OPEN LAB(8th Semester) FINAL REPORT
Mean life time of Muons.

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Abstract

Muons are created from Interaction of charged particle coming from cosmic rays with upper atmosphere of our planet.These Muons reach surface of our planet.Hence Muons are freely available to us for doing experiment.The Motivation in this open lab is to detect those muons in scintillator detector and measure their mean life.We found the mean life to be $2.108 \pm .086\mu$s.Using this We calculated Fermi coupling constant and found it to be $\frac{G_F}{(\hbar c)^3} = 1.163 \times 10^{-5} \pm 2.277 \times 10^{-7} GeV^{-2}$.

1 Theory:

1.1 Muons:

About 98 % of Primary Cosmic rays are protons or heavier nuclei and 2 % are electrons.The primary rays collide with the nuclei of air molecules and produce a shower of particles that include protons, neutrons, pions (both charged and neutral), kaons, photons, electrons and positrons. These secondary particles then undergo electromagnetic and nuclear interactions to produce yet additional particles in a cascade process.Some pions decay via weak interactions into muons.

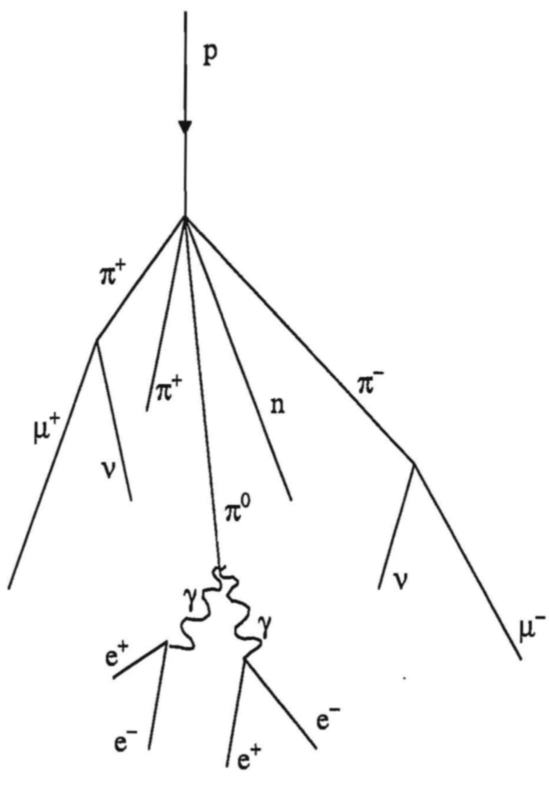

Figure 1: Cosmic ray cascade induced by a cosmic ray proton striking an air molecule nucleus.

- Charge:$\pm$e Mass:$206.78m_e$ spin:$\frac{1}{2}$
- Muon Decay:Via Weak force.

- $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$
- $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$

- The muon does not interact with matter via the strong force but only through the weak and electromagnetic forces.

- Literature Mean life time of free muon:2.197 μS

1.2 Muon decay:

Suppose muon decays are independent events.Suppose that a muon will decay during a time period dt is λdt. If We have N(t) number of muons at time t,the after dt interval the decrease in number of muons:$dN = -\lambda dt \times N(t)$.$\frac{dN}{dt} = -\lambda dt$

$$N(t) = N_0 \exp -\lambda t$$

$$\frac{-dN}{N_0} = \lambda. \exp -\lambda.tdt$$

So the fraction of muon that decays does not depend on how many muons we start with. Even if In our detector we observe decay time of single muons at random times and the make a histogram , it is same as if we had started with a large number of muons together in a sample and recorded number of muons left with time like in radioactive decays experiment.

- Some muons pass through the detector completely giving, us a single detector output pulse.

- Other muons are stopped inside dectector and decay into electron or positron.This event gives two consecutive pulse.

- It is the time interval between two consecutive pulse of this second type of event that we want to measure in order to calculate the lifetime.

1.3 Time Dilation effect of special relativity:

The muons are created high above the sky(15 km) ,given the life time of muon around 2 micro seconds ,they should not be able to reach the ground even at the speed of light.15km/c $\approx$ 50 micro seconds. If we are at rest w.r.t muon we measure a life time of around 2 us.But when muon is moving at relativistic speed that time length of around 2 us will be dilated w.r.t us according to special theory of relativity.

For a rough calculation(Not taking in to account how the muon will be affected when travelling thorugh a medium) if time was absolute. Fraction that would survive the decay $\approx \exp \frac{-50\mu S}{2\mu S} = \exp -25$ which is a very negligible quantity.But we see a muon flux rate of around 1 per cm^2 per minute on the ground. If we take relativity:$t = \frac{\tau}{\sqrt{1-\frac{v^2}{c^2}}}$ Muon of around 2Gev(with rest mass around 0.1Gev) $\gamma \approx 20$.so t=$\approx 40\mu S$. $\exp \frac{-50}{40}$ =exp $-5/4$ which is small but not negligible anymore.

2 Techspin apparatus:

The electronics module houses all electronics needed to run the experiment. Connections on the front panel allow students not only to examine the PMT signal itself, but also to monitor that signal as it moves along the readout chain.

PMT pulses are first amplified and compared against an adjustable threshold. Pulses above threshold are sent to timing circuitry implemented in a field programmable gate array (FPGA) chip. The first flash of the scintillator starts the timing system. If a second flash occurs within 20 microseconds of the first, the readout electronics measures the time between the two flashes and passes that time to the lifetime display software.

If no second flash occurs within 20 microseconds, the pulse is simply recorded as a charged particle that has passed through the detector. Communication circuitry transfers the data to a PC or laptop through either a serial or USB port.

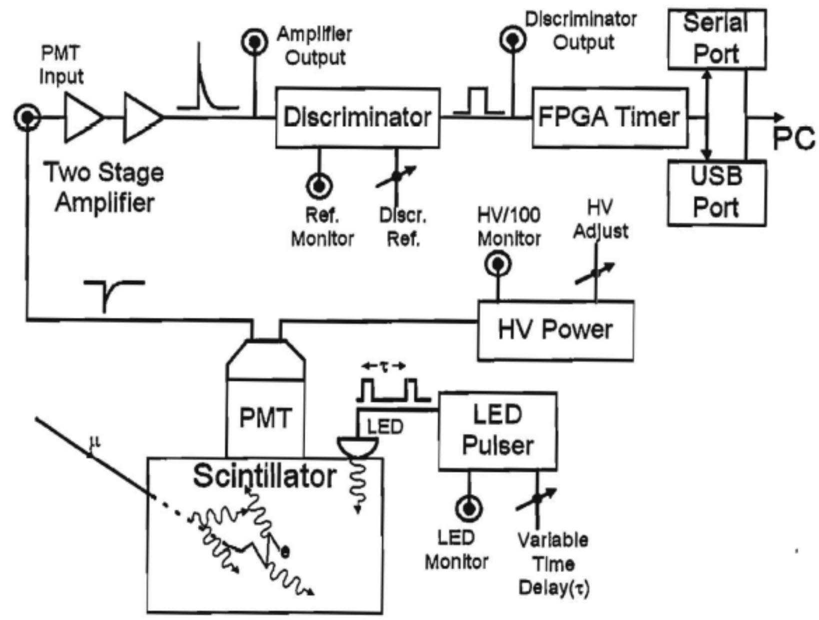

Figure 2: Full Schematics of the setup

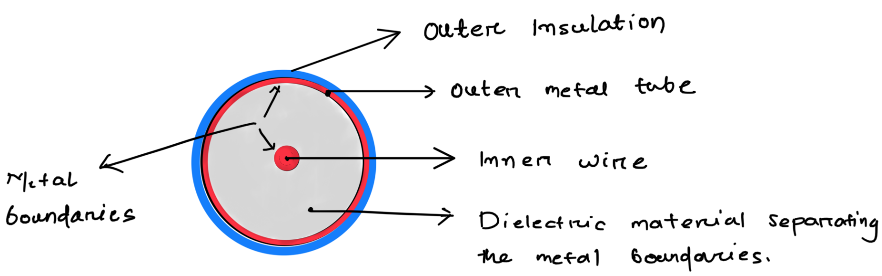

Figure 3: Coaxial cable cross section:They have impedance which depend on dielectric material and the radii.

3 Observation:

Impedance matching for Coaxial Cable

The connections from output of Discriminator and amplifier to oscilloscope are made through 50 ohm Coaxial Cable.Since the square pulses from discriminator are DC signals.The oscilloscope should be DC coupled with input impedance set to 50 ohm.Otherwise Signal loss and distortion due to reflection was observed. The reflection coefficient is given by:

$$R = \frac{Z_L - Z_0}{Z_L + Z_0}$$

If Z_L becomes very large compared to Z_0 then R$\approx$ 1.Then Signal is mostly reflected and and we get negligible transmitted signal.

Muon detection vs Discriminator voltage:

Following are observation of discriminator voltage.

Discussion on figure 5: When discriminator voltage is less than 100 mV the pulse counts are very high.This is because signals from noise for e.g gamma rays are also getting detected.When we took Discriminator voltage larger than 500 mV pulse counts are too low and experiment would take longer time although doable.So Discriminator voltage from above 100 mV up to 400 mV is optimum for the experiment.This data was taken at keeping the HV supply for PMT tube at constant 700 V.

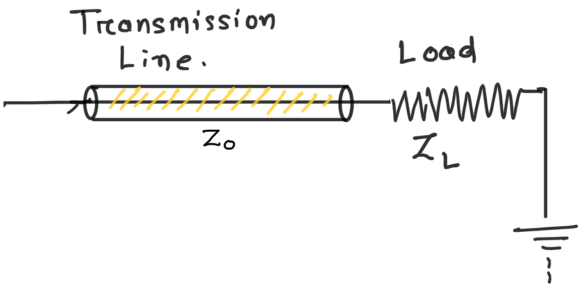

Figure 4

Discreminator Voltage(mV)	Number of Discriminator pulse detected/ 2 min
0	12396
26	12221
35	10495
52	2326
84	2500
102	1307
147	1169
204	1010
258	818
278	805
303	695
329	642
351	537
400	381
453	224
505	191
564	140
583	143

Table 1

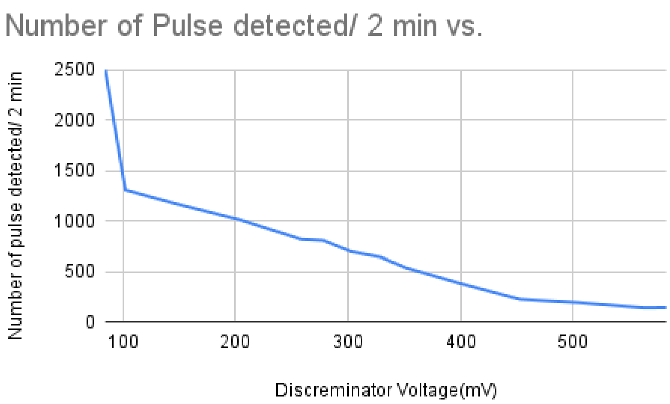

Figure 5: Number of pulse with varying Discriminator voltage

Mean life Calculation:

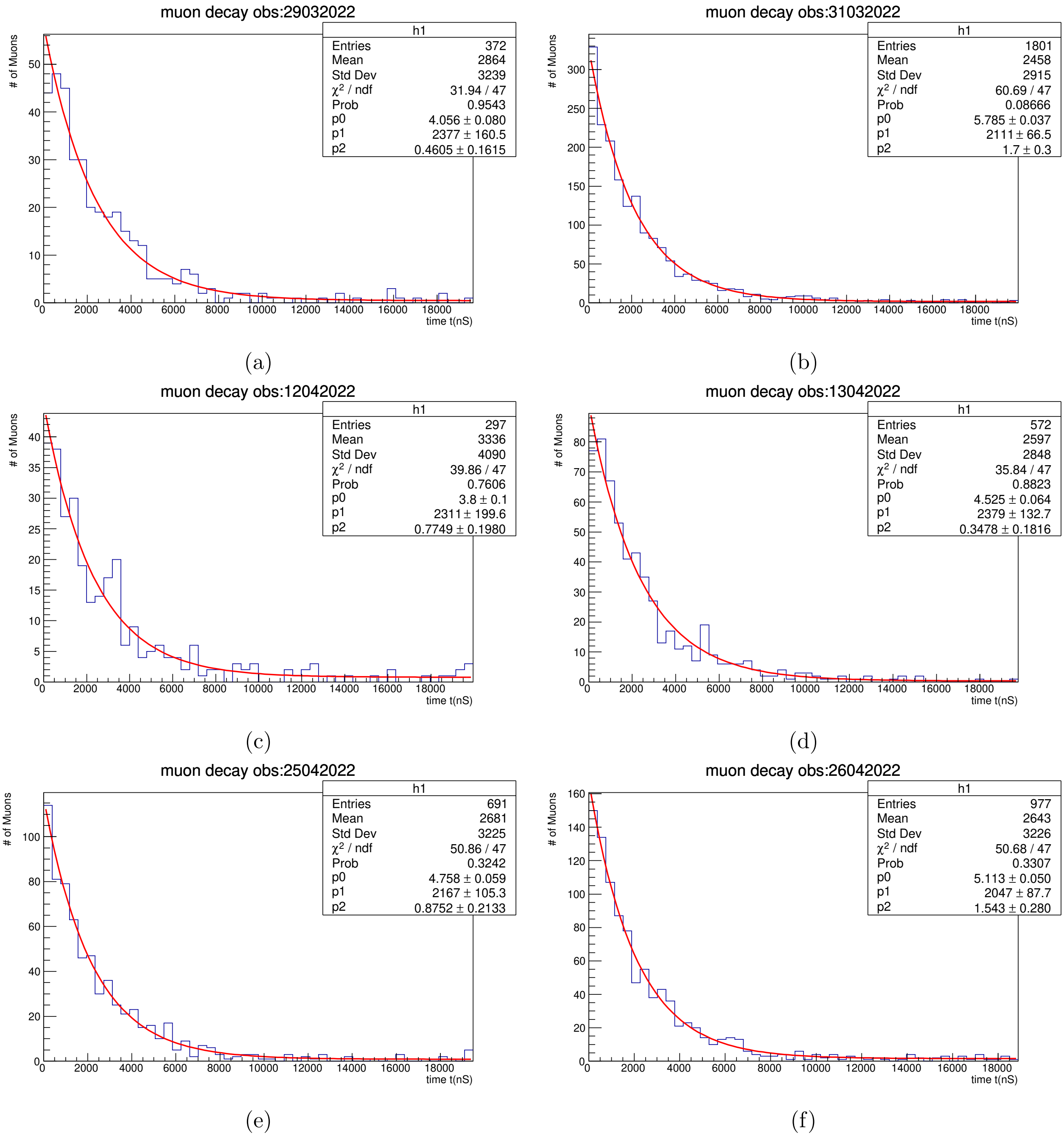

Figure 6: Muon Decay observation

Taking average of these 6 observations:we get $\tau_{obs} = 2.232 \pm .125\mu$S.But this is unexpected and erroneous probably because number of decay events on which exponential fitting is done are not sufficient,refer fig 3 and look at Entries in the histogram legend.We are supposed to get Less than 2.2 μS since negative muons are interacting with matter.Lets Discard Results having mean life greater than 2.2 μS.Now taking the average of observation 2, 5 and 6 in table-2 we get $\tau_{obs} = 2.108 \pm .086\mu$S.We will use this value to evaluate muon charge ratio and Fermi coupling constant.

4 Muon Charge Ratio Calculation:

The muon charge ratio $\rho = \frac{N^+}{N^-}$ is defined as the ratio of the number of positive- to negative-charge atmospheric muons arriving at the Earth's surface.

- The magnitude and momentum dependence of ρ are determined by the production and interaction Cross sections of mesons(mainly pions and kaons) and their decay lengths.As most cosmic rays and Nuclei

Experiment No	Mean life(ns)	error(ns)
1	2377	160.5
2	2111	66.5
3	2311	199.6
4	2379	132.7
5	2167	105.3
6	2047	87.7

Table 2

with which they interact are positively charged,positive meson production is favoured,hence more positive muons are expected[1].so ρ is expected to be >1.

Negative muons can get bound to the nuclei of Atoms in the material like the electron around the nucleus. Such bound negative muons can interact with protons:$\mu^- + p \rightarrow n + \nu_\mu$ Since Negative muons have two ways to dissapear,their life time in matter is effectively less than that of free muons. Life time of negative muons in carbon:$\tau_c = 2.043 \pm 0.003 \mu Sec$[Reiter ,1960].

The effective Decay constant or mean life can be written in terms of muon charge ratio.

$$\bar{\lambda} = \frac{1}{\tau_{obs}} = \frac{N^+\lambda^+ + N^-\lambda^-}{N^+ + N^-} = \frac{\rho\lambda^+ + \lambda^-}{1+\rho}$$
$$\implies \rho = -\frac{\tau^+}{\tau^-}\left(\frac{\tau^- - \tau_{obs}}{\tau^+ - \tau_{obs}}\right)$$

we put τ_{obs} that we got from experiment,$\tau^- = \tau_c$ and $\tau^+ = \tau = 2.197\mu S$(mean life of free muons) to get a estimte of the positive to negative muon ratio.

$$\rho = 1.785$$

Hence our estimation shows that there are more positive muons than negative muons in the incoming muon flux from sky.

5 Fermi's coupling Constant:

Once the muon lifetime is measured, a value of the Fermi coupling constant G_F, characterizing the strength of the weak interactions, is easily determined from the relation:

$$\tau = \frac{192\pi^3\hbar^7}{G_F^2 m_\mu^5 c^4}$$

Inverting the relation and putting $\tau = \tau_{obs}$ we got:

$$\frac{G_F}{(\hbar c)^3} = 1.163 \times 10^{-5} \pm 2.277 \times 10^{-7} GeV^{-2}$$

6 Time to Digital Converter(TDC):

TDC is essential for our experiment .This section is devoted to how TDC works. The simplest way to implement a TDC is to use a "coarse counter" that starts counting with the "start" pulse and ends with the "stop" pulse. If the counter is N bit the range of measurement is:

$$R = 2^{N_{bit}}.T_{clk}$$

The carry chain method , i.e. a delay line TDC is as follows. The START signal is propagated through a delay line, in ASIC implemented with either buffers or inverters (minimum propagation delay). On the arrival of the STOP signal, the propagated START signal is latched. This directly gives a thermometer time code. The number of stages flipped to one gives the timing information. The resolution of the TDC is given by the buffer or inverter propagation time τ figure-8.

[1]Measurement of charge ratio of atmospheric muons with CMS detector.CMS Collaboration / Physics Letters B 692 (2010) 83–104

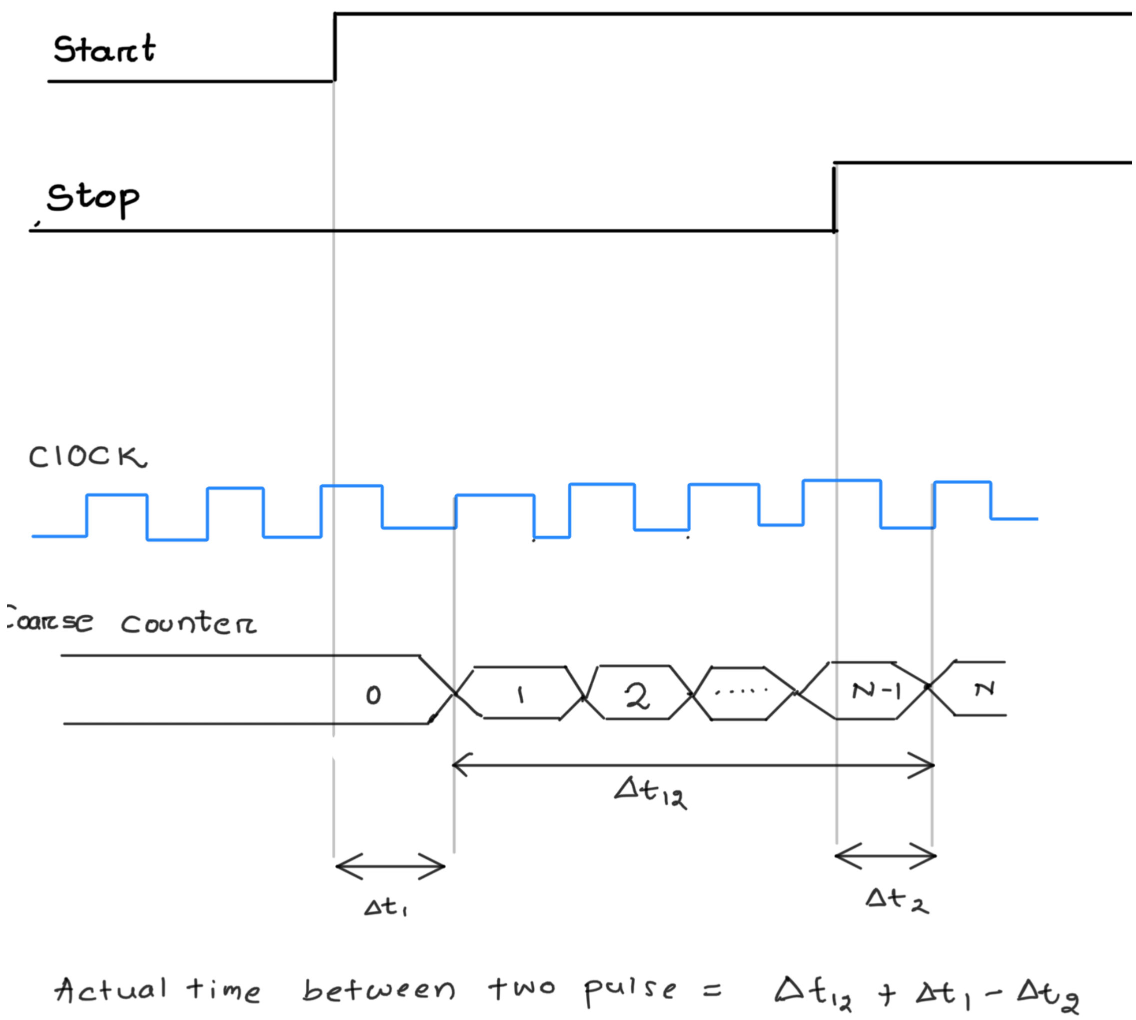

Figure 7

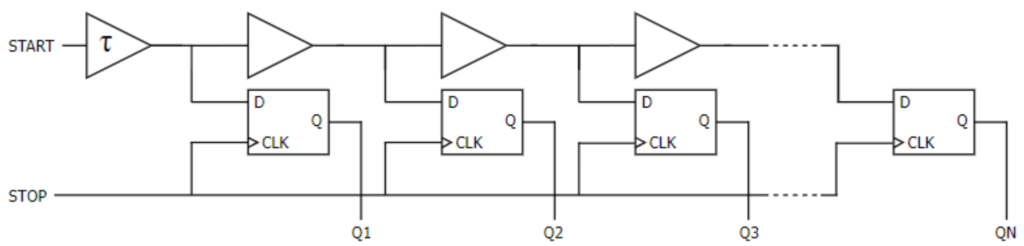

Figure 8: Delay line

7 FPGA Programming:

Field Programmable Gate Arrays (FPGAs) are semiconductor devices that are based around a matrix of configurable logic blocks (CLBs) connected via programmable interconnects. FPGAs can be reprogrammed to desired application or functionality requirements after manufacturing. To define the behavior of the FPGA, the user provides a design in a hardware description language (HDL) or as a schematic design.

We used Altera Cyclone V FPGA board and VHDL as the HDL language.For uploading the code and it's synthesis we used intel Quartus softaware.We carried out basic combinational and sequential circuits.We also tried TDC implementation which is based on coarse counter described in section 6, But we could not develop a read out system for FPGA.The code for TDC is attached in appendix.

8 Conclusion:

We found the mean life of muon to be $\tau_{obs} = 2.108 \pm .086\mu$S.From this we calculated Fermi coupling constant to be $\frac{G_F}{(\hbar c)^3} = 1.163 \times 10^{-5} \pm 2.277 \times 10^{-7} GeV^{-2}$.Litrature value is[2] $1.1663787(6) \times 10^{-5} GeV^{-2}$. Then we calculated muon charge ratio around 1.785.For comparison ,Muons have been detected[3]. in the momentum range from 5 GeV/c to 1 TeV/c. The surface flux ratio is measured to be around 1.277, independent of the muon momentum, below 100 GeV/c.For implementation of TDC in FPGA we have done the coding part,but the challenge was to make a readout system for FPGA and carrying out the experiment with that FPGA TDC which is incomplete.

9 Reference:

- Techspin Manual
- Altera Cyclone V User Manual
- Muon lifetime measurement – Franz Muheim.
- A conceptual introduction to Muon Physics-David Van Baak.

Appendix:

OR Gate

```
    --------------------------------------
-- OR gate
--
--------------------------------------

library ieee;
use ieee.std_logic_1164.all;

--------------------------------------

entity OR_ent is
port( x: in std_logic;
y: in std_logic;
F: out std_logic
);
end OR_ent;

---------------------------------------
--Method 1
architecture OR_arch of OR_ent is
begin
```

[2] https://pdg.lbl.gov/2014/reviews/rpp2014-rev-phys-constants.pdf

[3] Measurement of the charge ratio of atmospheric muons with the CMS detector:https://doi.org/10.1016/j.physletb.2010.07.033

```
    process(x, y)
    begin
        -- compare to truth table
        if ((x='0') and (y='0')) then
    F <= '0';
else
    F <= '1';
end if;
    end process;

end OR_arch;
---Method 2
architecture OR_beh of OR_ent is
begin

    F <= x or y;

end OR_beh;

---------------------------------------
```

D flipflop

```
---------------------------------------------
-- D Flip-Flop (ESD book Chapter 2.3.1)
-- by Weijun Zhang, 04/2001
--
-- Flip-flop is the basic component in
-- sequential logic design
-- we assign input signal to the output
-- at the clock rising edge
---------------------------------------------

library ieee ;
use ieee.std_logic_1164.all;
use work.all;

---------------------------------------------

entity dff is
port( data_in: in std_logic;
clock: in std_logic;
data_out: out std_logic
);
end dff;

----------------------------------------------

architecture behv of dff is
begin

    process(data_in, clock)
    begin

        -- clock rising edge
```

```
if (clock='1' and clock'event) then
    data_out <= data_in;
end if;

    end process;

end behv;

----------------------------------------------
```

VHDL code for TDC:

```
library ieee;
use ieee.std_logic_1164.all;
use STD.textio.all;
--------------------------------------

entity tac is
port(
a: in std_logic; --Signal pulse from muon event
clk : in std_logic; --clock
setter:out std_logic;
getter:in std_logic;
o1:out std_logic;
o2:out std_logic;
o3:out std_logic
);
end tac;
---------------------------------------
---------------------------------------

architecture BEV of tac is
signal b:std_logic:='0';
signal c:std_logic:='0';
signal count : integer range 0 to 10000000 :=0;
signal d:std_logic:='0';


begin
o1<=a;
process(a) is
begin
if(a'event and a='1') then
b<=not b;
setter<=b;
end if;
--setter<=b;
o2<=b;
end process;

counter :process(clk)
variable my_line :line;
begin
if (clk'event and clk = '1') then
if (d='1')then
```

```
count<=count+1;
end if;

if(getter='0')then
--print count value and make it zero
--write(my_line,string'(" counter ="));
--write(my_line,counter);
count<=0;
end if;

if(count=20000000)then
c <= not c;

count<=0;
end if;

end if;
end process;

process(getter) is
begin
if(getter = '1' and c ='0') then
d <= '1';
else
d<='0';
end if;
o3<= d;
end process;



end BEV;
---------------------------------------
---------------------------------------
```

In [8]:

```
import math
T=2.108e-6
t=2.043e-6
#from our experiment
t0=2.197e-6
del_t0=0.086e-06
#constants
m=.105658     #Gev/C^2
Pi=3.141
h_bar=6.582e-25  #GeV.S

r=-1*(T/t)*(t-t0)/(t0-T)
G=math.sqrt(192*math.pow(Pi,3)*h_bar/(math.pow(m,5)*t0))
del_G=0.5*(del_t0/t0)*G
print('Charge ratio:')
print(r)
print('G_F')
#Fermi coupling constant
print(G)
print('uncertainity')
print(del_G)
```

```
Charge ratio:
1.7853894086136806
G_F
1.1634841338483898e-05
uncertainity
2.2771878814511046e-07
```