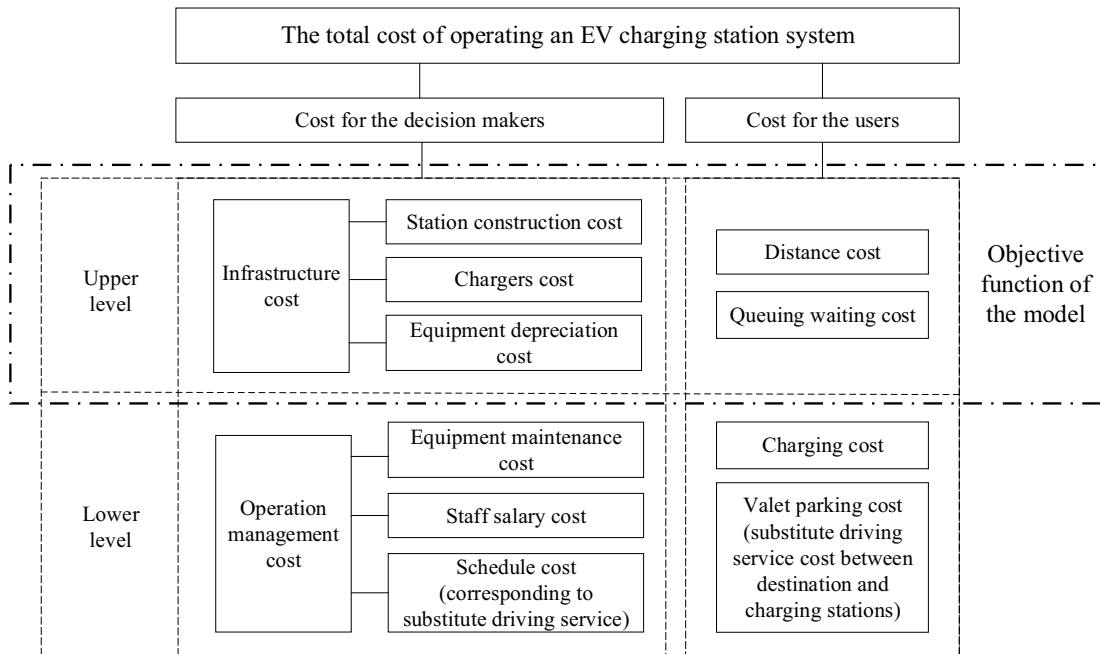


Fig. 1. Illustration of EV charging station total cost.

that the charging mode is uniform and it is slow charging in this paper. And the charging duration is not taken into account, either.

2.2. Model formulation

We assume that a set of candidate locations I distributes in an area covering workplaces, hospitals, schools, shopping malls, parks and so on, namely the set of destinations. If a certain candidate location $i \in I$ is selected to build a charging station, the chargers will be installed at this location. Other candidate locations not selected are only for EV parking.

In this model, for the decision makers, they need to afford the infrastructure cost, including the costs of charging stations construction, chargers installation and equipment depreciation. Among these costs, the charging stations construction cost is denoted as following:

$$C_{station} = \sum_{j \in I} c_j^{station} \times X_j \quad (1)$$

where $c_j^{station}$ denotes the fixed construction cost of building a charging station at location $j \in I$, and X_j is a binary decision variable indicating whether a charging station is built at station j . If it does, $X_j = 1$; otherwise, $X_j = 0$. Similarly, the chargers cost consisted of purchase cost and installation cost is formulated by Eq. (2):

$$C_{charger} = c^{charger} \times \sum_{j \in I} s_j \times X_j \quad (2)$$

where $c^{charger}$ is the cost of installing a charger including the unit price and installing fee. s_j represents the number of chargers installed at charging station j , which means it is a nonnegative integer. It can be converted to a formulation as

$$s_j \in \mathbb{Z}^+ \cup \{0\} \quad (3)$$

To ensure the service quality, s_j should not be too small; while s_j is not allowed to be too large, either, which is subject to the cost and available coverage area of charging stations. Thus, s_j is restricted as

$$A \times X_j \leq s_j \leq B \times X_j \quad (4)$$

where A and B represent a small positive number and a large positive number, respectively. Thus, the comprehensive infrastructure cost for the decision makers can be denoted as

$$C_{infrastructure} = \frac{r(1+r)^T}{(1+r)^T - 1} \times (C_{station} + C_{charger}) \quad (5)$$

where T represents the lifespan of the charging infrastructure and the equipment depreciates at a discount rate of r year by year.

For the users, the costs they need to bear are the distance cost (i.e. charging station access cost) and the queuing time cost. As mentioned above, if the destinations happen to build charging stations and still have available chargers, users can charge their EVs directly. However, if chargers are all occupied while waiting spaces are available at a charging station, users can still choose this station for charging, but they need to join into the queue to wait, namely charging congestion. The service may follow a certain order, such as first-come and first-served (FCFS), last-come and first-served (LCFS), priority service, and random service [37]. We assume that the EVs are charged in the FCFS order in this paper. The maximum capacity of charging station j is defined as N_j , and it can be calculated as

$$N_j = s_j + \left\lceil \frac{s_j}{\delta} \right\rceil \quad (6)$$

where δ denotes the ratio of the chargers' quantity to the maximum queue length at charging station j . In short, when there have been $N_j - s_j$ EVs queuing up at charging station j , the $N_j - s_j + 1$ th EV will not join into the queue and leave. That is to say that $\lceil \frac{s_j}{\delta} \rceil$ equates to the maximum queue length (i.e. maximum waiting spaces) at the charging station j . In conclusion, it's a multiple service finite queue length standard that is expressed as $M/M/S/N$. The first M stands for EV

arrivals following Poisson distribution with mean λ and the second M stands for charging service time following negative exponential distribution with mean μ . The number of chargers is S . The N means the maximum capacity of a charging station holding EVs (equals to S plus waiting spaces). The EVs charging service density ρ is the ratio of arrival rate λ and charging service rate μ , shown as Eq. (7).

$$\rho = \lambda/\mu \quad (7)$$

$$\rho/S = \lambda/\mu S \quad (8)$$

The value of Eq. (8) is restricted to be less than 1, otherwise, the queue length will never fade away and the system is unstable. Then, based on the above formulations and other queuing formulations given by references [37,38], through a series of mathematical derivations, we can obtain the effective queue length L_{jq} and the expected queuing waiting time of each EV W_j at charging station j ,

$$L_{jq} = \frac{\rho^{s_j+1}}{(s_j-1)!(s_j-\rho)^2} \times \left[\frac{1 - (\rho/s_j)^{N_j-s_j+1}}{-(N_j-s_j+1)(1-\rho/s_j)(\rho/s_j)^{N_j-s_j}} \right] \times p_j(0) \quad (9)$$

$$W_j = \left\{ \frac{\rho^{s_j+1}}{(s_j-1)!(s_j-\rho)^2} \times \left[\frac{1 - (\rho/s_j)^{N_j-s_j+1}}{-(N_j-s_j+1)(1-\rho/s_j)(\rho/s_j)^{N_j-s_j}} \right] \times p_j \right. \\ \left. (0) \times \frac{1}{\lambda \left[1 - \frac{\rho^{N_j}}{s_j! s_j^{N_j-s_j}} \times p_j(0) \right]} \right\} \times X_j \quad (10)$$

where $p_j(0)$ represents the probability of no EV at charging location j (i.e. neither charging nor queuing), and it equals

$$p_j(0) = \begin{cases} \left[\sum_{n=0}^{s_j-1} \frac{\rho^n}{n!} + \frac{\rho^{s_j} [1 - (\rho/s_j)^{N_j-s_j+1}]}{s_j! (1-\rho/s_j)} \right]^{-1} \times X_j, & \text{for } \rho/s_j \neq 1 \\ \left[\sum_{n=0}^{s_j-1} \frac{\rho^n}{n!} + \frac{\rho^{s_j}}{s_j!} (N_j - s_j + 1) \right]^{-1} \times X_j, & \text{for } \rho/s_j = 1 \end{cases} \quad (11)$$

Furthermore, we can calculate the queuing time cost for all users as

$$C_{waiting} = c^{waiting} \times \sum_{j \in I} (L_{jq} \times W_j) \quad (12)$$

where $c^{waiting}$ is the value of unit time.

On the other hand, if there is no charging station at the destination, users will need to drive to the nearest and available (still having unoccupied waiting spaces, or even unoccupied chargers) charging station within their tolerable range. We take the distance cost between the users' destinations and the satisfied charging stations (i.e. charging station access cost) into consideration, and it is denoted as

$$C_{distance} = c^{distance} \times \sum_{i \in I} \sum_{j \in I} d_{ij} \times m_i \times Y_{ij} \quad (13)$$

where $c^{distance}$ is the unit distance cost and m_i represents the number of EVs at a candidate location i , $\forall i \in I$. Y_{ij} is a binary decision variable that indicates whether the users drive to charge from location i to charging station j . If they do, $Y_{ij} = 1$; otherwise, $Y_{ij} = 0$. It is restricted that an EV can be charged at a certain location only if a charging station is built there and each EV can only be charged at one charging station, which can be expressed as Eq. (14) and Eq. (15), respectively.

$$Y_{ij} \leq X_j \quad (14)$$

$$\sum_{j \in I} Y_{ij} = 1 \quad (15)$$

Defining d_{ij} as the distance between location i and j , it is restricted within the users' tolerable range when the users drive to charge their EVs from location i to j , namely

$$\sum_{j \in I} d_{ij} \times Y_{ij} \leq D \quad (16)$$

where D is the maximum distance users can accept between their destinations and charging stations.

In order to unify the expense standard of decision makers and users on the same time magnitude, we adopt one day (24 h) as a cost cycle, that is, the minimum total daily cost of the system as the objective function:

$$\min \frac{1}{365} \times C_{\text{infrastructure}} + C_{\text{waiting}} + C_{\text{distance}} \quad (17)$$

subject to Eqs. (3),(4),(6),(10),(11),(14)–(16) and Eqs. (18) and (19)

$$X_j = \{0, 1\} \quad (18)$$

$$Y_{ij} = \{0, 1\} \quad (19)$$

3. Results and discussion

In this section, we apply the above proposed model to a case study to illustrate the validity of the model. The genetic algorithm (GA) is used to optimize the deployment of EV charging infrastructure, and the idea of solving integer programming is combined simultaneously.

3.1. Study area

This paper select a small area within 1 mile radius from the center of the Empire State Building in Manhattan midtown, New York as the study area. As of December 2019, there are about 80 EV charging stations installed in this area, which are all in operation and open to the public (see Fig. 2(a)). The locations of the existing EV charging stations in this area are acquired from the EvaluateNY [39]. It is a data analysis tool that provides access to the comprehensive data on the New York

State's EV market by collecting data from public and private activities.

Considering the land use and destination diversities, we first randomly group the densely distributed charging stations into 20 candidate locations (Fig. 2(b)), which distribute as evenly as possible and try to cover the hospitals, parks, transport hub stations, schools, office buildings and shopping malls, etc. (Fig. 2(c)). The 20 locations will be the candidate locations for constructing charging stations.

3.2. Model parameters

The fixed costs of each EV charging station c^{station} and each charger c^{charger} are \$900,000 and \$120,000, respectively [40]. The unit value of queuing time c^{waiting} is \$29.72/h computed based on the ratio of average income of \$62765/person/year [41], 12 months/year and 176 working hours/month in New York. We assume that, D , the maximum tolerable distance for users from the destinations to charging stations is 2 miles [4], namely the service range of charging station. Distances between each candidate location d_{ij} are derived according to the Google Map. And the unit distance cost for users c^{distance} is equal to \$4.72/mile which includes the EV electricity consumption cost of \$0.08/mile driving from the destination to charging station and the walking time cost of \$4.64/mile from charging station to the destination. Therein, the former is based on the EV electricity consumption rate of 0.35 kWh/mile [42] and New York electricity price of \$0.23/kWh, and the latter is based on a walking speed of 4 km/h [43] and the value of time of \$29.72/h. Default values of the key parameters are summarized in Table 1.

3.3. Optimal results of base scenario

Before the sensitivity analysis, we present the optimal results of base scenario, as shown in Table 2. The optimal total number of charging stations is four, and the optimal locations are the 2nd, 8th, 10th and

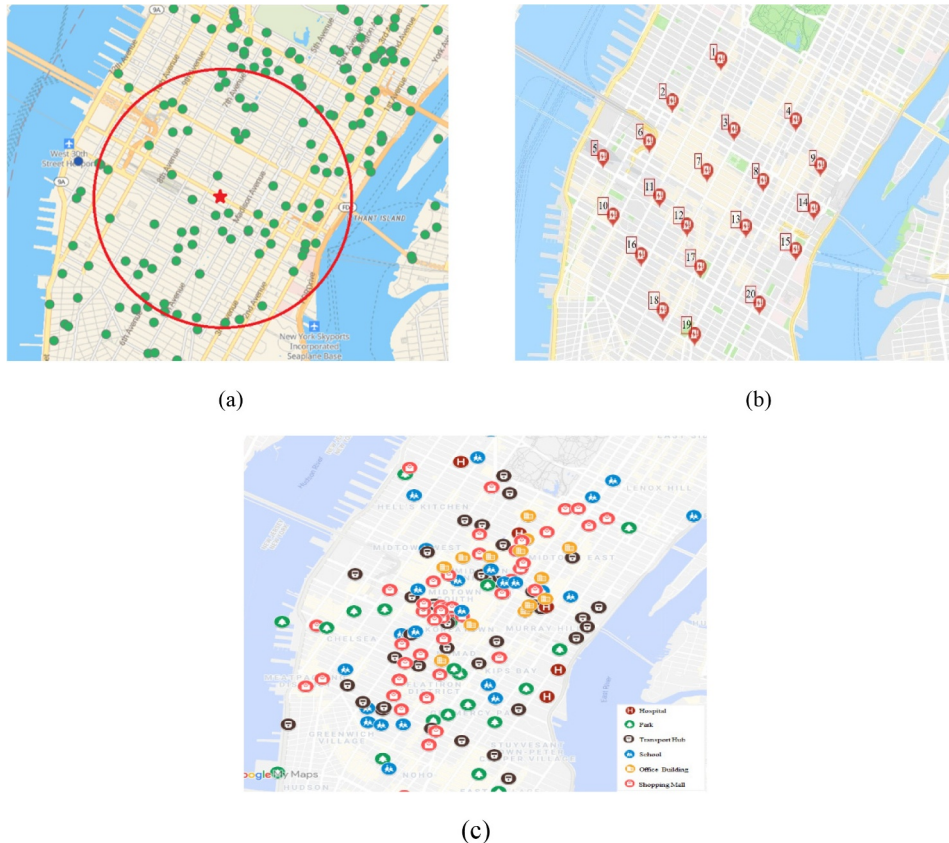


Fig. 2. The layout of existing EV charging stations (a), 20 candidate locations (b) and the distribution of various public places (c) in Manhattan midtown, New York.

Table 1
Default values of some parameters.

Parameters	Default value	Meaning
$c^{station}$	\$900,000	Fixed cost of each charging station
$c^{charger}$	\$120,000	Fixed cost of each charger
$c^{waiting}$	\$29.72/hour	Unit value of queuing time
$c^{distance}$	\$4.72/mile	Unit distance cost
T	10	Lifespan of charging infrastructure
r	0.1	Discount rate
D	2 miles	Maximum tolerable distance
$[A, B]$	[1,50]	Number interval of chargers
δ	5	Ratio of chargers' quantity to maximum queue length

17th candidate locations, respectively, namely $X_2 = 1$, $X_8 = 1$, $X_{10} = 1$ and $X_{17} = 1$. According to Fig. 2(b and c), it's not hard to find that the four charging stations are almost evenly distributed in the study area, basically covering parks, transport hub stations, schools, office buildings and shopping malls. And correspondingly, the optimal number of chargers allocated at each charging station are 4, 7, 6 and 6, respectively. The allowable maximum queuing lengths at these charging stations are calculated by ceil (rounded upwards to the next integer) and the results are 1, 2, 2 and 2 EVs, respectively. Further, the maximum capacity of each charging station are 5, 9, 8 and 8 EVs, which are equal to the sum of s_j and $\lceil \frac{s_j}{\delta} \rceil$. As shown in Fig. 2(a), the 2nd charging station is configured the fewest chargers obviously, which is consistent with the low distribution density of existing charging stations in the north-west of study area. This phenomenon indicates that uniform distribution of charging stations in the study area can ensure the maximum coverage, and variable quantities of chargers allocated at each charging station can meet various charging demands of different locations.

3.4. Sensitivity analysis

In this section, we will illustrate how the values of each sub-cost $C_{station}$, $C_{charger}$, $C_{waiting}$, $C_{distance}$, the total cost TC , the optimal quantity of charging stations N^* and chargers S^* vary with the several key unit price parameters $c^{station}$, $c^{charger}$, $c^{distance}$ and $c^{waiting}$.

3.4.1. Parameter: fixed cost of each charging station

In this part, Table 3 lists the comparisons of optimal results between the base scenario and variation scenarios of $c^{station}$, and Fig. 3(a)–(c) depicts the sensitivity of the cost components, the optimal quantity of charging stations and chargers to the unit fixed charging station construction cost $c^{station}$. We see that in Fig. 3(a), as $c^{station}$ increases, the total cost TC and the charging station construction cost $C_{station}$ almost both increase with a steady slope, while the queuing time cost $C_{waiting}$ and the charger cost $C_{charger}$ both decrease with gradually flat decreasing slopes. Combined with Fig. 3(b), this could be explained that when $c^{station}$ increases, the total number of charging stations N^* keeps decreasing, then system tries to allocate more chargers at each charging station (i.e. s_j increasing) on the original basis to meet charging demand but the total number of chargers S^* still decreases (see Fig. 3(c) and Table 3). Because the value of S^* is affected by both N^* and s_j , the above phenomenon indicates that N^* has more conspicuous influence on S^* than s_j . By installing more chargers at each charging station, the queuing waiting gets alleviated, and thus $C_{waiting}$ decreases. The distribution density of charging stations reduces with the decrease of N^* , so the distance cost $C_{distance}$ increases. When $c^{station}$ is equal to 1.2–2.5 million dollars and over 3 million dollars, N^* and $C_{distance}$ remain unchanged, as shown in Fig. 3(a and b). When $c^{station}$ exceeds 3 million dollars, $C_{charger}$, $C_{waiting}$ and $C_{distance}$ all keep constant values, that is, the growth of $C_{station}$ will directly lead to the growth of TC .

3.4.2. Parameter: fixed cost of each charger

The optimal results of the base scenario and variation scenarios are presented in Table 4, and Fig. 4(a)–(c) shows the sensitivity of the fixed cost of each charger $c^{charger}$. As shown in Fig. 4(a), TC , $C_{charger}$, $C_{waiting}$ and $C_{distance}$ all increase with the increasing of $c^{charger}$, while $C_{station}$ has the opposite variation trend. Fig. 4(b and c) plots that, as $c^{charger}$ increases, N^* and S^* reduce sharply first and then slow down into a smooth curve, but the decreasing slope of S^* is steeper than the slope of N^* . This indicates that not only the total number of chargers S^* decreases with the decreasing of N^* , but also the number of chargers allocated at each charging station s_j decreases synchronously (see Table 4). Thus, based on the variation of S^* , $C_{waiting}$ goes up by a distinct slope firstly and then it almost flattens out (see Fig. 4(a)). Due to the offset and complementarity between the growth of $c^{charger}$ and the reduction of S^* , $C_{charger}$ increases moderately. It is found from Fig. 4(b) that N^* experiences three different stages: firstly decreases faster, then decreases slightly slower and finally keeps steady. The probable reason is that the system will tend to decrease the quantity of charging stations and/or partially sacrifice users' benefits, consequently offsetting the increasing costs of $C_{charger}$ and $C_{waiting}$ resulted by the increase of $c^{charger}$. Obviously, the decrease of N^* results in the decrease of charging station distribution density, then further leads to the increase of $C_{distance}$. And the three variation stages of $C_{distance}$ match with the three stages of N^* .

3.4.3. Parameter: unit value of time

Table 5 shows how the optimal results of variation scenarios vary with the unit value of time $c^{waiting}$. Fig. 5(a)–(c) illustrates the impact of varying unit value of time $c^{waiting}$ on various costs and optimization results. TC and $C_{waiting}$ show a obvious near-linear growth trend, while the variations of $C_{station}$, $C_{charger}$ and $C_{distance}$ behave gently (see Fig. 5(a)). Increasing the total quantity of chargers S^* to enhance the whole system's service capacity can effectively restrain the excessive growth of $C_{waiting}$ with the increase of $c^{waiting}$. Further, N^* should be reduced appropriately to balance the increase of $C_{charger}$ resulted by the increase of S^* . From the above analyses, we know that, when N^* decreases, the more conspicuous the effect of N^* on S^* , the lower the growth slope of S^* , and even decreasing to a negative growth slope (see Fig. 5(a)–(c)). Naturally, $C_{distance}$ increases with the decrease of N^* , and the growth slope of $C_{distance}$ varies synchronously with the decrease slope of N^* (see Fig. 5(a and b)).

3.4.4. Parameter: unit distance cost

In this part, Table 6 lists the comparisons of optimal results between the base scenario and variation scenarios of $c^{distance}$, and Fig. 6(a)–(c) shows how the various costs and optimization results vary with the unit distance cost $c^{distance}$. We can see that various costs TC , $C_{station}$, $C_{charger}$, $C_{waiting}$ and the optimization results N^* , S^* all develop towards growth trend until reach stability, except for $C_{distance}$. As $c^{distance}$ increases, in order to restrain the rapid growth of $C_{distance}$ to guarantee users' benefits, the system will properly increase N^* to improve the distribution density and shorten the distance between charging stations as shown in Fig. 6(b). As we can see from Fig. 6(b and c), compared with N^* , the increase of S^* has a more gradual slope, that is, S^* increases while s_j

Table 2
Optimal results of the base scenario.

Base scenario	Optimal value				Meaning
N^*	4				Total number of charging stations
$j(X_j = 1)$	2	8	10	17	Each charging station's serial number
S^*	23				Total number of chargers
s_j	4	7	6	6	Number of chargers allocated at charging stations j
$\lceil \frac{s_j}{\delta} \rceil$	1	2	2	2	Maximum queuing length at charging station j
N_j	5	9	8	8	Maximum capacity of charging station j

Table 3
Optimal results of the base scenario and $c^{station}$ variation scenarios.

Parameter	Value (\$)	N^*	$j(X_j = 1)$	S^*	s_j	$[\frac{s_j}{\delta}]$	N_j
Base scenario		4	2,8,10,17	23	4,7,6,6	1,2,2,2	5,9,8,8
$c^{station}$	0	16	1,2,3,4,5,6,7,8,9,10,11,15,16,18,19,20	50	3,3,3,3,3,3,3,43,3,3,3,4,3,3,3	1,1,1,1,1,1,1,1,1,1,1,1,1,1,1	4,4,4,4,4,4,4,5,4,4,4,5,4,4,4
	1,000,000	3	6,8,17	19	6,7,6	2,2,2	8,9,8
	2,000,000	2	8,10	15	8,7	2,2	10,9
	3,000,000-6,000,000	1	13	11	11	3	14

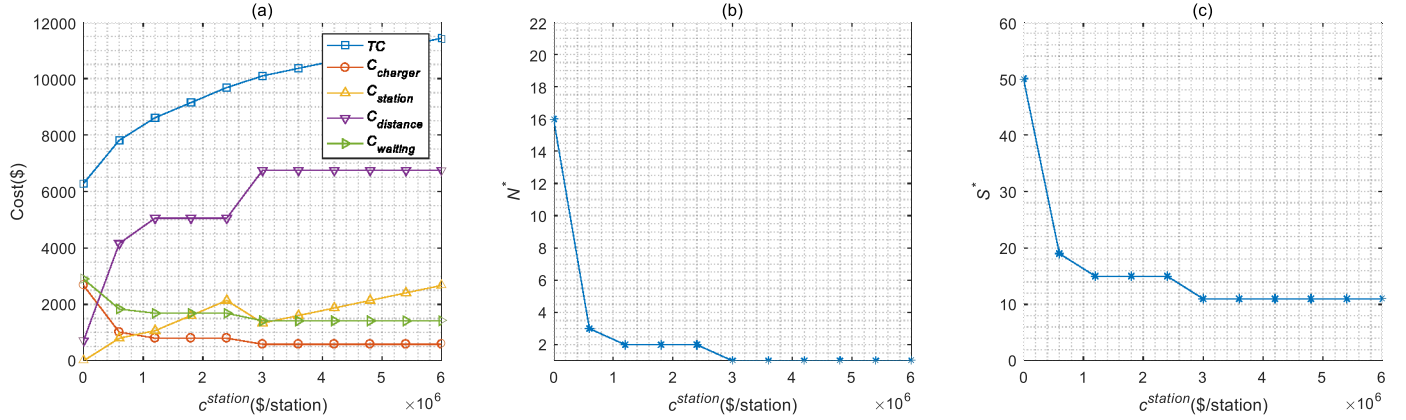


Fig. 3. Sensitivity of cost components and optimization results to parameters $c^{station}$ (a)–(c).

Table 4
Optimal results of the base scenario and $c^{charger}$ variation scenarios.

Parameter	Value (\$)	N^*	$j(X_j = 1)$	S^*	s_j	$[\frac{s_j}{\delta}]$	N_j
Base scenario		4	2,8,10,17	23	4,7,6,6	1,2,2,2	5,9,8,8
$c^{charger}$	0	20	1-20	1000	50/station	10/station	60/station
	50,000	11	1,2,3,5,6,8,9,11,17,18,20	29	2,2,2,2,2,3,3,3,4,3,3	1,1,1,1,1,1,1,1,1,1	3,3,3,3,3,4,4,4,5,4,4
	100,000	8	1,2,8,9,11,17,18,20	24	2,2,4,3,4,4,3,3	1,1,1,1,1,1,1,1	3,3,5,4,5,5,4,4
	150,000-250,000	4	2,8,10,17	16	3,5,4,4	1,1,1,1	4,6,5,5

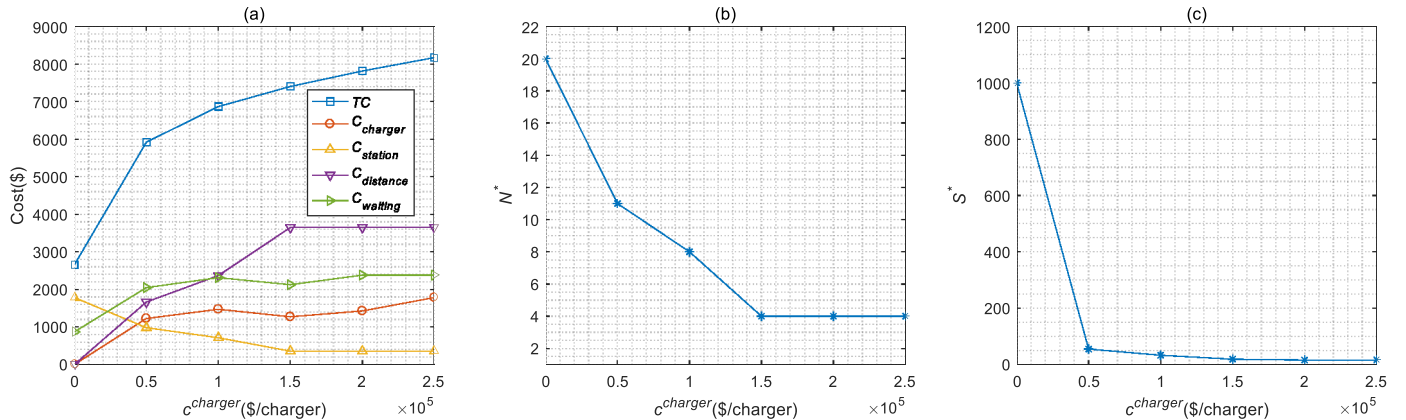


Fig. 4. Sensitivity of cost components and optimization results to parameters $c^{charger}$ (a)–(c).

decreases, which means the service capacity of each charging station declines. Accordingly, the queuing phenomenon aggravates and $C_{waiting}$ increases. We can see from Fig. 6(a), $C_{distance}$ increases at the first stage and then decreases to zero. The variation trend implies that, for $C_{distance}$, when $c^{distance}$ increases to over \$2.5/mile, the distribution density of charging stations has a more significant impact. As $c^{distance}$ approaches \$6/mile, the 20 candidate locations will all build charging stations, thus there is unnecessary to change charging stations for

charging and $C_{distance}$ goes to zero. Various cost components and the optimization results are all constant when N^* is equal to 20.

3.5. Total cost growth rate analysis

Based on the 3.4 section, in order to further investigate the parameters that have the greatest impacts on the deployment of EV charging infrastructure, we depict the growth rates of total cost TC with varying

Table 5
Optimal results of the base scenario and c^{waiting} variation scenarios.

Parameter	Value (\$/hour)	N^*	$j(X_j = 1)$	S^*	s_j	$[\frac{s_j}{\delta}]$	N_j
Base scenario		4	2,8,10,17	23	4,7,6,6	1,2,2,2	5,9,8,8
c^{waiting}	0	11	1,2,3,5,6,8,9,11,17,18,20	24	2,1,2,1,1,3,2,3,4,2,3	1,1,1,1,1,1,1,1,1,1	3,2,3,2,2,4,3,4,5,3,4
	5	4	2,8,10,17	24	5,11,9,9	1,3,2,2	6,14,11,11
	10	2	8,10	43	23,20	5,4	28,24
	15	1	13	40	40	8	48
	20	1	13	48	48	10	58
	25-30	1	13	50	50	10	60

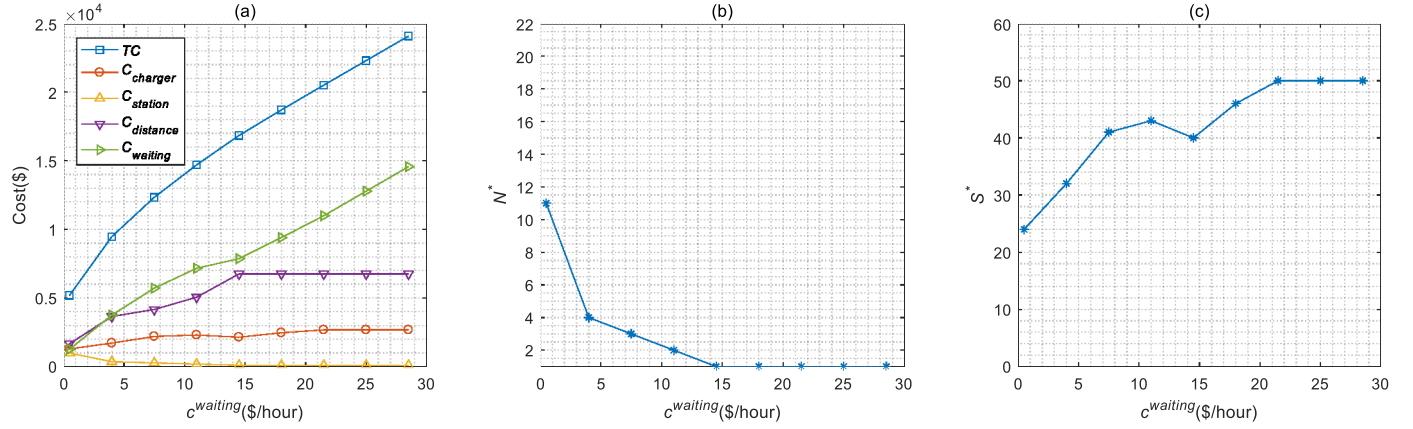


Fig. 5. Sensitivity of cost components and optimization results to parameters c^{waiting} (a)–(c).

Table 6
Optimal results of the base scenario and c^{distance} variation scenarios.

Parameter	Value (\$/mile)	N^*	$j(X_j = 1)$	S^*	s_j	$[\frac{s_j}{\delta}]$	N_j
Base scenario		4	2,8,10,17	23	4,7,6,6	1,2,2,2	5,9,8,8
c^{distance}	0	1	13	11	11	3	14
	2	3	6,8,17	19	6,7,6	2,2,2	8,9,8
	4	12	1,2,3,4,5,6,11,13,14,17,18,20	43	3,3,3,4,3,3,4,3,4,5,4,4	1,1,1,1,1,1,1,1,1,1,1	4,4,4,5,4,4,5,4,5,6,5,5
	6-10	20	1-20	60	3/station	1/station	4/station

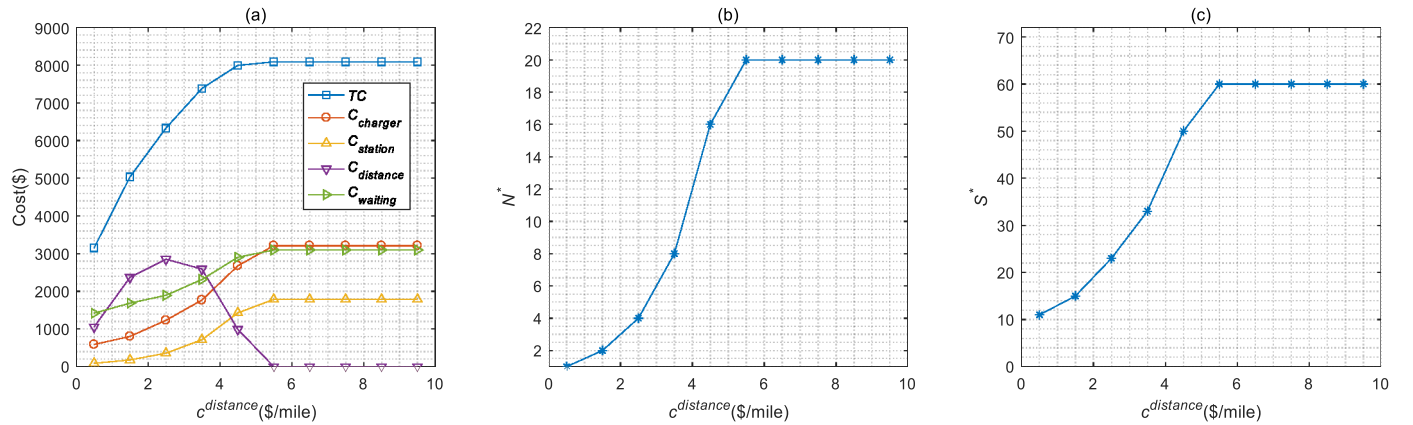


Fig. 6. Sensitivity of cost components and optimization results to parameters c^{distance} (a)–(c).

of c^{station} , c^{charger} , c^{distance} and c^{waiting} , as shown in Fig. 7(a)–(d). Taking c^{station} as an example, we evaluate the changing range of TC when c^{station} varies from c_1^{station} to c_2^{station} , and define the growth rate θ as follows:

$$\theta = \frac{\Delta TC}{\Delta c^{\text{station}}} = \frac{TC_2 - TC_1}{c_2^{\text{station}} - c_1^{\text{station}}} \quad (20)$$

where TC_2 and TC_1 are the corresponding values of the total cost when

c^{station} is equal to c_2^{station} and c_1^{station} . Eq. (20) is also applicable to other parameters.

As seen from Fig. 7(a)–(d), TC is the most sensitive to c^{distance} , followed by c^{waiting} , c^{charger} and c^{station} . The results are probably ascribed that the total cost of this optimal model is set as the daily cost. Initially, we assume the lifespan of charging infrastructure is 10 years. Calculated in terms of the daily cost, the daily fixed infrastructure cost is

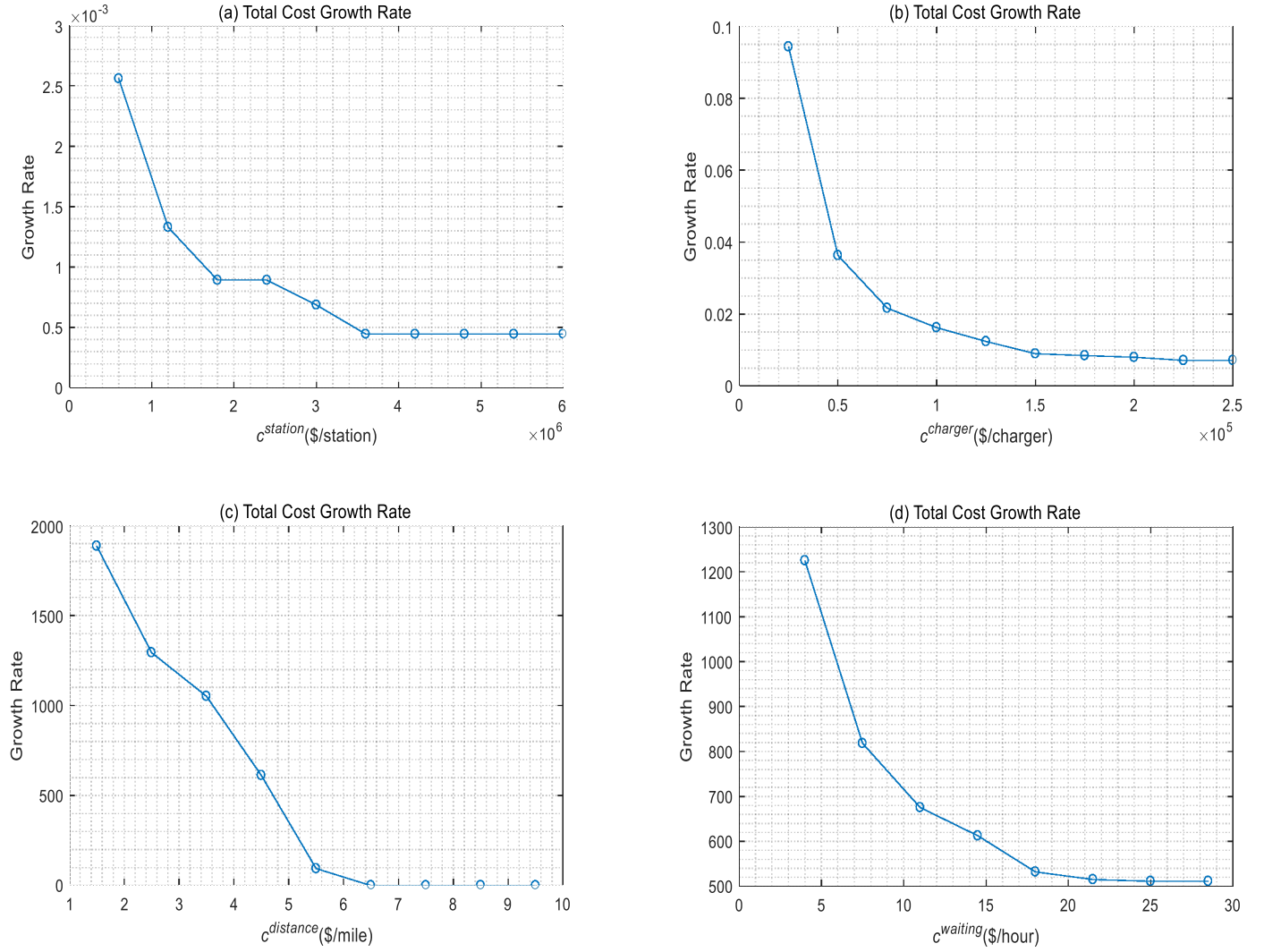


Fig. 7. Growth rates of total cost TC with various parameters increasing.

pretty small compared to the daily distance cost and the daily queuing time cost, and these cost values may not even be in the same order of magnitude. That is, the variations of $c_{distance}$ and $c_{waiting}$ have more significant and obvious influence on TC relatively. Therefore, the appropriate increases of s_j and distribution density of charging stations can effectively reduce the total cost TC, which has reference value for decision makers.

4. Conclusions and future works

This paper proposed an optimization location model to deploy the optimal locations and capacities of EV charging infrastructure under queuing behavior with finite queue length and various siting constraints. The objective function is to minimize the comprehensive total cost, including charging stations construction cost, chargers cost, equipment depreciation cost, users' queuing time cost and distance cost. This study has two major contributions to the existing literature: (1) we adopt the $M/M/s_j/N$ queuing model, considering finite queue length and finite charging service capacity of each charging station; (2) the quantity of chargers allocated at each charging station is set as variable, namely s_j in Section 2.2 and Section 3.2.

The main research achievements of this study are presented as follows:

(1) The proposed model can determine the optimal layout of EV

charging infrastructure, including the optimal total number of EV charging stations N^* and the corresponding optimal locations, the optimal total number of chargers S^* and the corresponding quantity of chargers allocated at each charging station s_j , and the optimal allowable maximum queuing length and the optimal maximum capacity of each charging station;

- (2) The optimal values of variable s_j and maximum queuing length $[\frac{s_j}{\delta}]$ are consistent with the distribution densities of existing charging stations at different locations. Generally, in the study area, the higher distribution density of existing charging stations means the greater charging demand. The simulation results show that at the charging stations with greater charging demand, the more chargers are allocated and the longer queuing length are allowed (i.e. the values of s_j , S^* and $[\frac{s_j}{\delta}]$ are greater);
- (3) The variations of $c_{waiting}$ and $c_{distance}$ have more obvious impacts on TC relatively, so the appropriate increases of s_j and distribution density of charging stations can effectively reduce the total cost TC, which has reference value for decision makers;
- (4) There is a phenomenon that uniform distribution of charging stations in the study area can ensure the maximum coverage, and different numbers of chargers allocated at each charging station can meet various charging demands of different locations.

For future study, this research can be extended to higher fidelity and accuracy by relaxing a series of assumptions. Generally, land prices

vary in different areas even if in the same city, which is caused by geographical locations, land uses and economic development levels. So it could be more realistic that the basic infrastructure construction costs of each charging station varies with different land prices. Moreover, this paper does not consider the operation process of the charging station system, such as operating maintenance, charging service process, various charging modes, real-time notifying the chargers' available rate of all charging stations and so on. It's interesting to integrate these into the optimal model under various complicated and realistic constrains (e.g., different charging time of each EV, different electricity price, etc.). Further, it's also a new research direction to combine car-sharing mode, namely charging infrastructure locations planning under EV sharing mode. It will be a much more complicated mathematical model with various complex constrains, which also will be our future work.

Declaration of Competing Interest

None.

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Supplementary materials

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