Charging Network Optimization with Nonlinear Station Size Effects

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1 Queuing at Stations

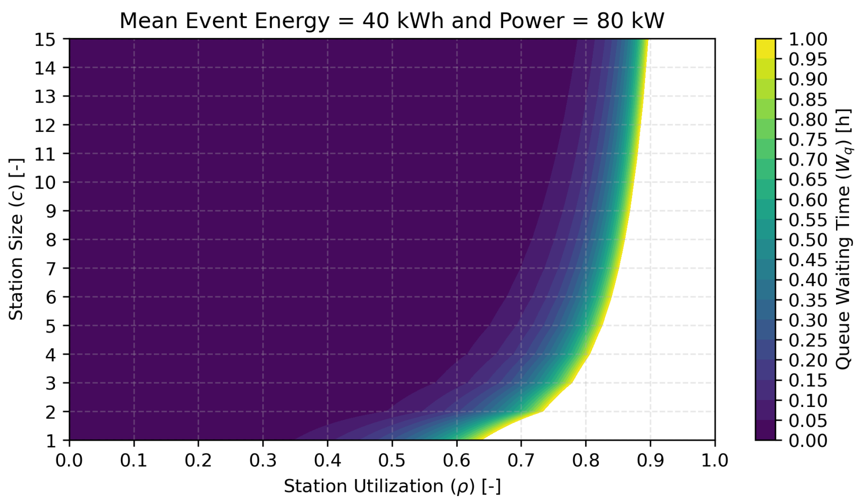

Figure 1: Queuing time with M/M/c queue model

2 Travel Time Minimization Formulation

The purpose of this formulation is to minimize total travel time in the system. Travel time minimization is accomplished by vehicle routing and charging station provision. For a given O/D pair there will, usually, be multiple viable charging paths of different lengths. As demand increases, chargers become increasingly congested leading to queuing time at stations. Queuing delays on shorter paths will push traffic to longer paths. Eventually, queuing will be sufficient to make the charging network no longer beneficial. This point is defined as when the marginal vehicle trip would take as much time using the network as it would take using level 1 charging. The goal is to place chargers and route vehicles to minimize total ravel time. As such, each O/D pair has a "failure" flow which vehicles can be assigned to and the penalty assigned for this is equal to the travel time with level 1 charging.

Delay at stations is modeled using the outputs of the M/M/c queue formula as discussed in Section [REF]. Specifically, the outputs are linearized. Stations are initialized with a vector of m

binary variables representing possible sizes (e.g. 1 charger, 3 chargers, 5 chargers). The vector of station size binary variables must sum to 1. For each size considered, the M/M/c model returns volumes and delays corresponding to a set of marginal utilization rates $R : |R| = n$. The utilization rates $\rho \in R$ are the bounds of a set of $n-1$ utilization intervals. Thus, m by $n-1$ matrices of marginal volumes and marginal delays are created. Additionally, a m by n matrix of unit-interval continuous variables are created and the sum of these variables multiplied by the corresponding marginal volumes must equal the flow passing through the station. the delay at the station is computed by summing the marginal utilization rates multiplied by the marginal delays.

- $G = \{V, E\}$: System graph containing nodes $v \in V$ and edges $(i, j) \in E$. Edge costs are defined by the following sets:
  - Y^T: The time required to traverse edge (i, j)
  - Y^E: The time required to charge at node i to successfully traverse edge (i, j)
- $O \subseteq V$: Set of origin nodes
- $D \subseteq V$: Set of destination nodes
- $S \subseteq V$: Set of nodes with charging stations (or the possibility of a station). Stations provide energy to vehicle flows at a given rate. Depending on the utilization level of the station, vehicles may experience delay. The relationship between utilization is linearized using the following sets:
  - C_s: Set of possible station sizes at station $s \in S$
  - $K_{s,c}$: Set of capacity intervals at station $s \in S$ for station size $c \in C_s$
  - Y^V: Set of volumes corresponding to each $c \in C_s$ and $k \in K_s$
  - Y^D: Set of delays corresponding to each $c \in C_s$ and $k \in K_s$
- $\hat{C}$: Maximum number of chargers which can be installed
- Q: Set of demand tuples of the form $\langle o, d, v, c, \hat{t} \rangle$ where o is the origin, d is the destination, v is the volume, c is the capacity of the Energy Storage System (ESS) capacity of vehicles, and $\hat{t}$ is the maximum travel time that is acceptable for the given demand.
  - Y^Q: Set of time penalties for failing to accommodate flow. Set so that y^q_q is equal to $\hat{t}$ in q.
- P: Set of paths corresponding to each demand $q \in Q$. Paths begin at $o \in O$ and end at $d \in D$. All intermediate nodes $i \in P \setminus \{o, d\}$ must be stations $s \in S$.
  - P^q: Paths that correspond to demand $q \in Q$
  - P^s: Paths that include station $s \in S$
- X: Set of continuous decision variables:
  - X^Q: Portion of demand flow not facilitated by the network

- X^P: Flow volumes along paths
- X^U: Portion of station capacity intervals utilized
- X^V: Volume seen at station
- X^D: Queuing delay seen at station

- U: Set of integer decision variables:
  - U^S: Booleans for station sizes corresponding to S and C

The objective of the optimization is

$$\min_{\overline{X},\overline{U}} \quad \underbrace{\sum_{q\in Q} x_q^q y_q^q}_{\text{Penalty Time}} + \underbrace{\sum_{q\in Q}\sum_{P^q\in P}\sum_{p\in P^q}\sum_{(i,j)\in p} x_p^p \left(y_{(i,j)}^t + y_{(i,j)}^e\right)}_{\text{Edge Traversal Time}} + \underbrace{\sum_{s\in S}\sum_{c\in C_s}\sum_{k\in K_{s,c}} u_{s,c}^s x_{s,c,k}^u y_{s,c,k}^d}_{\text{Queuing Time}} \tag{1}$$

subject to

$$Q[v] - x_q^q - \sum_{p\in P^q} x_p^p = 0 \quad \forall q \in Q \tag{2}$$

$$\sum_{p\in P^s} x_p^p - \sum_{c\in C_s}\sum_{k\in K_{s,c}} u_{s,c}^s x_{s,c,k}^u y_{s,c,k}^v = 0 \quad \forall s \in S \tag{3}$$

$$x_{s,c,k}^u - u_{s,c}^s \leq 0 \quad \forall s \in S,\ \forall c \in C_s,\ \forall k \in K_{s,c} \tag{4}$$

$$\sum_{c\in C_s} u_{s,c}^s - 1 = 0 \quad \forall s \in S \tag{5}$$

$$\sum_{s\in S}\sum_{c\in C_s} u_{s,c}^s - \hat{C} \leq 0 \tag{6}$$

The objective function (1) minimizes total travel time in three terms. The first term is the time penalties accrued for failing to accommodate demand. The theory is that, without dedicated charging infrastructure, vehicles could, theoretically, complete the trip using level 1 charging but this would be very slow if the trip is beyond full-charge range. The second term is the time spent driving along edges and charging to drive along edges. The third term is the time spend queuing for a charger. Constraint (2) forces the sum of flows and un-accommodated flows to be equal to total demand. Constraint (3) forces the sum of utilization intervals at a station to be equal to the sum of flows which pass through the station. Constraint (4) forces station utilization to only accrue for the selected station size. Constraint (5) forces only one station size to be selected per station and (6) limits the total number of chargers in the network.