

3)

We have that for constant v_{in} :

$v(t) = v_{in} \left(1 - e^{-\frac{t}{\tau}}\right)$ is a solution of

$$\tau \frac{dv}{dt} = v_{in} - v, \quad v(0) = 0$$

The interspike interval are the sum of refractory time and the time it take for v to climb from 0 to 1.

Let the time when v is 0 equals to 0.

Then we have

$$\begin{aligned} v(t) &= 1 \\ \Rightarrow v_{in} \left(1 - e^{-\frac{t}{\tau}}\right) &= 1 \end{aligned}$$

$$\Rightarrow e^{-\frac{t}{\tau}} = 1 - \frac{1}{v_{in}}$$

$$\Rightarrow -\frac{t}{\tau} = \ln \left(1 - \frac{1}{v_{in}}\right)$$

$$\Rightarrow t = -\tau \ln \left(1 - \frac{1}{v_{in}}\right)$$

Then we have that :

$$\begin{aligned} t_{isi} &= \tau_{ref} + \left(-\tau \ln \left(1 - \frac{1}{v_{in}}\right) - 0 \right) \\ &= \tau_{ref} - \tau \ln \left(1 - \frac{1}{v_{in}}\right) \end{aligned}$$

Then from the interspike interval we can have the firing rate of LIF is :

$$G(v_{in}) = \begin{cases} \frac{1}{\tau_{ref} - \tau \ln \left(1 - \frac{1}{v_{in}}\right)} & \text{for } v_{in} > 1 \\ 0 & \text{otherwise} \end{cases}$$