Tools and Algorithms for Deciding Relations on Timed Automata

B Tech project, supervised by S Arun-Kumar, verification group

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Outline

Automata without timing and relations on them

Timed automata and relations on them

Algorithms

Labeled transition systems

Definition

Labelled Transition System: A labelled transition system (LTS) [1] is an automaton which is described by

- S, a set of *states*
- Act, a set of *actions*
- $\rightarrow \subseteq S \times Act \times S$, a *transition relation*.
- optionally, $I \subseteq S$,a set of initial states. If there is exactly one initial state, then the LTS is said to be *rooted*.

Relations on LTS I

Definition

Strong bisimulation:A binary relation R on the states of an LTS is a strong bisimulation if and only if, for all $(s_1, s_2) \in R$ and $a \in Act$.
$\forall s_1'(s_1 \xrightarrow{a} s_1' \Rightarrow \exists s_2'.(s_2 \xrightarrow{a} s_2' \wedge (s_1', s_2') \in R))\wedge$
$\forall s_2'(s_2 \xrightarrow{a} s_2' \Rightarrow \exists s_1'.(s_1 \xrightarrow{a} s_1' \wedge (s_1', s_2') \in R))$

Definition

It can be shown that the union of all strong bisimulations over the set of states is a strong bisimulation. This binary relation is called *strong bisimilarity*, denoted by $\sim$.

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Timed Automata

Definition

Timed Automaton: A timed automaton [2] over a finite set of clocks C and a finite set of actions Act is a 4-tuple (L, l_0, E, I).

- L is a finite set of locations.
- l_0 is the initial location.
- $E \subseteq L \times B(C) \times Act \times 2^C \times L$ is a finite set of edges.
- $I : L \rightarrow B(C)$ assigns invariants to each edge location.
- $B(C)$ is the set of clock constraints over C.

Relations on timed automata

Definition

Strong time abstracted bisimulation: A binary relation R is a strong time abstracted bisimulation (STaB) if and only if, for all $(s_1, s_2) \in R$, $a \in Act$, $d \in R_{\geq 0}$

$$\forall s_1'(s_1 \xrightarrow{a} s_1' \Rightarrow \exists s_2'.(s_2 \xrightarrow{a} s_2' \wedge (s_1', s_2') \in R))\wedge$$
$$\forall s_2'(s_2 \xrightarrow{a} s_2' \Rightarrow \exists s_1'.(s_1 \xrightarrow{a} s_1' \wedge (s_1', s_2') \in R))\wedge$$
$$\forall s_1'(s_1 \xrightarrow{d} s_1' \Rightarrow \exists (s_2', d').(s_2 \xrightarrow{d'} s_2' \wedge (s_1', s_2') \in R))\wedge$$
$$\forall s_2'(s_2 \xrightarrow{d} s_2' \Rightarrow \exists (s_1', d').(s_1 \xrightarrow{d'} s_1' \wedge (s_1', s_2') \in R))$$

It can be shown that the union of all strong time abstracted bisimulations over the set of (location, valuation) pairs is a strong time abstracted bisimulation. This binary relation is called *strong time abstracted bisimilarity*.

Relations on timed automata

Definition

Time abstracted delay bisimulation: A binary relation R is a time abstracted delay bisimulation (TadB) if and only if, for all $(s_1, s_2) \in R$, $a \in Act$, $d \in R_{\geq 0}$

$$\forall s_1'(s_1 \xrightarrow{a} s_1' \Rightarrow \exists(s_2', d).(s_2 \xrightarrow{d}\xrightarrow{a} s_2' \wedge (s_1', s_2') \in R))\wedge$$
$$\forall s_2'(s_2 \xrightarrow{a} s_2' \Rightarrow \exists(s_1', d).(s_1 \xrightarrow{d}\xrightarrow{a} s_1' \wedge (s_1', s_2') \in R))\wedge$$
$$\forall s_1'(s_1 \xrightarrow{d} s_1' \Rightarrow \exists(s_2', d').(s_2 \xrightarrow{d'} s_2' \wedge (s_1', s_2') \in R))\wedge$$
$$\forall s_2'(s_2 \xrightarrow{d} s_2' \Rightarrow \exists(s_1', d').(s_1 \xrightarrow{d'} s_1' \wedge (s_1', s_2') \in R))$$

It can be shown that the union of all time abstracted delay bisimulations over the set of (location, valuation) pairs is a time abstracted delay bisimulation. This binary relation is called *time abstracted delay bisimilarity*.

Relations on timed automata

Definition

Time abstracted observational bisimulation: A binary relation R is a time abstracted observational bisimulation (TaoB) if and only if, for all $(s_1, s_2) \epsilon R$, $a \epsilon Act$, $d \epsilon R_{\geq 0}$

$$\forall s_1'(s_1 \xrightarrow{a} s_1' \Rightarrow \exists(s_2', d, d').(s_2 \xrightarrow{d}\xrightarrow{a}\xrightarrow{d'} s_2' \wedge (s_1', s_2') \epsilon R))\wedge$$
$$\forall s_2'(s_2 \xrightarrow{a} s_2' \Rightarrow \exists(s_1', d, d').(s_1 \xrightarrow{d}\xrightarrow{a}\xrightarrow{d'} s_1' \wedge (s_1', s_2') \epsilon R))\wedge$$
$$\forall s_1'(s_1 \xrightarrow{d} s_1' \Rightarrow \exists(s_2', d').(s_2 \xrightarrow{d'} s_2' \wedge (s_1', s_2') \epsilon R))\wedge$$
$$\forall s_2'(s_2 \xrightarrow{d} s_2' \Rightarrow \exists(s_1', d').(s_1 \xrightarrow{d'} s_1' \wedge (s_1', s_2') \epsilon R))$$

It can be shown that the union of all time abstracted observational bisimulations over the set of (location, valuation) pairs is a time abstracted observational bisimulation. This binary relation is called *time abstracted observational bisimilarity*.

Difference bound matrices

Definition

Difference bound matrix: A difference bound matrix (DBM) is a representation of a convex polyhedron on a set of clocks $\{x_1, \ldots x_n\}$ in the form of an $(n+1) \times (n+1)$ matrix M, each element of which takes the form $(m_{ij}, \smile_{ij})$, where m_{ij} is an integer and $\smile_{ij} \, \epsilon \, \{<, \leq\}$. Assuming x_0 to be a clock always valued at zero, the associated polyhedron is given by

$$\bigcap_{0 \leq i,j \leq n} (x_i - x_j \smile_{ij} m_{ij})$$

Abstractions

Abstractions serve to cap the number of zones in a zone graph by reducing zones to equivalent regions which contain them, thus ensuring termination of zone creation algorithms which use forward analysis.

In this implementation we use a simplified version of the *maximum constants* abstraction described in [3].

Algebraically, our abstraction can be thus described: given a set of clocks $\{x_i | 1 \leq i \leq n\}$, a maximum constant k over all clocks, and a DBM $M = \langle (m_{ij}, \smile_{ij}) \rangle_{0 \leq i,j \leq n}$, we can replace M with $M' = \langle m'_{ij}, \smile'_{ij} \rangle_{0 \leq i,j \leq n}$ where

$$(m'_{ij}, \smile'_{ij}) = \begin{cases} (\infty, <) & \text{if } m_{ij} > k \\ (-k, <) & \text{if } m_{ij} < -k \\ m_{ij}, \smile_{ij} & \text{if } -k \leq m_{ij} \leq k \end{cases}$$

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Creating the zone valuation graph - forward analysis

Initialise the queue Q with a single element $(null, null, l_0)$;
Initialise the graph *zone_graph* with a single node $(l_0, v_0 \uparrow)$ with an ϵ self-loop;
while *Q is not empty* **do**
    Dequeue $(l_{parent}, t, l_{child})$ from Q;
    if $l_{parent} \neq null$ **then**
        foreach *zone Z_{parent} of l_{parent}* **do**
            Add new zones to the zones of l_{child} so that all zones reachable from Z_{parent} are represented;
            Abstract if necessary;
            Update edges from Z_{parent} to the new zones of l_{child} **if** *new zones are created in l_{child} or l_{parent} is null* **then**
                foreach *outgoing transition t' of l_{child}* **do**
                    Enqueue $(l_{child}, t', \texttt{t'.target})$ in Q;
                end
            end
        end
    end
end

Creating the zone valuation graph - backward analysis

```
Set new_zone;
while new_zone do
    Reset new_zone;
    foreach transition t in the timed automaton do
        Split the zones of t.source to be stable with respect to the zones of
        t.target;
        Update edges accordingly;
        if new zones are created in t.source then
            Set new_zone;
        end
    end
end
Generate zone_valuation_graph by applying Fernandez' algorithm to zone_graph;
Return zone_graph;
```

Zone valuation graph example

Figure: Timed automaton. Here, the states are $\{0\}$, the actions are $\{a\}$, and the clocks are $\{X, Y\}$.

Zone valuation graph example

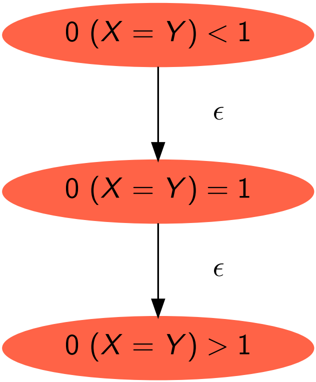


Figure: Zones after one iteration.

Zone valuation graph example

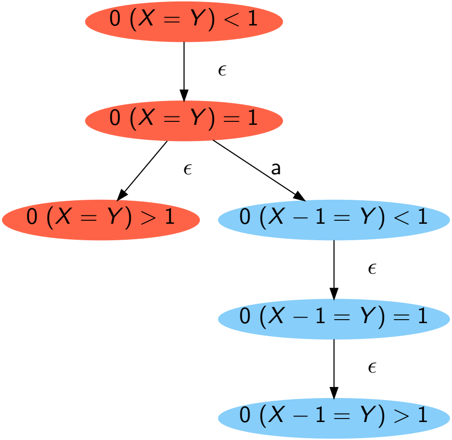


Figure: Zones after two iterations.

Zone valuation graph example

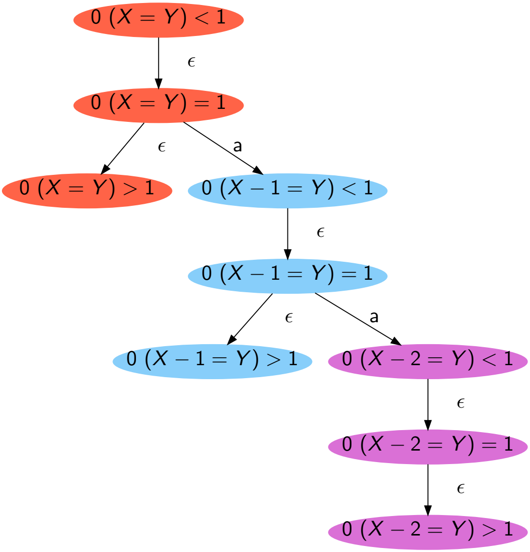


Figure: Zones after three iterations without abstraction.

Zone valuation graph example

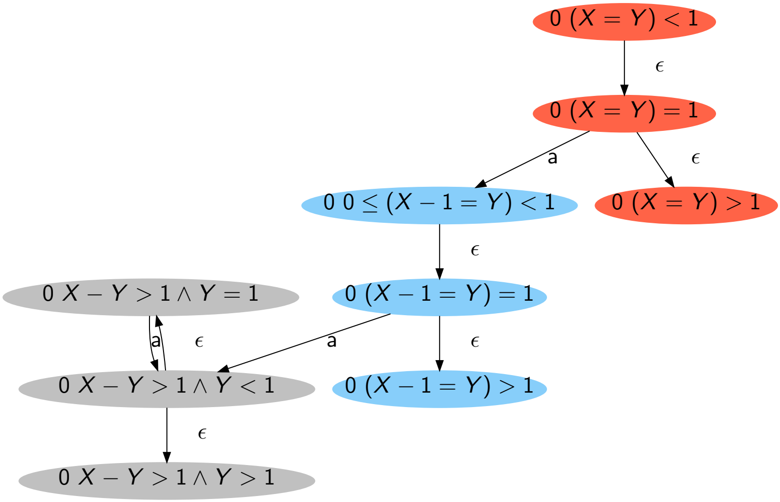


Figure: Zones after three iterations with abstraction.

Zone valuation graph example

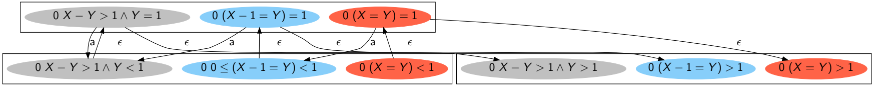


Figure: Zone graph with bisimilarity classes.

Verifying relations on pairs of timed automata.

- Origin for this algorithm - [4]
- General method to compute (ρ, σ)-bisimilarities on two LTS, starting from their initial locations.
- Can be adapted for a certain class of timed and time abstracted relations by using zone valuation graphs.
- For every relation R satisfying this property, functions f_P and f_Q must exist such that the proposition $s_P R s_Q$ resolves to one of these:
  - yes
  - no
  - if and only if

$$\forall (s'_P, L'_Q) \in f_P(s_P) : \exists s'_Q \in L'_Q : s'_P R s'_Q \quad \wedge$$
$$\forall (L'_P, s'_Q) \in f_Q(s_Q) : \exists s'_P \in L'_P : s'_P R s'_Q$$

Verifying relations on pairs of timed automata.

- For STaB, we define f_P and f_Q as

$$
\begin{aligned}
f_P(s_P) =& \{(s'_P, L'_Q) | s_P \xrightarrow{a} s'_P, L_Q = \{s'_Q | s_Q \xrightarrow{a} s'_Q\}\} \\
& \cup \{(s'_P, L'_Q) | s_P \xrightarrow{\epsilon} s'_P, L_Q = \{s'_Q | s_Q \xrightarrow{\epsilon} s'_Q\}\} \\
f_Q(s_Q) =& \{(L'_P, s'_Q) | s_Q \xrightarrow{a} s'_Q, L_P = \{s'_P | s_P \xrightarrow{a} s'_P\}\} \\
& \cup \{(L'_P, s'_Q) | s_Q \xrightarrow{\epsilon} s'_Q, L_P = \{s'_P | s_P \xrightarrow{\epsilon} s'_P\}\}
\end{aligned}
$$

- For TadB, we define f_P and f_Q as

$$
\begin{aligned}
f_P(s_P) =& \{(s'_P, L'_Q) | s_P \xrightarrow{a} s'_P, L_Q = \{s'_Q | s_Q \xrightarrow{\epsilon}\xrightarrow{a} s'_Q\}\} \\
& \cup \{(s'_P, L'_Q) | s_P \xrightarrow{\epsilon} s'_P, L_Q = \{s'_Q | s_Q \xrightarrow{\epsilon} s'_Q\}\} \\
f_Q(s_Q) =& \{(L'_P, s'_Q) | s_Q \xrightarrow{a} s'_Q, L_P = \{s'_P | s_P \xrightarrow{\epsilon}\xrightarrow{a} s'_P\}\} \\
& \cup \{(L'_P, s'_Q) | s_Q \xrightarrow{\epsilon} s'_Q, L_P = \{s'_P | s_P \xrightarrow{\epsilon} s'_P\}\}
\end{aligned}
$$

- For TaoB, we define f_P and f_Q as

$$
\begin{aligned}
f_P(s_P) =& \{(s'_P, L'_Q) | s_P \xrightarrow{a} s'_P, L_Q = \{s'_Q | s_Q \xrightarrow{\epsilon}\xrightarrow{a}\xrightarrow{\epsilon} s'_Q\}\} \\
& \cup \{(s'_P, L'_Q) | s_P \xrightarrow{\epsilon} s'_P, L_Q = \{s'_Q | s_Q \xrightarrow{\epsilon} s'_Q\}\} \\
f_Q(s_Q) =& \{(L'_P, s'_Q) | s_Q \xrightarrow{a} s'_Q, L_P = \{s'_P | s_P \xrightarrow{\epsilon}\xrightarrow{a}\xrightarrow{\epsilon} s'_P\}\} \\
& \cup \{(L'_P, s'_Q) | s_Q \xrightarrow{\epsilon} s'_Q, L_P = \{s'_P | s_P \xrightarrow{\epsilon} s'_P\}\}
\end{aligned}
$$

Verifying relations on pairs of timed automata.

```
begin
  if lookup(yes_table, s_P, s_Q) then
    return true;
  else
    if lookup(yes_table, s_P, s_Q) then
      return false;
    else
      insert(yes_table, s_P, s_Q);
      Set v_P;
      foreach (s'_P, L'_Q) in f_P(s_P) do
        Reset v_P;
        foreach s'_Q in L'_Q do
          if CheckStatesRelation(P, Q, s'_P, s'_Q, yes_table, no_table) then
            Set v_P;
          end
        end
      end
      Set v_Q;
      foreach (s'_Q, L'_P) in f_Q(s_Q) do
        Reset v_Q;
        foreach s'_P in L'_P do
          if CheckStatesRelation(P, Q, s'_P, s'_Q, yes_table, no_table) then
            Set v_Q;
          end
        end
      end
      if v_P ∧ v_Q then
        return true;
      else
        remove(yes_table, s_P, s_Q);
        insert(yes_table, s_P, s_Q);
        return false;
      end
    end
  end
end
```

Procedure CheckStatesRelation(P, Q, s_P, s_Q, *yes_table*, *no_table*)

Verifying relations on pairs of timed automata.

begin

Create zone valuation graphs G_P, G_Q of T_P, T_Q;
Find the zone s_P in G_P which contains the initial state of T_P;
Find the zone s_Q in G_Q which contains the initial state of T_Q;
Initialise *yes_table* and *no_table* to empty tables;
return `CheckStatesRelation`(G_P, G_Q, s_P, s_Q, *yes_table*, *no_table*);

end

Procedure CheckAutomataRelation(T_P, T_Q)

Verifying relations: example

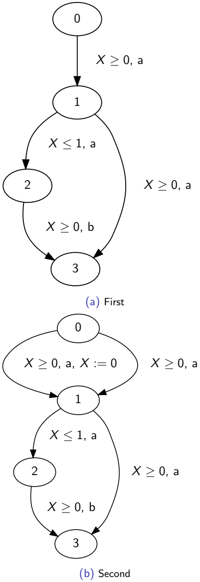
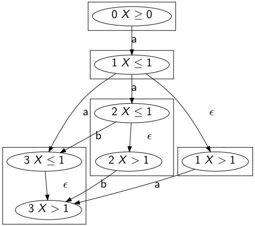
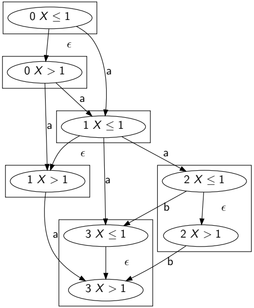


Figure: Timed automata.

Verifying relations: example



(a) First



(b) Second

Figure: Zone valuation graphs.

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