

Charging Network Optimization with Nonlinear Station Size Effects

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1 Introduction

Infrastructure networks tend to evolve in three phases; connecting, balancing, and hardening. In the first phase minimum service is provided for all users. In the second phase high usage elements are upgraded to meet observed demand. In the third phase, redundant elements are added in order to accommodate overflow demand and to maintain functionality in the case of element failure. Battery Electric Vehicle (BEV) charging infrastructure is relatively new when compared to other transportation infrastructure systems. Per Alternative Fuels Data Center (AFDC), the number of public charging stations in the US increased by more than 800% in the decade between 2013 and 2023 [SOURCE AFDC 10972]. The number is certainly much higher when factoring in private charging stations but less is known about these [1]. Most of these stations are small AC charging stations and serve primarily to meet daily travel needs. Of the stations currently tracked by AFDC, about 16.5% are DC charging stations which must, by necessity, serve as the backbone of long distance BEV travel.

The current charging network in the US is spatially unequal. In some areas of the country the DC charging network is quite sparse and, in others, quite dense. There are only 66 total DC stations in the state of Idaho with a total of 193 ports between them. The vast majority of these are on the interstate highway between the large population centers of Boise and Idaho Falls. By contrast, California contains nearly DC 2,500 stations and 14,000 ports. These networks are, plainly, in different phases of their development with different investment priorities. It is important to provide methodology which fits each phase.

Figure with cut sets for CA and ID

In the connecting phase, it is sufficient to ask where should stations be located such that all origin-destination pairs are connected. Where the DC charging network has guaranteed connectivity, the next priority is balancing demand and supply. It is pertinent to note that the amount of "supply" a network provides is not captured by a single number such as total stations or total chargers. The composition of a network in terms of locations, speeds, and port counts of stations matter but cannot be evaluated independent of demand. Much research has been performed in the field of charging network capacity expansion optimization. The vast majority of this research is connectivity-oriented and ignores the effects of station composition.

2 Literature Review

The the problem of optimal Energy Supply Network (ESN) expansion belongs to the larger field of strategic facility location optimization, a fundamental problem in operations research [2]. The optimal ESN expansion problem seeks to find the set of station locations and characteristics which most efficiently serves a set of demands. The underlying goal of the optimization is multi-objective: to find a solution which maximizes level-of-service while minimizing cost. In order to compute an optimal solution, one must either define an equivalence between cost and level-of-service or optimize for one subject to a constraint on the other. The constraint approach has two varieties. A service-constrained problem is one where cost is minimized subject to a minimum level-of-service. A cost-constrained problem is one where level-of-service is maximized subject to a maximum cost. Where level-of-service is defined as demand coverage service and cost constrained problems become set-covering and maximal-covering problems. Because much of the literature defines level-of-service in this way, the terms set-covering and maximal-covering are commonly used.

Demand may be defined by locations (nodal), or trips/tours (travel) [3]. Nodal demand formulations attempt to minimize the cost of relocating from locations where users will be when they desire to charge to charging locations. This is accomplished by assigning equipment to locations and demand nodes to equipment. Nodal demand studies have employed various formulations [4] to optimize around technical, economic, and environmental objectives and constraints [5–7]. Variations introduce stochasticity [8, 9], fleet operations [10, 11], nonlinear queuing [12] dynamics, and over-flow relocation [13]. Nodal demand problems are most relevant to planning on a municipal scale. Most travel is routine and short-distance [SOURCE], and most charging is accomplished during long-dwells [SOURCE]. Access to low-rate charging is an important aspect of BEV user convenience [14].

Travel demand formulations attempt to minimize the added cost of charging/fueling for itineraries which are likely to exceed vehicle range. This is accomplished by assigning equipment to locations and flows to equipment. Charging and fueling add time to trips because of the time required to charge/fuel, the time required to wait for available equipment, and the time required to deviate from one’s route to reach a station. Flow-capturing formulations optimize an ESN to best serve traffic flows along the routes they normally take, often the shortest path [15–17]. Flow-enabling formulations optimize an ESN to best enable traffic flows by dictating what routes they will take from a set of possible paths [18–24] [SOURCE - more and more recent]. Travel demand problems are most relevant to regional-scale planning.

Although each of the cited papers presents a different formulation, a basic synthesis of travel demand formulations is possible. The first step in each problem is defining the set of origin-destination demands and the set of alternative paths for each. Paths of sufficient range will require the use of charging/fueling stations. The problem, then, becomes finding a best allocation of equipment to stations and demands to paths. A set-covering formulation will require all demands to be connected by n paths where n is usually equal to one. A maximal-coverage formulation problem will seek to maximize the number of demands connected by at least n paths. The level of service aspect is handled implicitly in path selection by only considering paths whose cost is below a given limit.

This methodology is well suited to a scenario where the existing ESN is sparse. However, the focus on connectivity renders the methodology unable to effectively consider how a well connected

system might be improved. A philosophical shift is required to address this. Rather than viewing an ESN's function as being to connect, one can view it as to reduce travel costs. If no charging stations exist between two cities, that does not mean that a BEV driver cannot travel between the them. The BEV driver would have to shift modes or rent an Internal Combustion Engine Vehicle (ICEV) which would provide additional cost. It is further true that, if the charging ESN between the cities exists but is slow and expensive, the driver might still prefer an alternative. In this case, the ESN is insufficient even if the cities are technically connected. The alternatives available to the driver provide a ceiling on cost. Drivers will elect to use the ESN if the cost of doing so is lower. The goal of the optimal ESN expansion problem in such a scenario is to minimize driver costs.

3 Methods

Previous work considers the function of an ESN to be to connect origins and destinations for a set of demand flows whose distance exceeds vehicle range. This approach is valid but limits the scope of analysis. The approach in this paper considers the function of an ESN as being to minimize the time-cost imposed by the need to refuel/recharge on drivers of a given vehicle type. For a set of demands defined by origin, destination, and volume, the minimum overall travel time is the overall travel time if all vehicles take their shortest path. This forms a lower bound. Any time spent deviating to a station or at the station can be thought of as a cost imposed by the structure of the ESN. If travel between an origin and destination is impossible or sufficiently difficult, the driver might pick a different mode (such as air travel) or rent a vehicle of a different fuel type. This forms an upper bound. The method presented here is an optimal ESN expansion problem which minimizes the travel cost imposed by the ESN.

3.1 Supply Network Graphs

Consider a road network represented by a directed graph $G_R = \{V_R, E_R\}$ where V_R is a set of nodes and E_R is a set of edges. The set of nodes V_R contains places (cities and towns) at the nodes in $V_P \subseteq V_R$ and stations at the nodes in $V_S \subseteq V_R$. The set of edges E_R contains road links. Drivers may pass by places and stations without stopping and drivers who take the same road path will not necessarily stop at the same stations. Thus, it is useful to transform the graph as in [19]. Transformed graph $G = \{V, E\}$ contains nodes $V = V_S \cup V_P$ and the edges in E are paths taken along the road network. E contains edges for all pairs in V if the energy consumption required is less than the vehicle Energy Storage System (ESS) capacity. Routes along G_R are road-paths. Routes along G are supply-paths. It is not possible for any edge $(i, j) \in E$ to have negative cost thus only simple supply-paths need to be considered.

As is the case in prior work, the method presented requires a set of alternative paths for each origin-destination flow. These paths are found using a k-path routing algorithm [25–27]. In order to guarantee that the alternative paths are simple paths a second transformation is made. For every place $o \in V_P$ which serves as a demand origin, a level graph $\bar{G}_o = \{V, \bar{E}\}$ is constructed where $\bar{E} \subseteq E$ and all edges $(i, j) \in \bar{E}$ lead away from o . The graph transformations are demonstrated in Figure 1

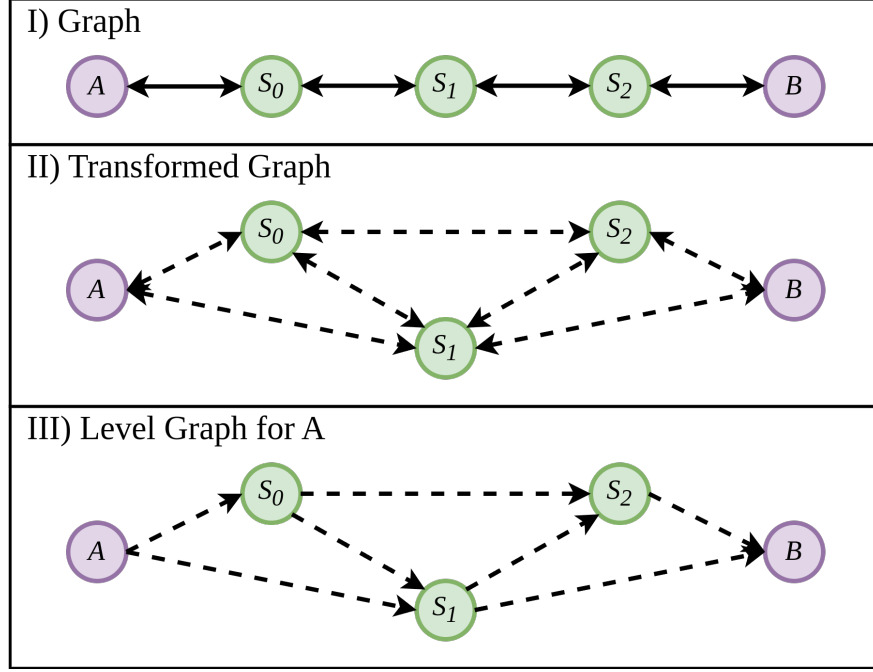


Figure 1: Graphs for simple vehicular transportation network. Panel I shows a graph G for a section of road from A to B with stations S_0 , S_1 , and S_2 . Panel II shows the transformed graph \hat{G} for a vehicle with sufficient range to reach S_1 from A and B . Panel III shows the level graph \bar{G}_A for the same vehicle leaving A . The simple supply-paths for pair $\langle A, B \rangle$ are $A - S_0 - S_1 - B$, $A - S_0 - S_2 - B$, $A - S_1 - B$, and $A - S_1 - S_2 - B$.

The time cost of edge $(i, j) \in \hat{E}$ is the total trip time. This includes the time required to traverse the edge and the time spent charging at a station before traversing the edge. For a vehicle with a usable ESS capacity of β at node i which contains a charger whose maximum rate is α the time cost for edge (i, j) is

$$Y_{(i,j)}^T = \tau_{(i,j)} + \begin{cases} f_c(\varepsilon_{(i,j)}, \beta, \alpha) & \varepsilon_{(i,j)} \leq \beta \\ \infty & \varepsilon_{(i,j)} > \beta \end{cases} \quad (1)$$

where τ is edge traversal time, ε is edge energy consumption, and f_c is a function relating energy to charging time. Charging is modeled using a CC-CV relationship. The first part of charging is linear and the second part follows an exponential decay function. The time required for a given charge event is

$$f_c(\varepsilon, \beta, \alpha) = \begin{cases} \frac{\varepsilon}{\alpha} & \varepsilon \leq \eta\beta \\ \frac{\eta\beta}{\alpha} \left(1 - \ln \left(1 - \frac{\varepsilon - \eta\beta}{\beta(1-\eta)} \right) \right) & \varepsilon > \eta\beta \end{cases} \quad (2)$$

where η is the inflection point separating linear and nonlinear charging. A typical value for η will be in the range of 0.7 to 0.8. Charging past η will be substantially slower than below η .

The time-cost of queuing depends on the presence or absence of other vehicles at a station and is modeled at a system level. Queue waiting time is modeled using the M/M/c queuing formula. The expected waiting time in an M/M/c queue is computed as

$$W_q = f_q(\lambda, \mu, c) = \pi_0 \frac{\rho (c\rho)^c}{\lambda (1-\rho)^2 c!} \quad (3)$$

$$\pi_0 = \left[\left(\sum_{k=0}^{c-1} \frac{(c\rho)^k}{k!} \right) + \frac{(c\rho)^c}{c! (1-\rho)} \right] \quad (4)$$

$$\rho = \frac{\lambda}{c\mu} \quad (5)$$

where λ is the mean arrival frequency, μ is the mean service frequency, c is the number of homogeneous servers, ρ is the ratio of arrival frequency to composite maximum service completion frequency, and π_0 is the probability of an empty system. The M/M/c formulation assumes exponential distributions for λ and μ . One can think of ρ as equivalent to "utilization". Where ρ is low the station has excess capacity and where high the station approaches saturation. W_q approaches infinity as ρ approaches one.

Queue formation is a combinatorial effect. In order to maintain a stable queue length, the rate of vehicle arrivals must match the rate of vehicle departures. This is extremely unlikely over a short time interval and the length of the queue should fluctuate. Over a long enough period, a stable non-zero mean queue length can emerge even when the rate of arrivals is less than the theoretical maximum capacity of the station. This happens because the random sequence of arrivals and departures will, often, produce scenarios where all ports are occupied when the next vehicle arrives. There will be some times when there are no vehicles at the station at all and some times when a long queue exists. **Scenarios where all ports are occupied are more likely at stations with fewer ports even at identical utilization rates.** This effect is shown in Figure 2.

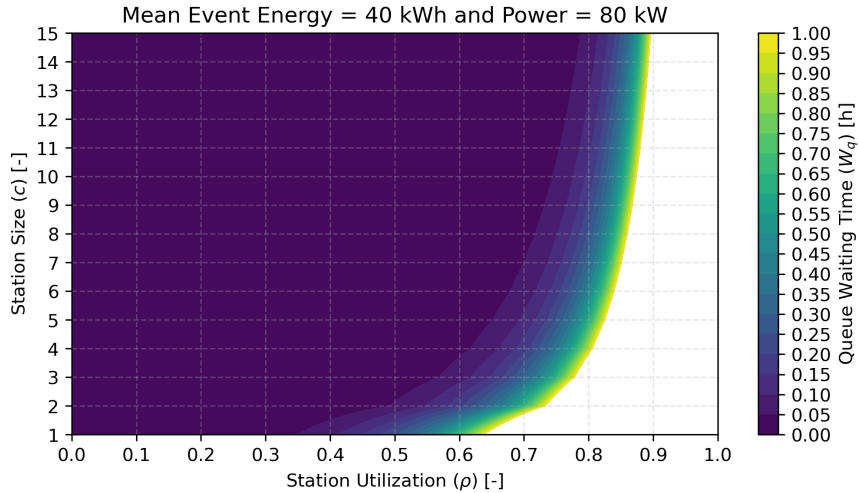


Figure 2: Queuing time with M/M/c queue model

Queuing dynamics mean that larger stations can handle higher utilization rates. As in Figure 2, a station with 15 ports can handle a rate of $\rho = 0.8$ before substantial queue formation equivalent to 24 vehicles per hour. A station of 4 chargers can handle a rate of $\rho = 0.6$ before substantial queue formation equivalent to 4.8 vehicles per hour. Thus, larger stations are more efficient on a

capacity-per-charger basis with this effect being very substantial below 10 ports and leveling off after 15 ports given current values for mean energy dispensed and charging rate.

3.2 Formulation

The purpose of this formulation is to minimize total travel time in the system subject to travel demand. Travel time minimization is accomplished by vehicle routing and charging station provision. For a given origin-destination pair there will, usually, be multiple viable charging paths of different lengths. As demand increases, chargers become increasingly congested leading to queuing time at stations. Queuing delays on shorter paths will push traffic to longer paths. Eventually, queuing will be sufficient to make the charging network no longer beneficial. This point is defined as when the marginal vehicle trip would take as much time using the network as it would take using level 1 charging. The goal is to place chargers and route vehicles to minimize total travel time. As such, each origin-destination pair has a "failure" flow which vehicles can be assigned to and the penalty assigned for this is equal to the travel time with level 1 charging.

Delay at stations is modeled using the outputs of the M/M/c queue formula as previously discussed. Specifically, the outputs are linearized. Stations are initialized with a vector of m binary variables representing possible sizes (e.g. 1 charger, 3 chargers, 5 chargers). The vector of station size binary variables must sum to 1. For each size considered, the M/M/c model returns volumes and delays corresponding to a set of marginal utilization rates $R : |R| = n$. The utilization rates $\rho \in R$ are the bounds of a set of $n - 1$ utilization intervals. Thus, m by $n - 1$ matrices of marginal volumes and marginal delays are created. Additionally, a m by n matrix of unit-interval continuous variables are created and the sum of these variables multiplied by the corresponding marginal volumes must equal the flow passing through the station. the delay at the station is computed by summing the marginal utilization rates multiplied by the marginal delays. In this formulation, travel demand is modeled as a continuous flow in units of vehicles per unit time and accrue at a station simultaneously. This is the same approximation as in the M/M/c queue model.

Optimization uses the following sets:

- $G = \{V, E\}$: System graph containing nodes $v \in V$ and edges $(i, j) \in E$. Edge costs are defined by the following sets:
 - Y^T : The time required to traverse edge (i, j)
- $O \subseteq V$: Set of origin nodes
- $D \subseteq V$: Set of destination nodes
- $S \subseteq V$: Set of nodes with charging stations (or the possibility of a station). Stations provide energy to vehicle flows at a given rate. Depending on the utilization level of the station, vehicles may experience delay. The relationship between utilization and delay is linearized using the following sets:
 - C_s : Set of possible station sizes at station $s \in S$
 - $K_{s,c}$: Set of capacity intervals at station $s \in S$ for station size $c \in C_s$
 - Y^V : Set of volumes corresponding to each $c \in C_s$ and $k \in K_s$

- Y^D : Set of delays corresponding to each $c \in C_s$ and $k \in K_s$
- Y^C : Set of expenditure requirements to install a given number of chargers at a given station.
- \hat{C} : Maximum number of chargers which can be installed
- Q : Set of demand tuples of the form $\langle o, d, v, c, \hat{t} \rangle$ where o is the origin, d is the destination, v is the volume, c is the capacity of the ESS capacity of vehicles, and \hat{t} is the maximum travel time that is acceptable for the given demand.
 - Y^Q : Set of time penalties for failing to accommodate flow. Set so that y_q^q is equal to \hat{t} in q .
- P : Set of paths corresponding to each demand $q \in Q$. Paths begin at $o \in O$ and end at $d \in D$. All intermediate nodes $i \in P \setminus \{o, d\}$ must be stations $s \in S$.
 - P^q : Paths that correspond to demand $q \in Q$
 - P^s : Paths that include station $s \in S$
- X : Set of continuous decision variables:
 - X^Q : Portion of demand flow not facilitated by the network
 - X^P : Flow volumes along paths
 - X^U : Portion of station capacity intervals utilized
 - X^V : Volume seen at station
 - X^D : Queuing delay seen at station
- U : Set of integer decision variables:
 - U^S : Booleans for station sizes corresponding to S and C

The objective of the optimization is

$$\min_{\bar{X}, \bar{U}} \underbrace{\sum_{q \in Q} x_q^q y_q^q}_{\text{Penalty Time}} + \underbrace{\sum_{q \in Q} \sum_{P^q \in P} \sum_{p \in P^q} \sum_{(i,j) \in p} x_p^p y_{(i,j)}^t}_{\text{Edge Traversal Time}} + \underbrace{\sum_{s \in S} \sum_{c \in C_s} \sum_{k \in K_{s,c}} u_{s,c}^s x_{s,c,k}^u y_{s,c,k}^d}_{\text{Queuing Time}} \quad (6)$$

subject to

$$Q[v] - x_q^q - \sum_{p \in P^q} x_p^p = 0 \quad \forall q \in Q \quad (7)$$

$$\sum_{p \in P^s} x_p^p - \sum_{c \in C_s} \sum_{k \in K_{s,c}} u_{s,c}^s x_{s,c,k}^u y_{s,c,k}^v = 0 \quad \forall s \in S \quad (8)$$

$$x_{s,c,k}^u - u_{s,c}^s \leq 0 \quad \forall s \in S, \forall c \in C_s, \forall k \in K_{s,c} \quad (9)$$

$$\sum_{c \in C_s} u_{s,c}^s - 1 = 0 \quad \forall s \in S \quad (10)$$

$$\sum_{s \in S} \sum_{c \in C_s} y_{s,c}^c u_{s,c}^s - \hat{C} \leq 0 \quad (11)$$

The objective function (6) minimizes total travel time in three terms. The first term is the time penalties accrued for failing to accommodate demand. The theory is that, without dedicated charging infrastructure, vehicles could, theoretically, complete the trip using level 1 charging but this would be very slow if the trip is beyond full-charge range. The second term is the time spent driving along edges and charging to drive along edges. The third term is the time spend queuing for a charger. Constraint (7) forces the sum of flows and un-accommodated flows to be equal to total demand. Constraint (8) forces the sum of utilization intervals at a station to be equal to the sum of flows which pass through the station. Constraint (9) forces station utilization to only accrue for the selected station size. Constraint (10) forces only one station size to be selected per station and (11) limits the total number of chargers in the network. This formulation can be used for two varieties of analysis.

Performance of Current System: If one fixes the values of all Booleans in U^S to reflect an existing system, the result of the optimization will be the best possible performance of the existing system subject to a given demand level.

Optimal Expansion: If one allows for the Booleans in U^S to be changed by the optimizer, then the result of the optimization will be the best possible allocation of resources to facilitate travel for a given demand level.

Taking the example simple network from Figure 1, one can intuit that, if each additional charger at each station is of the same cost ($Y_{s,c}^C = 1 \quad \forall s \in S, c \in C_s$), the optimal solution will be to place all available resources at the central station S_1 because only it can be reached from both A and B . Doing so will give the greatest benefit in terms of queuing efficiency. The costs of adding and removing chargers may scale in a nonlinear manner. It is easy to imagine a scenario where a network operator finds it easier to remove an existing charger than to install a new charger. Swapping equipment between stations in a network is unlikely to be a net-zero cost operation. In such a case, the optimal solution may or may not be altered.

4 Case Studies

4.1 Highway Corridor Capacity

The state of California has large population concentrations in its northern and southern regions. These regions are connected by a highway corridor which runs through mores sparsely populated

agricultural areas in the San Joaquin Valley in the center of the state. Most road traffic between the states densely populated regions will follow a road corridor defined by Interstate 5 to the west and CA-99 to the east. This corridor is referred to as the 5-99 Corridor. The 5-99 corridor and all California cities with a population of greater than or equal to 50 thousand are shown in Figure 3.

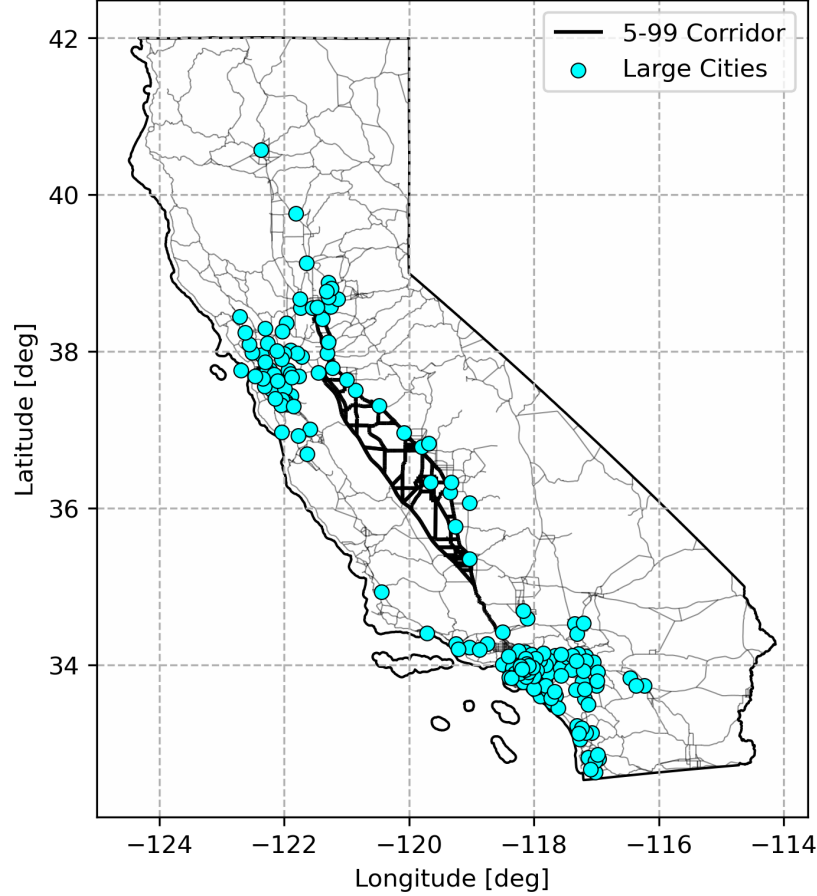


Figure 3: 5-99 Corridor and California Cities with $\geq 50,000$ population.

The 5-99 corridor is of vital importance to road traffic in the state of California. Because of the large distance covered by the corridor, most BEVs will have to charge at a DC charging station in order to complete their itineraries in a reasonable amount of time. DC Charging stations can be divided into those which use the CCS plug and those which use the Tesla/NACS plug. At present, there are 344 CCS and 47 NACS stations in the 5-99 corridor per AFDC [28] with 1,029 and 901 chargers respectively. The locations and port counts for the CCS and NACS stations are shown in Figure 4.

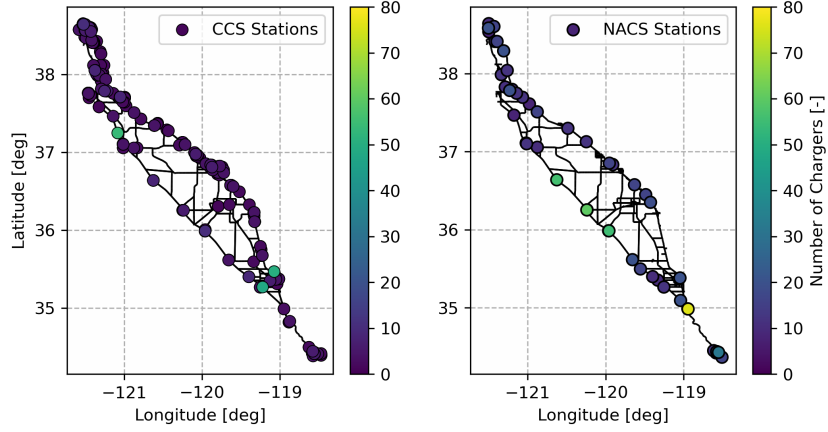


Figure 4: Locations and sizes of DC charging stations in 5-99 Corridor.

The NACS stations in the 5-99 corridor are, in general, larger and further apart than the CCS stations. Survival functions for station size by plug type are shown in Figure 5.

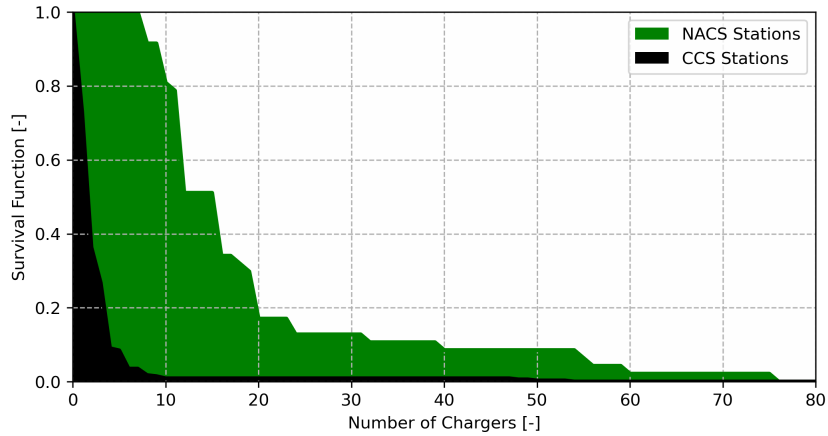


Figure 5: Survival functions for 5-99 corridor station size (port count) by plug type.

A second distance is charging speeds. Tesla chargers and vehicles are capable of charging at high rates, most commonly 150 kW or 250 kW. Non-Tesla chargers frequently have maximum charging rates in the range of 50 kW to 80 kW. In the 5-99 corridor, the vast majority of the Tesla chargers are of the 250 kW variety while 60% of CCS chargers have maximum rates less than 150 kW and 80% less than 250 kW. Vehicles which use the CCS infrastructure may also act as a charging speed bottleneck, often with maximum charging rates lower than that of the charger [SOURCE - EV Database].

A first question is how much travel can the current ESN accommodate. In order to answer this it is necessary to predict travel demand. Inter-city travel demand was generated using a travel gravity model [SOURCE]. Populations were taken from the US Census Bureau [SOURCE] and a friction function was fitted from NHTS Long Trip Survey data [SOURCE]. The distances used for the gravity model were based on the shortest-time-paths between cities. Where a shortest path did not go through the 5-99 corridor (between San Francisco and San Jose as an example), or the

shortest path was short enough to not require charging (Sacramento to Stockton for example) the demand for that pair was not considered in the optimization. The vehicle range used was 300 km, a typical practical range for a modern BEV [SOURCE - EV Database]. The remaining demands were converted into fractions of overall demand such that they could scale with overall demand. Following this, for those origin-destination pairs remaining, a maximum acceptable travel time was computed by computing the time the shortest path would take if the vehicle could charge at any location at a low rate (3.3 kW). If congestion causes travel times exceed this level, then the DC ESN is no longer providing benefit.

Using demand fractions generated by the gravity model it is possible to compare the performance of an ESN subject to varying levels of demand. Some vehicles will be limited to using only one plug type while some may use either. The optimal performances of the CCS, NACS, and combined ESNs with increasing demand is displayed in Figure 6.

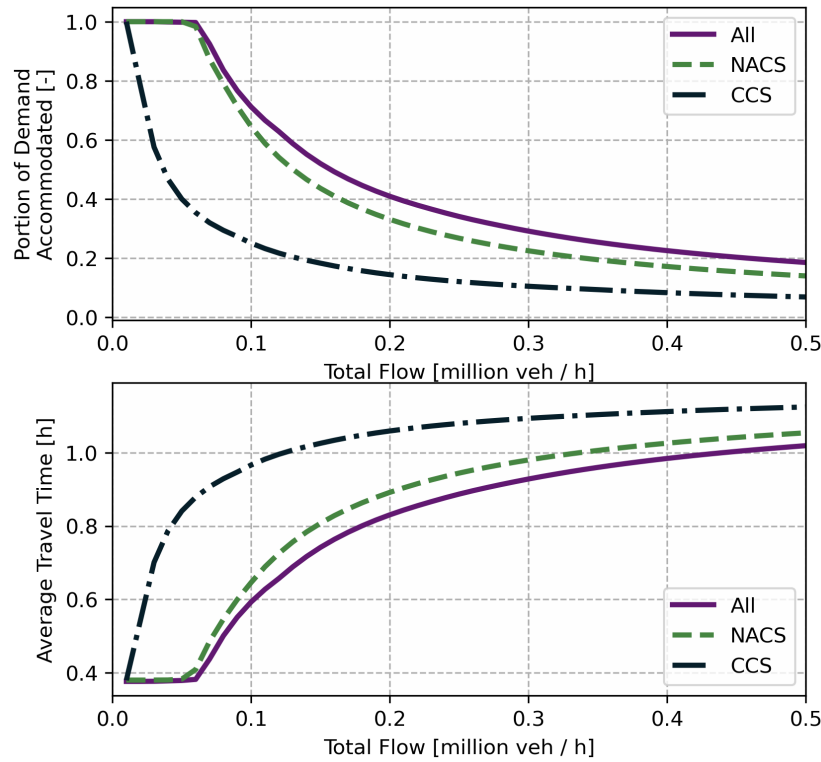


Figure 6: Performance of ESNs with increasing demand.

The range of demand used in Figure 6 is reflective of the range of hourly vehicle flows seen on Interstate 5 in the San Joaquin Valley [SOURCE <https://dot.ca.gov/programs/traffic-operations/census/traffic-volumes/2017/route-5-6>]. At the end of 2023, BEVs accounted for around 4% of the California light-duty vehicle fleet [29] and, likely a similar portion of light-duty vehicle trips. At this volume, some congestion should be seen at CCS stations but very little should be seen at NACS stations and this appears to be the case [SOURCE - <https://www.ucdavis.edu/blog/charging-not-range-becoming-top-concern-electric-car-drivers>]. The model presented, and what data can be found, suggest that the configuration of the NACS ESN is more efficient on a effective-capacity-per-charger basis. Some

of the delta is based on the difference in charger speeds which favors the NACS ESN. The majority of the difference is due to the more centralized structure of the NACS network, being composed of more efficient stations. The differences in station size distribution between the CCS and NACS networks are stark. Both networks have a small number of very large stations clearly intended for high volume usage. The main difference is that the remainder of NACS stations are mid-sized, in the range of 10 to 25 chargers, while the remainder of CCS stations are small, in the range of 1 to 4 chargers.

The benefit of a more distributed network is that, by virtue of having more station locations, it is more likely to have a station closer to a demand node and to require fewer and shorter deviations out-of-path for travel demand. There are also more candidate locations which can accommodate a smaller station. The stations located in the 5-99 corridor serve local demand as well as thru-traffic, local demand may even be their primary purpose. As seen in [12], larger stations are also favored for nodal demand if queuing is considered. However, drivers looking to charge their vehicles then return to their starting location can only go so far before the energy expended on the return leg reduces the utility of the trip. This means that more and better sited locations bring a real benefit. The preference for large stations is more pronounced for travel demand. Given modern BEV ranges and the concentration of long-distance traffic on limited access highways, long-distance travelers are almost entirely insensitive to the location advantages of smaller stations but benefit from the greater queue dissipation efficiency of larger stations.

Where might additional chargers prove most beneficial? For both the CCS and NACS networks, an additional 100 chargers were optimally allocated. The resulting networks are shown in Figure

Given the option to reconfigure the CCS network without changing the total number of chargers, could the CCS network be made to perform more comparably to the NACS network? And how would this alternate CCS network be configured?

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