A Model for Location of Capacitated Alternative-Fuel Stations

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Increased interest in alternative fuels is attributable, in part, to rising oil prices and increasing concern about global warming. A lack of a refueling infrastructure, however, has inhibited the adoption of alternative-fuel vehicles. Little economic incentive exists to mass-produce alternative-fuel vehicles until a network of stations exists that can refuel a reasonable number of trips. The flow refueling location model (FRLM) was developed to minimize the investment necessary to create a refueling infrastructure by optimizing the location of fueling stations. The original uncapacitated FRLM assumes that the presence of a refueling station is sufficient to serve all flows passing through a node, regardless of their volume. This article introduces the capacitated flow refueling location model that limits the number of vehicles refueled at each station. It also introduces a modified objective function maximizing vehicle-miles traveled instead of trips, applies both models to an intercity network for Arizona, and formulates several other extensions.

Introduction

The world's heavy dependence on petroleum for transportation fuel leads to a host of problems including high oil prices, pollution, political instability in the Middle East, and global climate change. This broad range of negative consequences has led to calls for a reduction in petroleum use. While increased efficiency can reduce energy consumption, in the long run overcoming petroleum dependence will require alternative energy sources, especially for automobiles and trucks.

While there is considerable impetus to develop alternative-fuel (alt-fuel) vehicles, such efforts face considerable challenges. In addition to potential technological hurdles, most forms of alternative fuel also face a significant, largely organizational, hurdle. Before alt-fuel vehicles can be deployed on a large scale, they require a refueling infrastructure. After more than a century of automobile use,

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gasoline stations are effectively ubiquitous. Drivers seldom have to worry about the locations of or intervals between gas stations. Refueling stations for most alternative fuels, however, can be few and far between (U.S. Department of Energy (DOE) 2006a, b). For example, there were only 140 hydrogen stations worldwide at the end of 2006, although the number had increased seven-fold since 2000 (Adamson and Crawley 2007). The lack of a refueling infrastructure limits the utility of these vehicles, effectively precluding their widespread adoption. Because the alt-fuel user base is very small, there is little demand for such fuels and little incentive to develop a refueling network.

Many have recognized this ''chicken and egg'' problem (CDOE 2002; Melaina 2003; National Research Council 2004; Melendez, M., and A. Milrandt 2005). In hopes of breaking this logjam, Kuby and Lim (2005) introduced the flow refueling location model (FRLM) to maximize the impact of a given investment in refueling infrastructure for alt-fuel vehicles. The model optimizes the location of a given number of refueling facilities within a network with the goal of enabling the maximum number of trips by vehicles with a limited range.

The FRLM is an uncapacitated model: it implicitly assumes that a single facility can refuel an infinite amount of flow. This is not a terribly unrealistic assumption for the earliest stages of an alt-fuel industry because the number of vehicles will be small. As alternative fuels begin to take off, however, this assumption will become increasingly tenuous. For instance, Argentina, Brazil, and Pakistan *each* had more than 1 million natural-gas vehicles and 900 refueling stations in 2006 and an average of over 1000 vehicles per station (International Association of Natural Gas Vehicles 2007). Also, some alt-fuel stations have very small capacities, such as hydrogen energy stations that generate their own hydrogen (California Hydrogen Highway Network 2005). To address this concern, this article introduces the capacitated flow refueling location model (CFRLM) that limits the amount of flow that any particular facility can refuel. We also introduce a new objective function—maximizing vehicle miles traveled (VMT) by alt-fuel vehicles, rather than maximizing vehicle trips—if the goal is to replace as much conventional fuel as possible.

The second section of this article reviews the literature on the FRLM and introduces its nomenclature. The third section formulates the CFRLM. The fourth and fifth sections lay out a case study using a simplified version of Arizona's highway network and present results. The sixth section introduces the VMT-maximizing objective and compares its results to those for maximizing trips. The seventh and eighth sections offer conclusions and directions for future research.

Literature review

The uncapacitated FRLM is based on the Flow Capturing Location Model (FCLM) developed by Hodgson (1990) and Berman, Larson, and Fouska (1992). Instead of the more traditional approach based on the distance between demand nodes and a facility, the FCLM optimizes facility locations based on the flows between origins and destinations. A flow is considered to be ''captured'' or ''intercepted'' if a single facility lies anywhere along the shortest path. Specifically, the FCLM locates p facilities so as to maximize the amount of flow captured. The FCLM is used to model facilities at which people stop on their way to somewhere else, such as convenience stores and automated teller machines.

The FRLM differs from the basic version of the FCLM in two fundamental ways—both a function of vehicle range limitations. First, multiple facilities may be required to capture a flow. If the trip is long enough, the vehicle may need to refuel more than once during its journey. Second, it matters where along the path the facility is located—as is also true of several other variants on the basic FCLM (Zeng and Hodgson 2004). If only one refueling station is required, that station must be within round-trip range of both the origin and destination. If a path requires multiple refuelings, not only are stations within range of the origin and destination needed, but stations must also be no further apart than the vehicle's maximum range. These factors make the FRLM a considerably more complex problem than the FCLM. If the vehicle's range is longer than any round-trip distance between an origin and a destination, the FRLM's new constraints will impose the same logic as the FCLM's. The FRLM, therefore, can be used as a seamless model for refueling both short- and long-distance trips.

The most basic input to the FRLM is a set of origin–destination (O–D) pairs and the flow volumes between them. For each O–D pair, one must calculate the shortest path between origin and destination and then determine all the combinations of facilities that can refuel a round trip along that path (Kuby and Lim 2005). These combinations of refueling stations, and the flow volume associated with each combination, are evaluated by the FRLM's mixed integer programing (MIP) formulation to determine optimal facility locations.

The formulation of the FRLM is as follows:

$$\text{Max } Z = \sum_{q \in Q} f_q y_q \tag{1}$$

Subject to:

$$\sum_{h \in H} b_{qh} v_h \geq y_q \quad \forall q \in Q \tag{2}$$

$$x_k \geq v_h \quad \forall h \in H; k | a_{hk} = 1 \tag{3}$$

$$\sum_{k \in K} x_k = p \tag{4}$$

$$x_k, v_h, y_q \in \{0, 1\} \quad \forall k, h, q \tag{5}$$

where the variables are $y_q = 1$ if f_q is captured, 0 otherwise, $x_k = 1$ if a facility is located at k, 0 otherwise, $v_h = 1$ if all facilities in combination h are open, 0 otherwise, and the constants are q is the index of O–D pairs (and, by implication, the shortest paths for each pair), Q the set of all O–D pairs, f_q the flow volume on the shortest path between O–D pair q, k the index of potential facility locations, K the set of all potential facility locations, p the number of facilities to be located, h the index of combinations of facilities, H the set of all potential facility combinations, a_{hk} a coefficient equal to 1 if facility k is in combination h and 0 otherwise, and b_{qh} a coefficient equal to 1 if facility combination h can refuel O–D pair q and 0 otherwise.

The objective function (1) seeks to maximize the total amount of flow that can be refueled. The constant f_q is the round-trip flow between each O–D pair q. The binary variable y_q will only take on a value of 1 if there is a combination of open facilities that can refuel trip q, as determined by constraints (2) and (3). The objective function sums the total flow captured by the facilities over all O–D pairs.

Constraint (2) ensures that, in order for y_q to be 1 (capturing the flow), there must be at least one combination h of open facilities capable of refueling the flow between O–D pair q. The constant b_{qh} is 1 if combination of facility locations h can refuel O–D pair q; otherwise it is 0. The variable v_h takes on a value of 1 only if all facilities in combination h are open (see constraint (3)). Unless b_{qh} and v_h are both equal to one for at least one combination of h, the inequality forces y_q to 0, meaning the flow is not captured, nor counted in the objective function.

The next constraint (3) ensures that all facilities in a combination are open before it can be used. The variable x_k takes on a value of 1 if facility k is open. Because this constraint only exists for facilities k in combination h (i.e., where $a_{hk} = 1$), v_h is effectively determined by the lowest value of x_k in a particular combination h, ensuring that any closed facilities will preclude this combination from being used for refueling.[1] Constraint (4) requires that p facilities be built. Constraint (5) is an integrality constraint, limiting x_k, v_h, and y_q to values of 0 and 1. In practice, however, only the x_k variables must be declared as binary; the others will naturally solve to 0 or 1 (Kuby and Lim 2005).

To summarize, the FRLM seeks to maximize the flow that can be refueled (1), using combinations of open facilities (3) that do not exceed the vehicle's range (2), and only p facilities may be built (4).

Kuby and Lim (2005) showed that the nodes of the network do not constitute a finite dominating set of candidate sites. It is not uncommon that no single node can refuel the round trip for a given path, but a single facility placed near the midpoint of the path, where no node exists, could succeed. Kuby and Lim (2007) compared two methods for generating additional candidate sites on arcs. The FRLM refueled the most trips with a given p using the network nodes plus candidate sites dispersed along arcs (Kuby, Lim, and Upchurch 2005), followed by nodes plus mid-path segments where a single facility can refuel a path, followed by the set of network nodes only.

Formulation of the CFRLM

The CFRLM incorporates two basic changes from the FRLM. First, it uses y_{qh} instead of y_q. To ensure that no facility refuels more than its capacity, the capacitated model requires precise knowledge of where each flow refuels. The CFRLM uses y_{qh} to indicate exactly which combination of facilities h is refueling the flow. In addition, because the capacity constraint may make it impossible for a single combination of facilities to refuel all of the flow of a particular O–D pair, y_{qh} is redefined from a binary to a continuous variable to indicate the fraction of the flow q refueled by facility combination h.

The second major change is to redefine the facility location variable x_k as an integer variable rather than a binary variable. In the capacitated model, it is possible for a node to have more flow than a single facility can refuel. In the basic CFRLM formulation, capacity is defined in interchangeable modular units. These modules can be thought of as single dispensers at one refueling station or as complete refueling stations in one town, depending on how the capacity constant c and spatial scale of analysis are defined. Whether multiple stations are actually required depends on the scale. In a statewide model, such as the one used in the accompanying case study, each node represents an entire city or town. It is not only possible, but also likely, that multiple stations will be required to serve this level of demand. In a model built on an urban scale, where each node might represent a major intersection and surrounding development, the need for multiple facilities at a single node is much less acute. Indeed, there is an upper bound on the number of refueling stations that can be located at an intersection.

The third major change is the addition of the capacity constraint. The CFRLM capacity constraint is similar to those used in capacitated fixed-charge location problems (Sá 1964; Marks 1967; Davis and Ray 1969). The left-hand side totals the use of the facility by all flows and combinations of facilities for each flow, while the right-hand side is the facility's capacity multiplied by the number of facilities located at k.

The formulation of the CFRLM is as follows.

$$\text{Max } Z = \sum_{q \in Q} \sum_{h \mid b_{qh}=1} f_q y_{qh} \tag{6*}$$

Subject to:

$$\sum_{q \in Q} \sum_{h \mid b_{qh}=1} e_q g_{qhk} f_q y_{qh} \leq c x_k \quad \forall k \in K \tag{7**}$$

$$\sum_{k \in K} x_k = p \tag{8}$$

$$\sum_{h \mid b_{qh}=1} y_{qh} \leq 1 \quad \forall q \in Q \tag{9**}$$

$$y_{qh} \geq 0 \quad \forall q \in Q,\ h \in H \tag{10**}$$

$$x_k \in \{nonnegative\ integers\} \qquad (11**)$$

where the following notation is the same as in the FRLM: q is the index of O–D pairs (and, by implication, the shortest paths for each pair), Q the set of all O–D pairs, f_q the flow volume on the shortest path between O–D pair q (in the same trips/period as c), k the potential facility location, K the set of all potential facility locations, h the index of combinations of facilities, H the set of all potential facility combinations, and b_{qh} a coefficient equal to 1 if facility combination h can refuel O–D pair q and 0 otherwise.

The following elements are changed or added in the CFRLM: Variable y_{qh} is the proportion of f_q being refueled by facility combination h, x_k is the number of modules located at site k.

Constants p is the number of modules of capacity to be located, c the number of vehicle stops that can be refueled by each module (in the same trips per time unit as f_q), $\frac{e_q}{1}$ the fraction of round trips, on average, that require refueling per time period on the shortest path for O–D pair q, calculated as:

$$e_q = \frac{1}{\max(1, \mathrm{int}\left(\frac{\text{range}}{\text{roundtrip distance}}\right)}$$

g_{qhk} is the average number of times a vehicle traveling on path q and being refueled by combination h stops at station k on each round trip that requires refueling. It is a coefficient equal to:

- 2 if facility k is in combination h but not at the origin or destination, meaning the vehicle must stop at the station to refuel in both directions;
- 1 if facility k is in combination h and at the origin or destination of flow q;
- 0 if facility k is not in combination h that can refuel path q.

Equations with a double asterisk (**) are new to the CFLRM, those with a single asterisk (*) are modified from the FRLM, while the one without an asterisk is taken verbatim from the FRLM.

The objective function (6) seeks to maximize the flow volume that can be refueled. The flow f_q for O–D pair q is multiplied by the proportion of the flow y_{qh} that combination h can refuel, summed over all flows and all combinations of refueling facilities.

Constraint (7) is a multipurpose constraint. Constraint (7) replaces constraints (2) and (3) from the original FRLM in an efficient formulation that eliminates the v_h combination variables. Recall that constraint (2) in the FRLM prevents a flow q from being refueled unless at least *one* of the v_h combination variables capable of refueling q is equal to one. Also recall that constraint (3) prevented a v_h combination variable from being open unless *all* of the facility variables x_k are open for all sites k in combination h. Our new constraint (7) eliminates the intermediate role of the v_h

variables and directly relates the flow-refueling variables y_{qh} to the facility-location variables x_k. Constraint (7) can simultaneously apply the logic of (2) and (3) because the left-hand side of (7) is summed over only those y_{qh} variables such that combination h can refuel flow q and facility k is in combination h. It does so by multiplying the y_{qh} variables by a new coefficient g_{qhk} that is set to zero if either combination h cannot refuel path q or facility k is not in combination h. Essentially, g_{qhk} combines the functions of the a_{hk} and b_{qh} coefficients of the uncapacitated FRLM (plus a third function that we introduce below). If any x_k variable is zero, (7) forces to zero all y_{qh} variables that depend on a facility at k being open. Likewise, if any y_{qh} variable is nonzero, (7) forces all x_k variables in combination h to at least one.

The new structure of (7) dramatically reduces the number of constraints. Only $|K|$ such constraints are needed, compared with $|Q|$ constraints of type (2) and $|H|$ multiplied by the average number of k per h for constraints of type (3). Assuming that all nodes are both candidate sites and origins/destinations and that the set Q includes only ij pairs and not ii or ji pairs, the number of constraints of type (2) is $k^*(k-1)/2$. The number of constraints of type (3) is idiosyncratic and depends on the range of the vehicle, the density of the network, and the lengths of the paths. In the 25-node Arizona network in ''Case study and solution procedures'' section, with 50 candidate sites, the CFRLM has 50 constraints of type (7) compared with the FRLM formulation, which would have 300 constraints of type (2) and 4723 of type (3). The savings would increase exponentially with bigger networks. Often, such a savings in the number of constraints would sacrifice the ''integer friendliness'' of the formulation (ReVelle 1993), but in this case, neither of the variables involved are binary. The facility location variable x_k is now integer, and the coverage variable y_{qh} is continuous.

Constraint (7) is also the capacity constraint that prevents more flow being allocated to a station k than its capacity allows. The left-hand side totals all use of the facility, while the right-hand side is the facility's capacity. For each facility k, the constraint sums the proportion (y_{qh}) of the flow (f_q) weighted by the frequency at which flow q stops at facility k in combination h ($e_q g_{qhk}$), and ensures that the total is less than or equal to the number of modules (x_k) times the capacity of each module (c). The unit of measurement for the left- and right-hand sides is the number of vehicles stopping at the station, assuming that vehicles are refueled only as often as necessary on each round trip to be able to make the same or a different trip afterwards,[2] and all refuelings take the same amount of time regardless of how much fuel must be dispensed (more on this assumption after the formulation is explained).

The g_{qhk} coefficient represents the number of stops a vehicle will make at each facility k during a round-trip based on its location. If a facility is located anywhere in the middle of a path, a round-trip flow will stop at the facility twice, once in each direction. At the endpoints of a path, the origin and destination, the flow will stop only once.[3] The g_{qhk} coefficient also replaces the a_{hk} coefficient from the FRLM. The coefficient g_{qhk} is 1 for sites at the origin and destination, 2 for sites at any other point along the path, and 0 for sites that are not part of a given combination.

The g_{qhk} coefficient alone functions quite adequately for longer trips. For shorter trips, however, it may be possible to make several round trips before refueling. The e_q coefficient takes into account the length of roundtrip q and represents the average frequency of refueling each q (e.g., every trip, every other trip, every third trip, etc.). For very short trips, a vehicle may be capable of making more than one round trip on a single tank of fuel. By dividing the range of the vehicle by the round-trip distance, and then rounding down to the nearest integer, we can determine the average number of complete round trips possible per fill-up. If this rounded number is greater than one, its reciprocal tells us the fraction of trips (1/2, 1/3, 1/4, etc.) that must be refueled in each time period. If this rounded number equals one, a driver could make one trip but not two, so we assume that every trip must be refueled in each time period. If the range of the vehicle is less than the round-trip distance, multiple facilities are required and we again assume that every trip must be refueled in each time period.

The overall frequency at which flow q stops at facility k in combination h is indicated by $e_q g_{qhk}$. For longer trips, $e_q = 1$ and g_{qhk} represents the actual number of refueling stops. For shorter trips e_q is <1 and reduces the average number of stops at a station. If $e_q = 0.5$ and the station is at the origin or destination ($g_{qhk} = 1$), the flow will stop every other trip ($e_q g_{qhk} = 0.5$). If the station is in the middle of the path ($g_{qhk} = 2$), then the flow will stop once per round trip, on average ($e_q g_{qhk} = 1$). The e_q and g_{qhk} coefficients could be combined into one, but with a loss of transparency.

Constraint (8) limits the number of units of capacity to be built. Constraint (9) prevents more than 100% of a flow from being allocated. Combined with constraint (10) it also means that y_{qh} varies from 0 to 1. Constraint (11) ensures that x_k is an integer.

The CFRLM incorporates several assumptions about the nature of the flows and the refueling stations. Flows are assumed to be infinitely divisible: a single vehicle is so small in relationship to total capacity that treating the flow as continuous is a reasonable assumption.

The model also assumes that refueling capacity is built in discrete, modular units, such as pumps. This is only one of several ways to represent capacity. Capacity could be represented continuously, assuming that a facility could be built to the exact size required. Making capacity continuous would remove the only integer variable from the model, completely avoiding branch-and-bound and perhaps making heuristic algorithms unnecessary. In some circumstances, continuously varying capacity may be a reasonable abstraction. It may, however, result in impossibly small facilities at some locations, representing a false economy of full utilization.

The model also assumes a uniform flow. In reality, automobile traffic has a large degree of temporal variation. Daily and weekly cycles produce large fluctuations in the number of vehicles per hour. In urban areas, traffic is highest during weekday peak periods and lower during weekends and off-peak times. Intercity highways may experience different cycles as traffic on a road to a popular recreation destination rises during the weekend. If a refueling facility is just barely able to handle the average refueling demands over the course of a day or week, this temporal variation would put it over its capacity during peak periods. Therefore, the most appropriate measurement for the model may be peak rather than average volumes.

The time and flow units used to measure peak volume and capacity will depend on fueling technology and behavioral assumptions. In a refueling station that operates similarly to gas stations (fuel delivered from off-site and stored in large quantities), the time it takes to dispense fuel to customers will be the bottleneck. Because much of a driver's time is taken with swiping a credit card, shopping for coffee or snacks, using restrooms, and cleaning windows, it is not unreasonable to assume that stops take the same amount of time regardless of the quantity dispensed. In this case, peak hourly number of vehicles stopping would be an appropriate unit for measuring capacity. For some alt-fuel stations, however, the total daily or weekly amount of fuel dispensed is the bottleneck. For instance, hydrogen, biofuel, and methane can be produced on-site in a ''distributed production'' system (California Hydrogen Highway Network 2005). In this case, alt-fuel production is the bottleneck, gallons or cubic-feet are the relevant unit of measurement, and constraint (7) would have to be rewritten using a miles-per-gallon factor and the distance since the last refueling (see below).

Finally, the model allocates trips to combinations of stations using the y_{qh} variables, which assumes that drivers will choose refueling stations in a system-optimal manner. In a situation with no excess capacity, every driver must stop at just the right set of stations, a set that will not only allow them to successfully reach their destination, but that will not interfere with anybody else's ability to refuel. Such a tightly integrated refueling ballet, though optimal to the model, is unlikely to transfer perfectly to the real world. We will return to this issue in the ''Directions for future research'' section.

Case study and solution procedures

This case study uses a simplified version of the Arizona state highway network previously used by Kuby, Lim, and Wang (2004) and shown in Fig. 1. The network includes 25 of Arizona's largest cities and major road junctions and links between them using major state, U.S., and Interstate highways. Candidate sites include the 25 nodes, as well as another 25 nodes distributed using the Added Node Dispersion Problem method (Kuby, Lim, and Upchurch 2005) to allow for stations between the major nodes. Flows between nodes were calculated using a gravity model based on the node populations. The Phoenix-Tucson flow comprises over 42% of Arizona's intercity traffic, followed by Phoenix-Casa Grande (9%) and Phoenix-Prescott (8%).

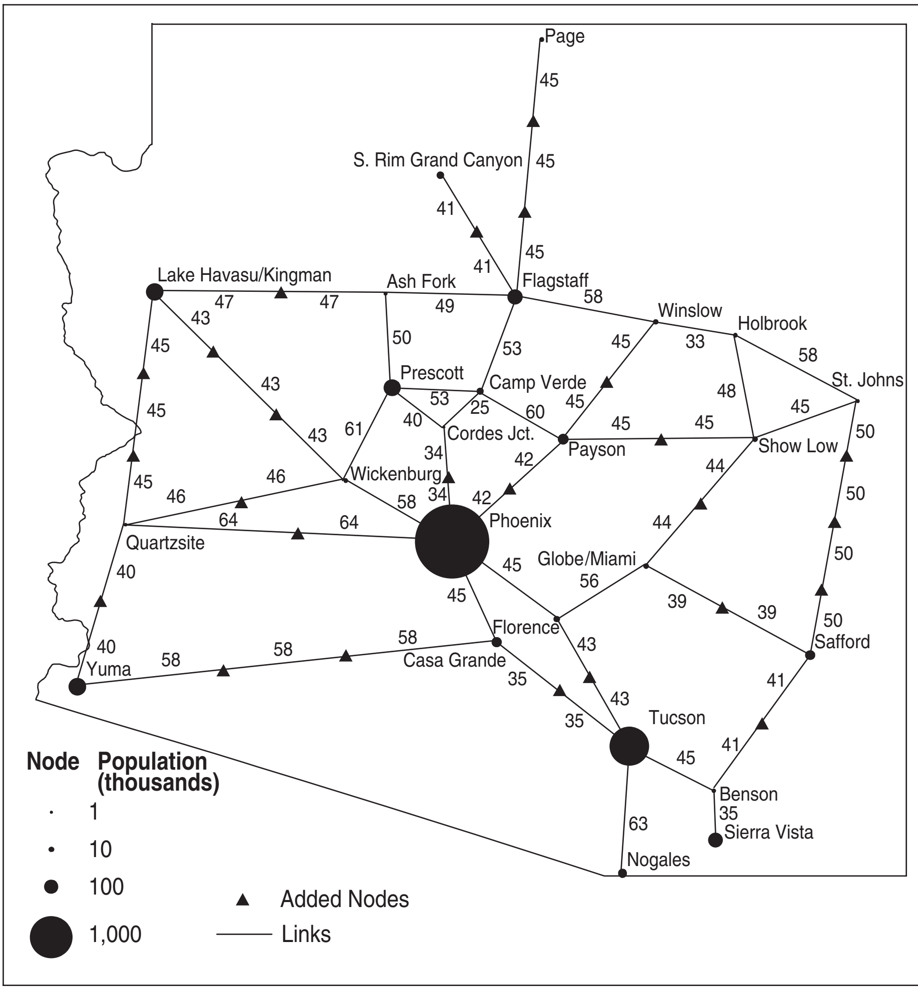

Figure 1. The network used for the Arizona case study.

The trip table does not include any flows to or from other states. External flows are excluded both because of a lack of data and because refueling only part of a flow would be ineffective unless refueling stations were also available to serve the out-of-state portion of the trip.

The driving range of the vehicles on a single tank of fuel is assumed to be 100 miles. The capacity should represent a prudent limit on when a driver should refuel rather than the technical limit of the vehicle assuming maximum storage, optimal performance, and no detours, side trips, or traffic jams. The facility capacity is arbitrarily set at 100,000 gravity-model units equivalent to almost 46% of the daily Tucson-Phoenix flow of alt-fuel vehicles, or 250% of the Phoenix-Prescott traffic. The range and capacity were chosen to be realistic but also to force the model to make interesting trade-offs. Keep in mind, however, that a minimum of three facilities—not one—would actually be required to refuel 46% of the Tucson-Phoenix trips, given that it is 115 miles one way and 230 miles round trip. A vehicle with a driving range of 100 miles would have to stop three times at two different facilities to complete the trip. One such combination is to refuel initially at a station in Tucson, again at Casa Grande 70 miles north, go 90 miles to Phoenix and back to Casa Grande, refuel again, and return to Tucson with 30 miles left in the tank. On the other hand, for the 90-mile round-trip Phoenix-Casa Grande flow, drivers would need to refuel only once at a station at either node.

The CFRLM was solved for the Arizona network using Xpress MP software on a HP laptop with a 1.5 GHz Pentium-M processor and 1280 MB RAM. It took 10 minutes and 48 seconds to solve for all values of p between 1 and 50. The model had 1477 variables (of which 50 were integer) and 322 constraints.

Results

When locating only one station ($p = 1$), it is placed in Phoenix and captures flows among Phoenix, Casa Grande, and Florence. This configuration covers 12.6% of total intercity flow. The limiting factor here is not the capacity of the Phoenix station, which uses only 63.3% of its capacity, but the limited number of flows that can be served by a single facility location. The 100-mile range of vehicles is too small to drive from Phoenix to (clockwise from due north) Cordes Jct., Payson, Globe, Tucson, Quartzsite, or Wickenburg on a single tank of fuel.

In the results for $p = 2$–5, stations come in and out of the optimal set, illustrating that a greedy-adding algorithm would not always find the optimal solution. The suboptimality of the greedy approach to the FRLM has already been demonstrated by Kuby and Lim (2005), who attributed it to need for a combination of facilities to refuel longer flows. In the CFRLM, an additional rationale is maximum utilization of capacity. Given that some trips require two or more refueling stops, and that the objective is to maximize the number of trips, the following results show that the CFRLM prefers to locate facilities in the dense-flow corridor to maximize capacity utilization, but also prefers to serve flows that require fewer refueling stops over those that require more stops to maximize the number of trips refueled.

When $p = 2$, stations are placed in Phoenix and on Interstate 10 between Casa Grande and Tucson, capturing 100% of the Tucson-Casa Grande flow and 16.9% of the Phoenix-Tucson flow in addition to the previous flows. Even though the trips from Phoenix to Tucson refuel twice on I-10 and only once in Phoenix, the Phoenix station operates at 100% of capacity while the station on I-10 operates at 86%. This is because the model prioritizes the other, *shorter* Phoenix-based trips that can be refueled with fewer stops, and any further utilization of the I-10 station would depend on the Phoenix station, which is at capacity.

With three stations, one is located in Tucson (operating at 84% of capacity), and two in Casa Grande (both at 100%). The Tucson station enables trips to Benson, which is a 90-mile round trip from Tucson. Casa Grande is favored over Phoenix because its shorter trips require fewer refuelings (one refueling stop per round trip to Phoenix, two per round trip to Tucson). The arrangement serves 100% of Casa Grande's and Benson's flows to each other and to Phoenix and Tucson, as well as 33.6% of Phoenix-Tucson flows.

The $p = 4$ solution is shown in Fig. 2. Stations are located at Cordes Jct. (99.5% utilization), Phoenix (85.3%), Casa Grande (100%), and on I-10 between Casa Grande and Tucson (100%). Why not add a fourth facility in the busy Phoenix-Tucson corridor, where much demand was still unserved in the $p = 3$ solution, instead of at Cordes Jct., the smallest town in the network? The reason lies in the aforementioned fact that three refueling stops are required for each Phoenix-Tucson trip. It therefore makes sense to add facilities in threes—a fourth facility in the corridor would be underutilized. The Cordes Jct. facility does a better job of balancing capacity with flows by connecting Prescott and Camp Verde to Cordes Jct., to Phoenix, and to points south.

The most efficient solution is for $p = 5$, with single stations in Phoenix, Tucson, and between Casa Grande and Tucson, and two in Casa Grande, all operating at 100% capacity. One set of three stations almost exclusively serve the Phoenix-Tucson traffic, while the two remaining facilities fully utilize their capacity serving several flows in and around the corridor.

Cordes Jct. reappears as the sixth facility, while the seventh through ninth facilities are again placed along the Phoenix-Tucson corridor, increasing the percentage of Phoenix-Tucson traffic being refueled to 89.5% at $p = 9$. The tenth

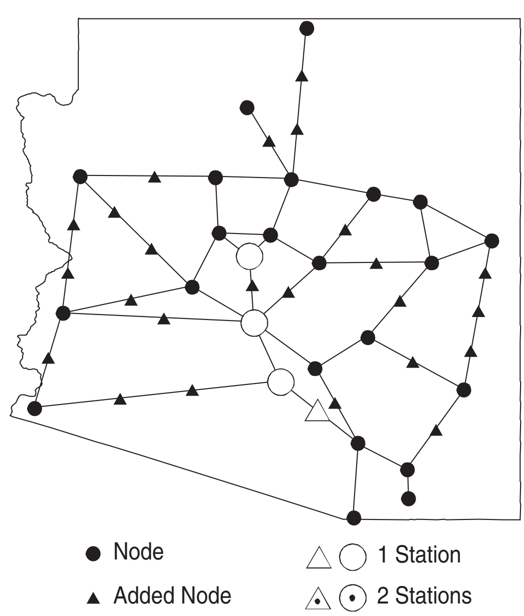

Figure 2. Solution results for the trips-maximizing objective for $p = 4$.

facility is in Benson, making it possible for vehicles with a range of 100 miles to go all the way from Prescott to Sierra Vista and back.

Increasing the number of facilities beyond ten expands the refueling network to cover increasingly larger portions of the state. Once again, certain cities may be added, then drop out of the solution. For instance, a station is located in Nogales at $p = 12$, but is dropped in favor of stations at Flagstaff and Camp Verde with $p = 13$. Nogales does not reappear in a solution until $p = 18$, once again illustrating the suboptimality of a simple greedy heuristic.

When p reaches 37, the number and arrangement of facilities are sufficient to refuel every flow in the network. As shown in Fig. 3, increasing the number of stations shows generally—but not strictly—diminishing returns. The best ''bang for the buck'' comes in the first eight stations, which increase total coverage an average of 7.8%. No station beyond the eighth increases coverage by more than 4.9%.

Fig. 3 also demonstrates that average capacity utilization follows a reverse-U shape. A single facility may not be able to achieve high utilization by itself because many paths require more than one refueling. Utilization peaks at 100% at $p = 5$, with local peaks at $p = 8$ and $p = 11$. Efficiency is maximized by adding facilities to the Phoenix-Tucson corridor in threes because it takes three stops to refuel a single Phoenix-Tucson trip. It remains above 90% through $p = 11$, at which point the entire Phoenix-Tucson flow is finally served. As smaller nodes are connected, utilization gradually declines. The shape of the average utilization curve will likely depend on the range of the vehicle and the structure of the network. With a longer-range vehicle, a dense network with heavy traffic, or if many short intraurban trips are included, the average utilization curve may start at 100% with $p = 1$ and plateau before declining.

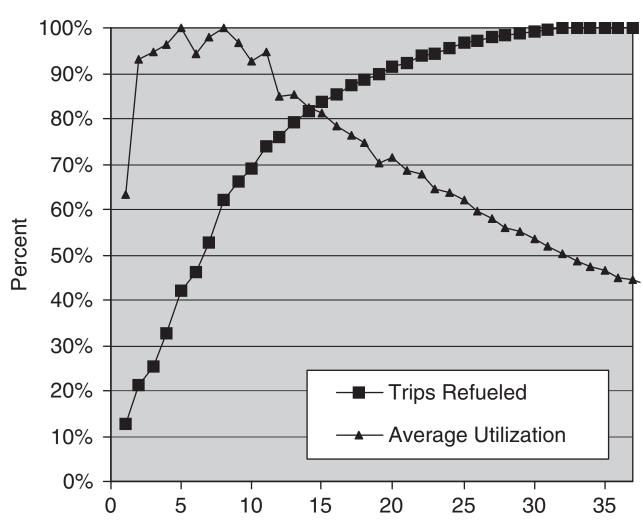

Figure 3. Percentage of trips and average capacity utilization for $p = 1$–37.

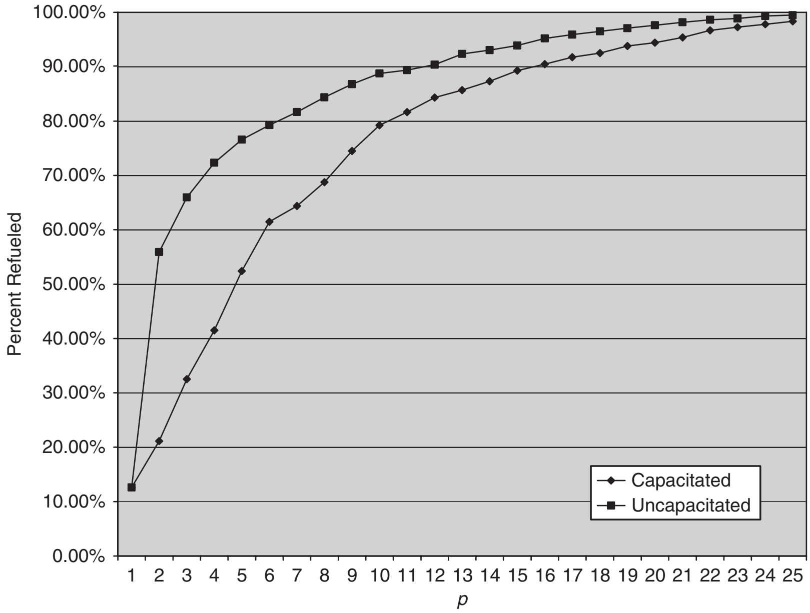

Figure 4. Percentage of trips refueled for the capacitated and uncapacitated models for $p = 1$–25.

The differences between the capacitated and uncapacitated versions of the FRLM are demonstrated by Fig. 4. Because capacity is not a limiting factor for $p = 1$, the uncapacitated and capacitated versions of the model serve exactly the same percentage of total demand with a single station. Once the second station is added, however, the uncapacitated model can serve a far greater proportion of total traffic. Stations at Phoenix and Casa Grande allow the model to capture all of the Phoenix-Tucson traffic, refueling as many trips with two stations as the capacitated model does with five. The marginal return of each new station decreases quickly, however, decreasing the difference between the two models with every additional station.

Comparing Figs. 2 and 5 illustrates the effect of the capacity constraint using the solutions for $p = 4$. The uncapacitated (Fig. 5) model places stations in Phoenix, Casa Grande, Florence, and Cordes Junction. The capacitated model (Fig. 2) locates all of its stations in the Phoenix-Tucson corridor.

Maximizing VMT

The CFRLM maximizes the number of trips that can be refueled. This may be an appropriate objective for breaking the chicken-and-egg dilemma if consumers are more likely to buy an alt-fuel car if they can complete more trips. If the objective is to reduce pollution, carbon emissions, or gasoline use directly, however, it may be more appropriate to target the longest distance trips first. The objective shown below maximizes the total VMT of refuelable flows of alt-fuel vehicles:

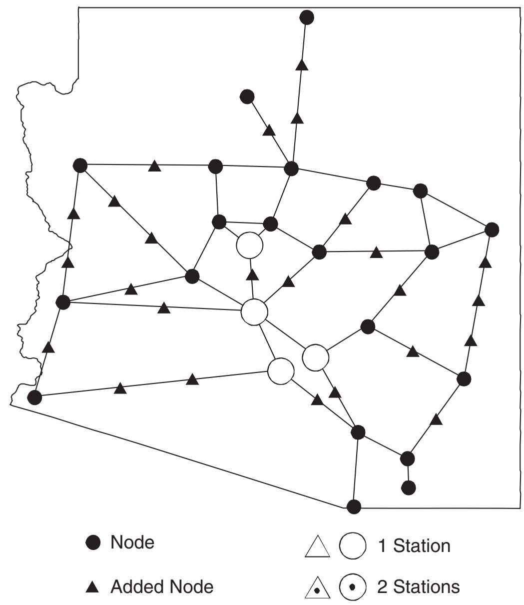

Figure 5. Solution results for the uncapacitated model for $p = 4$.

$$\text{Max } Z = \sum_{q \in Q} \sum_{h | b_{qh}=1} f_q d_q y_{qh} \tag{12}$$

where d_q is the shortest path distance between O–D pair q and all constraints and variables remain the same. The constant d_q acts as a weight, ensuring that longer trips will have priority. A multiobjective model, trading off trips and trip-distance, is also possible.

The station locations using the VMT objective are generally similar to the locations chosen using the trips objective. The first station is located in Phoenix, followed by a series of stations along the Phoenix-Tucson corridor. As with the results for the trips objective, there are several alternate optima.

The first real divergence between the station locations chosen by the trips and VMT objectives comes at $p = 4$. The trips objective placed the fourth station in Cordes Junction (Fig. 2), while the VMT objective has two in Phoenix and two on I-10 between Tucson and Casa Grande (Fig. 6). One Phoenix station and two I-10 stations are devoted exclusively to the Phoenix-Tucson flow, while the second Phoenix station connects Phoenix, Casa Grande, and Florence. That the trips objective, rather than the VMT objective, is the first to venture outside the Phoenix-Tucson corridor seems counterintuitive. The longer distances ought to make the more-distant station in Cordes Jct. better at maximizing VMT. However, given that both trips require three refueling stops but Prescott is 7 miles closer to Phoenix than Tucson is, the Phoenix-Prescott trip is less appealing for the VMT objective.

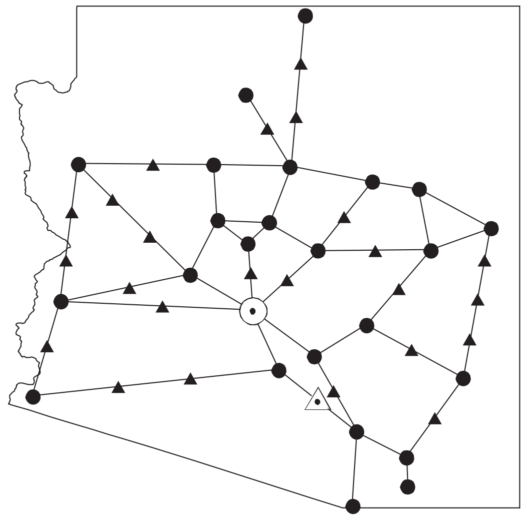

Figure 6. Solution results for the capacitated VMT-maximizing objective for $p = 4$.

This general pattern continues as p increases. At some values of p, the two objectives produce almost identical results. Overall, both objectives place their initial stations in the Phoenix-Tucson corridor. The results differ only at the margins, in which of Arizona's minor communities they choose to serve. These results beg the question, if the VMT objective weights trips by their length, why don't we see more differences due to prioritizing the longest trips? There are two reasons. First, longer trips require multiple refueling stations. If a long trip requires two refueling stations, while a short trip requires only one, the model could serve twice as many of the shorter trips. Second, in the Arizona case study, the only stations outside the Phoenix-Tucson corridor that ever reach 100% capacity are at Cordes Jct. and between Phoenix and Cordes Jct. Further from Phoenix, there just isn't enough traffic to use stations to their fullest. This economy of full utilization trumps the weight for longer distances. Results in other regions could be very different.

It is possible to track both number of trips and VMT that can be refueled as each objective is maximized (Fig. 7). As expected, the percentage of trips refueled is the same or slightly higher when trips are maximized than when VMT are maximized. Likewise, the opposite is true when VMT are maximized. The difference between the two objectives, however, is relatively small. For 12 of 37 values of p, there is no difference between the two objectives in terms of the amount of VMT refueled. The average difference between the two objectives is less than half a percent and the largest is 3.1%. The difference in the number of trips refueled is slightly larger, with an average difference of 0.9% and a maximum difference of 5.1%, and 13 values of p displaying no difference between the two objectives.

Regardless of the objective maximized, the percentage of trips refueled consistently runs ahead of the percentage of VMT for the same p. The two curves are closer together when maximizing the VMT, but the percentage of trips refueled is

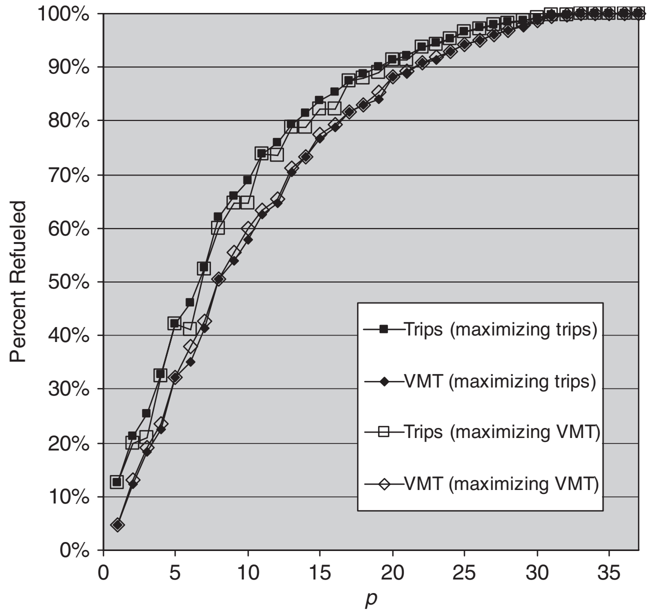

Figure 7. Total trips and VMT refueled using the trips objective, compared with the same using the VMT objective, for $p = 1$–37.

still noticeably higher. The model favors short trips even when maximizing VMT because short trips can be serviced by fewer facilities than longer trips. As both curves approach 100%, the gap gradually closes.

Fig. 8 tracks the number of unique candidate sites with facilities for both CFRLM objectives and the uncapacitated FRLM. The FRLM requires 27 facilities to satisfy all demand, and of course it needs only one per node because they are uncapacitated. The CFRLM models need 37 facilities spread over 29–31 unique nodes to refuel all flows, meaning at optimal 100% coverage, six to eight nodes have multiple facilities. Thus, if one had used the uncapacitated FRLM to solve for optimal locations, and then sized the facilities according to the flow volume served by each, one would have gotten a very different and suboptimal result.

Conclusions

By adding a capacity constraint, the CFRLM effectively addresses one of the major shortcomings of the FRLM. It provides a more realistic representation of refueling limitations. There are still many issues to address before the CFRLM becomes a truly realistic model. The addition of a capacity constraint is an important step in adapting the FRLM to real-world conditions.

The Arizona case study demonstrates the CFRLM's functionality, including its differences from the FRLM. Locations that were optimal to the uncapacitated FRLM locations may be suboptimal to the CFRLM. The case study also shows the use of

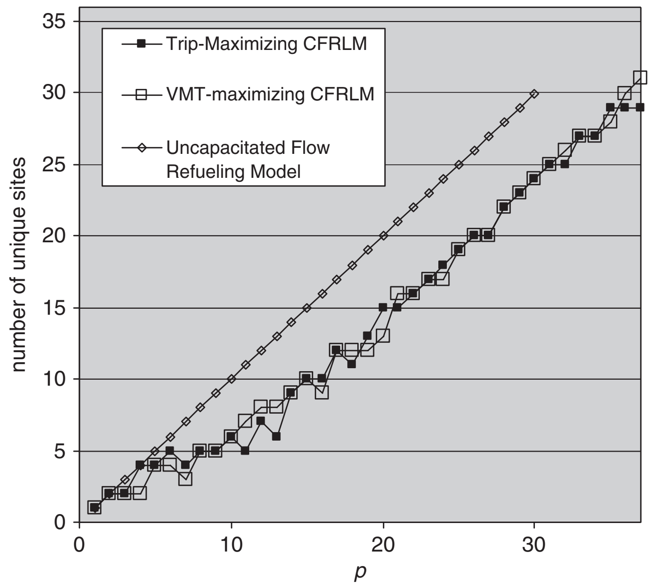

Figure 8. Comparison of unique sites with stations for the two CFRLM objectives and the uncapacitated FRLM, for all *p*.

multiple facilities at a single node, and the use of multiple facility combinations to serve flows for a single O–D pair.

A greedy algorithm can produce suboptimal results for the CFRLM—similar to the uncapacitated FRLM but different than the FCLM on which both are based. If we think of the $p = 1$ to n solutions as a plan for phasing in an alt-fuel infrastructure, this finding implies that there are difficult tradeoffs to evaluate between immediate and future performance when building out a network of refueling stations over time. A dynamic version of the CFRLM may be desirable for planning the phasing of alt-fuel stations.

As the two objective functions (trip-maximizing and VMT-maximizing) demonstrate, the motivation for creating a refueling network has a bearing on the formulation of the model and the ultimate location of the stations. Maximizing the number of trips that can be refueled may best address the chicken-and-egg problem because more refuelable trips may convince more drivers to purchase vehicles. Maximizing VMT, on the other hand, may best reduce the use of traditional fuels. While the two objectives are not one and the same, the case study demonstrates that the difference between them is relatively small for the Arizona network. An optimal solution for one objective is near-optimal for the other.

Directions for future research

There are many avenues available for future research using the CFRLM. The most obvious is to apply it to a more realistic network. The Arizona case study is sufficient to prove the functionality of the model, but it lacks realism in some areas. A more realistic case study could use actual origin and destination data instead of gravity model estimates. Dealing with out-of-state trips is more difficult, because the usefulness of refueling those flows depends on stations outside the study area. A relatively self-contained study area, with minimal flows to or from other regions, would be desirable.

The CFRLM assumes that all drivers will make their refueling choices in a system-optimal manner. This is clearly unrealistic, but determining just how much effect non-system-optimal behavior would have would require some other methodology. An agent-based model could be used to examine the effect of different refueling behaviors on a refueling network generated by the CFRLM. Alternatively, facility capacities in the CFRLM could simply be reduced by a cushioning factor to allow for driver behavior that is not system optimal, or the behavior of users could be separated from that of suppliers (Friesz, Gottfried, and Morlok 1986).

Other possibilities involve modifications to the model itself. The base model assumes that the amount of refueling capacity that could be built at each node is potentially infinite. This assumption could be modified to incorporate a maximum number of facilities at node *k*. A dynamic version of this model could be used to study phasing of refueling station construction, changes in vehicle range, growth of demand for the alt-fuel vehicles, and changes in population.

The current model places stations to maximize the number of trips refueled without regard for profitability. An economic component representing profitability could be developed to model the commercial incentive to build these facilities. Such a model could be used to examine the effects of subsidy policy on the location of fueling stations.

Earlier, we mentioned that the integer capacity variables could be treated as continuous variables for computational purposes, but it could lead to unrealistically small facilities being built. One way to partially overcome the problem of fractional facilities and still utilize a continuous capacity variable would be to represent both fixed and variable capacity costs calculated on a per-unit-of-capacity basis. Although costs are only included implicitly in the basic model (as a limitation on the number of facilities to be built), the *p*-facility constraint (8) could easily be rewritten as a budget constraint in terms of dollars.

The base CFRLM model assumes no economies of scale. The fact that the number of modules is fixed by constraint (8), regardless of whether they are placed singly at different locations or in groups, implies that the cost is constant regardless of the size of the installation. More realistic economies of scale could be introduced by defining capacity based on a number of fixed facility sizes (such as small, medium, and large) rather than modular units, as in Osleeb et al. (1986). The fixed cost of underground storage, for instance, would increase nonlinearly with scale. The larger sizes would provide increasingly more capacity per dollar of fixed cost. If more capacity than the largest station size were required, multiple facilities could be located.

Another extension would more accurately estimate the quantity of fuel needed at each station. Assuming tanks are always filled to capacity when refueling, the amount of fuel required can be calculated as the distance from the last refueling stop times the reciprocal of the fuel efficiency of the vehicle (e.g., gallons per mile). Because the possible refueling combinations for each flow are defined based on distance and vehicle range, the mileage since the last refueling stop is known.

While the above modifications apply exclusively to the CFRLM, there are a range of variants that apply to both the CFRLM and the FRLM. Both models assume that flows always follow the shortest path and do not allow deviations, even when a slightly longer route would allow a flow to be refueled. We are developing an extension incorporating the possibility of alternate routes, which could be applied to either model.

Some of the versions of the CFRLM proposed here could become extremely large and difficult to solve with branch-and-bound. As candidate sites are added along arcs, and multiple sizes of facilities are considered, the number of integer variables will mushroom. Add to this the fact that the number of O–D pairs q grows exponentially with the number of nodes, and if these are multiplied by several classes of vehicles and several detours off the shortest path, the size of the model may become unmanageable. We are in the process of exploring a variety of heuristic solution algorithms for the locating refueling stations.

Finally, with the introduction of alt-fuel stations and vehicles, other informational tools are needed for successful vehicle operation. Already, a Web site helps drivers or alt-fuel cars find stations along their shortest path. We tip our hats to an anonymous reviewer who suggested a GPS-based information system that would track a vehicle's remaining fuel and location and warn drivers before they travel beyond the reach of any surrounding stations.

Acknowledgements

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Notes

1 In the original formulation (Kuby and Lim 2005), the left-hand side of (3) contained $a_{hk}v_h$. The a_{hk} term is redundant given that the constraint is only written for h,k combinations for which $a_{hk} = 1$.

2 In Kuby and Lim (2005), valid facility combinations h for a given path q are determined assuming that vehicles begin each trip with a half-tank of fuel. If it can get from there to the next open station in the combination, then it can refuel at that same station on the way

back and return with at least half a tank, thus enabling it to make the same trip again or a different trip from the same origin.

3 The assumption that vehicles refuel at the same station(s) in each direction does not limit the model from splitting flows among different valid combinations to free up station capacity for other flows, because both capacity and the y_{qh} variables are infinitely divisible. For instance, if a flow could be refueled by stopping at station *X* in one direction and station *Y* in the other, the model could achieve an equivalent station usage by having half the traffic refuel at station *X* in both directions and the other half refuel at station *Y* in both directions. Because any valid combination of facilities must be able to start *and* finish its round trip with half a tank, if the origin-*X*-destination-*Y*-origin trip is refuelable, then the distance from origin to *X and* origin to *Y* must both be less than or equal to half the range. In that case, the origin-*X*-destination-*X*-origin trip and the origin-*Y*-destination-*Y*-origin trip should also be refuelable, and the model could split the traffic among them to achieve the same result.

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