Tools and Algorithms for Deciding Relations on Timed Automata

B Tech project, supervised by S Arun-Kumar, verification group

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Outline

Automata without timing and relations on them

Timed automata and relations on them

Algorithms

Labeled transition systems

Definition

Labelled Transition System: A labelled transition system (LTS) [1] is an automaton which is described by

- S, a set of *states*
- Act, a set of *actions*
- $\rightarrow \subseteq S \times Act \times S$, a *transition relation*.
- optionally, $I \subseteq S$,a set of initial states. If there is exactly one initial state, then the LTS is said to be *rooted*.

Relations on LTS I

Definition

Strong bisimulation:A binary relation R on the states of an LTS is a strong bisimulation if and only if, for all $(s_1, s_2) \in R$ and $a \in Act$.
$\forall s_1'(s_1 \xrightarrow{a} s_1' \Rightarrow \exists s_2'.(s_2 \xrightarrow{a} s_2' \wedge (s_1', s_2') \in R))\wedge$
$\forall s_2'(s_2 \xrightarrow{a} s_2' \Rightarrow \exists s_1'.(s_1 \xrightarrow{a} s_1' \wedge (s_1', s_2') \in R))$

Definition

It can be shown that the union of all strong bisimulations over the set of states is a strong bisimulation. This binary relation is called *strong bisimilarity*, denoted by $\sim$.

Outline

Timed Automata

Definition

Timed Automaton: A timed automaton [2] over a finite set of clocks C and a finite set of actions Act is a 4-tuple (L, l_0, E, I).

- L is a finite set of locations.
- l_0 is the initial location.
- $E \subseteq L \times B(C) \times Act \times 2^C \times L$ is a finite set of edges.
- $I : L \rightarrow B(C)$ assigns invariants to each edge location.
- $B(C)$ is the set of clock constraints over C.

Outline

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Algorithms

Creating the zone valuation graph

Initialise the queue Q with a single element $(\mathit{null}, \mathit{null}, l_0)$;
Initialise the graph *zone_graph* with a single node $(l_0, v_0 \uparrow)$ with an ϵ self-loop;
while *Q is not empty* **do**
  Dequeue $(l_{parent}, t, l_{child})$ from Q;
  if $l_{parent} \neq \mathit{null}$ **then**
    foreach *zone Z_{parent} of l_{parent}* **do**
      Add new zones to the zones of l_{child} so that all zones reachable from Z_{parent} are represented;
      Abstract if necessary;
      Update edges from Z_{parent} to the new zones of l_{child} **if** *new zones are created in l_{child} or l_{parent} is null* **then**
        foreach *outgoing transition t' of l_{child}* **do**
          Enqueue $(l_{child}, t',$ `t'.target`$)$ in Q;
        end
      end
    end
  end

Zone valuation graph example

Figure: Timed automaton. Here, the states are $\{0\}$, the actions are $\{a\}$, and the clocks are $\{X, Y\}$.

Zone valuation graph example

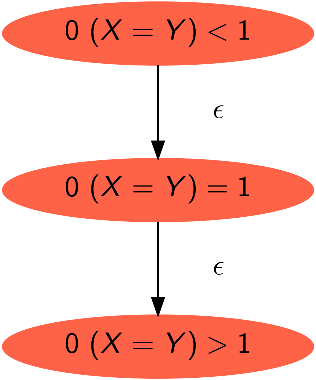

Figure: Zones after one iteration.

Zone valuation graph example

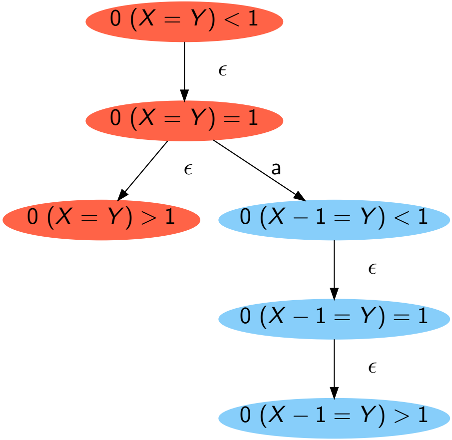

Figure: Zones after two iterations.

Zone valuation graph example

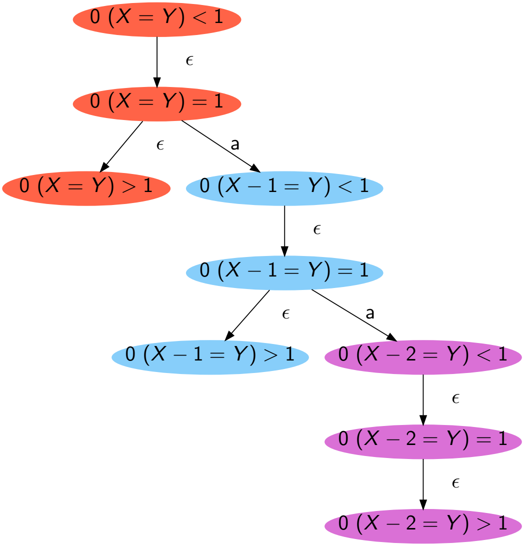

Figure: Zones after three iterations without abstraction.

Zone valuation graph example

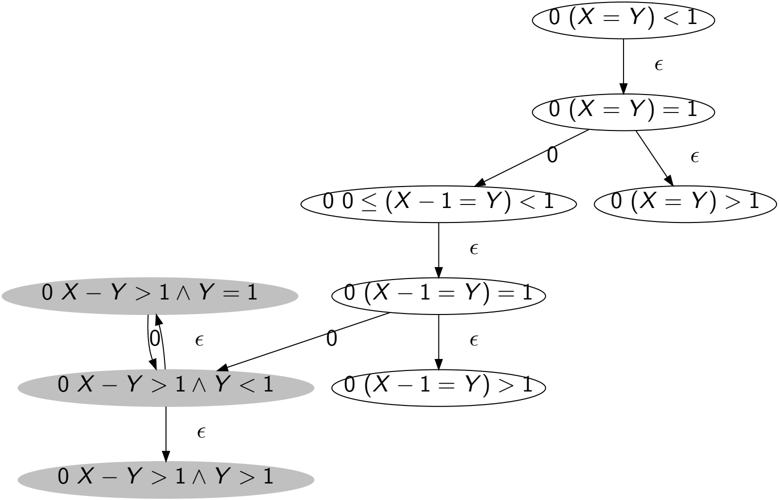

Figure: Zones after three iterations with abstraction.

References I

References II

Keller, R.M.:
Formal verification of parallel programs.
Commun. ACM **19**(7) (July 1976) 371–384

Alur, R., Dill, D.L.:
A theory of timed automata.
Theoretical Computer Science **126** (1994) 183–235