## Derivative of Logistic

$$\sigma(z) = \frac{1}{1+e^{-z}} = (1+e^{-z})^{-1}$$

$$\frac{d\sigma(z)}{dz} = -1(1+e^{-z})^{-2} \frac{d}{dz}(1+e^{-z})$$

$$= -(1+e^{-z})^{-2}(-e^{-z})$$

$$= \frac{e^{-z}}{(1+e^{-z})^2}$$

$$= \underline{e^{-z}} \, \sigma^2(z)$$

We need to find an expression for $e^{-z}$

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

Recip. $\rightarrow \frac{1}{\sigma(z)} = 1+e^{-z}$

$$\therefore e^{-z} = \frac{1}{\sigma(z)} - 1$$

Thus,

$$\frac{d\sigma(z)}{dz} = \left(\frac{1}{\sigma(z)} - 1\right) \sigma^2(z)$$

$$= \sigma(z)(1-\sigma(z)) \quad \text{as required.}$$