Tools and Algorithms for Deciding Timed Relations

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Mihir Mehta

Department of Computer Science and Engineering
Indian Institute of Technology, Delhi.

`cs1090197@cse.iitd.ac.in`

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Overview

- Bisimilarity and related notions
- Kanellakis and Smolka's algorithm
- Fernandez' algorithm
- Paige and Tarjan's algorithm
- Timed Automata
- Equivalences on Timed Automata
- Code written so far

Bisimilarity and related notions

- Labeled Transition System: This is a triple
  $(Proc, Act, \{\stackrel{a}{\rightarrow} | a\epsilon Act\})$ where
  - *Proc* is a set of states (also called processes or configurations.)
  - *Act* is a set of actions (also called labels.)
  - $\stackrel{a}{\rightarrow} \subseteq$ *Proc* $\times$ *Proc* is a transition relation.
- CCS expression: Defined by the following grammar:
  - $P ::= K$
  - $P ::= \alpha.P$
  - $P ::=_{i\epsilon I} P_i$
  - $P ::= P|Q$
  - $P ::= P[f]$
  - $P ::= P\text{Ł}$

Bisimilarity and related notions

- Equivalence for CCS processes.
- Trace equivalence: $Traces(P) = Traces(Q)$
- Unsatisfactory (differences in deadlock behaviour.)
- Strong bisimulation: A binary relation R is a `strong bisimulation` if and only if, for all $(s_1, s_2)\epsilon R$ and $a\epsilon Act$.
  $$\forall s_1'(s_1 \xrightarrow{a} s_1' \Rightarrow \exists s_2'.(s_2 \xrightarrow{a} s_2' \wedge (s_1', s_2')\epsilon R))\wedge$$
  $$\forall s_2'(s_2 \xrightarrow{a} s_2' \Rightarrow \exists s_1'.(s_1 \xrightarrow{a} s_1' \wedge (s_1', s_2')\epsilon R))$$
- It can be shown that the union of all strong bisimulations over the set of states is a strong bismulation. This binary relation is called `strong bisimilarity`, denoted by $\sim$.

Bisimilarity and related notions

- Better notion of equivalence than trace equivalence: picks up differences in the deadlock behaviour of processes under study.
- Failing: does not account for the invisible nature of τ transitions in CCS processes.
- Weak bisimulation: A binary relation R is a weak bisimulation if and only if, for all $(s_1, s_2)\epsilon R$ and $a\epsilon Act$.
  $$\forall s_1'(s_1 \xrightarrow{a} s_1' \Rightarrow \exists s_2'.(s_2 \xRightarrow{a} s_2' \wedge (s_1', s_2')\epsilon R))\wedge$$
  $$\forall s_2'(s_2 \xrightarrow{a} s_2' \Rightarrow \exists s_1'.(s_1 \xRightarrow{a} s_1' \wedge (s_1', s_2')\epsilon R))$$
- It can be shown that the union of all weak bisimulations over the set of states is a weak bismulation. This binary relation is called weak bisimilarity, denoted by $\approx$.
- Better suited to CCS processes, as it ignores τ transitions, thus disregarding hidden behaviour within a process.

Kanellakis and Smolka's algorithm

- This is a naive algorithm for determining the bisimilarity relation for the set of processes in a labelled transition system.
- This relies on the notion of a splitter.
- Let $\pi = \{B_0, ..., B_k\}$, $k \geq 0$ be a partition of the set of states Pr in a labeled transition system.
- A splitter for a block $B_i \; \epsilon \; \pi$ is a block $B_j \; \epsilon \; \pi$ such that for some action $a \; \epsilon \; Act$, some states in B_i have a-labelled transitions whose targets lie in B_j while other states in B_i do not.
- This suggests a refinement of π: replace block B_i with
  $B_i^1 = B_i \cap T_a^{-1}[B_j]$
  $B_i^2 = B_i - B_i^1$
- Refinements of this kind constitute the steps of this algorithm.

Kanellakis and Smolka's algorithm

Initialise π to Pr;
while *there exist splitters among the elements of π* **do**
    Pick a splitter B;
    foreach $B_j \ \epsilon \ \pi$ **do**
        foreach $a \ \epsilon \ Act$ **do**
            Split B_j with respect to B for action a;
        end
    end
end
Return π;

- The time complexity of this algorithm is $O(mn)$, since there can be at most n iterations, and all m edges are scanned in each iteration.

Fernandez' algorithm

- More efficient algorithm (O(m log n)).
- Relies on Paige and Tarjan's technique of three-way splitting.
- Splitters can now be 'simple' or 'compound'.
- Stability: A partition π is said to be stable with respect to a compound block S if S is not a splitter for any block in π for any action.
- For a compound block S, having a constituent simple block B satisfying $n(B) \leq 0.5 * n(S)$, and with respect to which π is stable, we can split a block B_i on an action a as follows:
  $B_i^1 = (B_i \cap T_a^{-1}[B]) - T_a^{-1}[S - B]$
  $B_i^2 = (B_i \cap T_a^{-1}[S - B]) - T_a^{-1}[B]$
  $B_i^3 = B_i \cap T_a^{-1}[B] \cap T_a^{-1}[S - B]$

Fernandez' algorithm

Initialise $\pi = \{Pr\}$;
Initialise W = $\{Pr\}$ **while** *W is not empty* **do**

> Choose a splitter B from W, removing it;
> **if** *B is a simple splitter* **then**
>
> > Perform a two-way split on each action with respect to B and update W;
>
> **else**
>
> > Perform a three-way split on each action with respect to B and update W;
>
> **end**

end

Paige and Tarjan's Algorithm

- Technique of three way splitting came from here.
- Special case of Fernandez' algorithm when there's only one kind of action.

Timed Automata

▶ Formally, a timed automaton over a finite set of clocks C and a finite set of actions Act is a 4-tuple (L, l_0, E, I).

▶ L is a finite set of locations.

▶ l_0 is the initial location.

▶ $E \subseteq L \times B(C) \times Act \times 2^C \times L$ is a finite set of edges.

▶ $I : L \rightarrow B(C)$ assigns invariants to each edge location.

▶ $B(C)$ is the set of clock constraints over C. An element of $B(C)$ can be an equality, a slack inequality, a strict inequality, or an AND combination of such constraints.

Equivalences on Timed Automata

- *Timed bisimulation*: A binary relation R is a timed
  bisimulation if and only if, for all $(s_1, s_2)\epsilon R$, $a\epsilon Act$, $d\epsilon R_{\geq 0}$
  $$\forall s_1'(s_1 \xrightarrow{a} s_1' \Rightarrow \exists s_2'.(s_2 \xrightarrow{a} s_2' \wedge (s_1', s_2')\epsilon R))\wedge$$
  $$\forall s_2'(s_2 \xrightarrow{a} s_2' \Rightarrow \exists s_1'.(s_1 \xrightarrow{a} s_1' \wedge (s_1', s_2')\epsilon R))\wedge$$
  $$\forall s_1'(s_1 \xrightarrow{d} s_1' \Rightarrow \exists s_2'.(s_2 \xrightarrow{d} s_2' \wedge (s_1', s_2')\epsilon R))\wedge$$
  $$\forall s_2'(s_2 \xrightarrow{d} s_2' \Rightarrow \exists s_1'.(s_1 \xrightarrow{d} s_1' \wedge (s_1', s_2')\epsilon R))$$

- It can be shown that the union of all timed bisimulations over
  the set of (location, valuation) pairs is a timed bisimulation.
  This binary relation is called `timed bisimilarity`, denoted
  by $\sim$.

Equivalences on Timed Automata

Time abstracted bisimulation: A binary relation R is a time abstracted bisimulation if and only if, for all $(s_1, s_2) \epsilon R$, $a \epsilon Act$, $d \epsilon R_{\geq 0}$

$\forall s_1'(s_1 \xrightarrow{a} s_1' \Rightarrow \exists s_2'.(s_2 \xrightarrow{a} s_2' \wedge (s_1', s_2') \epsilon R)) \wedge$

$\forall s_2'(s_2 \xrightarrow{a} s_2' \Rightarrow \exists s_1'.(s_1 \xrightarrow{a} s_1' \wedge (s_1', s_2') \epsilon R)) \wedge$

$\forall s_1'(s_1 \xrightarrow{d} s_1' \Rightarrow \exists (s_2', d').(s_2 \xrightarrow{d} s_2' \wedge (s_1', s_2') \epsilon R)) \wedge$

$\forall s_2'(s_2 \xrightarrow{d} s_2' \Rightarrow \exists (s_1', d').(s_1 \xrightarrow{d} s_1' \wedge (s_1', s_2') \epsilon R))$

It can be shown that the union of all time abstracted bisimulations over the set of (location, valuation) pairs is a time abstracted bisimulation. This binary relation is called `time abstracted bisimilarity`, denoted by $\sim_u$.

Equivalences on Timed Automata

- *Equivalence of valuations*: Two valuations v and v' of a timed automaton are said to be equivalent ($v \equiv v'$) if and only if:
    - For each $x \epsilon C$, either both $v(x)$ and $v'(x)$ are greater than c_x or $\lfloor v(x) \rfloor = \lfloor v'(x) \rfloor$
    - For each $x \epsilon C$ such that $v(x) \leq c_x$, $frac(v(x)) = 0$ if and only if $frac(v'(x)) = 0$.
    - For all $x, y \epsilon C$ such that $v(x) \leq c_x$ and $v(y) \leq c_y$, we have $frac(v(x)) \leq frac(v(y))$ if and only if $frac(v'(x)) \leq frac(v'(y))$.

- Under this notion of equivalence, an equivalence class is known as a *region*. The equivalence class containing a valuation v is denoted by $[v]_{\equiv}$.

Equivalences on Timed Automata

- *Region graph*: the region graph of a timed automaton A with clock set C and action set Act is an LTS
  $T_r(A) = (Proc, Act \cup \{\varepsilon\}, \{\overset{a}{\to} | a \epsilon Act \cup \{\varepsilon\}\})$
  where $Proc = \{(l, [v]_{\equiv}) | l \epsilon L, v : C \to R_{\geq 0}\}$ (these states are called symbolic states)
  The transitions are defined as follows:
    - For each $a \epsilon A$, $(l, [v]_{\equiv}) \overset{a}{\Rightarrow} (l', [v']_{\equiv})$ iff $(l, v) \overset{a}{\to} (l', v')$
    - $(l, [v]_{\equiv}) \overset{a}{\to} (l, [v']_{\equiv})$ if for some $d \epsilon R_{\geq 0}$, $(l, v) \overset{d}{\to} (l, v')$

Equivalences on Timed Automata

- *Extended Clock Constraints*: These are described by the grammar

  $g ::= x \bowtie n | x - y \bowtie n | g_1 \wedge g_2$

- The set of extended clock constraints is denoted by $B^+(C)$

- *Zones*: A zone is a set of valuations represented by an extended clock constraint.

  $Z = v | v \epsilon g_Z$

- *Symbolic state*: For a timed automaton A, a symbolic state is an ordered pair of a location and a zone, that is, (l, Z).

Equivalences on Timed Automata

- *Reachability graph*: The reachability graph of a timed automaton is a directed graph in which the nodes are symbolic states and the edges are given by the *reachability relation*:
  - (l, Z') is reachable from (l, Z) if there exists a valuation belonging Z' which can be obtained from Z by applying a time delay which does not violate the invariant at l.
  - (l', Z') is reachable from (l, Z) if l has an action transition to l' and there exists a valuation in Z' which violates neither the guard of the transition nor the invariant at l'.

Code written so far

- *Fernandez' algorithm*: We expect to need this algorithm while building the software, so we built a module implementing it in Ocaml (primary language for this project.)
- *Grammar*: We implemented a lexer and parser in Ocaml to build a representation of a timed automaton from a specification in a text file.

References

- Reactive Systems: Modeling, Specification and Verification - Luca Aceto, Anna Ingolfsdottir, Kim Larsen, Jiri Srba.