

# Statistics with Spa OWS

## Lecture 13

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# Outline

- Repeatability
- Pitfalls

# Repeatability

- How consistent something is within a group, compared to the whole sample

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$$R = \frac{\sigma_A^2}{\sigma_A^2 + \sigma_W^2}$$

- It's got lots of interesting uses in biology.

# Observer repeatability

- Measuring tarsus consistently is not easy
- Some people measure it 3 times and take the mean
- Are observers consistent in their measures?





# Individual behaviour - personality

- Do birds always behave the same way?
- Different from others?

# Ecology

- If you calculate diversity indices, are the one in one region more alike than those between regions?

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- If you calculate diversity indices, are the one in one region more alike than those between regions?
- Is earthworm abundance consistent between day in the same plots?

Repeatability calculated from ANOVA output

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- Balanced dataset

$$a_i = 1, 1, 2, 1, 1, 1$$

$$b_i = 3, 3, 4, 3, 3, 3$$

$$c_i = 5, 5, 4, 5, 5, 1$$

# Repeatability calculated from ANOVA output

- $N_0$  not sample size
- Among-group variance difficult to compute (we've had trouble with SS before, remember!)

- Unbalanced dataset
- Heterogeneous dataset

$$a_i = 1, 1, 2, 1, 1, 1$$

$$b_i = 3, 3, 4, 3, 3, 3, 4, 2, 5, 5, 7$$

$$c_i = 5, 5$$



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$$b_i = 3, 3, 4, 3, 3, 3, 4, 2, 5, 5, 7$$

$$c_i = 5, 5$$

- In ecology we have those often. In exact sciences less so (medicine, any planned experiments)

# Repeatability calculated from ANOVA output

- $N_0$  not sample size
- Among-group variance difficult to compute (we've had trouble with SS before, remember!)

$$n_0 = \left[ \frac{1}{a-1} \left[ \sum_{i=1}^a n_i - \left( \frac{\sum_{i=1}^a n_i^2}{\sum_{i=1}^a n_i} \right) \right] \right]$$

$a$  = number of groups

$n_i$  = sample size in each group  $i$

# LMMs

- Linear mixed models
- Combine linear models and variance analysis

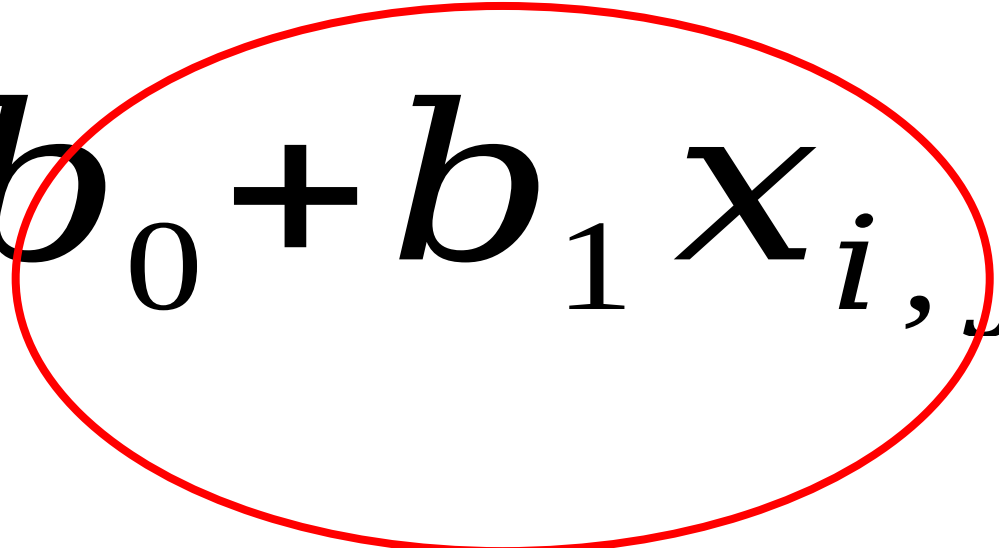
# Nested data structure

- Repeated measures
- Offspring in families
- ...

LMMs

$$y_{i,j} = b_0 + b_1 x_{i,j} + \alpha_j + \varepsilon_i,$$

# LMMs

$$y_{i,j} = b_0 + b_1 x_{i,j} + \alpha_j + \varepsilon_i,$$


Linear model bit that we know

Estimates FIXED intercept, covariates and factors

LMMs

$$y_{i,j} = b_0 + b_1 x_{i,j} + \alpha_j + \varepsilon_{i,j}$$

**Random** factor for a group  $j$  (i.e. BirdID)

Estimate variance component AMONG BIRDS

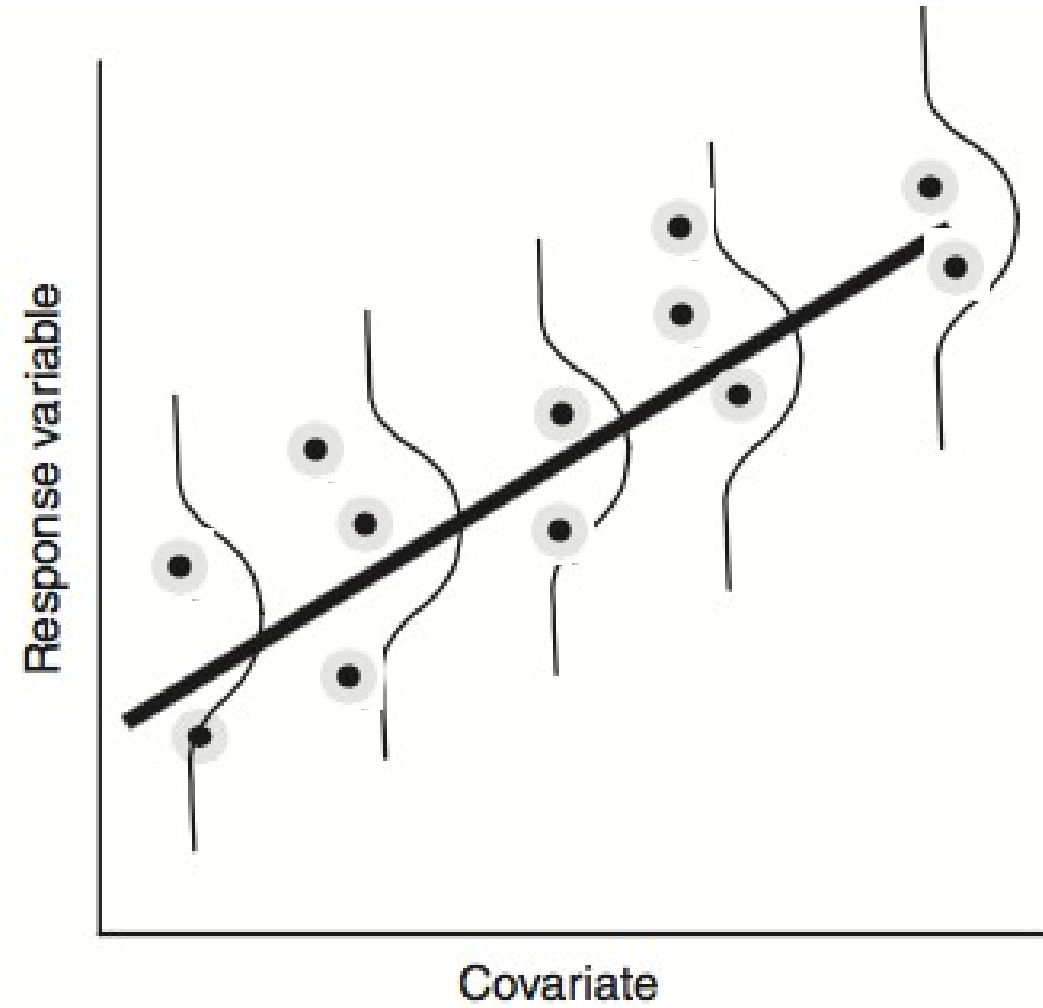
LMMs

$$y_{i,j} = b_0 + b_1 x_{i,j} + \alpha_j + \varepsilon_{i,j}$$

Residual variance

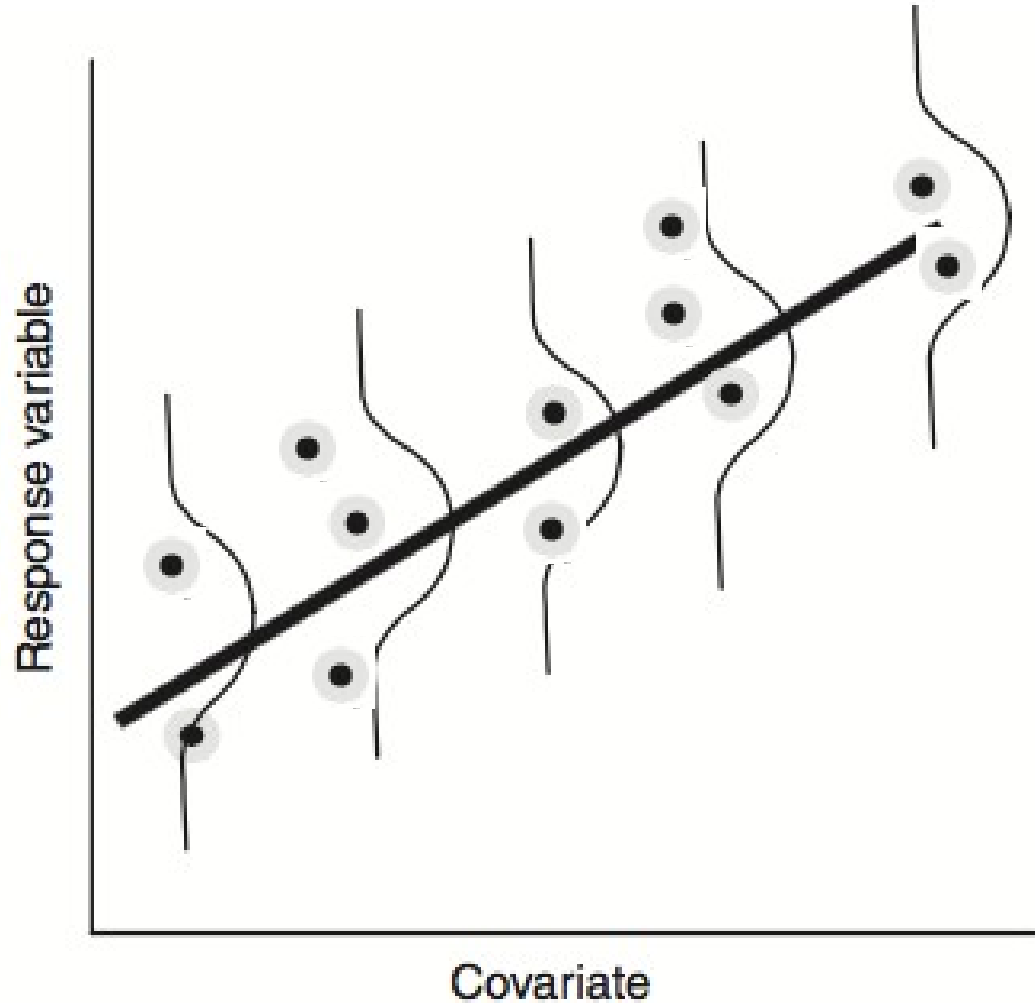


# LMMs



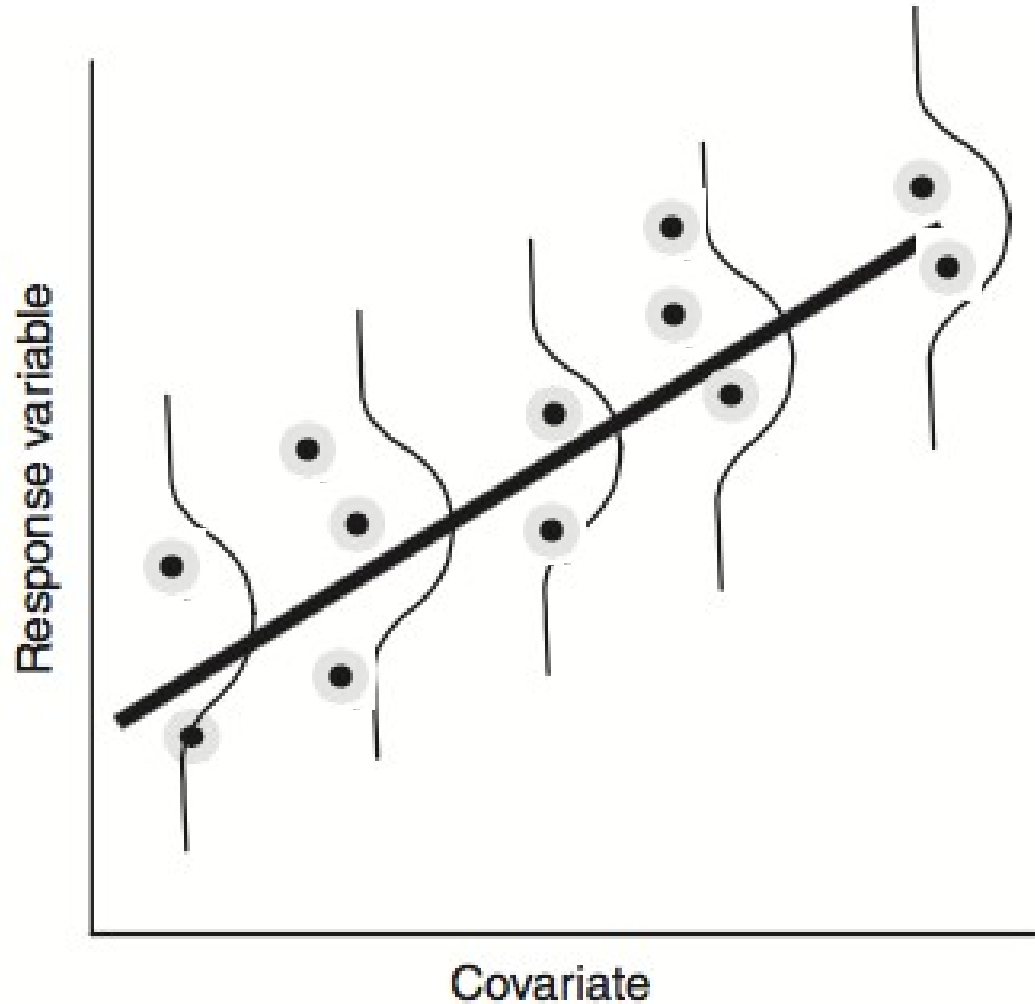
# LMMs

- Estimate variance components and fixed parameter estimates simultaneously



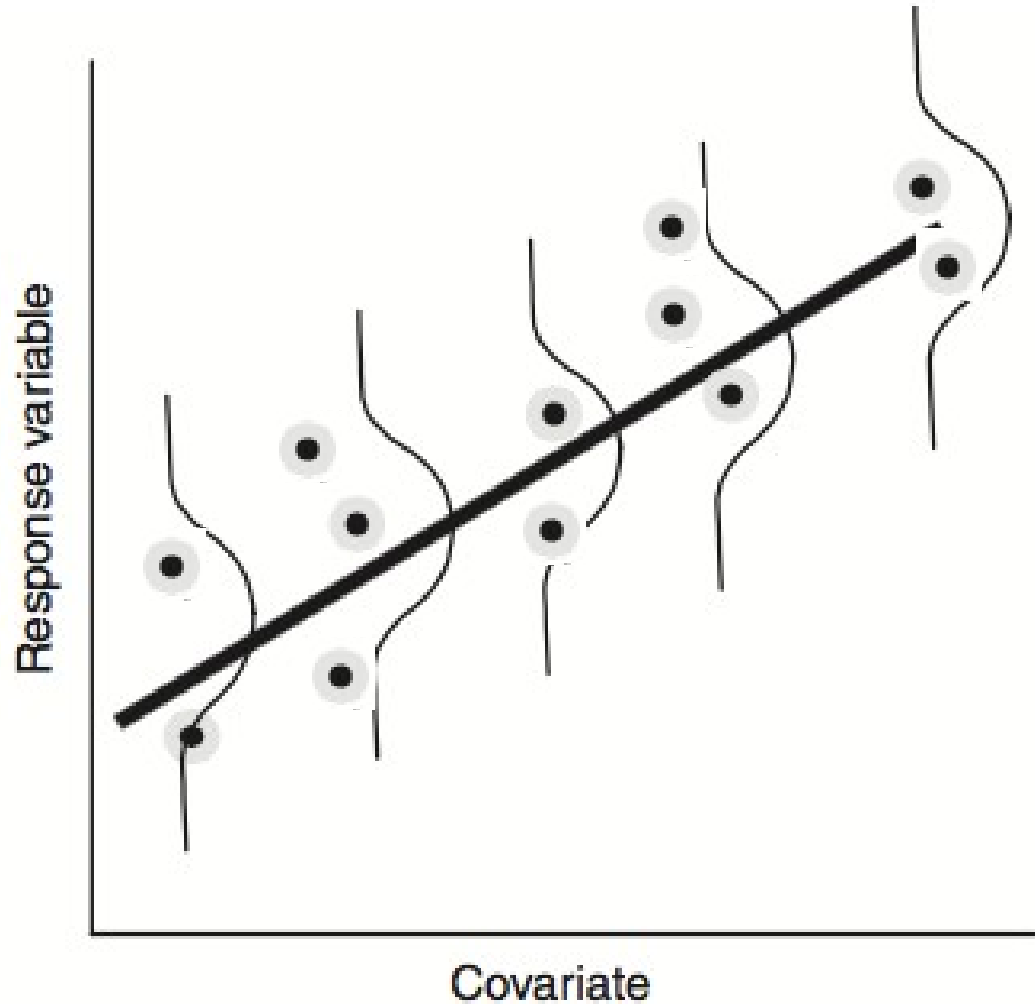
# LMMs

- Estimate variance components and fixed parameter estimates simultaneously
- Complicated but very useful



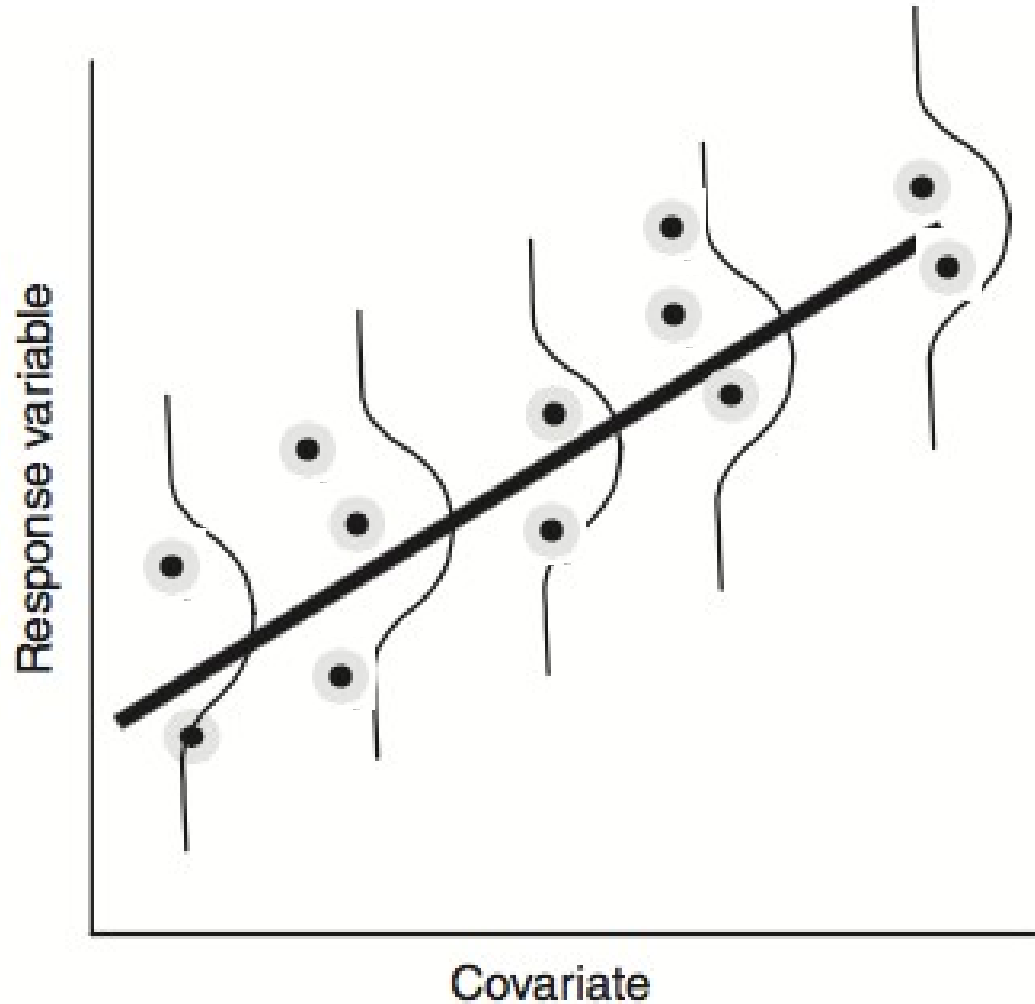
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# Linear mixed models:

- Can deal with heterogenous datasets (unbalanced groups)
- Estimates variance components and fixed effects *at the same time*

# Comparing ANOVA vs Linear mixed model

- multiple students measuring the tarus of one bird
- → observer repeatability describes how much variation is explained by different observers
- Measurement of methodological precision
- $R = \text{variance explained by student ID} / \text{total variance}$

# ANOVA

VS.

# LMM

```
> lm1<-lm(Tarsus~StudentID, data=b)
> anova(lm1)
```

Analysis of Variance Table

Response: Tarsus

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
StudentID	79	561.95	7.1133	4.2004	2.095e-09 ***
Residuals	71	120.24	1.6935		

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

```
> lmm1<-lmer(Tarsus~1+(1|StudentID), data=b)
```

```
> summary(lmm1)
```

Linear mixed model fit by REML ['lmerMod']

Formula: Tarsus ~ 1 + (1 | StudentID)

Data: b

REML criterion at convergence: 620.7

Scaled residuals:

	Min	1Q	Median	3Q	Max
	-3.1168	-0.3591	0.0201	0.4431	3.3629

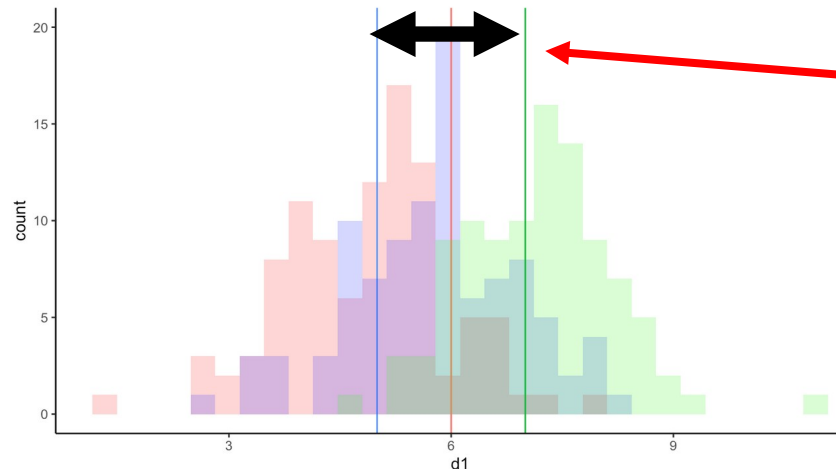
Random effects:

Groups	Name	Variance	Std.Dev.
StudentID	(Intercept)	2.963	1.721
	Residual	1.684	1.298

Number of obs: 151, groups: StudentID, 80

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	18.7101	0.2228	83.99



Sum  
or  
Var(Tarsus)



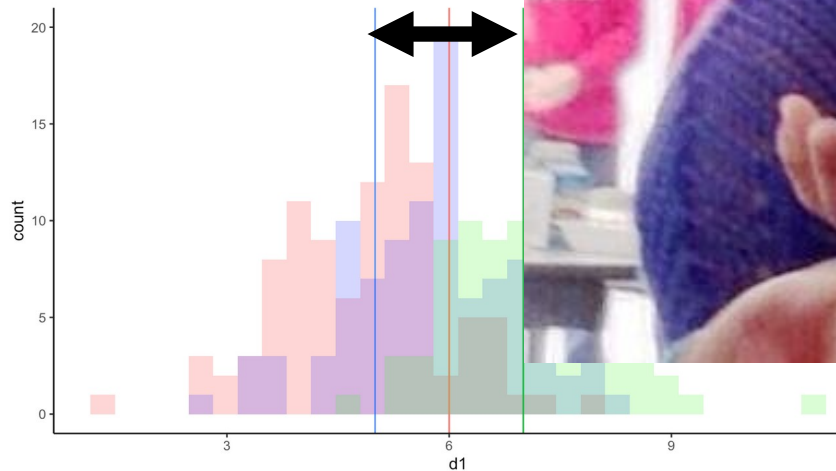
# ANOVA

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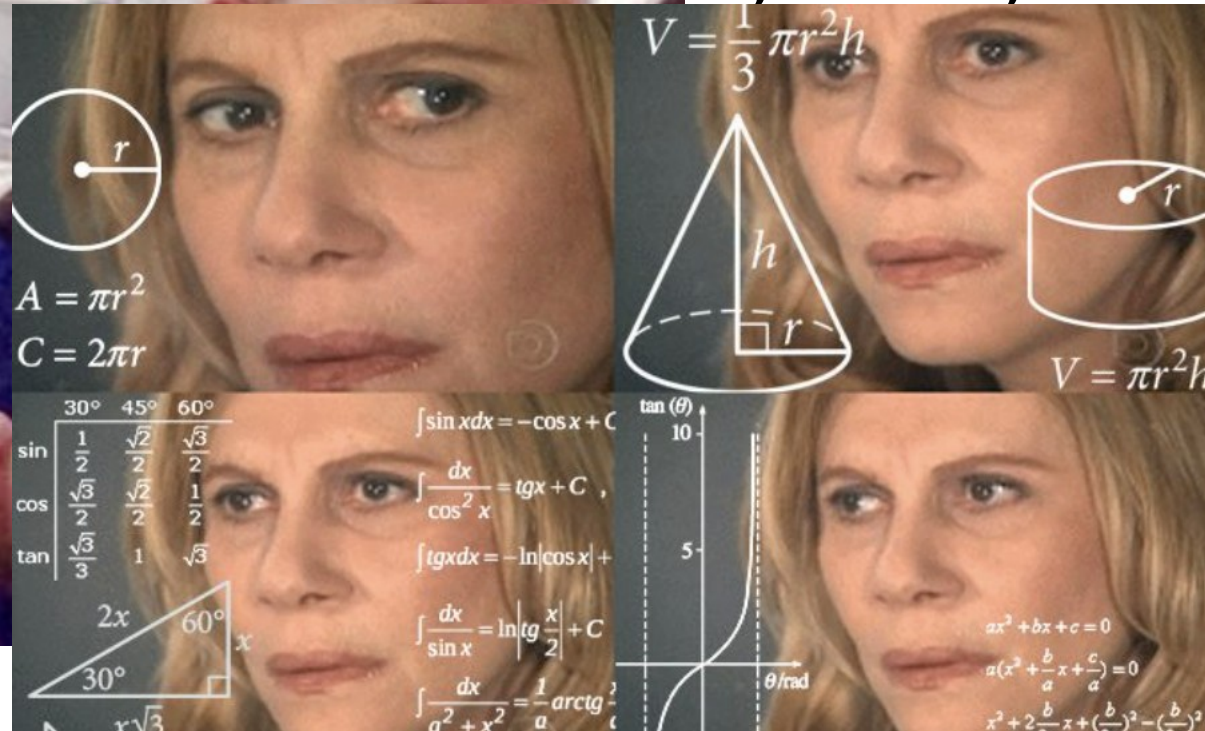
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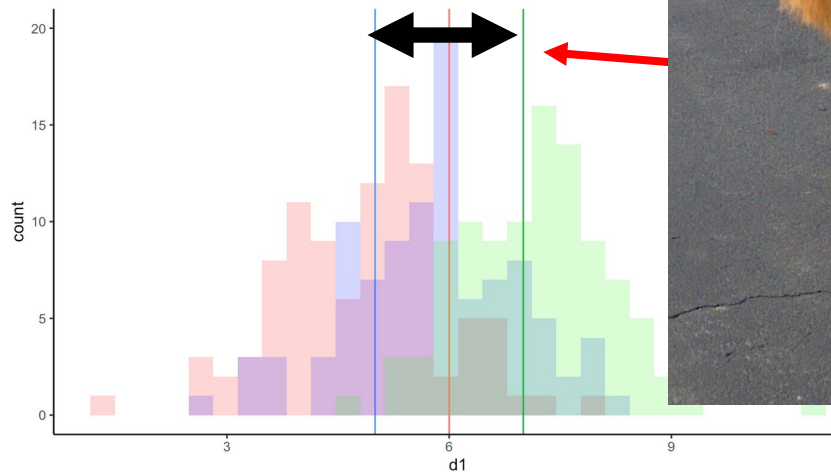
$$\sigma_A^2$$



$$S_w$$

# LMM

- Repeatability:  $\text{Var}_A$
- $2.96/(2.96+1.68) =$
- 64% of variance in  
is explained by diff  
student measuring



ID), data=b)

'lmerMod']  
ntID)

20.7

Max  
3.3629

Std.Dev.  
1.721  
1.298

dentID, 80

t value  
83.99

Var(Tarsus)

# Learning aim

- Repeatability is intra-class correlation coefficient
- Ratio of how much variance is explained by groups
- $N_0$  is horrible
- Linear mixed models are better at estimating variance components than ANOVA