

Show that

$$v(t) = v_{in} - (v_{in} - v_0) e^{\frac{t_0 - t}{\tau}}$$

is a solution of

$$\tau \frac{dv}{dt} = v_{in} - v \quad v(t_0) = v_0$$

Pf: Simply plug the solution in & show that the DE is satisfied.

$$\begin{aligned} \text{LHS} &= \tau \frac{dv}{dt} \\ &= -\tau (v_{in} - v_0) \left(-\frac{1}{\tau}\right) e^{\frac{t_0 - t}{\tau}} \\ &= (v_{in} - v_0) e^{\frac{t_0 - t}{\tau}} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= v_{in} - v \\ &= v_{in} - \left[ v_{in} - (v_{in} - v_0) e^{\frac{t_0 - t}{\tau}} \right] \\ &= (v_{in} - v_0) e^{\frac{t_0 - t}{\tau}} \end{aligned}$$

$\therefore \text{LHS} = \text{RHS}$ , as required.

