

Tools and Algorithms for Deciding Relations on Timed Automata

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Abstract. This describes the author's work in implementing the construction of zone-valuation graphs for timed automata and using these to verify certain relations on pairs of timed automata. Timed automata are a widely-used formalism, in verification, for describing systems where the execution is subject to timing constraints. One area of interest is the verification of certain timed and time abstracted relations on pairs of timed automata, which are generally of either an equivalence nature or a pre-ordering nature. Among existing tools, `minim` [1] serves to verify certain of these relations, but it is limited in its scope and fails to meet certain requirements of reachability in its output. We build upon the work of [2], in which algorithms for building zone-valuation graphs (minimal, discrete equivalent representations of timed automata) are given and used to verify certain pre-ordering relations on pairs of timed automata. We present an modified version of the zone-valuation graph construction algorithm, and a generalisation of the algorithm which uses zone-valuation graphs to verify a larger set of timed and time-abstracted relations on timed automata, where this generalisation builds upon the work of [3]. We also describe a reference implementation of these algorithms in Ocaml.

Table of Contents

Tools and Algorithms for Deciding Relations on Timed Automata	1
<i>Mihir Mehta</i>	
1 Introduction	4
2 Labelled transition systems	4
3 Equivalences on labelled transition systems	6
3.1 Strong bisimilarity	6
4 Clocks, valuations and related operations	6
5 Timed automata	7
6 Zone graphs	8
7 Difference bound matrices	9
8 Abstraction	9
9 Timed relations on timed automata	11
9.1 Time abstracted bisimilarity	11
10 Algorithms	12
10.1 Fernandez' algorithm	12
10.2 Creation of the zone valuation graph	12
Overview	12
Proof of correctness	14
10.3 Checking timed and untimed relations on timed automata	15
Overview	15
Proof of correctness	16

List of Figures

1	An example of a labelled transition system. Here, the states are $\{0, 1, 2, \dots, 7\}$ and the actions are $\{0, 1\}$	5
2	Strong bisimilarity quotient of the LTS in Figure 1.	5
3	Timed automaton representing a light bulb with two brightness settings, example taken from [4]	8
a	Timed automaton with potentially infinite state space.	10
b	Zones of Figure 4a after 1 iteration.....	10
c	Zones of Figure 4a after 2 iterations.	10
d	Zones of Figure 4a after 3 iterations.	10
4	Timed automaton with a potentially infinite set of zones, example taken from [5].....	10

List of Tables

1 Introduction

The work described in this thesis builds on the work of [2] in which Guha et al described an algorithm to generate *zone valuation graphs* for timed automata and an algorithm to use such zone-valuation graphs to determine *timed performance prebisimilarity* on pairs of timed automata. Our aim was to implement these algorithms in a generalised manner in order to verify various other time abstracted relations, such as time abstracted bisimulations [1] and time abstracted simulation equivalence. Towards this end, we studied the literature about timed automata as well as various existing tools for verifying these equivalences (such as `minim`, described in [1]). Our implementation, in OCaml, implements several of these relations and leaves some scope for implementing others.

This document proceeds by developing the relevant theory for labelled transition systems and strong bisimilarity, and then continues with timed automata and time abstracted bisimilarity in analogous fashion. We then detail several methods to discretise the state space of timed automata, including region graphs, zone graphs, and zone valuation graphs, and use region graphs to show that a finite representation of the state space of a timed automaton is always decidable. We mention difference bound matrices (DBM), a widely used representation for the time constraints (i. e. convex polyhedra) that are used to describe zones. Then, we explain abstractions, which are transformations on DBM that replace each DBM by a DBM which is its superset and which is equivalent to it modulo strong time abstracted bisimilarity. This helps us prevent state space explosion in our exploration algorithms which build the state space of a timed automaton. We then explain the common features of certain time abstracted relations which allow us to verify them in a generalised manner, and explain our implementation.

2 Labelled transition systems

Definition 1. Labelled Transition System: *A labelled transition system (LTS) [6] is an automaton which is described by*

- S , a set of states
- Act , a set of actions
- $\rightarrow \subseteq S \times Act \times S$, a transition relation.
- optionally, $I \subseteq S$, a set of initial states.

LTS are useful for describing the behaviour of untimed systems, and serve as the foundation for the development of more complex models such as CCS and timed automata. Thus, equivalences on LTS serve as the theoretical foundation for many timed and time abstracted equivalences on timed automata, and also have direct applications in determining some of these equivalences in cases where timed automata can be reduced to equivalent LTS.

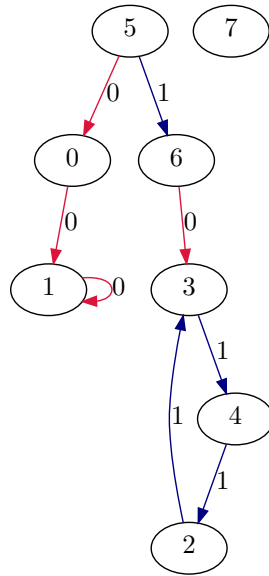


Fig. 1: An example of a labelled transition system. Here, the states are $\{0, 1, 2, \dots, 7\}$ and the actions are $\{0, 1\}$.

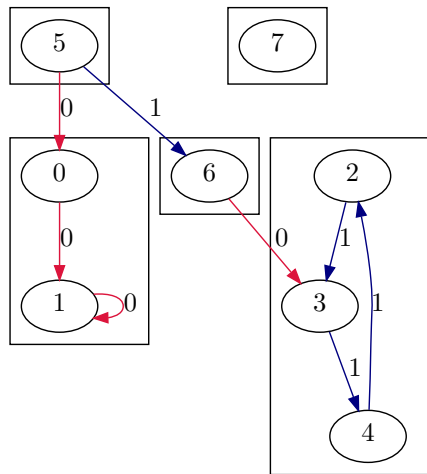


Fig. 2: Strong bisimilarity quotient of the LTS in Figure 1.

3 Equivalences on labelled transition systems

3.1 Strong bisimilarity

Definition 2. Strong bisimulation: *A binary relation R is a strong bisimulation if and only if, for all $(s_1, s_2) \in R$ and $a \in \text{Act}$.*

$$\begin{aligned} \forall s'_1 (s_1 \xrightarrow{a} s'_1 \Rightarrow \exists s'_2. (s_2 \xrightarrow{a} s'_2 \wedge (s'_1, s'_2) \in R)) \wedge \\ \forall s'_2 (s_2 \xrightarrow{a} s'_2 \Rightarrow \exists s'_1. (s_1 \xrightarrow{a} s'_1 \wedge (s'_1, s'_2) \in R)) \end{aligned}$$

Definition 3. Strong bisimilarity: *It can be shown that the union of all strong bisimulations over the set of states is a strong bisimulation. This binary relation is called strong bisimilarity, denoted by \sim .*

Strong bisimilarity between two states in an LTS implies, intuitively, that any action performed by the one can be performed by the other and vice versa.

An algorithm to evaluate strong bisimilarity was first given by Kanellakis and Smolka [7]. A faster algorithm for the special case having just one kind of action was given by Paige and Tarjan [8] and made general by Fernandez [9].

4 Clocks, valuations and related operations

The timing model we use relies on the notions of clocks and valuations. The timing information about the possible executions of a timed automaton can, intuitively, be expressed in terms of constraints on clocks. We will make this idea more precise as we go on.

Definition 4. Clock: *A clock is a variable ranging over the set of non-negative real numbers, $R_{\geq 0}$.*

Definition 5. Valuation: *A valuation over a set of clocks C is a function $C \rightarrow R_{\geq 0}$, assigning non-negative values to each of the clocks.*

Two important operations on clock valuations are clock resets and delays. For a subset X of C , the clock reset of a valuation v is given by

$$v[X := 0](x) = \begin{cases} 0 & \text{if } x \in X \\ v(x) & \text{if } x \in C - X \end{cases}$$

For a real delay $d \in R_{\geq 0}$, the delay of a valuation v is given by

$$(v + d)(x) = v(x) + d \quad \forall x \in C$$

Definition 6. Hyperplane: *Expressions of the forms $x \smile c$ and $x - y \smile c$, where $x, y \in C$, c is an integer, and $\smile \in \{<, \leq, \geq, >\}$ are known as atomic clock constraints. A valuation v over C is said to satisfy the atomic clock constraint $x \smile c$ if $v(x) \smile c$ and $x - y \smile c$ if $v(x) - v(y) \smile c$. A hyperplane is a set of valuations satisfying an atomic clock constraint.*

Definition 7. Polyhedron: *The set of polyhedra over a set of clocks C is the smallest subset of the set of all valuations over C that*

- *contains all hyperplanes over C*
- *is closed over intersection, union and complementation.*

Definition 8. Convex polyhedron: *A convex polyhedron is a polyhedron that can be represented as the intersection of a set of hyperplanes.*

It should be noted that this definition does not coincide exactly with the notion of a convex set from algebra. For instance, for $C = \{x_1, x_2\}$, the set $\{(x_1, x_2) | x_1 \geq 0, x_2 \geq 0\} - \{(x_1, x_2) | x_1 = x_2 = 0\}$ is convex but not a convex polyhedron.

Union, intersection, complementation and set difference are well defined operations on polyhedra.

The *pre-reset* and *post-reset* of a set $X \in C$ on a polyhedron ζ over the set C of clocks are respectively defined by

$$\begin{aligned} [X := 0]\zeta &= \{v | v[X := 0] \in \zeta\} \\ \zeta[X := 0] &= \{v[X := 0] | v \in \zeta\} \end{aligned}$$

The *future* of a polyhedron ζ over a set C of clocks is given by

$$\zeta \uparrow = \{v + d | v \in \zeta, d \in R_{\geq 0}\}$$

The operators intersection, future, pre-reset, post-reset and future preserve convexity of polyhedra.

Definition 9. Clock constraint: *A clock constraint is a polyhedron which can be expressed as an intersection of hyperplanes of the form $x \smile c$.*

A related term, *extended clock constraint*, can be used interchangeably with *polyhedron*.

5 Timed automata

Definition 10. Timed Automaton: *A timed automaton [10] over a finite set of clocks C and a finite set of actions Act is a 4-tuple (L, l_0, E, I) .*

- *L is a finite set of locations.*
- *l_0 is the initial location.*
- *$E \subseteq L \times B(C) \times Act \times 2^C \times L$ is a finite set of edges.*
- *$I : L \rightarrow B(C)$ assigns invariants to each edge location.*
- *$B(C)$ is the set of clock constraints over C .*

This is a useful formalism for describing the behaviour of a system with timing requirements on its behaviour. For a better understanding of its semantics, it is useful to describe a labeled transition system corresponding to it (called a timed LTS, or TLTS) that describes the behaviour of this timed automaton.

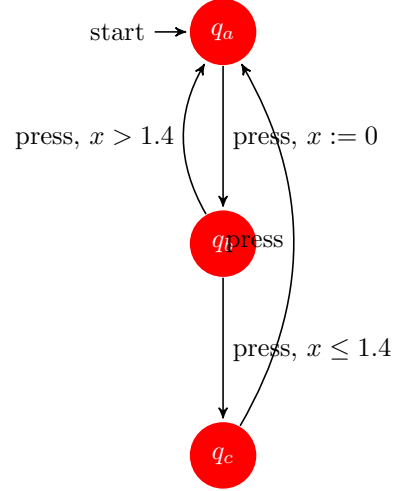


Fig. 3: Timed automaton representing a light bulb with two brightness settings, example taken from [4]

Definition 11. Timed LTS: We define the timed LTS $T(A)$ defined by a timed automaton (L, l_0, E, I) over a set of clocks C and a set of actions Act as

$$T(A) = (Proc, Lab, \rightarrow)$$

where

- $Proc = \{(l, v) | l \in L, v \in I(l)\}$
- $Lab = Act \cup R_{\geq 0}$
- \rightarrow is given by
 - $(l, v) \xrightarrow{a} (l', v')$ iff $a \in Act$, $\exists (l \xrightarrow{g, a, r} l') \in E$ such that $v \models g \wedge v' = v[r := 0] \wedge v' \models I(l')$
 - $(l, v) \xrightarrow{d} (l, v)$ iff $d \in R_{\geq 0} \wedge v \models I(l) \wedge v' \models I(l)$

We will use the object-oriented notation `l.invar` to refer to the invariant $I(l)$ of a location l and `e.source`, `e.guard`, `e.action`, `e.resets` and `e.target` to refer to l , g , a , r , l' of an edge $e = l \xrightarrow{g, a, r} l'$ unless otherwise noted.

6 Zone graphs

It is evident that the state space in a timed automaton is, in general, uncountably infinite. This makes it impossible for any algorithm to terminate which explores the state space in a naive manner. However, some standard techniques to discretise the state space of timed automata exist, which we elaborate on, below.

Definition 12. State: A state of a timed automaton is a pair (l, v) where l is a location in the automaton and v is a clock valuation satisfying $l.invar$.

Definition 13. Symbolic state: A symbolic state is a set of states in the timed automaton. The constituent states do not necessarily share a location.

Definition 14. Zone: A zone is a symbolic state where all the constituent states share a location and the set of valuations of these states forms a convex polyhedron on the valuation space.

Definition 15. Zone graph: A zone graph of a timed automaton is an LTS, where the states are zones, the actions consist of the actions of the timed automaton and an ϵ action, which represents a timed transition, and the transition relation respects the invariants, guards and clock resets of the original timed automaton.

Definition 16. Region graph: A region graph is a zone graph in which the zones of each location are the same, and given by the equivalence classes under region equivalence.

Valuations v_1 and v_2 are related under region equivalence if

- $\forall x \in C: \lfloor v_1(x) \rfloor = \lfloor v_2(x) \rfloor$ or $v_1(x) > c_x$ and $v_2(x) > c_x$ where c_x is the greatest constant compared to x over all guards and invariants in the automaton.
- $\forall x \in C: v_1(x) = 0$ iff $v_2(x) = 0$.
- \forall pairs of clocks $(x, y): \text{frac}(v_1(x)) \leq \text{frac}(v_1(y))$ iff $\text{frac}(v_2(x)) \leq \text{frac}(v_2(y))$.

7 Difference bound matrices

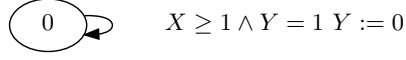
Definition 17. Difference bound matrix: A difference bound matrix (DBM) is a representation of a convex polyhedron on a set of clocks $\{x_1, \dots, x_n\}$ in the form of an $(n+1) \times (n+1)$ matrix M , each element of which takes the form (m_{ij}, \smile_{ij}) , where m_{ij} is an integer and $\smile_{ij} \in \{<, \leq\}$. Assuming x_0 to be a clock always valued at zero, the associated polyhedron is given by

$$\bigcap_{0 \leq i, j \leq n} (x_i - x_j \smile_{ij} m_{ij})$$

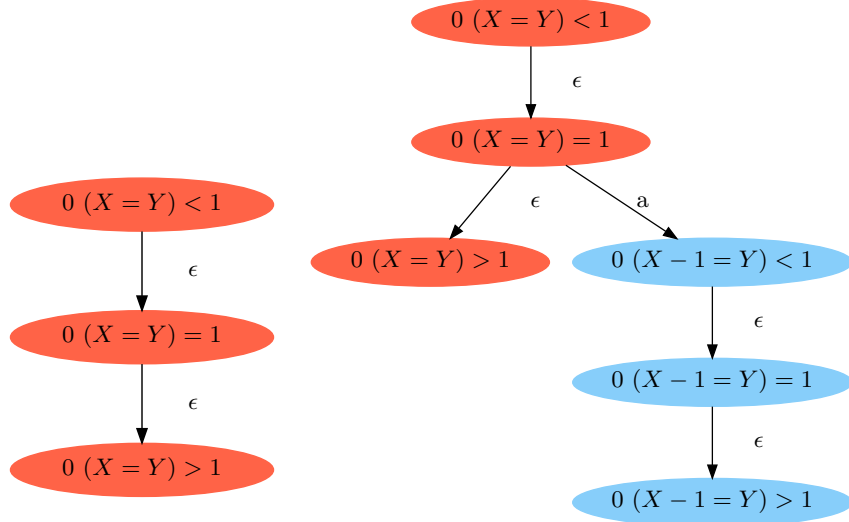
DBM offer a convenient method to represent polyhedra for most of the common operations required on zones, including intersection, clock resets, future, and abstraction. The UPPAAL DBM libraries [11] implement many common functions on DBM and have been extensively used in our implementation.

8 Abstraction

A common feature of algorithms that generate zone graphs for timed automata is a *forward propagation* step in which the algorithm attempts to create reachable zones in reachable locations by traversing the timed automaton. However, this introduces a vulnerability to certain pathological cases in which the number of

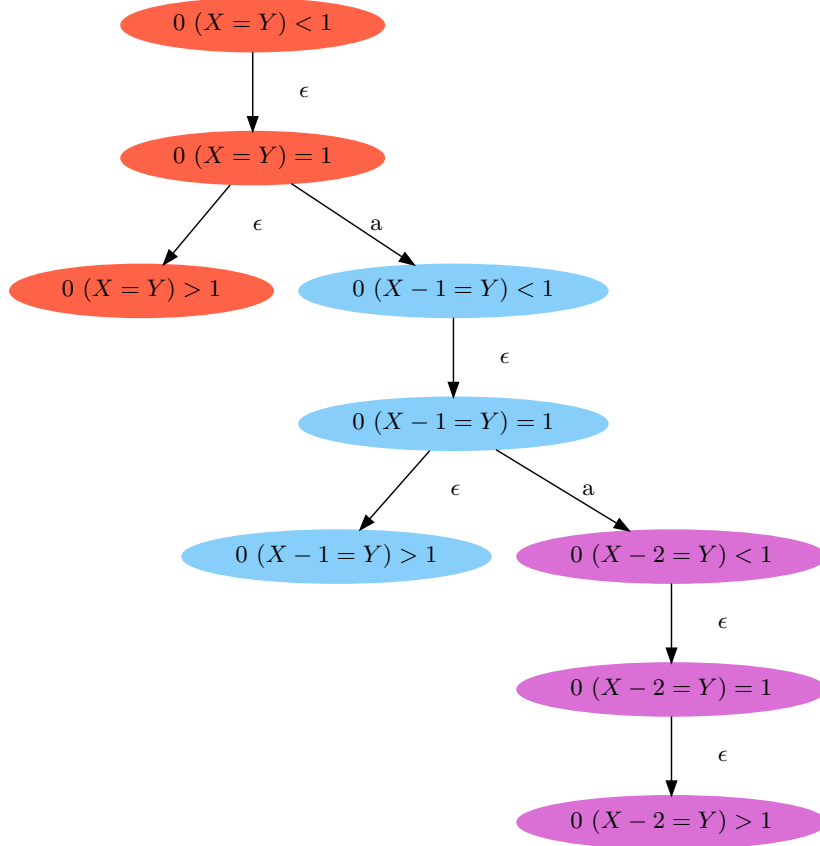


Timed automaton with potentially infinite state space.



Zones of Figure 4a after 1 iteration.

Zones of Figure 4a after 2 iterations.



Zones of Figure 4a after 3 iterations.

Fig. 4: Timed automaton with a potentially infinite set of zones, example taken from [5].

zones expands indefinitely, preventing termination of the algorithm. For example, in the automaton in Figure 4a, the number of zones may expand in each iteration, as shown in Figure 4b, Figure 4c, Figure 4d.

However, this is inconsistent with what we know about region graphs [10] and their implication of finiteness for the state spaces of timed automata, thus, we have abstractions which serve to cap the number of zones in a zone graph by reducing zones to equivalent regions which contain them, thus ensuring termination of zone creation algorithms.

In this implementation we use a simplified version of the *maximum constants* abstraction described in [5].

Algebraically, our abstraction can be thus described: given a set of clocks $\{x_i | 1 \leq i \leq n\}$, a maximum constant k over all clocks, and a DBM $M = \langle (m_{ij}, \smile_{ij}) \rangle_{0 \leq i, j \leq n}$, we can replace M with $M' = \langle m'_{ij}, \smile'_{ij} \rangle_{0 \leq i, j \leq n}$ where

$$(m'_{ij}, \smile'_{ij}) = \begin{cases} (\infty, <) & \text{if } m_{ij} > k \\ (-k, <) & \text{if } m_{ij} < -k \\ m_{ij}, \smile_{ij} & \text{if } -k \leq m_{ij} \leq k \end{cases}$$

9 Timed relations on timed automata

9.1 Time abstracted bisimilarity

Definition 18. Strong time abstracted bisimulation: A binary relation R is a strong time abstracted bisimulation (STaB) if and only if, for all $(s_1, s_2) \in R$, $a \in \text{Act}$, $d \in R_{\geq 0}$

$$\begin{aligned} \forall s'_1 (s_1 \xrightarrow{a} s'_1 \Rightarrow \exists s'_2. (s_2 \xrightarrow{a} s'_2 \wedge (s'_1, s'_2) \in R)) \wedge \\ \forall s'_2 (s_2 \xrightarrow{a} s'_2 \Rightarrow \exists s'_1. (s_1 \xrightarrow{a} s'_1 \wedge (s'_1, s'_2) \in R)) \wedge \\ \forall s'_1 (s_1 \xrightarrow{d} s'_1 \Rightarrow \exists (s'_2, d'). (s_2 \xrightarrow{d'} s'_2 \wedge (s'_1, s'_2) \in R)) \wedge \\ \forall s'_2 (s_2 \xrightarrow{d} s'_2 \Rightarrow \exists (s'_1, d'). (s_1 \xrightarrow{d'} s'_1 \wedge (s'_1, s'_2) \in R)) \end{aligned}$$

Definition 19. Strong time abstracted bisimilarity: It can be shown that the union of all strong time abstracted bisimulations over the set of (location, valuation) pairs is a strong time abstracted bisimulation. This binary relation is called strong time abstracted bisimilarity.

Definition 20. Time abstracted delay bisimulation: A binary relation R is a time abstracted delay bisimulation (TadB) if and only if, for all $(s_1, s_2) \in R$, $a \in \text{Act}$, $d \in R_{\geq 0}$

$$\begin{aligned} \forall s'_1 (s_1 \xrightarrow{a} s'_1 \Rightarrow \exists (s'_2, d). (s_2 \xrightarrow{d} s'_2 \wedge (s'_1, s'_2) \in R)) \wedge \\ \forall s'_2 (s_2 \xrightarrow{a} s'_2 \Rightarrow \exists (s'_1, d). (s_1 \xrightarrow{d} s'_1 \wedge (s'_1, s'_2) \in R)) \wedge \\ \forall s'_1 (s_1 \xrightarrow{d} s'_1 \Rightarrow \exists (s'_2, d'). (s_2 \xrightarrow{d'} s'_2 \wedge (s'_1, s'_2) \in R)) \wedge \\ \forall s'_2 (s_2 \xrightarrow{d} s'_2 \Rightarrow \exists (s'_1, d'). (s_1 \xrightarrow{d'} s'_1 \wedge (s'_1, s'_2) \in R)) \end{aligned}$$

Definition 21. Time abstracted delay bisimilarity: *It can be shown that the union of all time abstracted delay bisimulations over the set of (location, valuation) pairs is a time abstracted delay bisimulation. This binary relation is called time abstracted delay bisimilarity.*

Definition 22. Time abstracted observational bisimulation: *A binary relation R is a time abstracted observational bisimulation (TaoB) if and only if, for all $(s_1, s_2) \in R$, $a \in \text{Act}$, $d \in R_{\geq 0}$*

$$\begin{aligned} & \forall s'_1 (s_1 \xrightarrow{a} s'_1 \Rightarrow \exists (s'_2, d, d'). (s_2 \xrightarrow{d} \xrightarrow{a} \xrightarrow{d'} s'_2 \wedge (s'_1, s'_2) \in R)) \wedge \\ & \forall s'_2 (s_2 \xrightarrow{a} s'_2 \Rightarrow \exists (s'_1, d, d'). (s_1 \xrightarrow{d} \xrightarrow{a} \xrightarrow{d'} s'_1 \wedge (s'_1, s'_2) \in R)) \wedge \\ & \forall s'_1 (s_1 \xrightarrow{d} s'_1 \Rightarrow \exists (s'_2, d'). (s_2 \xrightarrow{d'} s'_2 \wedge (s'_1, s'_2) \in R)) \wedge \\ & \forall s'_2 (s_2 \xrightarrow{d} s'_2 \Rightarrow \exists (s'_1, d'). (s_1 \xrightarrow{d'} s'_1 \wedge (s'_1, s'_2) \in R)) \end{aligned}$$

Definition 23. Time abstracted observational bisimilarity: *It can be shown that the union of all time abstracted observational bisimulations over the set of (location, valuation) pairs is a time abstracted observational bisimulation. This binary relation is called time abstracted observational bisimilarity.*

10 Algorithms

10.1 Fernandez' algorithm

This algorithm is summarised here from [9]. Details of the splitting operations and the proof of correctness are omitted for brevity.

```

Initialise  $\pi = \{Pr\}$ ;
Initialise  $W = \{Pr\}$  while  $W$  is not empty do
  Choose a splitter  $B$  from  $W$ , removing it;
  if  $B$  is a simple splitter then
    Perform a two-way split on each action with respect to  $B$  and
    update  $W$ ;
  else
    Perform a three-way split on each action with respect to  $B$  and
    update  $W$ ;
  end
end

```

10.2 Creation of the zone valuation graph

Overview This algorithm is adapted from the algorithms in [2] and [12]. We changed it to make the correctness more evident. This algorithm consists of a *forward propagation* which ensures that all *reachable* zones are created, and a *backward propagation* which ensures that each zone is *stable* with respect to its successors.

For the forward propagation, we use a queue, akin to the queue of the classic *breadth-first search* algorithm for graphs, to traverse the timed automaton, starting from the initial location, to ensure that all reachable zones are created. In each queue element (with the exception of the first element containing the initial location, which has no predecessor), we store a location l_{succ} , its predecessor in the current path l_{pred} , and the transition t from l_{pred} to l_{succ} . It should be noted that this may result in some locations being visited multiple times, and in unreachable locations never being visited. Each time a location is visited, we create therein, new zones which are reachable from the zones of the predecessor. Thus, for each predecessor zone $(l_{pred}, \zeta_{pred_i})$, the derived successor zone is $(l_{succ}, \zeta_{succ_i})$, where

$$\zeta_{succ_i} = ((\zeta_{pred_i} \uparrow \mathbf{t.guard})[\mathbf{t.reset} := 0]) \uparrow$$

Thus, if we let $(l_{pred}, \zeta_{pred_i})$ range over the existing zones of l_{pred} , and if we let $(l_{succ}, \zeta_{succ_j})$ range over the existing zones of l_{succ} , then the new zones in l_{succ} will cover

$$\zeta_{succ_{new}} = \left(\bigcup_i \zeta_{succ_i} \right) - \left(\bigcup_j \zeta_{succ_j} \right)$$

Since $\zeta_{succ_{new}}$ is not necessarily convex, we may need to split it into multiple convex polyhedra before creating the corresponding zones in the l_{succ} . Then, we split the zones of l_{succ} to ensure stability with respect to its invariant and outgoing edge constraints. If any new zones are thus created, we enqueue each of the location's successors, in order to ensure that all reachable zones are created. The forward propagation ends when the queue is empty.

In the backward propagation, we iterate through the transitions of the timed automaton, multiple times if necessary, splitting the zones of the source of each transition with respect to the zones of the transition's target, until we achieve stability of each zone of each location. We recall that for stability, whenever we have an edge in the zone valuation graph from a zone (l_{pred}, ζ_{pred}) to (l_{succ}, ζ_{succ}) corresponding to a transition t in the timed automaton, we require

$$\zeta_{pred} \subseteq \mathbf{t.guard} \wedge \zeta_{pred}[\mathbf{t.reset} := 0] \subseteq \zeta_{succ}$$

Thus, when this does not hold, we split (l_{pred}, ζ_{pred}) into

$$(l_{pred}, \zeta_{pred} \cap (\mathbf{t.guard} \cap [\mathbf{t.reset} := 0] \zeta_{succ}))$$

(which is convex and has an edge to (l_{succ}, ζ_{succ})) and

$$(l_{pred}, \zeta_{pred} - (\mathbf{t.guard} \cap [\mathbf{t.reset} := 0] \zeta_{succ}))$$

(which does not have an edge to (l_{succ}, ζ_{succ}) and may need to be split into convex zones.)

This generates the zone valuation graph.

Initialise the queue Q with a single element $(null, null, l_0)$;
 Initialise the graph $zone_graph$ with a single node $(l_0, v_0 \uparrow)$ with an ϵ self-loop;
while Q is not empty **do**
 Dequeue $(l_{parent}, t, l_{child})$ from Q ;
 if $l_{parent} \neq null$ **then**
 foreach zone Z_{parent} of l_{parent} **do**
 Add new zones to the zones of l_{child} so that all zones reachable from Z_{parent} are represented;
 Abstract if necessary;
 Update edges from Z_{parent} to the new zones of l_{child} **if** new zones are created in l_{child} or l_{parent} is null **then**
 foreach outgoing transition t' of l_{child} **do**
 | Enqueue $(l_{child}, t', t'.target)$ in Q ;
 end
 end
 end
 end
 Set new_zone ;
 while new_zone **do**
 Reset new_zone ;
 foreach transition t in the timed automaton **do**
 Split the zones of $t.source$ to be stable with respect to the zones of $t.target$;
 Update edges accordingly;
 if new zones are created in $t.source$ **then**
 | Set new_zone ;
 end
 end
 end
end
 Generate $zone_valuation_graph$ by applying Fernandez' algorithm to $zone_graph$. Return $zone_graph$;

Proof of correctness

- **Termination:** The algorithm, in the worst case, will create as many zones as there are regions in the region graph, as the region graph abstraction ensures that the number of zones cannot exceed the number of regions. Since the number of regions is known to be bounded, termination of the algorithm is guaranteed.
- **Reachability:** Since the initial zone is the future of the zero valuation in the initial zone, it is reachable by definition. A new zone is only created when it is reachable from some zone which has already been created, thus each zone which is created is reachable by induction.

- **Stability:** Since the termination of the backward propagation step only occurs after an iteration in which all the edges of the timed automaton are traversed without causing any splitting of states, it follows that the zone graph is stable with respect to itself after this last iteration.

10.3 Checking timed and untimed relations on timed automata

Overview Many important relations on timed automata can be verified by arguing about the existence of winning strategies on two-player games. The algorithm described here uses this fact to verify these relations by simulating the two-player games, using a memoisation approach.

The relations which can be verified in this fashion are those in which functions f_P, f_Q exist such that the question ‘Are symbolic states s_P and s_Q in timed automata P and Q related under R ? ‘ can have three possible answers:

1. yes
2. no
3. if and only if

$$\begin{aligned} \forall (s'_P, L'_Q) \in f_P(s_P) : \exists s'_Q \in L'_Q : s'_P R s'_Q \quad \wedge \\ \forall (L'_P, s'_Q) \in f_Q(s_Q) : \exists s'_P \in L'_P : s'_P R s'_Q \end{aligned}$$

This property is satisfied by timed bisimulation, STaB, TadB, TaoB, and the corresponding simulation equivalences, among others.

So, for STaB, we define f_P and f_Q as

$$\begin{aligned} f_P(s_P) &= \{(s'_P, L'_Q) | s_P \xrightarrow{a} s'_P, L_Q = \{s'_Q | s_Q \xrightarrow{a} s'_Q\}\} \\ &\quad \cup \{(s'_P, L'_Q) | s_P \xrightarrow{\epsilon} s'_P, L_Q = \{s'_Q | s_Q \xrightarrow{\epsilon} s'_Q\}\} \\ f_Q(s_Q) &= \{(L'_P, s'_Q) | s_Q \xrightarrow{a} s'_Q, L_P = \{s'_P | s_P \xrightarrow{a} s'_P\}\} \\ &\quad \cup \{(L'_P, s'_Q) | s_Q \xrightarrow{\epsilon} s'_Q, L_P = \{s'_P | s_P \xrightarrow{\epsilon} s'_P\}\} \end{aligned}$$

For TadB, we define f_P and f_Q as

$$\begin{aligned} f_P(s_P) &= \{(s'_P, L'_Q) | s_P \xrightarrow{a} s'_P, L_Q = \{s'_Q | s_Q \xrightarrow{\epsilon} \xrightarrow{a} s'_Q\}\} \\ &\quad \cup \{(s'_P, L'_Q) | s_P \xrightarrow{\epsilon} s'_P, L_Q = \{s'_Q | s_Q \xrightarrow{\epsilon} s'_Q\}\} \\ f_Q(s_Q) &= \{(L'_P, s'_Q) | s_Q \xrightarrow{a} s'_Q, L_P = \{s'_P | s_P \xrightarrow{\epsilon} \xrightarrow{a} s'_P\}\} \\ &\quad \cup \{(L'_P, s'_Q) | s_Q \xrightarrow{\epsilon} s'_Q, L_P = \{s'_P | s_P \xrightarrow{\epsilon} s'_P\}\} \end{aligned}$$

For TaoB, we define f_P and f_Q as

$$\begin{aligned} f_P(s_P) &= \{(s'_P, L'_Q) | s_P \xrightarrow{a} s'_P, L_Q = \{s'_Q | s_Q \xrightarrow{\epsilon} \xrightarrow{a} \xrightarrow{\epsilon} s'_Q\}\} \\ &\quad \cup \{(s'_P, L'_Q) | s_P \xrightarrow{\epsilon} s'_P, L_Q = \{s'_Q | s_Q \xrightarrow{\epsilon} s'_Q\}\} \\ f_Q(s_Q) &= \{(L'_P, s'_Q) | s_Q \xrightarrow{a} s'_Q, L_P = \{s'_P | s_P \xrightarrow{\epsilon} \xrightarrow{a} \xrightarrow{\epsilon} s'_P\}\} \\ &\quad \cup \{(L'_P, s'_Q) | s_Q \xrightarrow{\epsilon} s'_Q, L_P = \{s'_P | s_P \xrightarrow{\epsilon} s'_P\}\} \end{aligned}$$

This formulation naturally suggests an approach: use tables to store pairs of states in the two timed automata which are known to be related or known to be unrelated, and use a dynamic programming approach to verify the relation for any given pair of states.

Proof of correctness Since a zone-valuation graph is minimal modulo strong time-abstracted bisimilarity, verifying a relation on two timed automata is equivalent to verifying it on the symbolic states containing the initial states of the timed automata in their respective zone-valuation graphs.

When a zone is

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```

begin
  if lookup(yes_table,  $s_P$ ,  $s_Q$ ) then
    | return true;
  else
    if lookup(yes_table,  $s_P$ ,  $s_Q$ ) then
      | return false;
    else
      insert(yes_table,  $s_P$ ,  $s_Q$ );
      Set  $v_P$ ;
      foreach ( $s'_P, L'_Q$ ) in  $f_P(s_P)$  do
        | Reset  $v_P$ ;
        | foreach  $s'_Q$  in  $L'_Q$  do
          | if CheckStatesRelation( $P, Q, s'_P, s'_Q, \text{yes\_table}, \text{no\_table}$ )
          | then
          | | Set  $v_P$ ;
          | end
        | end
      end
      Set  $v_Q$ ;
      foreach ( $s'_Q, L'_P$ ) in  $f_Q(s_Q)$  do
        | Reset  $v_Q$ ;
        | foreach  $s'_P$  in  $L'_P$  do
          | if CheckStatesRelation( $P, Q, s'_P, s'_Q, \text{yes\_table}, \text{no\_table}$ )
          | then
          | | Set  $v_Q$ ;
          | end
        | end
      end
      if  $v_P \wedge v_Q$  then
        | return true;
      else
        remove(yes_table,  $s_P$ ,  $s_Q$ );
        insert(yes_table,  $s_P$ ,  $s_Q$ );
        return false;
      end
    end
  end
end

```

Procedure CheckStatesRelation($P, Q, s_P, s_Q, \text{yes_table}, \text{no_table}$)

```

begin
  Create zone valuation graphs  $G_P, G_Q$  of  $T_P, T_Q$ ;
  Find the zone  $s_P$  in  $G_P$  which contains the initial state of  $T_P$ ;
  Find the zone  $s_Q$  in  $G_Q$  which contains the initial state of  $T_Q$ ;
  Initialise yes_table and no_table to empty tables;
  return CheckStatesRelation( $G_P, G_Q, s_P, s_Q, \text{yes\_table}, \text{no\_table}$ );
end

```

Procedure CheckAutomataRelation(T_P, T_Q)