Question 3: LIF Firing Rate

[3 marks]

Recall that the sub-threshold membrane potential for a LIF neuron is governed by the DE,

$$\tau \frac{dv}{dt} = v_{\mathrm{in}} - v \ . \tag{1}$$

Show that if v_{in} is held constant, then the firing rate of a LIF neuron can be computed using

$$G(v_{\mathrm{in}}) = \begin{cases} \frac{1}{\tau_{\mathrm{ref}} - \tau \ln\left(1 - \frac{1}{v_{\mathrm{in}}}\right)} & \text{for } v_{\mathrm{in}} > 1 \\ 0 & \text{otherwise} \end{cases}$$

Hint: The time between spikes (t_{isi}, the "inter-spike interval") is the reciprocal of the firing rate, and is also the sum of the refractory time and the time it takes for v to climb from 0 to the threshold of 1.

3. Notice $\tau \frac{dv}{dt} = v_{in} - v$

Then we can solve this DE to get t

$$\Rightarrow \frac{dv}{v_{in} - v} = \frac{dt}{\tau}$$

$$\Rightarrow \int \frac{dv}{v_{in} - v} = \int \frac{dt}{\tau}$$

$$\Rightarrow -\ln|v_{in} - v| = \frac{t}{\tau}$$

$$t = -\tau \ln|v_{in} - v|$$

Now we want time for v to climb from 0 to 1, thus

$$t_{0,1} = -\tau \ln|v_{in} - 1| - \tau \ln|v_{in} - 0|$$

$$= -\tau \left(\ln|v_{in} - 1| - \ln|v_{in}|\right)$$

$$= -\tau \ln\left(\frac{v_{in} - 1}{v_{in}}\right) \quad \text{since } v_{in} > 1$$

$$= -\tau \ln\left(1 - \frac{1}{v_{in}}\right)$$

Notice $t_{isi} = t_{0,1} + \tau_{ref}$ and t_{isi} is reciprocal of firing rate, thus

$$G(v_{in}) = \begin{cases} \dfrac{1}{\tau_{ref} - \tau \ln\left(1 - \frac{1}{v_{in}}\right)} & \text{for } v_{in} > 1 \\ 0 & \text{otherwise} \end{cases}$$