1 Integration by Parts

EXAMPLE 1.0.1.

EXAMPLE 1.0.2.

$$\begin{aligned}
\int x\ln(x)\,dx &= & u &= \ln(x) & dv &= x\,dx \\
&= uv - \int v\,du & du &= \frac{dx}{x} & v &= \frac{x^2}{2} \\
&= (\ln(x))(\frac{x^2}{2}) - \int \frac{x^2}{2}\frac{dx}{x} \\
&= \frac{x^2\ln(x)}{2} - \frac{1}{2}\int x\,dx \\
&= \frac{x^2\ln(x)}{2} - \frac{1}{2}\cdot\frac{x^2}{2} + C \\
&= \frac{x^2\ln(x)}{2} - \frac{x^2}{4} + C
\end{aligned}$$

EXAMPLE 1.0.3.

$$\begin{aligned}
\int x\sec(x)\tan(x)\,dx &= x\sec(x) - \int \sec(x)\,dx & u &= x & dv &= \sec(x)\tan(x)\,dx \\
&= x\sec(x) - \ln|\sec(x)+\tan(x)| + C & du &= dx & v &= \sec(x)
\end{aligned}$$

EXAMPLE 1.0.3.

$$\begin{aligned}
\int \ln(x)\,dx &= x\ln(x) - \int x\frac{dx}{x} & u &= \ln(x) & dv &= dx \\
&= x\ln(x) - x + C & du &= \frac{dx}{x} & v &= x
\end{aligned}$$

EXAMPLE 1.0.4.

$$\begin{aligned}
\int x\cos(x)\,dx &= uv - \int vdu & u &= x & dv &= \cos(x)\,dx \\
&= x\sin(x) - \int \sin(x)dx & du &= dx & v &= \sin(x) \\
&= x\sin(x) + \cos(x) + C
\end{aligned}$$

EXAMPLE 1.0.5.

$$\int \sin(x)\ln(\cos(x))\,dx = -\cos(x)\ln(\cos(x)) + \cos(x) + C \qquad u = \ln(\cos(x)) \qquad dv = x$$

EXAMPLE 1.0.6.

$$\begin{aligned}
\int_1^2 \frac{\ln(x)}{x^2}\,dx &= \int_1^2 \ln(x)\,x^{-2}\,dx && u = \ln(x) && dv = x^{-2}\,dx \\
&= \left[(\ln(x))(-x^{-1})\right]_1^2 - \int_1^2 -x^{-1}\frac{dx}{x} && du = \frac{dx}{x} && v = -x^{-1} \\
&= -\left[\frac{\ln(x)}{x}\right]_1^2 + \int_1^2 x^{-2}\,dx \\
&= \left[-\frac{\ln(x)}{x} + \frac{x^{-1}}{-1}\right]_1^2 \\
&= \left[\frac{-\ln(x)-1}{x}\right]_1^2 \\
&= \frac{-\ln(2)-1}{2} - \frac{-1}{1} \\
&= \frac{-\ln(2)-1}{2} + 1 \\
&= \frac{-\ln(2)+1}{2}
\end{aligned}$$

EXAMPLE 1.0.7.

$$\begin{aligned}
\int x^2 e^x\,dx &= x^2e^x - \int e^x(2xdx) && = u = x^2 && dv = e^x\,dx \\
&= x^2e^x - 2\int xe^x\,dx && du = 2x\,dx && v = e^x
\end{aligned}$$

EXAMPLE 1.0.8.

$$\begin{aligned}
\int e^x\sin(x)\,dx &= e^x\sin(x) - \int e^x\cos(x)\,dx && u = \sin(x) && dv = e^x\,dx \\
&= && du_1 = \cos(x)\,dx && v_1 = e^x
\end{aligned}$$

EXAMPLE 1.0.9.

$$\begin{aligned}
\int e^x \cos(x)\,dx &= e^x\cos(x) - \int e^x(-\sin(x)\,dx) && u_1 = \cos(x) \qquad dv_1 = e^x\,dx \\
& && du_1 = -\sin(x)\,dx \quad v_1 = e^x \\
&= e^x\cos(x) + \int e^x \sin(x)\,dx && u_2 = \sin(x) \qquad dv_2 = e^x\,dx \\
& && du_2 = \cos(x) \qquad v_2 = e^x \\
&= e^x\cos(x) + (e^x\sin(x) - \int e^x(\cos(x))) \\
e^x\cos(x)\,dx &= e^x\cos(x) + e^x\sin(x) - \int e^x\cos(x) \\
\frac{2\int e^x\cos(x)\,dx}{2} &= \frac{e^x\cos(x) + e^x\sin(x)}{2} + C \\
\int e^x\cos(x)\,dx &= \frac{e^x\cos(x) + e^x\sin(x)}{2} + C
\end{aligned}$$