Tools and Algorithms for Deciding Timed Relations

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Overview

- Bisimilarity and related notions
- Kanellakis and Smolka's algorithm
- Fernandez' algorithm
- Paige and Tarjan's algorithm
- Timed Automata
- Equivalences on Timed Automata
- Code written so far

Bisimilarity and related notions

- Labeled Transition System: This is a triple $(Proc, Act, \{\xrightarrow{a} | a \epsilon Act\})$ where
  - $Proc$ is a set of states (also called processes or configurations.)
  - Act is a set of actions (also called labels.)
  - $\xrightarrow{a} \subseteq Proc \times Proc$ is a transition relation.
- CCS expression: Defined by the following grammar:
  - $P ::= K$
  - $P ::= \alpha.P$
  - $P ::=_{i \epsilon I} P_i$
  - $P ::= P|Q$
  - $P ::= P[f]$
  - $P ::= P$Ł

Bisimilarity and related notions

- Equivalence for CCS processes.
- Trace equivalence: $Traces(P) = Traces(Q)$
- Unsatisfactory (differences in deadlock behaviour.)
- Strong bisimulation: A binary relation R is a `strong bisimulation` if and only if, for all $(s_1, s_2) \epsilon R$ and $a \epsilon Act$.
  $\forall s_1'(s_1 \xrightarrow{a} s_1' \Rightarrow \exists s_2'.(s_2 \xrightarrow{a} s_2' \wedge (s_1', s_2') \epsilon R)) \wedge$
  $\forall s_2'(s_2 \xrightarrow{a} s_2' \Rightarrow \exists s_1'.(s_1 \xrightarrow{a} s_1' \wedge (s_1', s_2') \epsilon R))$
- It can be shown that the union of all strong bisimulations over the set of states is a strong bismulation. This binary relation is called `strong bisimilarity`, denoted by $\sim$.

Bisimilarity and related notions

- Better notion of equivalence than trace equivalence: picks up differences in the deadlock behaviour of processes under study.
- Failing: does not account for the invisible nature of τ transitions in CCS processes.
- Weak bisimulation: A binary relation R is a `weak bisimulation` if and only if, for all $(s_1, s_2) \epsilon R$ and $a \epsilon Act$.
  $\forall s_1'(s_1 \xrightarrow{a} s_1' \Rightarrow \exists s_2'.(s_2 \overset{a}{\Rightarrow} s_2' \wedge (s_1', s_2') \epsilon R)) \wedge$
  $\forall s_2'(s_2 \xrightarrow{a} s_2' \Rightarrow \exists s_1'.(s_1 \overset{a}{\Rightarrow} s_1' \wedge (s_1', s_2') \epsilon R))$
- It can be shown that the union of all weak bisimulations over the set of states is a weak bismulation. This binary relation is called `weak bisimilarity`, denoted by $\approx$.
- Better suited to CCS processes, as it ignores τ transitions, thus disregarding hidden behaviour within a process.

Kanellakis and Smolka's algorithm

- This is a naive algorithm for determining the bisimilarity relation for the set of processes in a labelled transition system.
- This relies on the notion of a splitter.
- Let $\pi = \{B_0, ..., B_k\}, k \geq 0$ be a partition of the set of states Pr in a labeled transition system.
- A splitter for a block $B_i \epsilon \pi$ is a block $B_j \epsilon \pi$ such that for some action $a \epsilon Act$, some states in B_i have a-labelled transitions whose targets lie in B_j while other states in B_i do not.
- This suggests a refinement of π: replace block B_i with
  $B_i^1 = B_i \cap T_a^{-1}[B_j]$
  $B_i^2 = B_i - B_i^1$
- Refinements of this kind constitute the steps of this algorithm.

Kanellakis and Smolka's algorithm

```
Initialise π to Pr;
while there exist splitters among the elements of π do
    Pick a splitter B;
    foreach B_j ϵ π do
        foreach a ϵ Act do
            Split B_j with respect to B for action a;
        end
    end
end
Return π;
```

- The time complexity of this algorithm is $O(mn)$, since there can be at most n iterations, and all m edges are scanned in each iteration.

Fernandez' algorithm

- More efficient algorithm (O(m log n)).
- Relies on Paige and Tarjan's technique of three-way splitting.
- Splitters can now be 'simple' or 'compound'.
- Stability: A partition π is said to be stable with respect to a compound block S if S is not a splitter for any block in π for any action.
- For a compound block S, having a constituent simple block B satisfying $n(B) \leq 0.5 * n(S)$, and with respect to which π is stable, we can split a block B_i on an action a as follows:
  $B_i^1 = (B_i \cap T_a^{-1}[B]) - T_a^{-1}[S - B]$
  $B_i^2 = (B_i \cap T_a^{-1}[S - B]) - T_a^{-1}[B]$
  $B_i^3 = B_i \cap T_a^{-1}[B] \cap T_a^{-1}[S - B]$

Fernandez' algorithm

Initialise $\pi = \{Pr\}$;
Initialise W $= \{Pr\}$ **while** *W is not empty* **do**
  Choose a splitter B from W, removing it;
  if *B is a simple splitter* **then**
    Perform a two-way split on each action with respect to B and update W;
  else
    Perform a three-way split on each action with respect to B and update W;
  end
end

Paige and Tarjan's Algorithm

- Technique of three way splitting came from here.
- Special case of Fernandez' algorithm when there's only one kind of action.

Timed Automata

- Formally, a timed automaton over a finite set of clocks C and a finite set of actions Act is a 4-tuple (L, l_0, E, I).
- L is a finite set of locations.
- l_0 is the initial location.
- $E \subseteq L \times B(C) \times Act \times 2^C \times L$ is a finite set of edges.
- $I : L \rightarrow B(C)$ assigns invariants to each edge location.
- $B(C)$ is the set of clock constraints over C. An element of $B(C)$ can be an equality, a slack inequality, a strict inequality, or an AND combination of such constraints.

Equivalences on Timed Automata