Tools and Algorithms for Deciding Timed Relations

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Mihir Mehta

Department of Computer Science and Engineering
Indian Institute of Technology, Delhi.

cs1090197@cse.iitd.ac.in

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Overview

- Bisimilarity and related notions
- Kanellakis and Smolka's algorithm
- Fernandez' algorithm
- Paige and Tarjan's algorithm
- Timed Automata
- Equivalences on Timed Automata
- Code written so far

Bisimilarity and related notions

- Labeled Transition System: This is a triple $(Proc, Act, \{\xrightarrow{a} | a \epsilon Act\})$ where
  - $Proc$ is a set of states (also called processes or configurations.)
  - Act is a set of actions (also called labels.)
  - $\xrightarrow{a} \subseteq Proc \times Proc$ is a transition relation.
- CCS expression: Defined by the following grammar:
  - $P ::= K$
  - $P ::= \alpha.P$
  - $P ::=_{i \epsilon I} P_i$
  - $P ::= P|Q$
  - $P ::= P[f]$
  - $P ::= P$Ł

Bisimilarity and related notions

- Equivalence for CCS processes.
- Trace equivalence: $Traces(P) = Traces(Q)$
- Unsatisfactory (differences in deadlock behaviour.)
- Strong bisimulation: A binary relation R is a *strong bisimulation* if and only if, for all $(s_1, s_2) \epsilon R$ and $a \epsilon Act$.
  $\forall s_1'(s_1 \xrightarrow{a} s_1' \Rightarrow \exists s_2'.(s_2 \xrightarrow{a} s_2' \wedge (s_1', s_2') \epsilon R)) \wedge$
  $\forall s_2'(s_2 \xrightarrow{a} s_2' \Rightarrow \exists s_1'.(s_1 \xrightarrow{a} s_1' \wedge (s_1', s_2') \epsilon R))$
- It can be shown that the union of all strong bisimulations over the set of states is a strong bisimulation. This binary relation is called *strong bisimilarity*, denoted by $\sim$.

Bisimilarity and related notions

- Better notion of equivalence than trace equivalence: picks up differences in the deadlock behaviour of processes under study.
- Failing: does not account for the invisible nature of τ transitions in CCS processes.
- Weak bisimulation: A binary relation R is a *weak bisimulation* if and only if, for all $(s_1, s_2) \epsilon R$ and $a \epsilon Act$.
  $\forall s_1'(s_1 \xrightarrow{a} s_1' \Rightarrow \exists s_2'.(s_2 \stackrel{a}{\Rightarrow} s_2' \wedge (s_1', s_2') \epsilon R)) \wedge$
  $\forall s_2'(s_2 \xrightarrow{a} s_2' \Rightarrow \exists s_1'.(s_1 \stackrel{a}{\Rightarrow} s_1' \wedge (s_1', s_2') \epsilon R))$
- It can be shown that the union of all weak bisimulations over the set of states is a weak bisimulation. This binary relation is called *weak bisimilarity*, denoted by $\approx$.
- Better suited to CCS processes, as it ignores τ transitions, thus disregarding hidden behaviour within a process.

Kanellakis and Smolka's algorithm

- This is a naive algorithm for determining the bisimilarity relation for the set of processes in a labelled transition system.
- This relies on the notion of a splitter.
- Let $\pi = \{B_0, ..., B_k\}, k \geq 0$ be a partition of the set of states Pr in a labeled transition system.
- A splitter for a block $B_i \epsilon \pi$ is a block $B_j \epsilon \pi$ such that for some action $a \epsilon Act$, some states in B_i have a-labelled transitions whose targets lie in B_j while other states in B_i do not.
- This suggests a refinement of π: replace block B_i with
  $B_i^1 = B_i \cap T_a^{-1}[B_j]$
  $B_i^2 = B_i - B_i^1$
- Refinements of this kind constitute the steps of this algorithm.

Kanellakis and Smolka's algorithm

```
Initialise π to Pr;
while there exist splitters among the elements of π do
    Pick a splitter B;
    foreach B_j ϵ π do
        foreach a ϵ Act do
            Split B_j with respect to B for action a;
        end
    end
end
Return π;
```

- The time complexity of this algorithm is $O(mn)$, since there can be at most n iterations, and all m edges are scanned in each iteration.

Fernandez' algorithm

- More efficient algorithm (O(m log n)).
- Relies on Paige and Tarjan's technique of three-way splitting.
- Splitters can now be 'simple' or 'compound'.
- Stability: A partition π is said to be stable with respect to a compound block S if S is not a splitter for any block in π for any action.
- For a compound block S, having a constituent simple block B satisfying $n(B) \leq 0.5 * n(S)$, and with respect to which π is stable, we can split a block B_i on an action a as follows:
  $B_i^1 = (B_i \cap T_a^{-1}[B]) - T_a^{-1}[S - B]$
  $B_i^2 = (B_i \cap T_a^{-1}[S - B]) - T_a^{-1}[B]$
  $B_i^3 = B_i \cap T_a^{-1}[B] \cap T_a^{-1}[S - B]$

Fernandez' algorithm

Initialise $\pi = \{Pr\}$;
Initialise W $= \{Pr\}$ **while** *W is not empty* **do**

  Choose a splitter B from W, removing it;
  if *B is a simple splitter* **then**
    Perform a two-way split on each action with respect to B and update W;
  else
    Perform a three-way split on each action with respect to B and update W;
  end
end

Paige and Tarjan's Algorithm

- Technique of three way splitting came from here.
- Special case of Fernandez' algorithm when there's only one kind of action.

Timed Automata

- Formally, a timed automaton over a finite set of clocks C and a finite set of actions Act is a 4-tuple (L, l_0, E, I).
- L is a finite set of locations.
- l_0 is the initial location.
- $E \subseteq L \times B(C) \times Act \times 2^C \times L$ is a finite set of edges.
- $I : L \to B(C)$ assigns invariants to each edge location.
- $B(C)$ is the set of clock constraints over C. An element of $B(C)$ can be an equality, a slack inequality, a strict inequality, or an AND combination of such constraints which involve clock names and natural numbers.
  Example: $x < 5 \wedge y \geq 7 \wedge z = 3$, where $C = \{x, y, z\}$

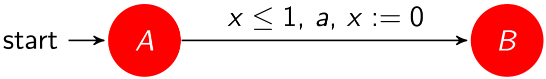
Equivalences on Timed Automata

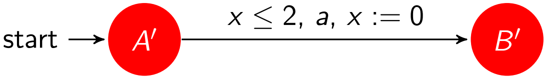
- *Timed bisimulation*: A binary relation R is a timed bisimulation if and only if, for all $(s_1, s_2) \in R$, $a \in Act$, $d \in R_{\geq 0}$
$\forall s_1'(s_1 \xrightarrow{a} s_1' \Rightarrow \exists s_2'.(s_2 \xrightarrow{a} s_2' \wedge (s_1', s_2') \in R))\wedge$
$\forall s_2'(s_2 \xrightarrow{a} s_2' \Rightarrow \exists s_1'.(s_1 \xrightarrow{a} s_1' \wedge (s_1', s_2') \in R))\wedge$
$\forall s_1'(s_1 \xrightarrow{d} s_1' \Rightarrow \exists s_2'.(s_2 \xrightarrow{d} s_2' \wedge (s_1', s_2') \in R))\wedge$
$\forall s_2'(s_2 \xrightarrow{d} s_2' \Rightarrow \exists s_1'.(s_1 \xrightarrow{d} s_1' \wedge (s_1', s_2') \in R))$
- It can be shown that the union of all timed bisimulations over the set of (location, valuation) pairs is a timed bisimulation. This binary relation is called *timed bisimilarity*, denoted by $\sim$.

Equivalences on Timed Automata

- *Time abstracted bisimulation*: A binary relation R is a time abstracted bisimulation if and only if, for all $(s_1, s_2) \in R$, $a \in Act$, $d \in R_{\geq 0}$
  $\forall s_1'(s_1 \xrightarrow{a} s_1' \Rightarrow \exists s_2'.(s_2 \xrightarrow{a} s_2' \wedge (s_1', s_2') \in R))\wedge$
  $\forall s_2'(s_2 \xrightarrow{a} s_2' \Rightarrow \exists s_1'.(s_1 \xrightarrow{a} s_1' \wedge (s_1', s_2') \in R))\wedge$
  $\forall s_1'(s_1 \xrightarrow{d} s_1' \Rightarrow \exists (s_2', d').(s_2 \xrightarrow{d'} s_2' \wedge (s_1', s_2') \in R))\wedge$
  $\forall s_2'(s_2 \xrightarrow{d} s_2' \Rightarrow \exists (s_1', d').(s_1 \xrightarrow{d'} s_1' \wedge (s_1', s_2') \in R))$
- It can be shown that the union of all time abstracted bisimulations over the set of (location, valuation) pairs is a time abstracted bisimulation. This binary relation is called *time abstracted bisimilarity*, denoted by $\sim_u$.

Equivalences on Timed Automata





Equivalences on Timed Automata

- *Equivalence of valuations*: Two valuations v and v' of a timed automaton are said to be equivalent ($v \equiv v'$) if and only if:
  - For each $x \in C$, either both $v(x)$ and $v'(x)$ are greater than c_x or $\lfloor v(x) \rfloor = \lfloor v'(x) \rfloor$
  - For each $x \in C$ such that $v(x) \leq c_x$, $\mathit{frac}(v(x)) = 0$ if and only if $\mathit{frac}(v'(x)) = 0$.
  - For all $x, y \in C$ such that $v(x) \leq c_x$ and $v(y) \leq c_y$, we have $\mathit{frac}(v(x)) \leq \mathit{frac}(v(y))$ if and only if $\mathit{frac}(v'(x)) \leq \mathit{frac}(v'(y))$.
- Under this notion of equivalence, an equivalence class is known as a *region*. The equivalence class containing a valuation v is denoted by $[v]_{\equiv}$.

Equivalences on Timed Automata

- *Region graph*: the region graph of a timed automaton A with clock set C and action set Act is an LTS
  $T_r(A) = (Proc, Act \cup \{\varepsilon\}, \{\xrightarrow{a} | a \in Act \cup \{\varepsilon\}\})$
  where $Proc = \{(l, [v]_\equiv) | l \in L, v : C \to R_{\geq 0}\}$ (these states are called symbolic states)
  The transitions are defined as follows:
  - For each $a \in A$, $(l, [v]_\equiv) \overset{a}{\Rightarrow} (l', [v']_\equiv)$ iff $(l, v) \xrightarrow{a} (l', v')$
  - $(l, [v]_\equiv) \overset{\varepsilon}{\Rightarrow} (l, [v']_\equiv)$ iff for some $d \in R_{\geq 0}$, $(l, v) \xrightarrow{d} (l, v')$

Equivalences on Timed Automata

- *Extended Clock Constraints*: These are described by the grammar
  $g ::= x \bowtie n | x - y \bowtie n | g_1 \wedge g_2$
  where $\bowtie \epsilon \{<, \leq, =, \geq, >\}$
- The set of extended clock constraints is denoted by $B^+(C)$
- *Zones*: A zone is a set of valuations represented by an extended clock constraint.
  $Z = \{v | v \epsilon g_Z\}$
- *Symbolic state*: For a timed automaton A, a symbolic state is an ordered pair of a location and a zone, that is, (l, Z).

Equivalences on Timed Automata

- *Reachability graph*: The reachability graph of a timed automaton is a directed graph in which the nodes are symbolic states and the edges are given by the *reachability relation*:
  - (l, Z') is reachable from (l, Z) if there exists a valuation belonging to Z' which can be obtained from Z by applying a time delay which does not violate the invariant at l.
  - (l', Z') is reachable from (l, Z) if l has an action transition to l' and there exists a valuation in Z' which violates neither the guard of the transition nor the invariant at l' after performing the clock resets required by the transition.

Code written so far

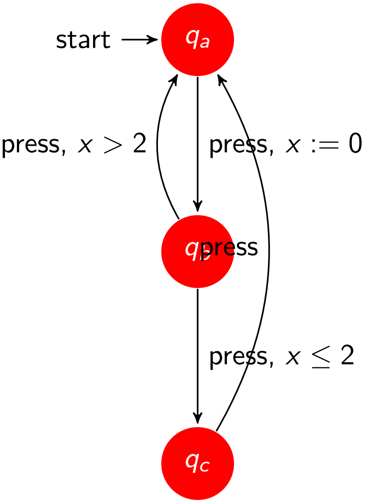
- *Fernandez' algorithm*: We expect to need this algorithm while building the software, so we built a module implementing it in Ocaml (primary language for this project.)
- *Grammar*: We implemented a lexer and parser in Ocaml to build a representation of a timed automaton from a specification in a text file.

Timed automaton example

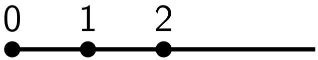
```
#states 3
#trans 4
#clocks 1
X

state: 0
invar: TRUE
trans:
TRUE => RESET { X }; goto 1

state: 1
invar: TRUE
trans:
X <= 2 => RESET { }; goto 0
X > 2 => RESET { }; goto 2

state: 2
invar: TRUE
trans:
TRUE => RESET { }; goto 0
```

Example taken from Reactive Systems

Timed automaton example



Timed automaton example



Regions in this graph:

- $[x = 0]_{\equiv}$, $[x = 1]_{\equiv}$, $[x = 2]_{\equiv}$
- $[0 < x < 1]_{\equiv}$, $[1 < x < 2]_{\equiv}$
- $[2 < x]_{\equiv}$

References

- Reactive Systems: Modeling, Specification and Verification - Luca Aceto, Anna Ingolfsdottir, Kim Larsen, Jiri Srba.