AST221H

Python 2

Due: Oct. 29, 2021

You may work together with other students to solve these problem sets, but all solutions must be written and submitted independently. Show your work clearly for full marks, including intermediate steps, and diagrams if necessary! Check the syllabus for reading recommendations. Include units!

A simple stellar model

This assignment combines a theoretical and a plotting portion to be completed in the associated Jupyter notebook.

As we discussed in class, there are four equations of stellar structure that need to be solved simultaneously to calculate an accurate stellar model of density, temperature, luminosity and pressure as a function of radius. In the theoretical component, we will calculate a simple model of the Sun using a simple expression for density ρ and the equations of hydrostatic equilibrium and mass continuity to solve for M, P, and T as a function of r.

Recall that hydrostatic equilibrium and mass continuity are given by:

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2} \tag{1}$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r) \tag{2}$$

In the python component, you will compare your derived model with the Standard Solar Model, which is a complete integration of the equations of stellar structure.

Assume that $\rho(r)$ can be approximated by a simple linear density profile,

$$\rho(r) = \rho_c(1 - r/R), \tag{3}$$

where $\rho_c = \rho(r = 0)$ is the central density. Assume further that $\rho(r) = 0$ at the the radius of the star, R.

a (3 points) Integrate the equation of mass continuity (Equation 2) using the above expression for $\rho(r)$ and solve for $m(r)$. Solve for the total mass of the star M at $r = R$, and show that

$$m(r) = M\left[4\left(\frac{r}{R}\right)^3 - 3\left(\frac{r}{R}\right)^4\right]. \tag{4}$$

b (4 points) Use your expression from for $m(r)$ above and the general expression for the density in the equation of hydrostatic equilibrium to show that

$$P = \pi G\rho_c^2 R^2\left[\frac{5}{36} - \frac{2}{3}\left(\frac{r}{R}\right)^2 + \frac{7}{9}\left(\frac{r}{R}\right)^3 - \frac{1}{4}\left(\frac{r}{R}\right)^4\right] \tag{5}$$

where $P = 0$ at the surface $r = R$. It may be useful to find an expression for ρ_c in terms of M and R and substitute where possible.

c (3 points) Assuming an ideal gas law with no radiation pressure, $P = \rho kT/\mu m_H$, find an expression for $T(r)$.

Given $mu = 0.61$ for the Sun (you can use the Sun's mass and radius from C&O), what is the central temperature of the Sun in this model? Compare it to the known value.

Once you have solved these problems, complete the python plotting notebook to compare your results with a full stellar model of the Sun.