Statistics with Sparrows - 07

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HO 07d

Simple linear functions

Linear functions come in the form:

$$y = a + bx$$

where b and m are single numbers (the intercept a, and slope b, respectively), and y and x are vectors. The latter means lists of multiple numbers, like in r. It's important the both, y and x have the same number of elements - both vectors need to be of the same length. To indicate that x and y aren't single numbers we sometimes use indices:

$$y_i$$

$$x_i$$

This is shorthand for: There are many y's - from y_1 to y_n, where n is the length of the vector and we want the ith element. The nice thing about this notation is it is also how R keeps track of its vectors, - but it uses double brackets [[]] to indicate an element:

```
rm(list=ls())
x<-seq(from = -5, to = 5, by = 1)
x

## [1] -5 -4 -3 -2 -1  0  1  2  3  4  5

x[[1]]

## [1] -5

x[[2]]

## [1] -4

x[[9]]

## [1] 3

x[[length(x)]]

## [1] 5
```

Neat, huh? We can even use a function inside the double brackets. This can be useful when we don't know the absolute length, or when it changes dynamically within our code. We can even use an i:

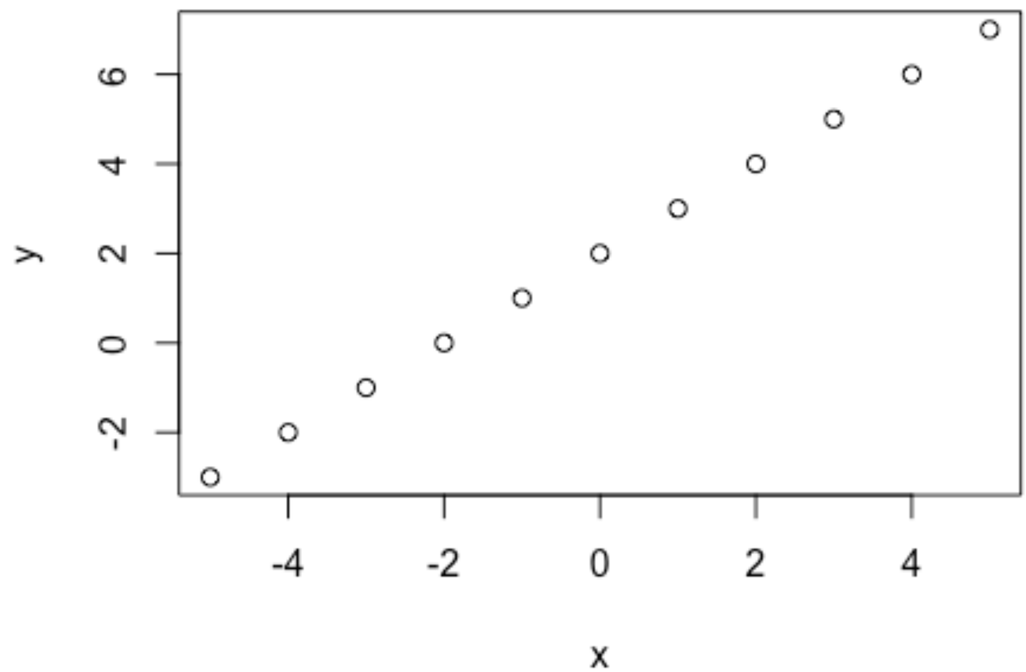
```
i<-1
x[[i]]

## [1] -5

i<- seq(0,10,1)
i

##  [1]  0  1  2  3  4  5  6  7  8  9 10

x[[i[[2]]]]

## [1] -5
```

Brain freeze much? The last example wasn't particularly applicable, but it will be helpful for your understanding if you can wrap your head around it. But we digress. Back to:

$$y \sim a + bx$$

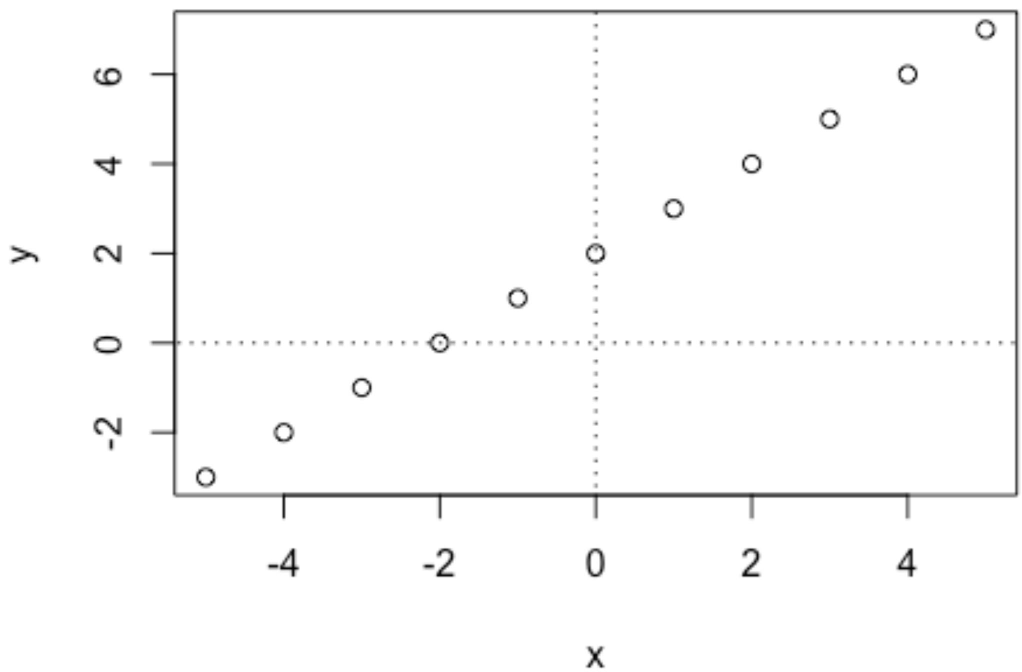
Now, we can come up with any sort of line. Say we want a line with an intercept of 2 (a=2 - meaning the line intercepts the y axis at 2), and a slope of 1 (b=1, meaning the line increases 1 y for each x), we can r to calculate the y's for us:

```
a<-2
b<-1
y<-a+b*x
plot(x,y)
```



This looks good, doesn't it? We'll add the cartesian axes so we can see the intercept better:

```
plot(x,y)
segments(0,-10,0,10, lty=3)
segments(-10,0,10,0,lty=3)
```

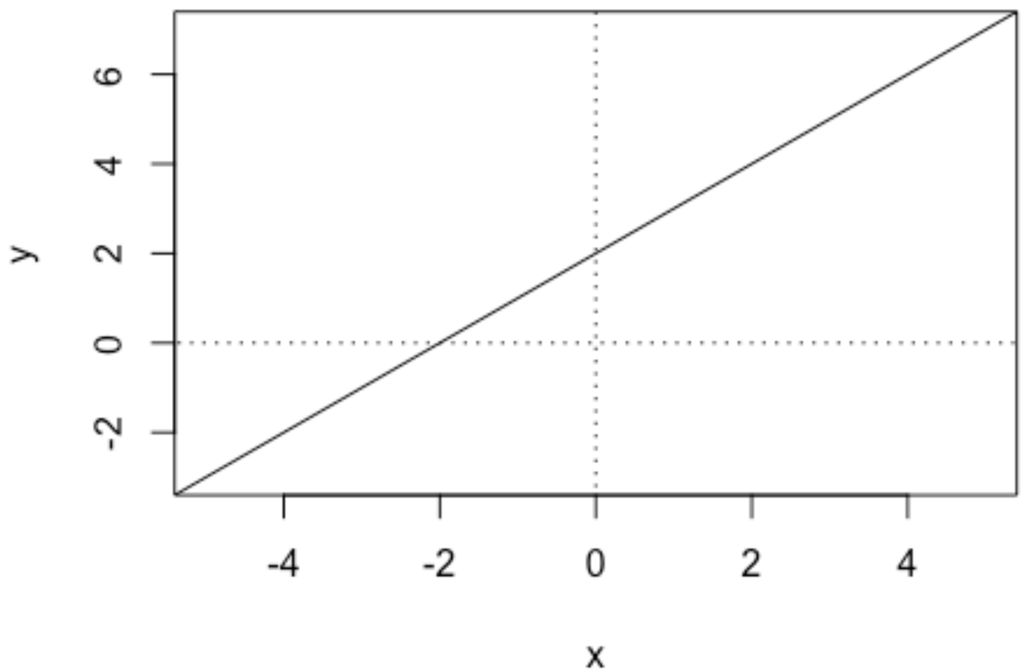


Better. But it's not a line, it's a number of dots. That's because x, and therefore also y, are not continuous - remember we told r to make the x-vector to only hold 11 data points. R plots lines using linear functions using `abline()`:

```
?abline
```

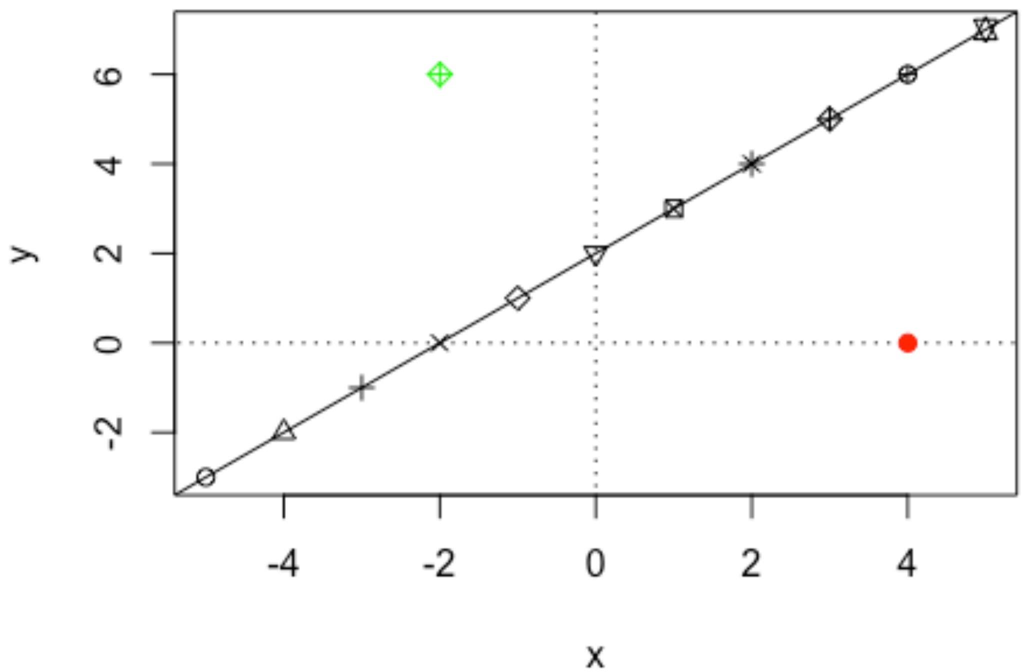
Read through the help text. It clearly tells us how to specify the intercept and the slope. Before we give it a go, we need to notice a small detail under the description of abline. It says *This function* **adds** *one or more straight lines through the current plot.* The **adds** is the key word here - it adds a line to an existing plot. So we first have to make a plot, then add to it. If we do not want the dots, we have a number of options. Today, we plot it but tell R to plot the dots in white, so we can't see them (it's a bit like cheating, but in coding ends often justify the means).

```
plot(x,y, col="white")
segments(0,-10,0,10, lty=3)
segments(-10,0,10,0,lty=3)
abline(a = 2, b=1)
```



Neat, huh? So we only need two values (intercept and slope) to plot any line. It's rather nice. Now you can plot points into this cartesian coordinate system anywhere you want, using the function `points`. Check it out in R:

```
plot(x,y, col="white")
segments(0,-10,0,10, lty=3)
segments(-10,0,10,0,lty=3)
abline(a = 2, b=1)

points(4,0, col="red", pch=19)
points(-2,6, col="green", pch=9)
points(x,y, pch=c(1,2,3,4,5,6,7,8,9,10,11))
```

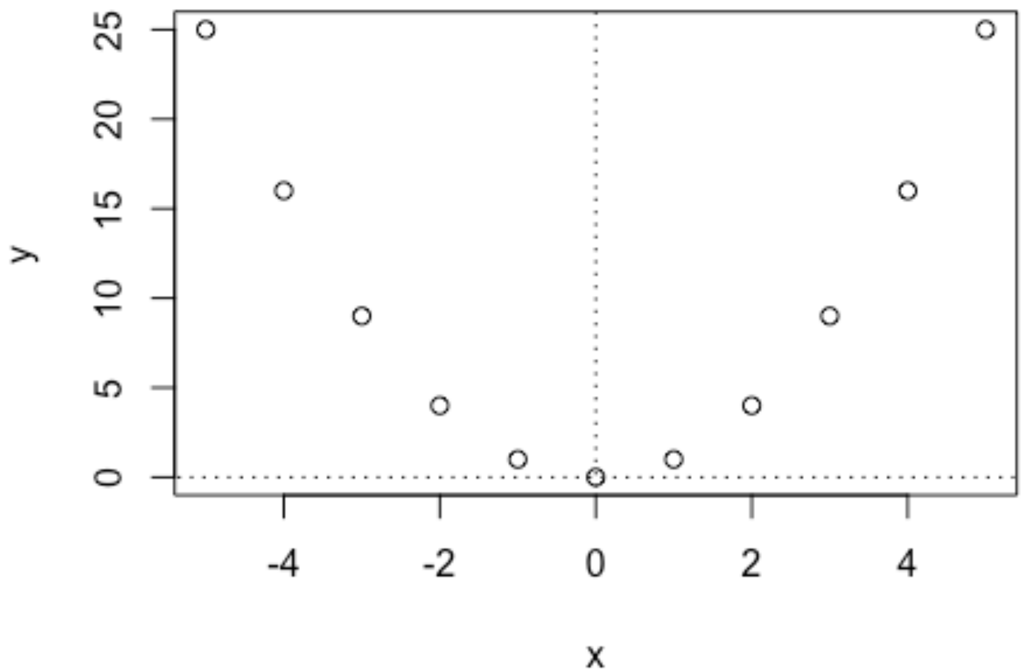


You can even plot a quadratic function. That's where x is squared. If you recall the quadratic function from school, it might have looked something like:

$$y = x^2$$

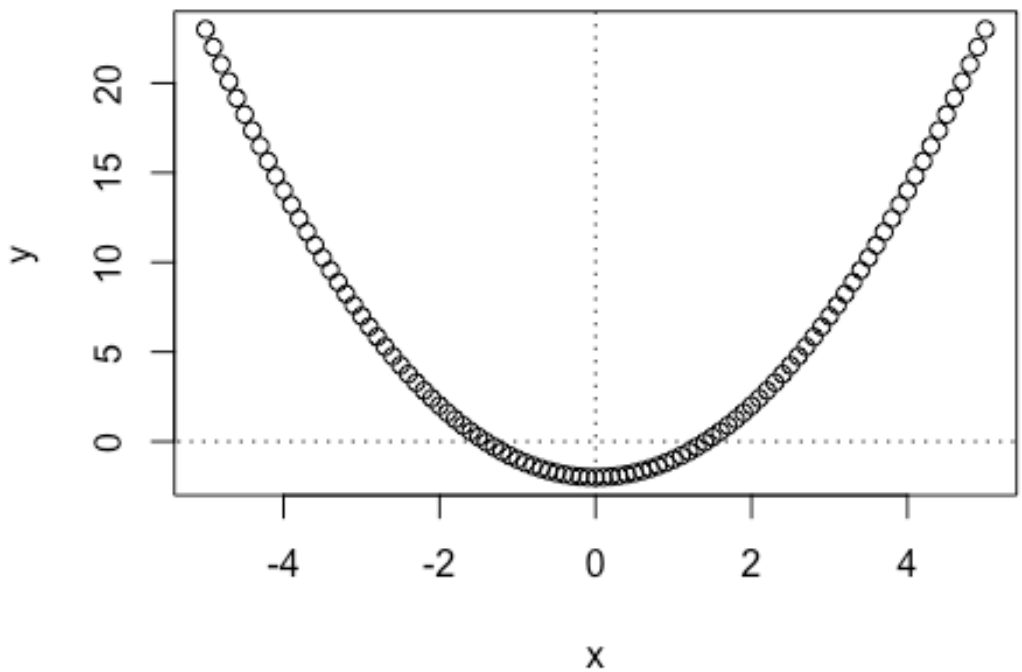
Let's see

```
y<-x^2
plot(x,y)
segments(0,-30,0,30, lty=3)
segments(-30,0,30,0,lty=3)
```



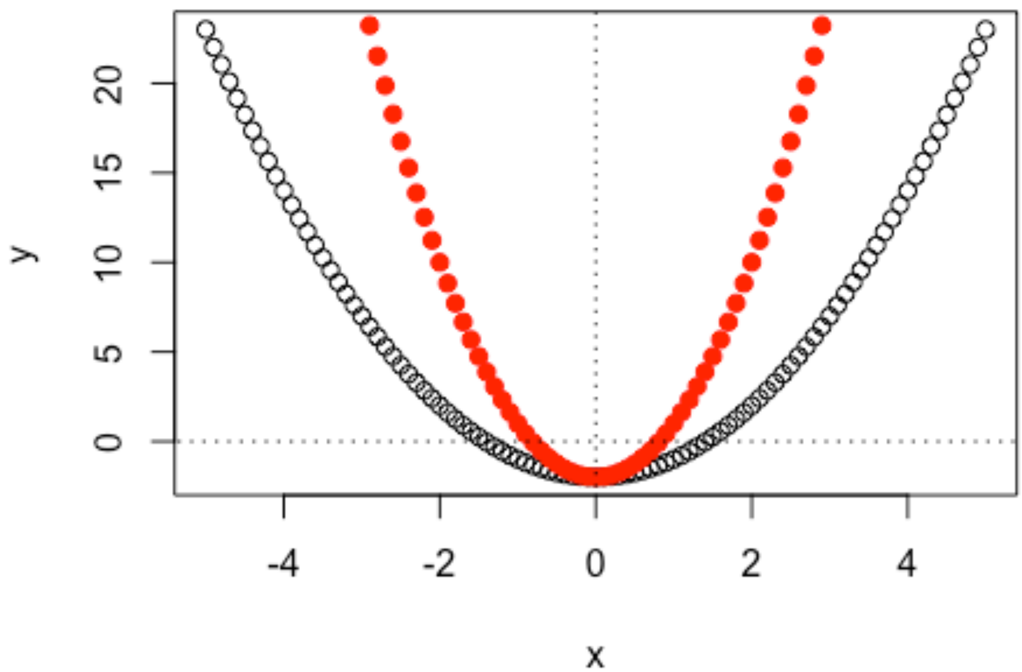
This is a quadratic curve, but it's still called a linear function. It goes through the origin - meaning the intercept is zero. Ok, now we can add an intercept *a* to the function. Let's set it to minus 2! We also make more x points (like, many!) so it is easier to see:

```
x<-seq(from = -5, to = 5, by = 0.1)
a<- -2
y<-a+x^2
plot(x,y)
segments(0,-30,0,30, lty=3)
segments(-30,0,30,0,lty=3)
```



What happens when we give the quadratic part a slope b? To see better what happens, we'll add it to the previous plot (we have to replot it, though).

```
plot(x,y)
a<- -2
b<-3
y<-a+b*x^2
points(x,y, pch=19, col="red")
segments(0,-30,0,30, lty=3)
segments(-30,0,30,0,lty=3)
```

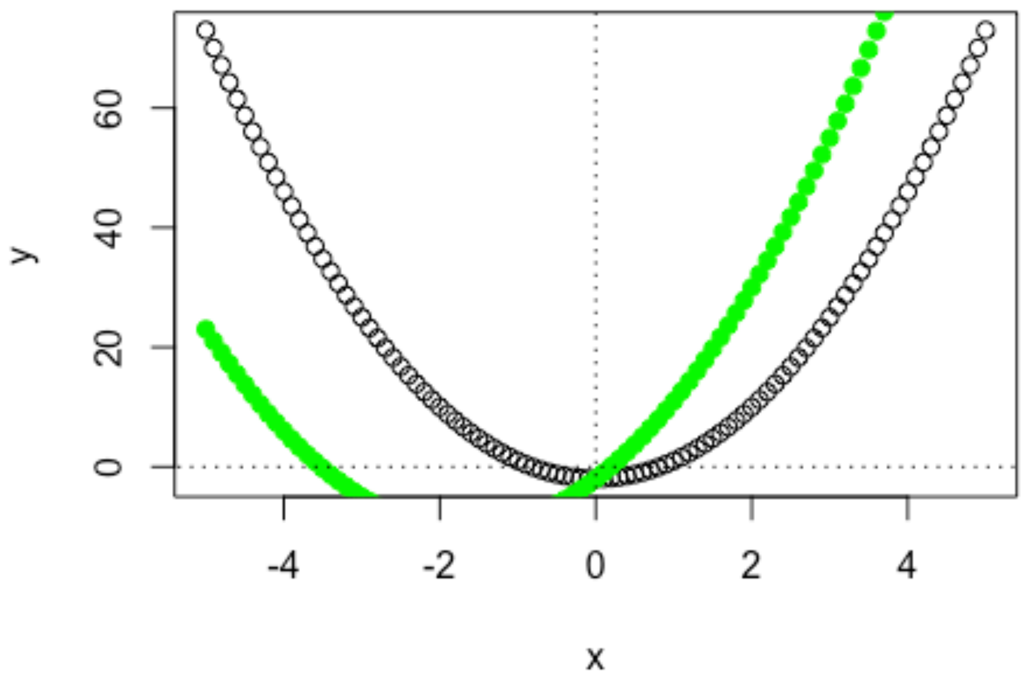


It gets steeper. So, now you can guess what happens when you make the slope smaller than 1, or even smaller than 0. It's an exercise, actually.

I want to take this one step further. What happens when we add a non-quadradic effect?

$$y = a + (b1 * x) + (b2 * x^2)$$

```
plot(x,y)
a<- -2
b1<- 10
b2<-3
y<-a+b1*x+b2*x^2
points(x,y, pch=19, col="green")
segments(0,-100,0,100, lty=3)
segments(-100,0,100,0,lty=3)
```



It moves the whole curve! How cool is that?

Exercises

1) The response variable (y) is body mass in kg of Romanian Longhorn dragons. The explanatory variable (x) is the amount of sheep they've eaten in the last hour. The slope is 12. You don't know the intercept (but you don't need to know it to answer this question). How much kg does a Longhorn dragon gain on average per sheep eaten in the last hour?

2) The slope is 8, and the intercept is 0. This relationship describes the number of new moth species in my garden per day of catching in July. How many species can I expect to have seen after catching 10 days in July?

3) The quadratic relationship y~-1+2x-0.15x^2 describes how a phoenix's' reproductive success first improves with age (in years), and then declines as they senesce. At what age are these birds in their prime year? How many offspring do they produce at their prime, before reproductive output starts declining again?

4) You tested for the relationship between nature reserve area (explanatory variable, in ha), and species richness (in number of species). You find a linear slope b1 of 2, a quadratic slope b2 of -0.08, and an intercept of -1. Describe the change of the number of species seen as reserve area size changes.

5) **Finish the Linear Functions – A Primer – Exit Quiz before proceeding to the next handout. If you can't answer some of these questions, contact a member of staff to help you get there.**