P305 Assignment 3

Roll NO : 1811074

1.a.

$$V(r) = 0 \qquad r \leq a$$
$$\infty \quad , \quad r > a$$

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi$$

$$-\frac{\hbar^2}{2m} \nabla^2 \psi = E\psi$$

In spherical coordinates, when we solve this we get solution of the form,

$$= (\text{radial part}) \, Y_{lm}(\theta, \phi)$$
$$\Downarrow$$
Bessel function.

$$\psi_{nlm}(r, \theta, \phi) = A_{nl} \, j_l\left(\frac{\beta_{nl} r}{a}\right) Y_{lm}(\theta, \phi)$$

β_{nl} is the n^{th} zero of the l^{th} spherical Bessel function.

Selection rules:

From dipole approximation $W_{ab} \propto |\hat{\epsilon}\cdot r_{ab}|^2$

$$\hat{\epsilon}\cdot r_{ab} = \left\langle \Psi_{n'l'm'} \middle| r\left(\frac{4\pi}{3}\right)^{1/2} Y_{1q}(\theta,\phi) \middle| \Psi_{nlm} \right\rangle$$

$$= \int \left(\frac{4\pi}{3}\right)^{1/2} r^3 j_l\left(\frac{\beta_{n'l'} r}{a}\right) j_l\left(\frac{\beta_{nl} r}{a}\right) dr$$

$$\times \int Y^*_{l'm'} Y_{1q} Y_{lm} \, d\Omega$$

For polarisation parallel to $\hat{z}$, $q = 0$

$\hat{\epsilon}\cdot r_{ab} \neq 0$, for transition to occur

$$\Rightarrow \int Y_{\ell m} Y^*_{\ell' m'} Y_{10} \, d\Omega \neq 0 \quad \Rightarrow \int_0 e^{i(m-m')} d\phi \neq 0$$

$$\Rightarrow \quad m = m' \quad \text{or} \quad \Delta m = 0$$

For polarisation $\perp \hat{z}$, $\quad q = \pm 1$

$$\int Y_{\ell m} Y^*_{\ell m'} Y_{1,\pm 1} \, d\Omega \neq 0 \quad \Rightarrow \int e^{i(m \pm 1 - m')} d\phi \neq 0$$

$$\Rightarrow \Delta m = \pm 1$$

$$\Delta \ell = \pm 1$$

Selection rule : $\Delta \ell = \pm 1$, $\Delta m = 0, \pm 1$