Statistics with Spa  ows

Lecture 10

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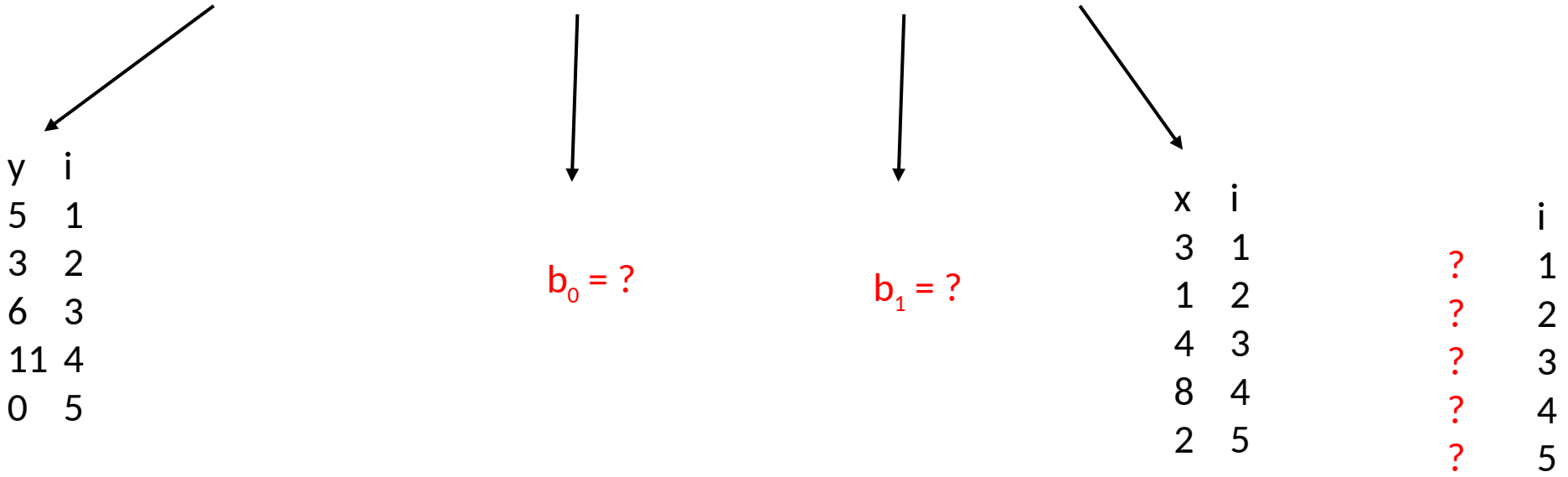
Outline

- Hypothesis testing in linear models
- Interpretation (a bit)
- Standardizing
- Reporting

Hypothesis testing in linear models

- What do we actually test?

$$y_i = b_0 + b_1 x_i + \varepsilon_i$$



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> y<-c(5,3,6,11,0)
> x<-c(3,1,4,8,2)
> mod<-lm(y~x)
> summary(mod)

Call:
lm(formula = y ~ x)

Residuals:
      1       2       3       4       5
 0.8219  1.5616  0.4521 -0.0274 -2.8082

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.06849    1.55075   0.044   0.9675
x            1.36986    0.35765   3.830   0.0314 *
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Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.933 on 3 degrees of freedom
Multiple R-squared:  0.8302,    Adjusted R-squared:  0.7736
F-statistic: 14.67 on 1 and 3 DF,  p-value: 0.03136
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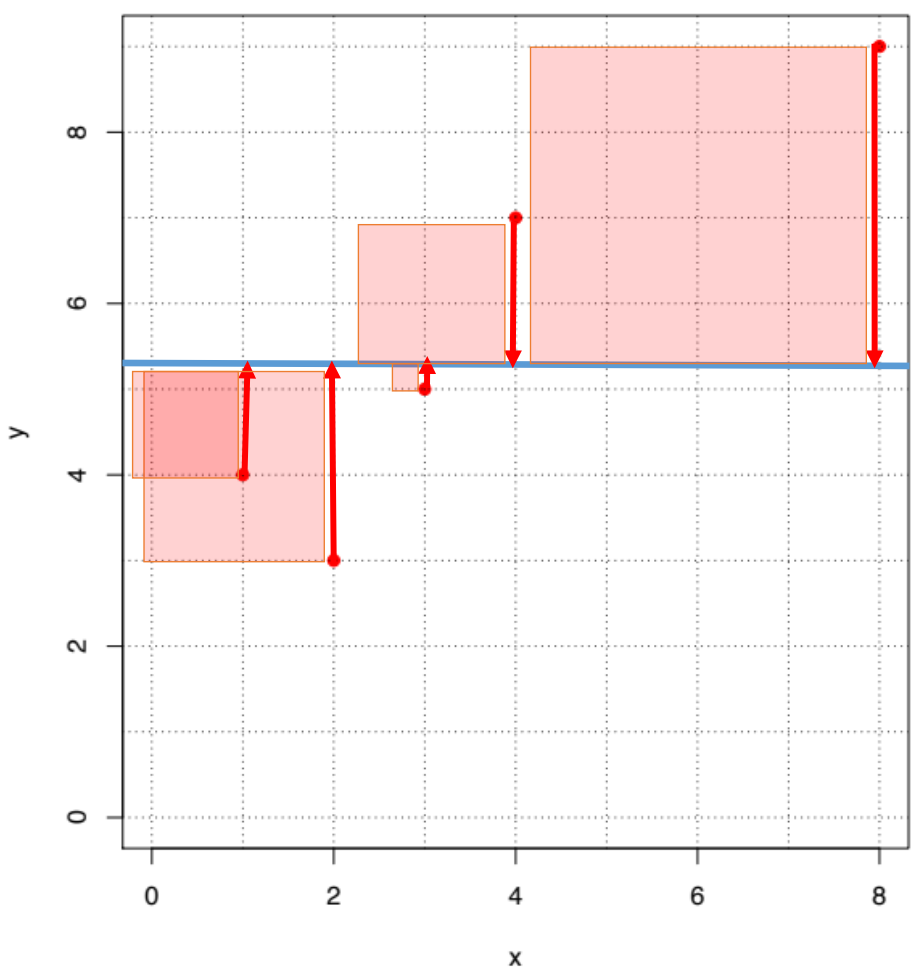

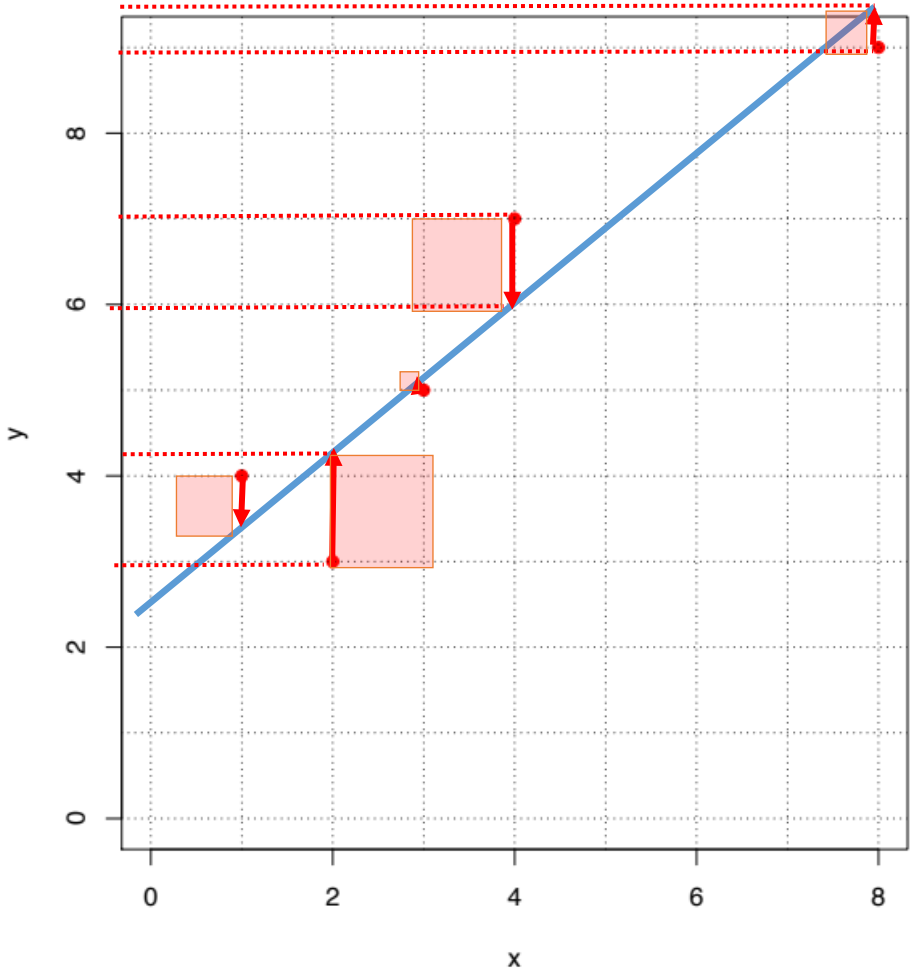

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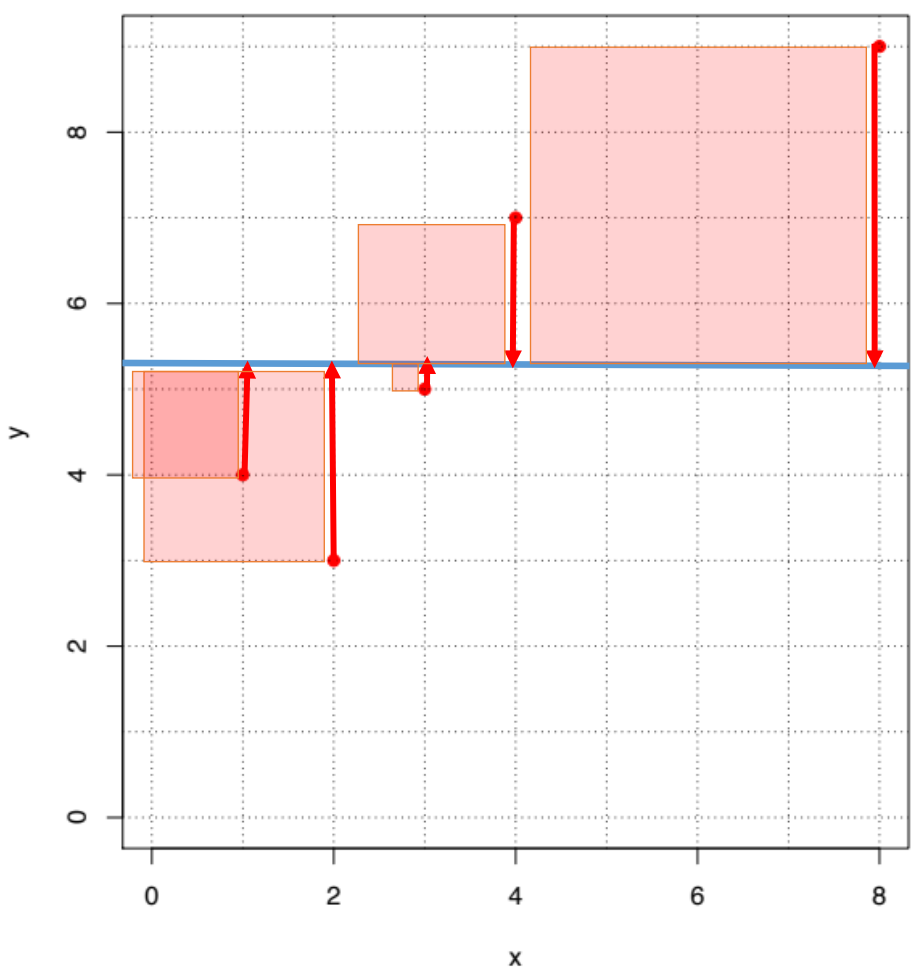

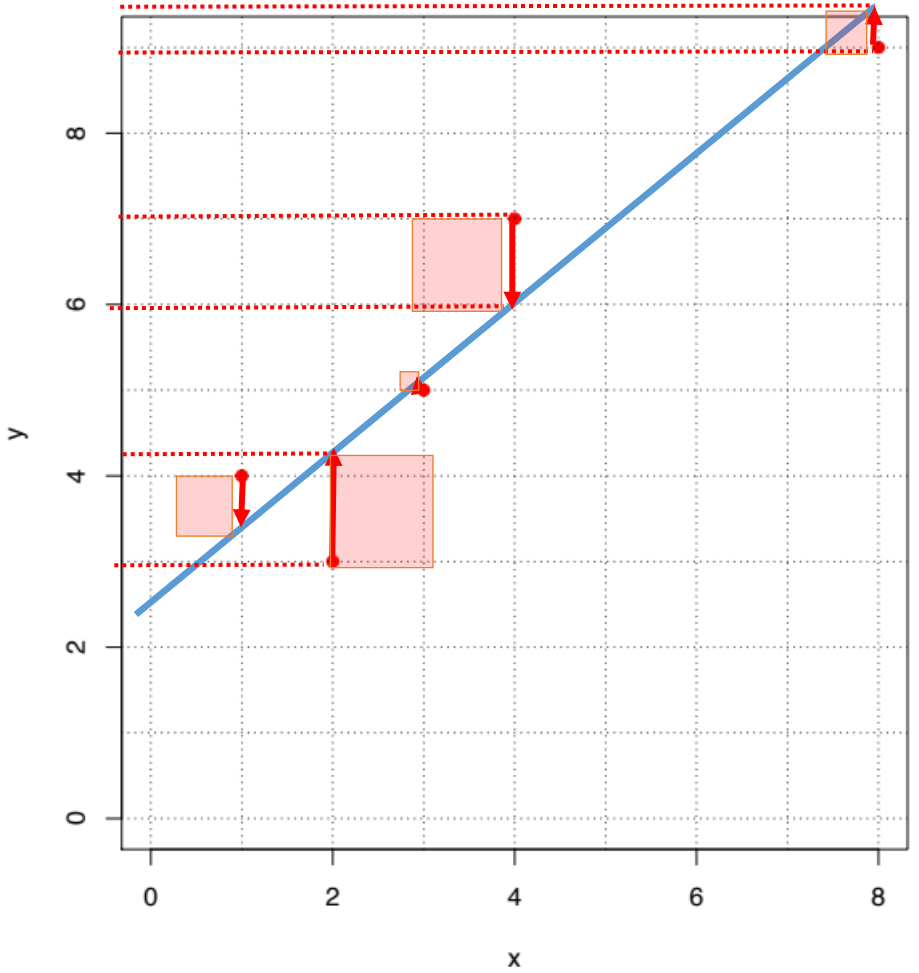

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→ Test if slope (b_1) estimate is different from 0

→ T-test!

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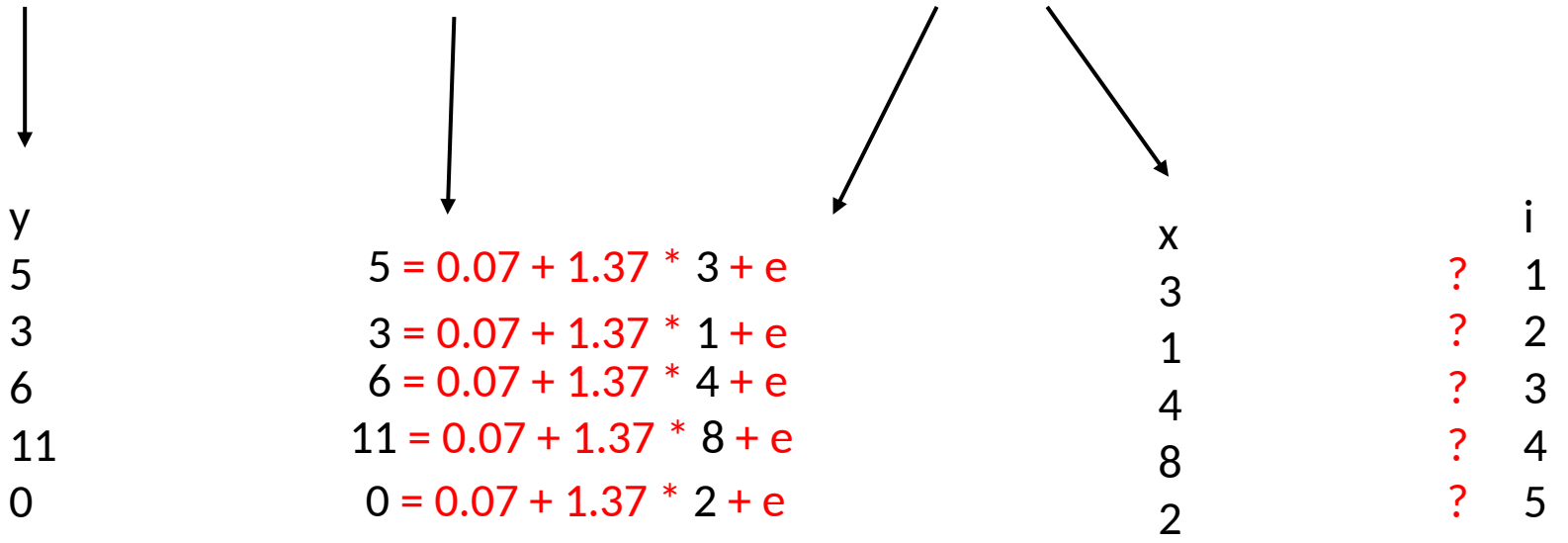
$$y_i = 0.07 + 1.37\, x_i + \varepsilon_i$$



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y		x		i
5	5 = 0.07 + 1.37 * 3 -0.82	3	?	1
3	3 = 0.07 + 1.37 * 1 + e	1	?	2
6	6 = 0.07 + 1.37 * 4 + e	4	?	3
11	11 = 0.07 + 1.37 * 8 + e	8	?	4
0	0 = 0.07 + 1.37 * 2 + e	2	?	5

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```
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          1           2           3           4           5
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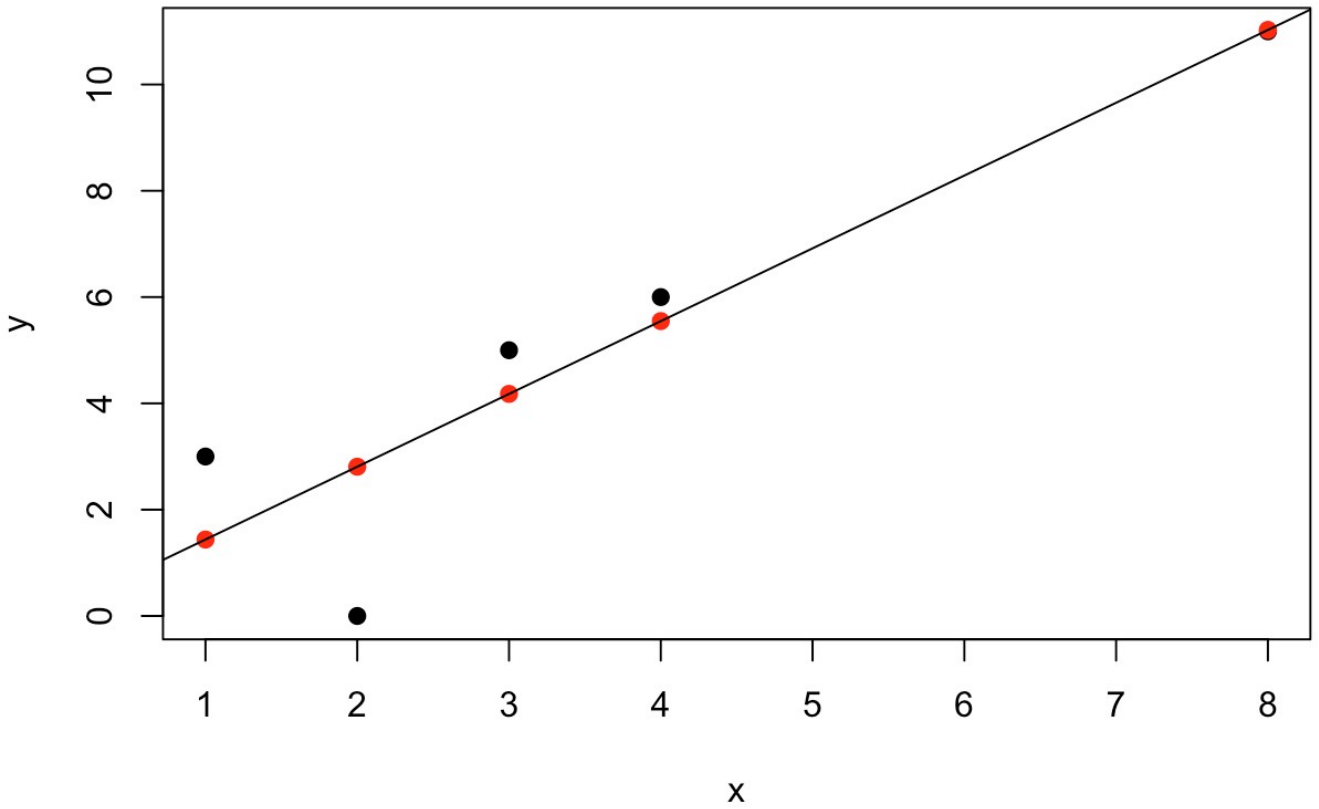
11	11 = 0.07 + 1.37 * 8 – 0.03	8	?	4
0	0 = 0.07 + 1.37 * 2 – 2.81	2	?	5



y	x	Y predicted
5	3	4.18
3	1	1.44
6	4	5.55
11	8	11.03
0	2	2.81

```
> predict(mod)
       1        2        3        4        5
4.178082 1.438356 5.547945 11.027397 2.808219
```

Hypothesis testing in linear models

- Measure precision of estimates:

$$y_i \qquad i + \varepsilon_i$$

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Tests null hypotheses that

$b_0 = 0$

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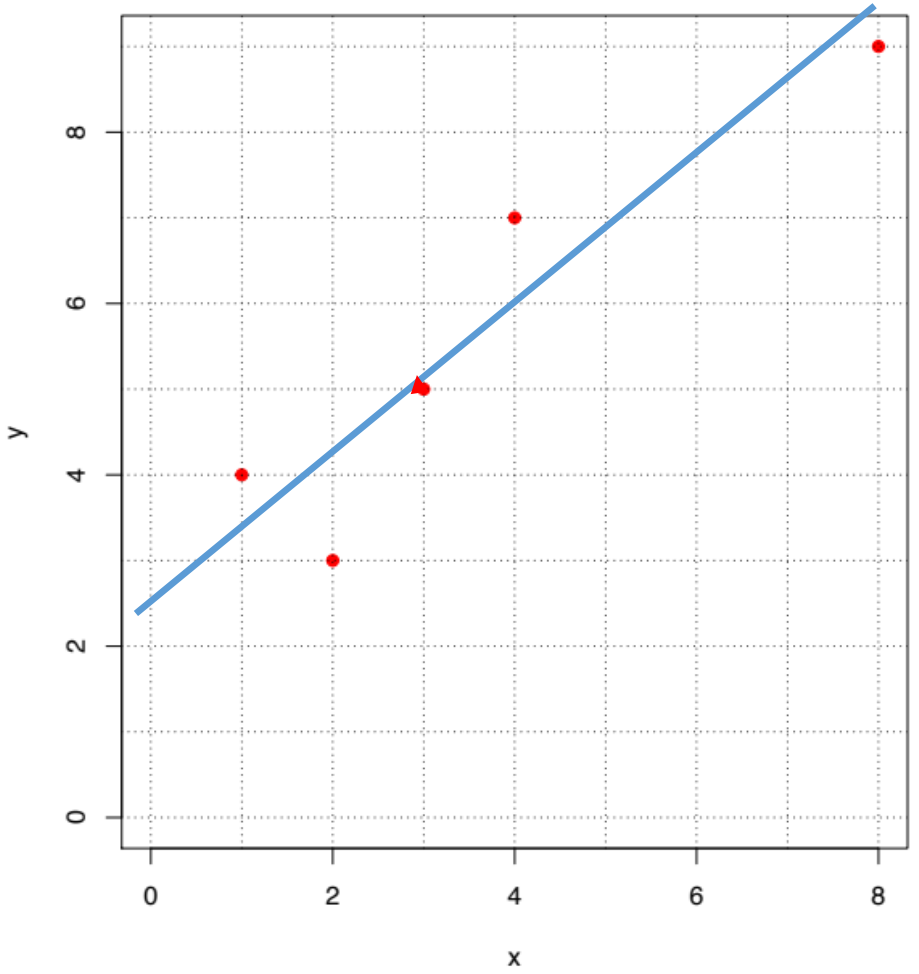
What does it mean if the null hypothesis is rejected?



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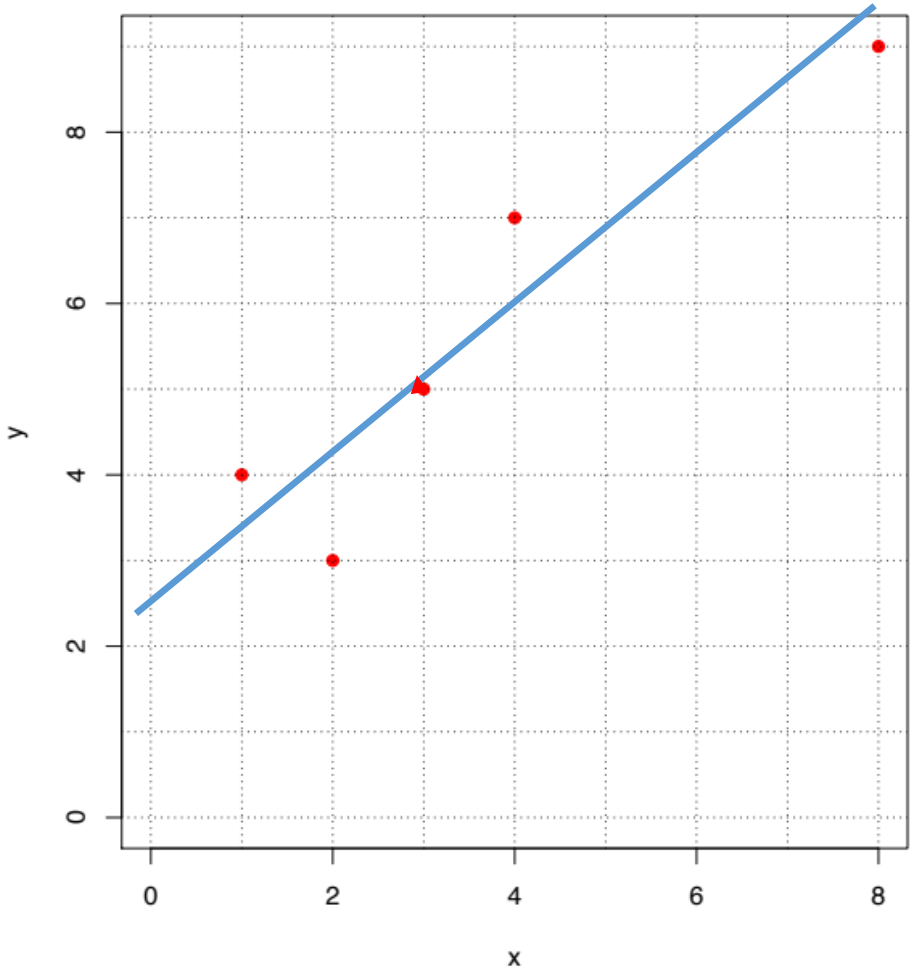
What does it mean if the null hypothesis is rejected?

Usually not much for b_0

The slope is b_1, this gives information about the relationship between y and x.



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The slope is 5.8
For each mm longer tarsus, a sparrow weights 1.18g more.

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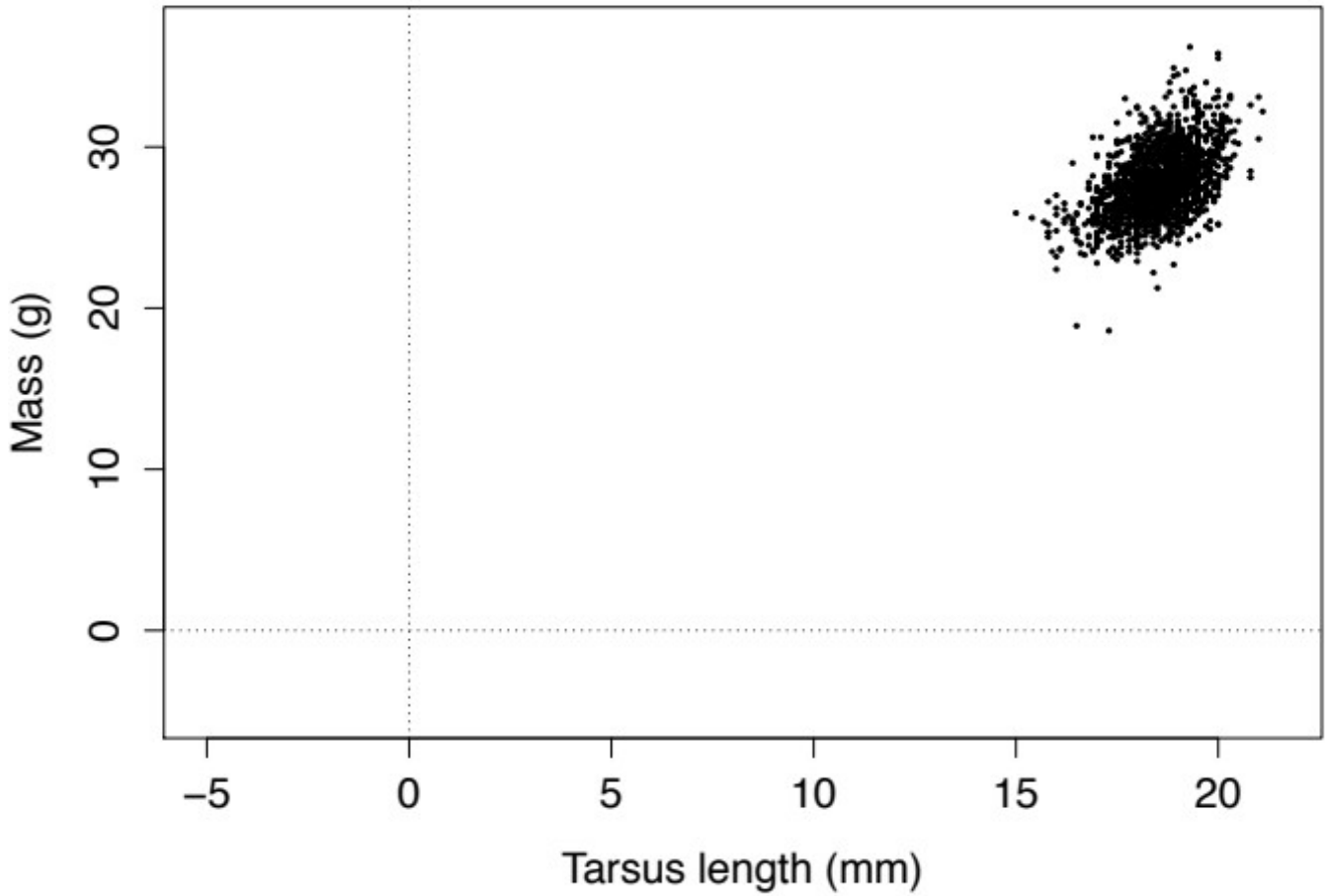

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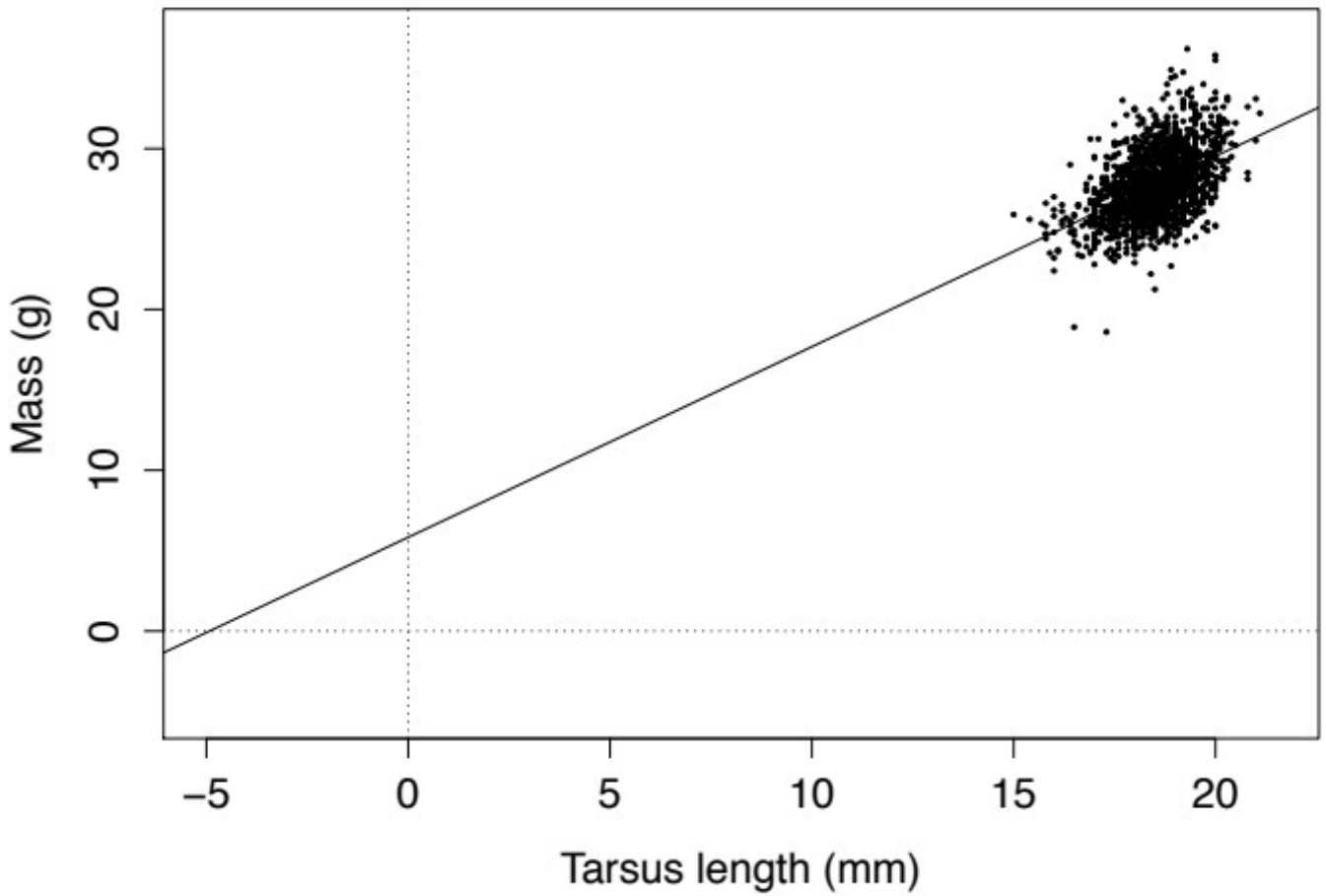
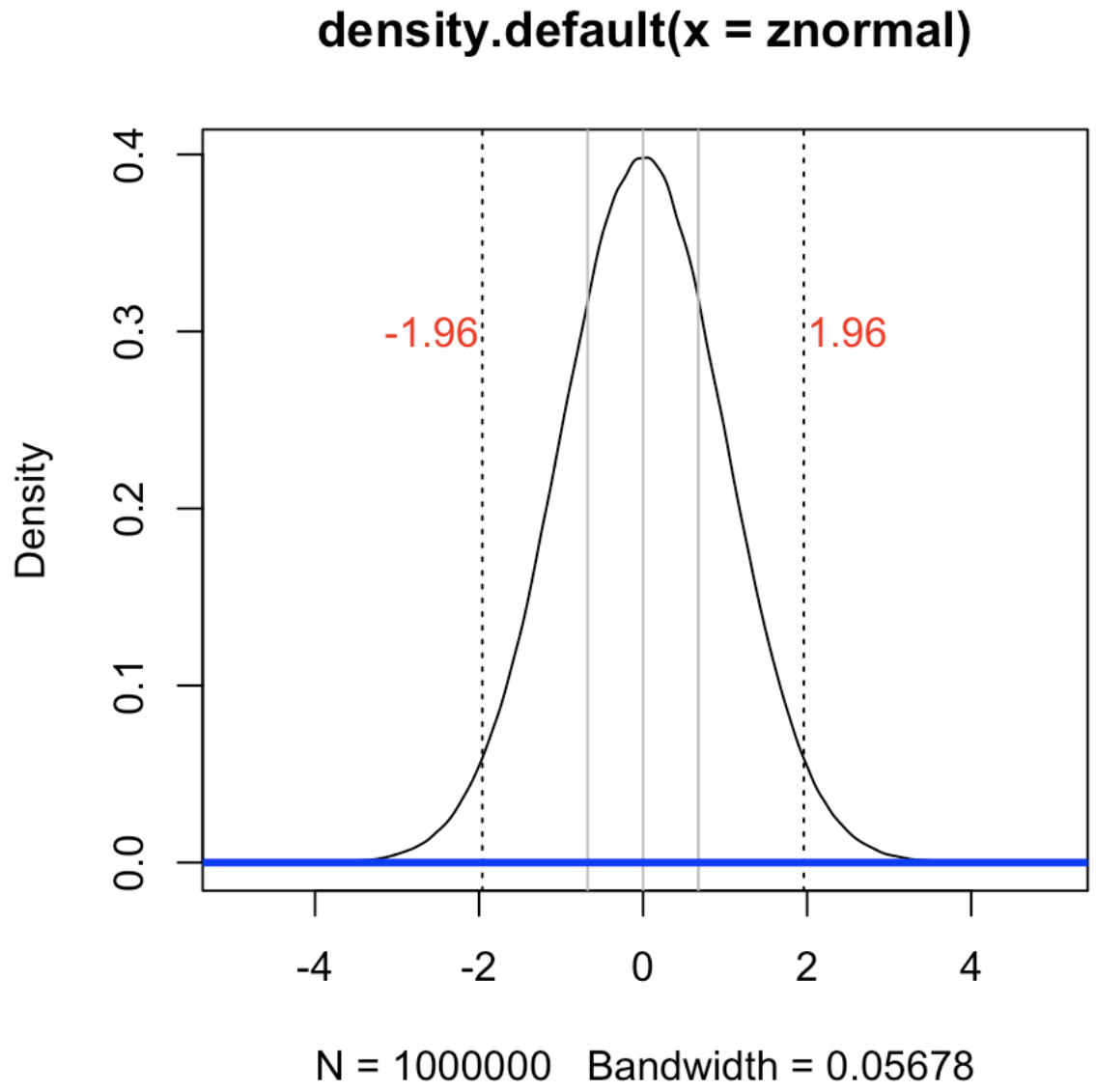


Why standardize da

- z-scores:
- Normal distributed
- Mean of 0
- Sd of 1



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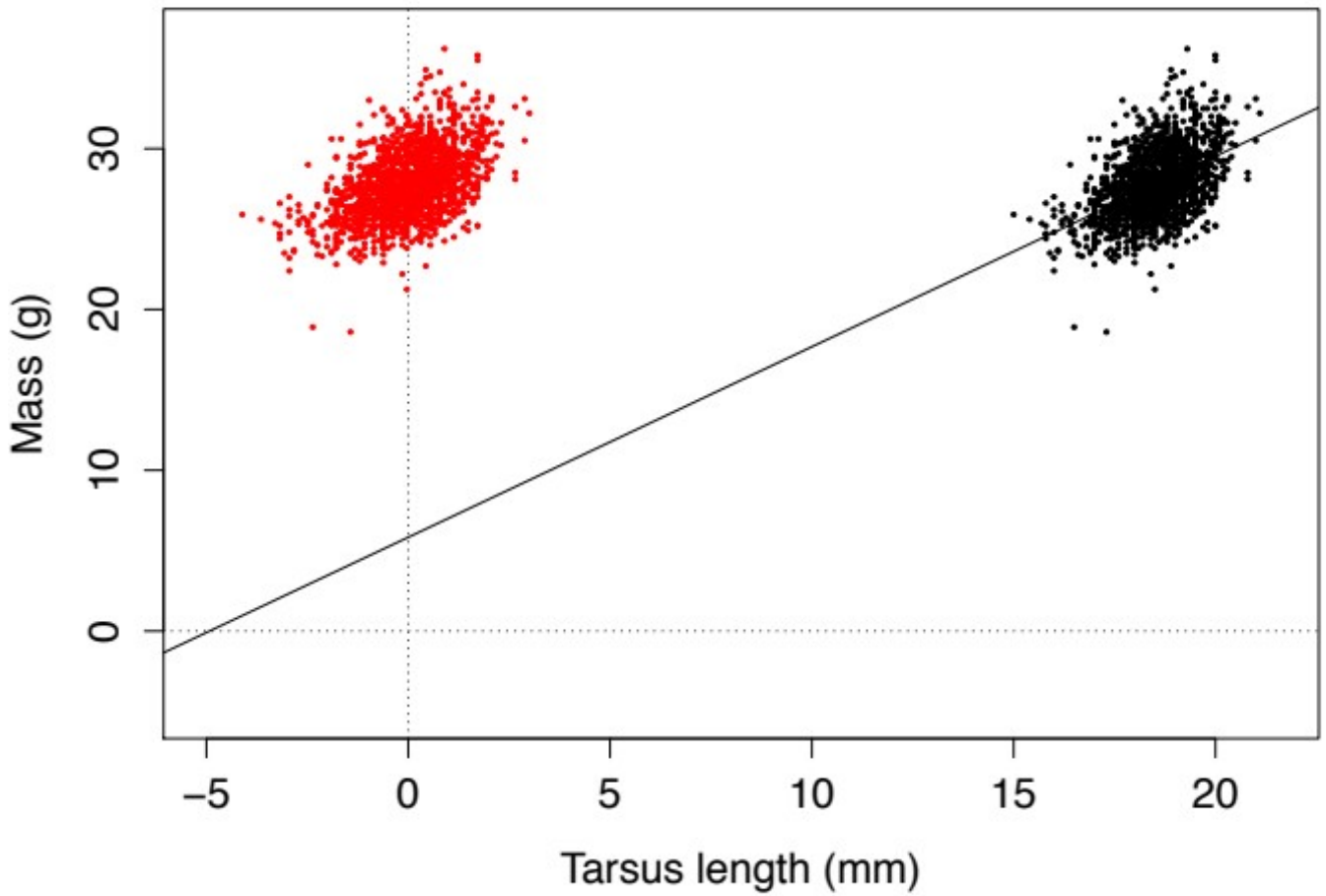

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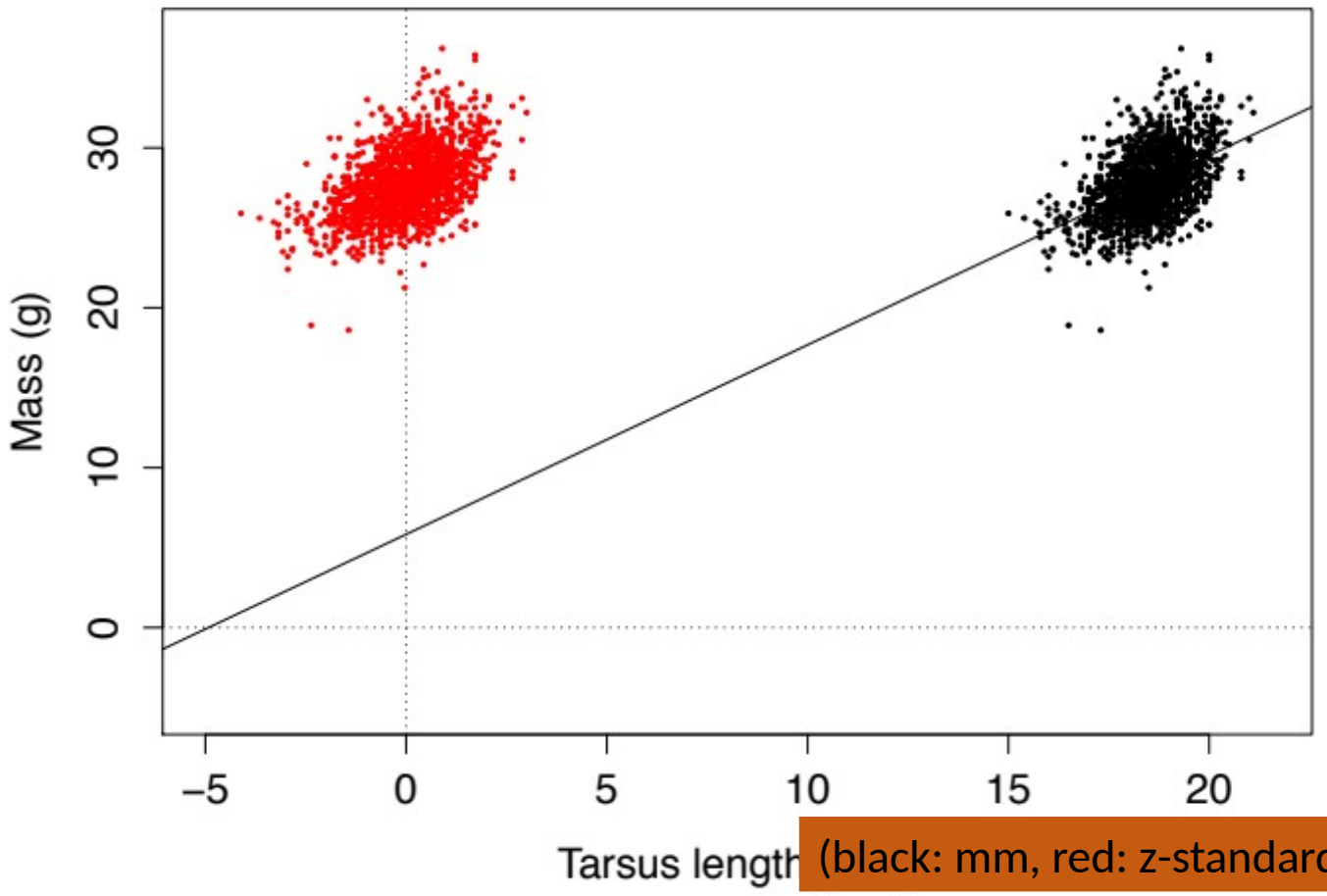

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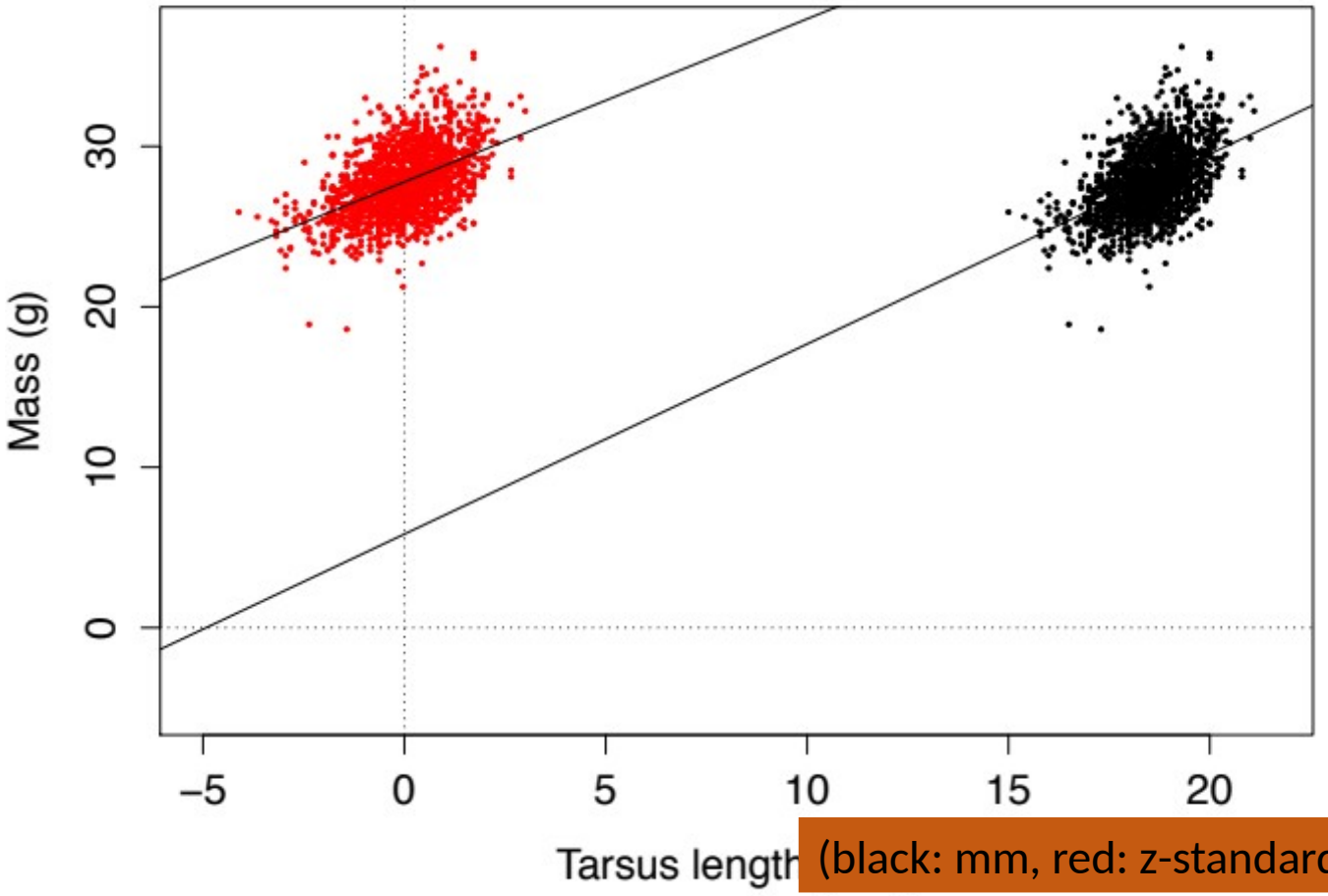

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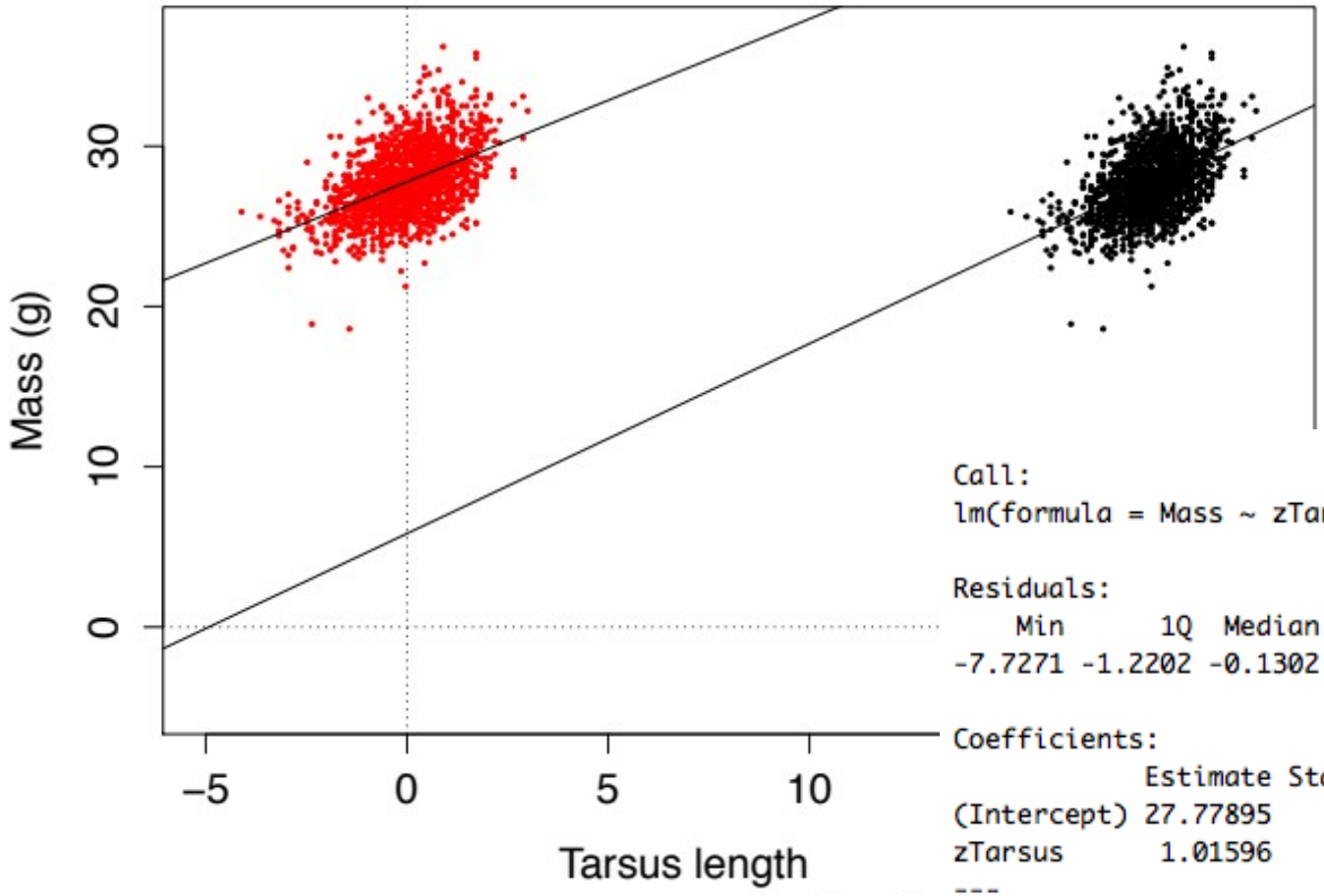

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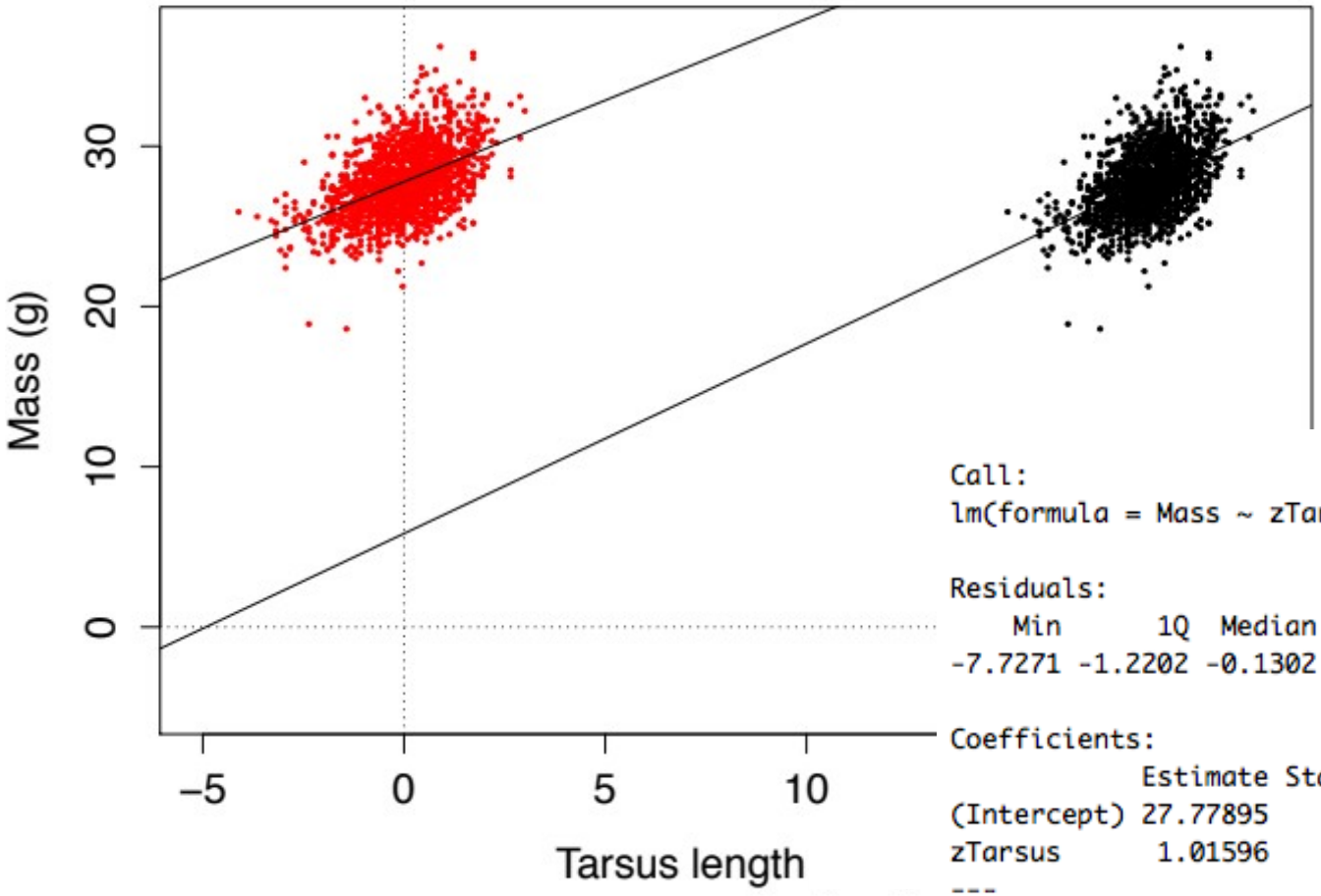

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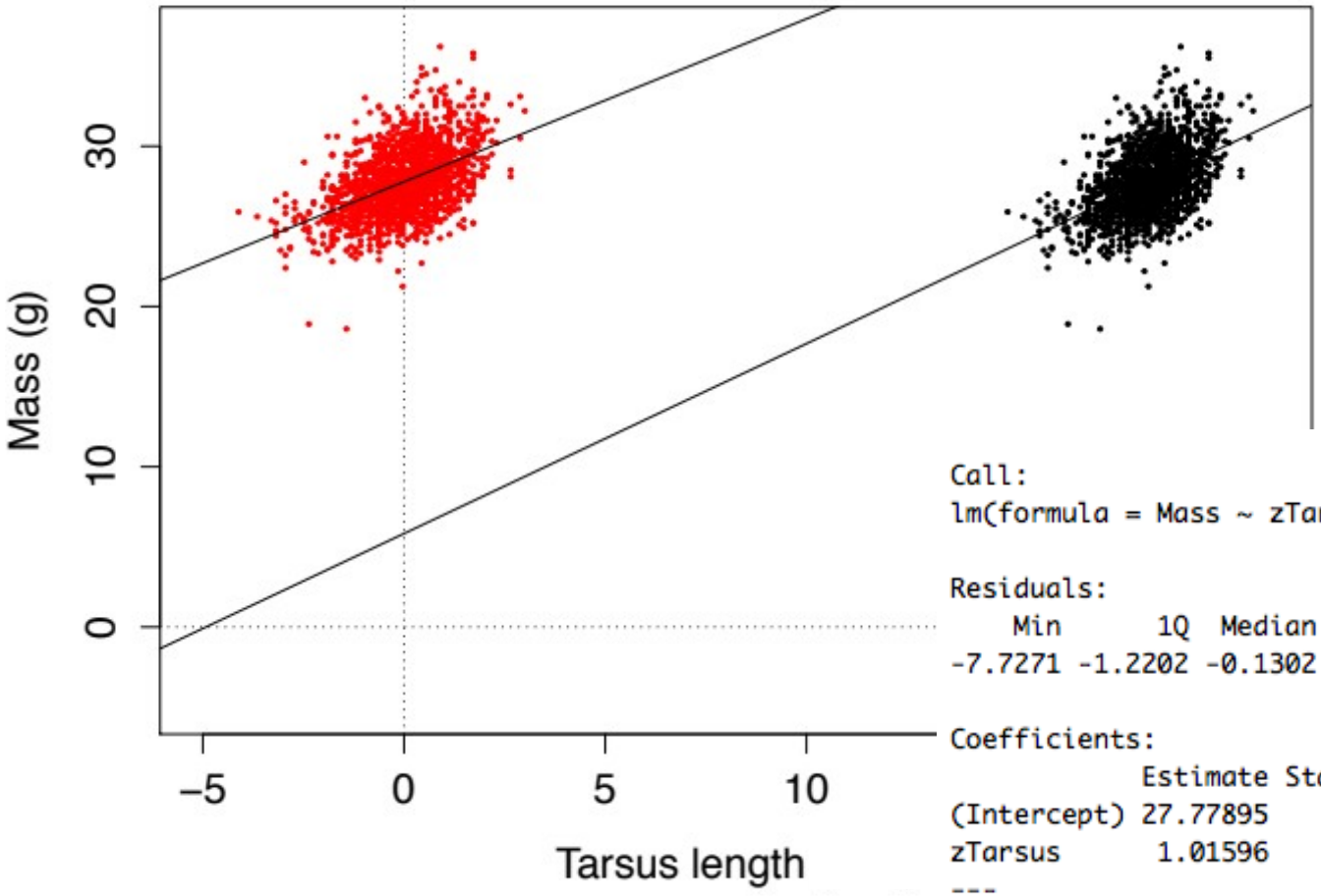

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Interpretation of statistics – take home

- Always consider units!
- Standardize to make intercept meaningful
- T-tests test for null hypothesis that parameter estimates equal –

- Always think of the biological meaning in units!

How to report?

Methods:

To test whether heavier birds also had longer tarsi, I used a linear model, where body mass (g) was the response variable, and tarsus length (mm) the explanatory variable.

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Table 1: Results from a linear model explaining body mass of Lundy sparrows with tarsus length. Tarsus length was z-standardized. N = 1646.

Variable	*b*	*SE*	*t*	*p*
Intercept	27.78	0.05	611.94	<0.001
Tarsus	1.02	0.05	22.37	<0.001

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- Headers of columns center aligned!

How to report?

Results:

I used data from 1646 observations. The sparrows weighed on average 27.78 g (SD 2.10, range: 18.60–36.20). The tarsi of the sparrows were on average 18.5mm long (SD 0.86, range: 15.00–21.10). I found a positive, statistically significant association between mass and tarsus (Table 1). An increase in 1SD Tarsus length meant an increase of 1.02g in body mass.

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How to report – take home

- Methods:
- Always describe *all analyses you present in results. Be precise and specific. Describe what's the response variable. Say what is the explanatory variable, and say WHY you used those. Give the units. Say when you standardize, and WHY.
- Justify, justify, justify.

How to report – take home

- Results:
- Start with describing the dataset.
- Sample size, mean, range, missing values ect.
- Explain results of each analysis
- Make nice tables and think before copy/pasting values from R!
- Legends need be self-explanatory!