

Tools and Algorithms for Deciding Relations on Timed Automata

B Tech project, supervised by S Arun-Kumar, verification group

Mihir Mehta

Department of Computer Science and Engineering
Indian Institute of Technology, Delhi

cs1090197

May 12, 2013

Outline

Automata without timing and relations on them

Timed automata and relations on them

Algorithms

Labeled transition systems

Definition

Labelled Transition System: A labelled transition system (LTS) [1] is an automaton which is described by

- ▶ S , a set of *states*
- ▶ Act , a set of *actions*
- ▶ $\rightarrow \subseteq S \times Act \times S$, a *transition relation*.
- ▶ optionally, $I \subseteq S$, a set of initial states. If there is exactly one initial state, then the LTS is said to be *rooted*.

Relations on LTS I

Definition

Strong bisimulation: A binary relation R on the states of an LTS is a strong bisimulation if and only if, for all $(s_1, s_2) \in R$ and $a \in Act$.

$$\forall s'_1 (s_1 \xrightarrow{a} s'_1 \Rightarrow \exists s'_2. (s_2 \xrightarrow{a} s'_2 \wedge (s'_1, s'_2) \in R)) \wedge$$

$$\forall s'_2 (s_2 \xrightarrow{a} s'_2 \Rightarrow \exists s'_1. (s_1 \xrightarrow{a} s'_1 \wedge (s'_1, s'_2) \in R))$$

Definition

It can be shown that the union of all strong bisimulations over the set of states is a strong bisimulation. This binary relation is called *strong bisimilarity*, denoted by \sim .

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Timed Automata

Definition

Timed Automaton: A timed automaton [2] over a finite set of clocks C and a finite set of actions Act is a 4-tuple (L, l_0, E, I) .

- ▶ L is a finite set of locations.
- ▶ l_0 is the initial location.
- ▶ $E \subseteq L \times B(C) \times Act \times 2^C \times L$ is a finite set of edges.
- ▶ $I : L \rightarrow B(C)$ assigns invariants to each edge location.
- ▶ $B(C)$ is the set of clock constraints over C .

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Creating the zone valuation graph

Initialise the queue Q with a single element $(null, null, l_0)$;

Initialise the graph $zone_graph$ with a single node $(l_0, v_0 \uparrow)$ with an ϵ self-loop;

while Q is not empty **do**

 Dequeue $(l_{parent}, t, l_{child})$ from Q ;

if $l_{parent} \neq null$ **then**

foreach zone Z_{parent} of l_{parent} **do**

 Add new zones to the zones of l_{child} so that all zones reachable from Z_{parent} are represented;

 Abstract if necessary;

 Update edges from Z_{parent} to the new zones of l_{child} **if**
 new zones are created in l_{child} or l_{parent} is null **then**

foreach outgoing transition t' of l_{child} **do**

 Enqueue $(l_{child}, t', t'.target)$ in Q ;

end

end

end

end

Zone valuation graph example

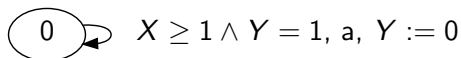


Figure: Timed automaton. Here, the states are $\{0\}$, the actions are $\{a\}$, and the clocks are $\{X, Y\}$.

Zone valuation graph example

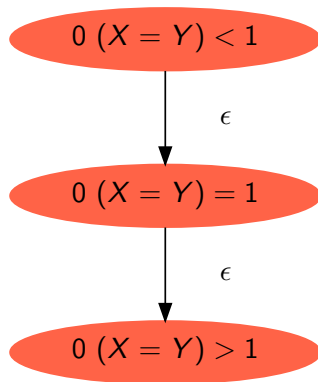


Figure: Zones after one iteration.

Zone valuation graph example

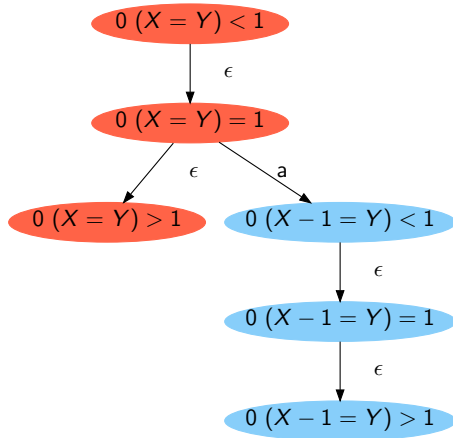


Figure: Zones after two iterations.

Zone valuation graph example

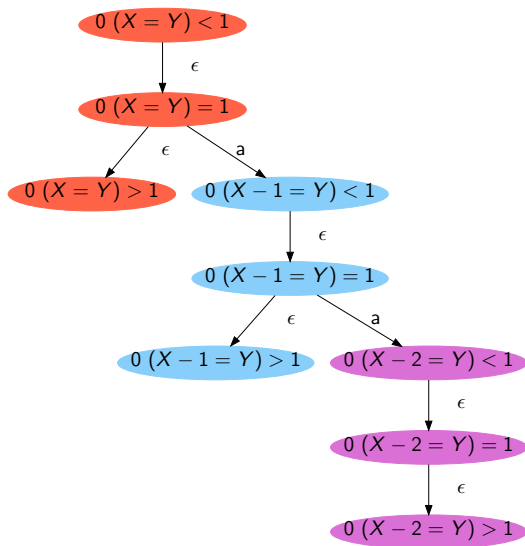


Figure: Zones after three iterations without abstraction.

Zone valuation graph example

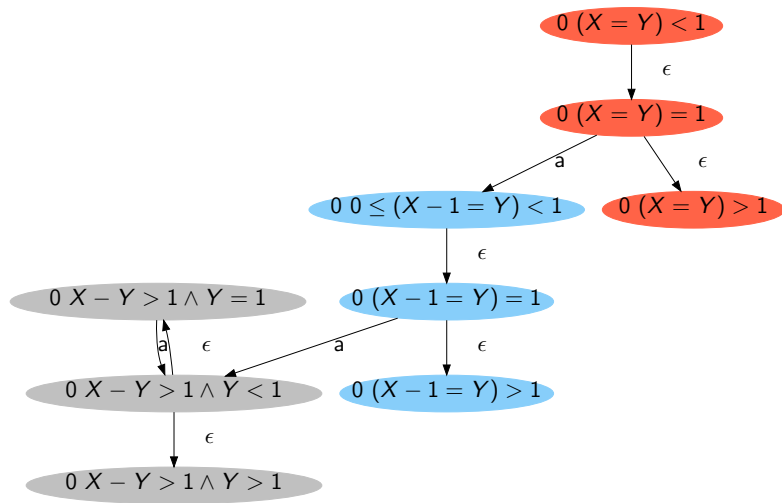


Figure: Zones after three iterations with abstraction.

Zone valuation graph example

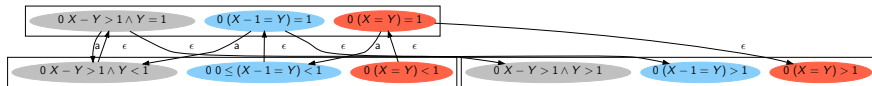


Figure: Zone graph with bisimilarity classes.

References I

References II



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