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Socio-Economic Planning Sciences 39 (2005) 125–145

SOCIO-ECONOMIC PLANNING SCIENCES

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The flow-refueling location problem for alternative-fuel vehicles

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Available online 17 June 2004

Abstract

Beginning with Hodgson (Geogr.Anal.22(1990) 270), several researchers have been developing a new kind of location-allocation model for "flow capturing." Instead of locating central facilities to serve demand at fixed points in space, their models aim to serve demand consisting of origin-destination flows along their shortest paths. This paper extends flow-capturing models to optimal location of refueling facilities for alternative-fuel (alt-fuel) vehicles, such as hydrogen fuel cells or natural gas. Existing flow-capturing models assume that if a flow passes just one facility along its path, it is covered. This assumption does not carry over to vehicle refueling because of the limited range of vehicles. For refueling, it may be necessary to stop at more than one facility in order to successfully refuel the entire path, depending on the vehicle range, the path length, and the node spacing. The Flow Refueling Location Model (FRLM) optimally locates p refueling stations on a network so as to maximize the total flow volume refueled. This paper presents a mixed-integer programming formulation for the nodes-only version of the problem, as well as an algorithm for determining all combinations of nodes that can refuel a given path. A greedy-adding approach is demonstrated to be suboptimal, and the tradeoff curve between number of facilities and flow volume refueled is shown to be nonconvex.

Keywords: Facility location; Allocation; Optimization; Flow refueling; Flow capturing; Integer program; Network; Shortest path; Alt-fuel vehicle

1. Introduction

Despite widespread concern about environmental sustainability, climate change, and Middle East terrorism, most countries remain dangerously dependent on imported petroleum. In the US.,

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0038-0121/$ - see front matter
doi:10.1016/j.seps.2004.03.001

oil imports accounted for 23% of primary energy supply in 2000, while renewable energy, mainly hydropower, made up only 6.4% [1]. Europe and Japan are forging ahead of the US. in renewable energy use, but are even more dependent on imported oil. Meanwhile, urban air pollution caused by vehicle emissions is a serious problem in large metropolitan areas around the globe.

For these reasons, there has been tremendous interest and substantial public and private research into alternative-fuel (alt-fuel) vehicles. Electric cars are being produced for sale to the general public, and some utility fleets and mass-transit vehicles have been converted to run on natural gas. Large corporations are developing and testing fuel cells for use in automobiles, buses, and railways. Depending on the fuel type, fuel cells can be a low- or zero-emission technology. With hydrogen as the fuel, the main emission from the engine is water vapor. Of course, the ultimate sustainability of the complete hydrogen-vehicle system depends upon the source of the hydrogen. Many authors have discussed the ways in which hydrogen can be derived from water using renewable energy sources [2,3] although currently it can be derived less expensively from hydrogen-rich fossil fuels.

One critical issue holding back the widespread diffusion of alt-fuel vehicles is their limited range and the scarcity of refueling facilities in convenient locations [4–6]. The huge cost of developing a dispersed refueling or recharging infrastructure is a significant barrier to developing and adopting alt-fuel vehicles, which is one reason why the auto industry currently markets gasoline-electric hybrid vehicles as the low-emissions vehicle of choice. However, while hybrids can always be refueled at ubiquitous gasoline stations, hybrid engines may not lower emissions and oil imports significantly. Therefore, it is important to develop methods to minimize the costs of developing an alt-fuel refueling infrastructure.

The objective of this paper is to develop a location–allocation model for optimally locating refueling facilities for range-limited vehicles. The intention is to apply it to alt-fuel vehicles, where refueling infrastructure does not yet exist. The model, however, could also work for any new network-based, range-limited surface transport mode. Most location–allocation models are designed to optimally locate a set of facilities and allocate demand to those facilities. Typically, demand is considered as a set of nodes. Examples include the p-median [7,8], location set cover [9], maximum cover [10], and the fixed-charge [11] models. All four basic models assume that demand is expressed at nodes, and therefore the important distance to be concerned with is the direct path from node to facility. Whether or not the customer travels from demand node to facility or the service provider delivers in the opposite direction, and whether or not distance is measured as the Manhattan, Euclidean, or as the shortest path along a network, these models assume that a special-purpose trip is made between the node and the facility to satisfy demand.

Hodgson [12] was one of the first researchers to challenge this assumption. He argued that many types of facilities, such as convenience stores, automated teller machines (ATM), and gasoline stations, serve demand in the form of traffic flows that pass by the facility. The unit of demand to be covered is the path from an origin to a destination. At any point on the network, there are many flows between many origin-destination pairs passing by, some with higher flow volumes than others. Hodgson coined the term "flow capturing" to represent the goal of locating facilities to serve passing flows. Any flow that uses a path that passes through the location of the facility was considered to be captured. He conceptualized his initial model as a maximum-cover problem: locate p facilities so as to "capture" as many of passing flows as possible.

Although flow-capturing models provide a solid starting point for the flow-refueling location problem, they leave out one crucial element: a vehicle's range. The capacity of its fuel tank or battery and its rate of energy consumption limits its range. Depending upon the length of the path, it may be necessary to refuel more than once. This need to refuel a single flow more than once stands in sharp contrast to Hodgson's initial Flow Capturing Location Model (FCLM), which was designed not to doublecount flows captured by more than one facility along their path, because the driver will presumably stop only once. The Flow Refueling Location Model (FRLM), on the other hand, should consider a flow refueled only if an adequate number of stations are spaced appropriately along the path. Like the FCLM, captured flows should be counted only once, regardless of how many stations were necessary to refuel them.

Section 2 of this paper introduces the nomenclature of the basic FCLM and interesting extensions of that model. Section 3 explores the spatial logic of refueling network paths and develops an algorithm for determining feasible combinations of facility sites for each path. This section demonstrates why it may be suboptimal to restrict the search to the nodes of the network. Section 4 introduces integer and mixed-integer programming models for the discrete FRLM using the combinations of locations that are outputs of the algorithm in Section 3. Section 5 explains the case study, while Section 6 presents initial results on a 25-node network. The final two sections offers conclusions and directions for future research.

2. Literature review: flow capturing and refueling

In Hodgson's FCLM [12], each origin–destination (O–D) pair exerts a unique demand to be covered, not each node. Hodgson introduced the single index q to replace the double-subscript ij representing an O–D pair. If n is the number of origin and destination nodes, and each node can serve as both origin and destination, then q potentially ranges from 1 to n^2. Hodgson, however, made the simplifying assumptions that: (1) all flows are O–D flows, not circular tours; (2) the entire flow for a given O–D pair follows the same route or path through the network; (3) O–D flow matrices are symmetrical, meaning that the ordered pairs i,j and j,i are identical and thus can be treated as the unordered pair i,j; and (4) intrazonal flows from a node to itself do not have to be covered. With these assumptions, the maximum number of q to be captured is $n(n-1)/2$.

Once the notation change from ij to q is made, the formulation of the problem turns out to mirror that of the maximum-covering location problem:

$$\text{Max } Z = \sum_{q \in Q} f_q y_q \tag{1}$$

Subject to

$$\sum_{k \in N_q} x_k \geqslant y_q \quad \forall\, q \tag{2}$$

$$\sum_{k \in K} x_k = p \tag{3}$$

$$x_k, y_q \in \{0, 1\} \quad \forall\, k, q \tag{4}$$

where:

q = index of O–D pairs (and, by implication, the shortest paths for each pair)
Q = set of all O–D pairs
f_q = flow volume on the shortest path between O–D pair q
y_q = 1 if f_q is captured
0 otherwise
k = potential facility location
K = set of all potential facility locations
x_k = 1 if a facility is located at k
0 otherwise
N_q = the set of potential facility locations capable of capturing q
(that is, the set of potential facility locations on the path q)
p = the number of facilities to be located.

It is important to note that the set of potential facility locations can be limited to the set of network intersections. Hodgson argued that facilities located at junction nodes rather than midway along arcs can capture all flows that use that arc plus all crossing flows, which enabled him to limit the search considerably. Berman et al. [13] independently studied the problem and reached similar conclusions about the ability to limit the search to junction nodes and also about the robustness of the greedy-adding heuristic. They used the term "discretionary service facilities" to refer to facilities that capture passing flows. The term "discretionary" will not be adopted here because of the obvious *necessity* of refueling vehicles before they run out of fuel.

The FCLM was designed by Hodgson to prevent double counting of flows captured by more than one facility along its path. In the integer programming formulation of the FCLM, double counting is prevented by constraint (2) and the definition of y_q as a binary variable: even if the left-hand side of an instance of (2) were to contain several x_k variables equal to one, y_q cannot be greater than one. A related problem is that of "flow cannibalization," a term that Hodgson used to describe the location of a facility that captures flows that previous facilities have already captured. Flow cannibalization is not an issue when the FCLM is solved to its global optimum by branch-and-bound, but it can be a problem for a heuristic solution that locates facilities one at a time. Hodgson avoided flow cannibalization by using a greedy algorithm that removed already-captured flows before determining which new facility would capture the most uncaptured flows.

The basic FCLM has been extended in a number of ways [14]. Hodgson and Rosing [15] considered the fact that demands in the real-world can be exerted by stationary nodes or by passing flows. For instance, many people go to convenience stores or ATMs on their way to somewhere else, but some will make a single-purpose trip from home. They developed a hybrid model with the dual objective of serving both kinds of demand and use a p-median objective to represent the demand at nodes. The same facilities serve both demands. They found that the p-median objective is fairly tolerant of minor locational shifts when facilities are shifted to locations that better capture passing flows, but minor locational shifts to locations that better serve the nodal demands seriously degrade the FCLM objective.

Berman et al. [16] extended the FCLM in a number of ways, some of which are relevant to the refueling problem. Four extensions deal with flow capturing when flows are allowed to deviate from the existing or shortest path by a maximum deviation of Δ. Customers are counted as captured if their path passes within Δ of the facility at its closest point. In their second extension, the percentage of customers captured is considered to be a decreasing function of the deviation. In the third, an objective of minimizing total deviations replaces the maximum deviation Δ. All three problems obviously pertain to the refueling behavior of drivers, who certainly can deviate from a path to refuel, but these extensions still do not ensure that multiple refuelings occur if the vehicle range is less than the path length. All three extensions are solvable by exogenously modifying the input data to the FCLM or its extensions, that is, by defining the cover sets and weights differently. In the same paper, they also extended the deviation models by adding the deviation time to the delay due to congestion at the facility and developed several probabilistic models. For our purposes, however, there is no need to build probabilistic versions of the FRLM in this, its early stage of development, so these models will not be further reviewed here.

Hodgson and Berman [17] reformulated the FCLM for locating billboards for passing motorists to see. Unlike in the FCLM, in which there is no benefit to more than one facility capturing a flow, there is a benefit to displaying an advertisement to a driver multiple times. Therefore, they relaxed the single-counting requirement that prevented double-counting of flows captured by more than one facility in the FCLM. Instead, they defined a concave, nondecreasing function to define the diminishing marginal impact of each subsequent viewing.

The optimal location of vehicle inspection stations for intercepting dangerous vehicles was modeled by Hodgson et al. [18]. Two cited threats were hazardous waste and drunk drivers. This model extension departs from the others in that the facilities only cover that part of q that is ‘‘downstream’’ of the facility. The portion of the path before it reaches the inspection facility is not considered to be captured or covered. This notion of partial coverage introduces an idea that may have some relevance to locating refueling stations. The greedy heuristic for this problem was found to produce generally suboptimal solutions, while the mixed-integer solution by branch-and-bound was typically inefficient. Other researchers have applied the FCLM to real networks [19].

Even though Hodgson's original paper mentions gasoline stations as a potential application, neither the original model nor the extensions are designed to handle the location of refueling facilities. In all likelihood, the flow refueling location problem has not appeared yet in the literature because of the dominance of internal combustion vehicles and the ubiquity of gasoline stations throughout the developed world. Only two location-allocation models were found that dealt specifically with gasoline retailing. In the first, Goodchild and Noronha [20] maximized market share based on two types of demand. Demand based on node-to-facility trips was weighted and combined with traffic flows, similar to [15] except the flow volumes were link flows, not O–D flows. The model was applied to the problem of rationalizing a company's set of gasoline stations after merger with another company. The advantage of their link-based approach is the greater availability of traffic count data than the O–D data needed for flow-capturing. The disadvantages (for alt-fuel refueling purposes) are its doublecounting and cannibalization of flows that pass over many links and its inability to assess whether the entire path can be refueled. Goodchild and Noronha's approach seems better suited for a mature, saturated stage of the industry.

Bapna et al. [4] wrote the only earlier paper focusing upon infrastructure development for alt-fuels, which in their case was conversion from leaded to unleaded gasoline in India. They call their multiobjective approach the Maximum Covering/Shortest Spanning Subgraph (MC3SP) problem. Their approach tackles similar issues as ours, such as vehicle range, but in a different way. Their decision variable X_{ij} is equal to 1 if arc ij is *enabled* by adding and/or upgrading enough stations so that a vehicle of standard range can traverse the arc using alt-fuels. This number of stations is determined exogenously for each arc based on a maximum spacing and the location of existing gas stations. One objective is to minimize the fixed cost associated with enabling arcs plus the variable costs of traffic along that arc; the other is to maximize the population on or near enabled arcs. Constraints ensure that the resultant subgraph be able to span the network and connect all cities.

To summarize the prior research, Bapna et al.'s approach [4] has been used for choosing arcs to enable with alt-fuel stations, but two factors limit its applicability. Depending on the weights on the objectives, the spanning subgraph may necessitate extremely roundabout paths. Also, because all long-distance paths are necessarily covered due to the spanning constraint, the maximum-cover objective maximizes the population along the enabled arcs, not the volume of O–D flows that can refuel along their shortest path. Goodchild and Noronha's model [20], meanwhile, double counts flows that pass over several links. The FCLM provides a more appropriate framework for single-counting the preferred paths of travelers whose demand is summarized in a standard origin-destination trip table. However, neither the FCLM nor its extensions can handle the multiple refueling stops needed for paths longer than the vehicle's range. The basic model cannot handle multiple captures, the billboard model diminishes returns for multiple captures, and the dangerous vehicle inspection model captures only the downstream part of the path. Multiple refueling stops is the key issue that the following section addresses.

3. The network logic of vehicle range

3.1. An example with six cases

Vehicle range is the key issue that sets this problem apart from the flow-capturing problem. The simple example in Fig. 1 illustrates the spatial characteristics of refueling, showing the shortest path of 500 miles between an origin O and a destination D with two nodes along the way. We assume that all flows are repeated on a regular basis. This simple example will demonstrate several key points through several cases assuming different vehicle ranges.

In Case 1, the range of the vehicle is over 1000 miles. In this case, a location anywhere on the path will be able to refuel either the one-way or round-trip journeys. If vehicles had

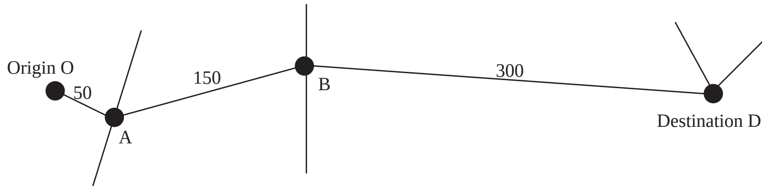

Fig. 1. Example round-trip flow for an origin–destination pair. Case 1 (range > 1000): any location can refuel the round trip. Case 2 (range = 700): B is the only node that can refuel the round trip. Case 3 (range = 550): no single node can refuel the round trip, but a midlink location can. Case 4 (range = 400): even a single midlink location cannot refuel the round trip—multiple facilities needed. Case 5 ($300 \leqslant$ range $\leqslant 399$): more than two facilities required if location is restricted to nodes. Case 6 (range < 300): no possible combination of nodes can refuel the round trip.

a range larger than the length of the longest path, the refueling problem would reduce to the FCLM.

Case 2 illustrates why it is essential to consider the complete round trip instead of the separate one-way flows and that it is not just the number of facilities along a path but their spacing that matters. In Case 2, the vehicle's range is 700 miles. If only the one-way trip is considered, a location at any node would refuel the vehicle because the total distance of 500 miles is less than the range of 700 miles. But when the round trip is considered, one realizes that the vehicle cannot refuel at origin O or node A, travel to destination D, and return to the refueling facility without running out of fuel. The same goes for a facility at destination D. However, if there were a facility at node B, the path would be covered as long as the vehicle refueled in both directions. It is 600 miles from B to D and back and 400 miles from B to O and back, both of which are less than the vehicle's 700-mile range.

Once one realizes that the model must consider round-trip paths, it follows that one must make careful assumptions about the starting level of the fuel tank. If there is a facility at the origin of the path, it is logical to assume that the tank will start out full. If not, it is safe to assume that the tank starts out half full, because if one can start at node O with a half tank and reach a refueling station along the path without running out of fuel, it follows that one could have also made the reverse trip from the station to node O with a half tank or less. In a mountainous region with large elevation changes that affect fuel consumption, this assumption may not be valid, but in this paper we assume that fuel consumption is strictly a function of distance. The more complex case is left for future work.

Case 3 illustrates a third important point, namely that nodal locations are not always optimal. In Case 3, the vehicle range is 550 miles, and no single nodal location can enable the vehicle to complete the round trip. However, if a facility were located at the exact midpoint of the path (50 miles east of B), a vehicle starting at the origin with a half tank could proceed to the midpoint, fill up, drive the 500 miles to D and back, fill up again, drive 500 miles to O and back, fill up again, and so on. In fact, any location within 25 miles of the exact midpoint could succeed in refueling the vehicle in both directions. More generally (vehicle range minus one-way distance)/2 equals the allowable departure from the exact midpoint.

Generally speaking, midlink locations cannot be ruled out in covering problems, which also deal with an explicit distance limit. Church and Meadows [21] recognized this fact and suggested that, in the set of potential facility locations, the set of nodes be augmented by the set of "network intersection points," which were all points on links that are exactly the cover distance from each demand node. A similar idea in the FRLM would be to divide the vehicle range by two and place "path midpoints" at that distance from any origins and destinations that need it. The need to add path midpoints could arise for any O–D pair with a round-trip path longer than the vehicle's range. On the other hand, while midlink locations might be optimal for a single path, their benefits may be offset by the fact that midlink locations are unable to refuel crossing flows. This problem will be left for a future paper. This paper will explore the basic properties of the discrete flow-refueling problem in which locations are restricted to the nodes of the network.

Returning to Fig. 1, still to be considered is Case 4, in which one refueling facility, even perfectly placed, will not be able to cover the entire path because the vehicle range is less than half the round-trip distance. In this case, multiple facilities along the path are necessary. In contrast to the billboard location problem [17], the facilities must be properly spaced. In Fig. 1, if the vehicle's range is 400 miles, the only combination of nodes that can refuel the round-trip path are B and D. On longer paths with many crossing links, many combinations of adequately spaced pairs of facilities will be able to refuel them.

Next, we can consider Case 5 as indicative of all paths for which more than two facilities are required. In Fig. 1, if the vehicle's range is any number between 300 and 399 miles, facilities are needed at B, D, and either A or O. Generally speaking, as the range continues to drop, more and more facilities would need to be added along each path.

Once the range of the vehicle drops below the maximum link length on the path (Case 6), it becomes impossible to refuel vehicles on the path in the nodes-only version of the problem. In reality, a driver may be able to detour from their shortest path onto one that contains shorter links to reach the same destination, as in [16]. This more complex problem, the flow-refueling problem with detours, is left for future work. Without dealing with this possibility, the results of this paper are likely to underestimate the covered demand in reality.

3.2. An algorithm for determining the combinations of nodes that can refuel a path

With so many possible O–D pairs in most real networks, and six possible cases for each path, a submodel is needed to determine which case (or cases) fit each path. Recall that Hodgson defined the index q as the cumulative index for the shortest paths for each unordered origin–destination pair i,j ($j<i$). In the algorithm below, the subscript h is introduced to represent a combination of facilities k able to refuel path q. There is a master list of combinations h, shared by all q, that is indexed sequentially beginning with 1, i.e., it is possible for the same combination h to refuel more than one path q. The following algorithm generates and records all viable combinations h that can refuel each path q.

The assumption of a fixed vehicle range assumes that fuel consumption is a constant per mile on all links and that all vehicles have the same fuel consumption rate and fuel tank size. This assumption also allows us to measure fuel as its distance equivalent, e.g., 100 miles of fuel remaining in the tank. We refer to this range as the *remaining fuel range*, which is not to be confused with the vehicle range.

Step 1: Initializations.

1.1. Generate the shortest path for all origin–destination pairs q. Store the nodes and links of the path.

1.2. Establish an empty master list of all combinations h.

Step 2: Beginning with the next path q on the list, generate a list of all possible combinations h of the nodes on the path. For example, for the path in Fig. 1, the possible combinations are {O}, {A}, {B}, {D}, {O,A}, {O,B}, {O,D}, {A,B}, {A,D}, {B,D}, {O,A,B}, {O,A,D}, {O,B,D}, {A,B,D}, {O,A,B,D}.

Step 3: Remove facility combinations that cannot refuel a vehicle of the given range on the given path. For each combination:

3.1. Begin at the origin node of path q. If there is a facility at the origin, set the remaining fuel range equal to the vehicle range, presuming that the vehicle is fully refueled there. Else, at the origin, set the remaining fuel range equal to half the vehicle range. It is set at half because, as Case 2 showed above, one must consider the round-trip. If the vehicle was last refueled at the nearest refueling station to the origin, there must be at least half a tank remaining in order for the vehicle to reach the origin, turn around, and return to that same facility. There could be more than half a tank remaining upon reaching the origin, but if there is, it can be ignored, because the excess will not be needed to return to the nearest facility.

3.2. Move to the next node on the round-trip path and subtract the distance traveled from the remaining fuel range, and check the following four conditions in the order given.
- If the remaining fuel range is less than zero, fuel would have run out before the node was reached. Remove this combination of facilities from the list of possible combinations for path q and go back to the start of Step 3 for the next combination.
- If the node is the destination, then:
  - If the destination node has a refueling station, this combination of facilities can refuel this path. Keep it in the list of combinations for path q, and go back to the start of Step 3 for the next combination.
  - Else, return to the top of Step 3.2. (In this case, the vehicle will be doubling back towards the origin.)
- If the node is the origin, the vehicle has made it back without running out of fuel. Keep the combination in the list of combinations for path q, and go back to the start of Step 3 for the next combination.
- If the node has a refueling station, set the remaining fuel range equal to the vehicle range, and go to the top of Step 3.2.
- Else, return to top of Step 3.2.

3.3. When all possible combinations have been evaluated for path q, go to Step 4.

Step 4: Remove combinations that are supersets of any other remaining combination, that is, there is another valid combination that is a strict subset of it. For example, if a path can be refueled by facilities at nodes 3, 5, and 7, and also by facilities at 3 and 7 only, there is no need to consider the larger combination. This is similar to Toregas and ReVelle's [22] column-reduction technique for the set-covering problem.

4.1. Sort the combinations based on the number of nodes in the combination in descending order, so that combinations with more nodes are above the combinations with less nodes.
4.2. Start with the first combination. If it is a superset of any combination after itself, remove it from the combinations list for path q.
4.3. Go to the next combination, and repeat the process until reaching the end of the combinations list for path q.

Step 5: Record the facilities k in each viable combination h, and the viable combinations h for path q. These relationships are stored in two sets of coefficients. The first coefficient, b_{qh}, equals 1 if facility combination h can refuel path q and 0 otherwise. The second, a_{hk}, equals 1 if facility k is in combination h, and 0 otherwise.[1] For all combinations remaining on the list for path q:

- If the combination does not already exist in the master list of combinations, add it to the master list using the next available h value and set $a_{hk} = 1$ for all k in h. Use the new h value to set $b_{qh} = 1$.

[1] Note that a minor difference between Hodgson's and the FRLM formulations is that we use incidence coefficients instead of set notation. This difference is mostly a matter of semantics—varying ways of expressing "all combinations h that can refuel path q." Using set notation consistent with Hodgson, we would have defined H_q as the set of potential facility combinations h capable of refueling path q. However, to be consistent with the way the model is set up in the math programming software Xpress-MP, we say the same thing using incidence coefficients.

- If the combination already exists in the master list of combinations, use the existing h value and set $b_{qh} = 1$.

Step 6: Repeat Steps 2–5 for all paths q.

4. Formulation of the flow refueling location model

Section 3 showed how to determine exogenously which *facility combinations h* could successfully refuel vehicles on each path q. These combinations are used in the following integer programming formulation of the FRLM. A new variable v_h is introduced which equals 1 if all the facilities in combination h are open, and 0 otherwise. This formulation is the most compact, in terms of number of variables and constraints, of the several with which we experimented.

$$\text{Max } Z = \sum_{q \in Q} f_q y_q \tag{5}$$

Subject to

$$\sum_{h \in H} b_{qh} v_h \geq y_q \quad \forall q \in Q \tag{6}$$

$$a_{hk} x_k \geqslant v_h \quad \forall h \in H; k | a_{hk} = 1 \tag{7}$$

$$\sum_{k \in K} x_k = p \tag{8}$$

$$x_k, v_h, y_q \in \{0, 1\} \quad \forall k, h, q \tag{9}$$

where the following notation is the same as in the FCLM:

q	=	index of O–D pairs (and, by implication, the shortest paths for each pair)
Q	=	set of all O–D pairs
f_q	=	flow volume on the shortest path between O–D pair q
y_q	=	1 if f_q is captured 0 otherwise
k	=	potential facility location
K	=	set of all potential facility locations
x_k	=	1 if a facility is located at k 0 otherwise
p	=	the number of facilities to be located

and the remaining notation is new to the FRLM:

h	=	index of combinations of facilities
H	=	set of all potential facility combinations
a_{hk}	=	a coefficient equal to 1 if facility k is in combination h and 0 otherwise
b_{qh}	=	a coefficient equal to 1 if facility combination h can refuel O–D pair q and 0 otherwise
v_h	=	1 if all facilities in combination h are open 0 otherwise

The objective function (5), which maximizes the total flow that can be refueled with p facilities, is the same as in the FCLM. Constraints (6) are similar to constraints (2) in the FCLM, except instead of requiring at least one eligible *facility i* to be open on path q, they require at least one eligible *combination of facilities h* to be open for paths to be refueled. One such constraint is written for each path q. The eligible combinations may consist of single facilities that can refuel the entire path, pairs of facilities, or more, as explained in the previous section. Constraints (7) hold v_h to zero unless all the facilities in combination h are open. Constraint (8) requires exactly p facilities to be built, and constraints (9) are the integrality requirements for the variables, which are all binary.

Although v_h and y_q are both defined as 0–1 variables, they can be relaxed in a mixed-integer program as continuous variables with an upper bound of 1, and still yield an all-integer solution, as in (5)-(8) plus (10), (11), (12), and (13):

$$x_k \in \{0,1\} \quad \forall k \tag{10}$$

$$v_h \leqslant 1 \quad \forall h \tag{11}$$

$$y_q \leqslant 1 \quad \forall q \tag{12}$$

$$v_h, y_q \geqslant 0 \quad \forall h, q \tag{13}$$

The reason is, the y_q are in a maximization objective function, and therefore are driven to their highest possible value. In constraint (6), that value is bounded from above by b_{qh} times $v_{h.}$ The former coefficient, b_{qh}, is a 0 or 1. The latter variable can be defined as $0 \leqslant v_h \leqslant 1$ because it too is driven towards 1 because of its relationship with y_q in (6). Following this "domino effect," constraint (7) in turn bounds v_h from above by a_{hk} times x_k, both of which are defined as 0 s or 1 s. In other words, y_q "wants" to be 1, so it will not stop at a fractional value unless limited by v_h. In turn, v_h also "wants" to be 1 unless limited by x_k, and will not end up fractional. In problem instances where p is so large that some combinations h may be redundant on all paths they refuel, it is possible that those v_h could solve as fractions in an alternate optima, but this will not affect the integrality of the coverage variables or the global optimality of the rest of the solution. Therefore, only x_k needs to be enforced as a 0–1 variable in branch-and-bound. In the discrete version of the FRLM, this greatly reduces the number of required 0–1 variables.

5. Case study and solution procedures

The FRLM was tested on the same 25-node sample network used by Hodgson [12] and before that by Berman and Simchi-Levi [23] (Fig. 2). Following Hodgson, the flow volumes f_q in the 25×25 O–D matrix were estimated using a gravity model. Flows were then assigned to their shortest paths. The candidate facility sites in this paper are limited to the 25 nodes of the network.

The FRLM mixed-integer programming model was generated using the Mosel modeling language and solved with the XPress-MP 12.0 solver on a Dell Precision 340 with 2.53 GHz Pentium-4 processor and 1024 MB of RAM. The problem size varies with the vehicle range: the

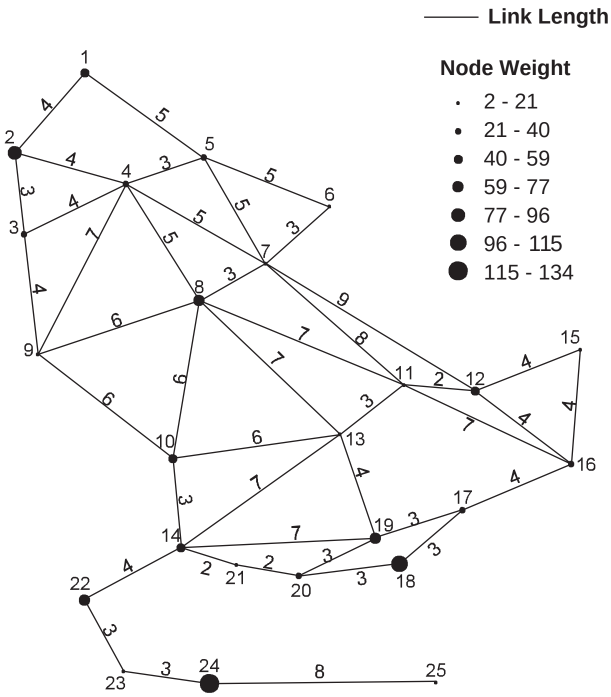

Fig. 2. The test network, based on Hodgson [12], originally from Berman and Simchi-Levi [23], has been redrawn for greater spatial accuracy, although the node weights and link lengths are identical.

longer the range, the more combinations can potentially refuel a given path. With a range of 4, the model has 300 continuous fractional flow-coverage variables y_q, 75 continuous fractional variables v_h for combinations of facilities, and 25 integer variables x_k. The model also has 300 flow refueling constraints (6), 223 combination constraints (7), and one p-facility constraint (8). With a vehicle range of 12, the number of combination variables goes up to 412, and the number of combination constraints rises to 1274.

Although the model was solved with LP and branch-and-bound, it was still possible to assess how a greedy algorithm would perform in terms of global optimality. Using XPress-MP, the model can be solved in a do-loop starting with $p = 1$ facilities. Within the loop, the x_k solution can be read and, if equal to 1, can trigger the setting of an $x_k \geqslant 1$ constraint. The model then augments p to $p + 1$ and returns to the start of the loop where it is re-solved to optimally locate the $(p + 1)th$ facility given the locations of the first p facilities. This procedure continues until facilities are allocated to all nodes, thus mimicking a greedy algorithm.

6. Results

6.1. Effect of vehicle range

Fig. 3 summarizes the results when the model was solved globally (exactly) and greedily with vehicle ranges of 4, 8, and 12 distance units, for p from 1 to 25. Solution times are given in Table 1.

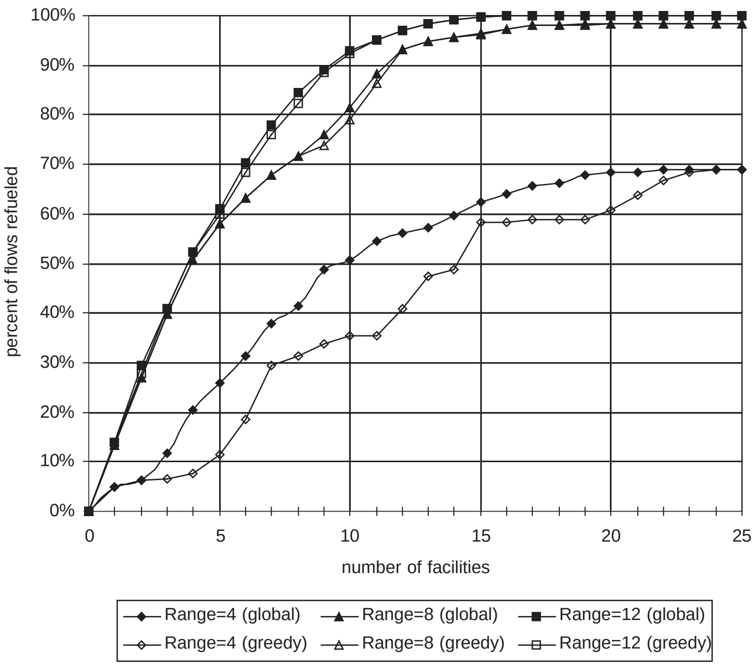

Fig. 3. Tradeoff graph between number of facilities and percentage of flows refueled, for different vehicle ranges and solution methods.

Table 1
Solution Times for $p = 1$–25 (seconds)

	Vehicle range		
	4	8	12
Global reoptimization at each p	0.6	3.8	6.5
Greedy optimization	0.5	1.2	1.4

A number of important characteristics of the model are illustrated by these 150 solutions. First, the longer the vehicle range, the fewer facilities it takes to refuel any given percentage of flows. For instance, 60% or more of the flow volume can be refueled with 5 optimally located facilities if the vehicle range is 12. With a range of 8, it takes 6 facilities to surpass 60%, and with a range of 4 it takes 15.

In the discrete, nodes-only version of the FRLM, it is not possible to refuel all flows if the vehicle range is too small, even with all facilities opened. If on a given path the range is shorter than one of the links between nodes, the vehicle would run out of fuel before making it to the next node. For instance, with a range of 8, all but 1.67% of the flow volume can be refueled, a plateau that is reached at 20 facilities. All paths that use the link from nodes 7 to 12, with a length of 9, cannot be refueled with a vehicle range of 8. The shortest paths between nodes 1–7 and nodes 12 and 15 use this link and therefore are not be refuelable with vehicles of range less than 9. With a vehicle range of 4, the maximum volume of flow that can be reached is 68.94%, attainable with 23 facilities (global) or 24 facilities (greedy). With a vehicle range of 12, however, 100% of the flows can be refueled with 17 facilities. The benefits of being able to refuel all paths may go beyond the difference between, say, 90% and 100% coverage, because participation in the alt-fuel industry may be depressed by consumers' inability to refuel their vehicles on routes they drive only occasionally.

6.2. Lack of convexity

The tradeoff curve between the number of facilities sited and the volume of flow refueled is monotonically non-decreasing, but not convex even when globally optimized. There are many concavities where the percentage of flow volume refueled increases at a slower rate until two or more facilities are added, and then speeds up again. This can occur if the uncovered paths require a combination of facilities to refuel them. Not until all facilities in the combination are sited does the objective function increase.

Lack of convexity in the exact solution is best illustrated with a short vehicle range of 4. With $p = 1$, the FRLM locates a facility at node 21 because it has links on either side with a length of 2. A vehicle could leave node 21, go to node 14, and return to node 21 without running out of fuel (just barely). The other paths that can be refueled with this short-range vehicle are those between O–D pairs 20–21 and 14–20, for total volume of 87,007 (Table 2). With $p = 2$, facilities are located at 18 and 20, which enables the large demand on the flow from nodes 18 to 20 to be served. This large flow was impossible to serve with only one facility, but it is now possible to serve with two. With a distance of 3 between nodes 18 and 20, facilities are needed at both ends because otherwise the round-trip distance between refuelings would be 6, which would exceed the vehicle range of 4. The flows between pairs 20–21 and 18–21 can also be refueled. These three flows can serve a flow volume of 111,706, which is 24,699 more than could be served with 1 facility. The concavity in the tradeoff curve occurs when p is increased to 3, node 21 is added, and the total flow volume increases by 97,228. This large increase is made possible partly because of the paths that node 21 can refuel alone (i.e., paths 14–21 and 14–21–20) and also because of the paths that the combination {18,20,21} can refuel (i.e., path 14–21–20–18).

6.3. Greedy solution

As for many location problems, a greedy-adding approach will sometimes be suboptimal for the FRLM. In the case of the FRLM, however, the fact that many paths cannot be refueled by a single facility compounds the usual problem of myopic location choice. The optimality gap between the global and greedy approaches can be quite large for very small vehicle ranges (Fig. 3). For a vehicle range of 4 and $p = 4$, the greedy objective reaches a maximum of 62% below the global optimum. The gap is of course nonexistent for $p = 1$ and 25 (all nodes), but for the other 23 pairs of solutions, the gap averages a startlingly high 21%. The optimality gap is much lower for the vehicle ranges of 8 and 12, reaching a maximum of 5.9% for a range of 12 and $p = 2$.

Table 2 illustrates the main reason for the poor performance of the greedy algorithm—namely, its inability to anticipate the benefits of combinations of facilities. We have already discussed the

Table 2
Solution results for a vehicle range of four

	Global (Exact)			Greedy		
	Facilities	O–D pairs refueled	Flow volume	Facilities	O–D pairs refueled	Flow volume
$p=1$	21	14–20	26,550	21	14–20	26,550
		14–21	37,547		14–21	37,547
		20–21	22,910		20–21	22,910
			87,007			87,007
$p=2$	18,20	18–20	72,053	11,21	11–12	23,900
		18–21	16,743		14–20	26,550
		20–21	22,910		14–21	37,547
			111,706		20–21	22,910
						110,907
$p=3$	18,20,21	14–18	33,131	11,13,21	11–12	23,900
		14–20	26,550		11–13	1,250
		14–21	37,547		12–13	2,325
		18–20	72,053		14–20	26,550
		18–21	16,743		14–21	37,547
		20–21	22,910		20–21	22,910
			208,934			114,482

reasons why node 21 is chosen first with a vehicle range of 4. With node 21 locked in, node 11 is added when p is increased to 2. Nodes 11 and 12 are the only two nodes in the entire network that can increase the volume refueled, because they are the only nodes next to a link of length 2. Either one would have been capable of refueling the round-trip of length 4 between them. It is unlucky that the model arbitrarily chose node 11, because if it had chosen node 12, it could have added node 16 in round 3 because the combination {12,16} would have been able to refuel the large-volume flow between O–D pair 12–16 and the smaller-volume flow between 11 and 16. As it happened, after adding node 11 in round 2, it added node 13 in round 3, which only enabled refueling of the very small-volume flows 11–13 and 12–13.

Why could not the greedy FRLM *see* the greater benefit of adding facilities at 18 and 20 instead of 11 and 13? The reason is because adding node 20 by itself does not refuel any O–D pairs that node 21 was not already able to refuel. Adding node 20 without nodes 18 or 19 achieves nothing because the round-trip distance from node 20 to those nodes and back is 6, exceeding the vehicle range of 4. This analysis strongly suggests that an effective greedy algorithm for the FRLM will have to be able to look at least 2 facilities beyond the current solution before adding a facility, give partial credit to partially refueled paths, or include a substitution routine [24]. Future research will explore these alternatives.

6.4. Comparison with flow capturing

Comparing the flow-refueling solutions in Fig. 3 with those of the flow-capturing model solved for the same network, substantial differences appear (Table 3). Obviously, with only one facility required to cover a flow, the FCLM captures more flow volume for each p than the FRLM is able to refuel for

Table 3
Comparison of solution results between FRLM and Hodgson's FCLM for the same network: percent of demand satisfied for each p

Number of facilities	Flow refueling location model (vehicle range = 12)		Flow capturing location model	
(p)	Global (exact)	Greedy	Global (exact)	Greedy
1	13.8	13.8	38.1	38.1
2	29.5	27.8	53.8	53.8
3	40.8	40.8	65.9	65.9
4	52.3	52.3	76.3	75.9
5	61.2	59.9	82.9	82.8
6	70.4	68.5	89.2	89.1
7	78.0	75.9	94.6	94.6
8	84.4	82.2	96.6	96.6
9	89.0	88.6	98.1	98.1
10	92.8	92.4	99.2	99.1
11	95.2	95.1	99.6	99.5
12	97.0	97.0	99.9	99.8
13	98.3	98.3	99.9	99.9
14	99.3	99.3	100.0	100.0
15	99.8	99.8	100.0	100.0
16	99.9	99.9	100.0	100.0
17	100.0	100.0	100.0	100.0

a range of 12, which was the longest vehicle range tested. However, with extremely long-range vehicles—longer than the longest round-trip path in the network—the two problems become identical. Hodgson's tradeoff curve for the globally optimal FCLM was strictly convex, with each additional facility covering fewer flows than the previous one added. In addition, the greedy heuristic performed exceptionally well for the FCLM, never falling more than 0.4% below the global optimum. In Hodgson's greedy heuristic, the first facility was added at the most heavily traveled node. Then, at each iteration, a new facility was added that can capture the greatest volume of previously uncaptured flows, thus minimizing cannibalization of previously captured flows and minimizing the optimality gap. The FRLM's optimality gap was more than two orders of magnitude larger than the FCLM's in the worst-case scenario, and it is worth remembering that these results were for the exact same network.

6.5. *Alternate optima*

It is well-known that covering models tend to have many alternate optima [24]. Because the FCLM reduces to the maximum covering location model (MCLM), and because the FRLM is an extension of the FCLM, it is important to ask whether the property of having many alternate optima is true here as well. To test for alternate optima, the FRLM was first solved for all p for a vehicle range of 12. Then, for each p, the objective function value was entered as a constraint, and the model was re-solved with a different objective [25]. In this case, the secondary objective was not a realistic one, but a dummy objective: the sum of the flow ID numbers that are refueled. Each feasible solution to the modified problem is also optimal to the original FRLM. If it has a different set of facilities opened, it constitutes an alternate optima to the original FRLM. For each

p, this secondary objective was both maximized and minimized to make sure that no alternate optima were missed.

The results showed that, unlike the MCLM, the FRLM does not often have alternate optima. In fact, from $p = 1$ to 16, only one solution ($p = 14$) had an alternate optima. There could be several reasons for the lack of alternate optima. First, in the FRLM, there are many more items to cover: $n(n-1)/2$ flows in the FRLM vs. n nodes in the MCLM. Second, for each particular item to be covered, the FRLM is more sensitive to the location of facilities than is the MCLM because sometimes, only some of the nodes along a path are capable of refueling that path because of the network logic of vehicle range (see Section 3). Third, in the FRLM, alternate optima may be rare because combinations of facilities are refueling many of the flows in a given solution. If there are multiple combinations active in the solution, it may not be possible to substitute for any of the optimal facilities. The FCLM would likely fall in between the FRLM and the MCLM in the number of alternate optima.

Once the FRLM plateaus at its maximum flow volume, alternate optima become commonplace. In the test runs for a vehicle range of 12, alternate optima were found for all solutions from $p = 17$, when the maximum flow volume refueled was first achieved, to $p = 24$. This is because all the refuelable flows for a given range have already been refueled, so it matters not which redundant facilities are built. It is a moot point, however, because redundant facilities should not be opened at all.

7. Conclusions

The problem of optimally locating facilities for refueling network paths has been overlooked in the literature. More than likely, this lack of attention is because O–D path data are less available than link data and because gasoline stations were already ubiquitous in the developed world by the time the methods of location analysis had been developed. Until the recent development of alt-fuel vehicles, there was little need for such a model. Bapna et al. [4] addressed the issue of environmentally friendly fueling infrastructure, but took the approach of a spanning subgraph, which could force trips onto highly roundabout paths and maximizes coverage of local population along arcs instead of O–D flows.

Flow-capturing models provide a realistic starting point for the flow-refueling location problem. The demand exerted by alt-fuel vehicles consists largely of O–D flows as opposed to special purpose trips from home (or some other base) to refueling stations and back. Flow refueling differs from flow capturing, however, because of limited vehicle range. In flow capturing, a key issue was to not double-count flows that could be captured by more than one facility. In flow refueling, on the other hand, the limited vehicle range requires multiple facilities on longer paths.

The major innovations in this model were the use of facility combinations to refuel network paths and an algorithm for determining which combinations are feasible each path. This appears to be one of the first instances in the literature where the cover set consists of combinations of facilities, rather than a simple list of individual facilities.[2] Various hierarchical or emergency service covering models [26–28] require coverage by more than one type of facility, but

[2] An anonymous referee suggested that this might be the first use of combinations in a single-level covering problem.

combinations in those models are necessary because they provide different types of service. Likewise, backup coverage models [29] require one facility for primary service and one for backup service, but the pair of facilities need not be chosen from a limited list of valid combinations. Unlike in these other models, valid combinations in the FRLM are determined not only by their location relative to the spatial demand but also relative to each other.

Although this initial paper limits location to network nodes, it has been shown that such an assumption might be suboptimal. Under certain conditions, a particular path might require two facilities to refuel it if facilities can be located only at the nodes of the network, and yet might be refueled by a single facility if located midlink. This depends not only on the total round-trip length of the path relative to the vehicle range, but also on the spacing of nodes along the path.

Test-problem results indicate that as the vehicle range is lowered, fewer flows can be refueled with a given number of facilities. If the range is short enough, it may not be possible to refuel all the flows, even with facilities located at every node; the range of the vehicle must be long enough to traverse the longest link on any shortest path. Results have shown that the tradeoff curve between the percentage of flows refueled and the number of facilities is not convex. It often happens that the pth facility adds more flows refueled than the previous facility ($p-1$), primarily because of the combinations of facilities required to refuel some paths. Finally, although no heuristic solution methods were developed in this paper, mixed-integer programming software was used to simulate how a greedy-adding algorithm would perform in terms of optimality. Results showed that sequentially and myopically adding the facility at each iteration that fully refuels the most additional flow volume can produce highly suboptimal results, especially when the vehicle range is small. The greedy approach, which performed extremely well in the flow-capturing problem, most likely does worse in the flow-refueling problem because it fails to consider combinations of facilities.

8. Directions for future research

A maximum-cover type of objective is indicated over a set-cover one in cases where a limited budget makes it impossible to achieve 100% coverage, or where coverage is optional and not a matter of life or death. Both reasons justify the use of maximum cover in the FCLM, and both are relevant to the FRLM as well. Locating alt-fuel facilities to refuel as many vehicle flows as possible implies maximizing return on investment in refueling infrastructure. It may be fruitful, however, to explore extensions of the flow-refueling problem based on other fundamental location models. A modeling approach based on the fixed-charge or p-median problems would imply that drivers make reasonable detours from the shortest paths to access refueling stations, as in [16]. Such behavior is likely to occur, especially in the early days of alt-fuel infrastructure development. The fixed-charge approach, however, may only be suitable in cases where the facility owners are also the vehicle owners, such as fleet owners or railways, and thus are interested in trading off investment costs and transport costs. Other promising directions for future research include relaxing the assumption of location at nodes, dealing with multiple vehicle types with different ranges, and developing a heuristic solution technique that efficiently handles combinations of facilities. A different application or extension of the FRLM could serve the problem of locating nature reserves along migratory flyways of birds, which have maximum distance or time limits before they need to stop to feed and rest.

This research has important implications for energy sustainability. Alt-fuel vehicles can help reduce dependence on imported oil, improve local air quality and, possibly, help avert global-climate change. However, a lack of a refueling infrastructure is a major barrier to widespread adoption of alt-fuel vehicles. Another valuable direction of future research is to extend the model for answering the question: how do we get to where we want to be (complete conversion to a hydrogen fuel-cell transport system) from where we are (limited infrastructure and limited participation)? Because individual drivers travel to multiple destinations, they may not purchase alt-fuel vehicles until they can refuel on all or most of their trips. Phasing in infrastructure to maximize participation is an important dynamic aspect of the problem. In the last 15 years, a number of researchers have emphasized the importance of considering technology choice when optimizing spatial networks and considering spatial networks when optimizing technology choice [30–33]. New technologies such as hydrogen fuel-cell vehicles cannot be adequately assessed based solely on their laboratory or test-track performance.

Railways represent a unique opportunity to implement fuel-cell technology safely on a large scale. Several research teams have found fuel cells and rail locomotives to be a promising and technologically feasible combination [34,35]. Railways could provide a major foothold for fuel-cell and hydrogen technologies that might allow them to achieve the necessary economies of scale to benefit other applications of hydrogen and fuel cells [36]. Because there are fewer path-miles in the rail network, and trains can go farther than automobiles without refueling, the cost of a distributed alt-fuel refueling infrastructure for railways would be much less than for motor vehicles. Railways also have very good O–D flow data. For these reasons, railways may be an ideal application of the FRLM.

Acknowledgements

This research was supported by the Decision, Risk, and Management Science program of the National Science Foundation, Proposal #0214630. We would like to thank Dash Optimization, Ltd. for the use of Xpress-MP under the Academic Partnership Program. We thank M. John Hodgson for his helpful comments and permission to use the network used in his paper, Charles ReVelle for comments and suggestions, and Lauren Kuby for editorial assistance. Finally, we thank Max Wyman for his initial inspiration to investigate refueling of hydrogen fuel-cell locomotives.

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