1 Derivatives of Logarithmic Functions

1. Find the differential of the given function.

   1.1. $h(x) = \ln \sqrt{4 + 5x}$

   1.2. $f(x) = \ln (3x + 1)^2$

   1.3. $g(x) = \ln^2 (3x + 1)$

   1.4. $F(x) = \ln (\sin 5x)$

   1.5. $f(x) = \cos (\ln x)$

   1.6. $f(x) = x \ln x$

   1.7. $f(x) = \ln [(5x - 3)^4 (2x^2 + 7)^3]$

   1.8. $h(x) = \frac{x}{\ln x}$

   1.9. $g(x) = \ln \sqrt[3]{\frac{x+1}{x^2+1}}$

   1.10. $h(x) = \frac{\log x}{x}$

   1.11. $f(x) = \log \left(\frac{x}{x+1}\right)$

   1.12. $f(x) = \sqrt{\log_a x}$

2. Find the $\frac{dy}{dx}$ of the given equations by implicit differentiation.

   2.1. $\ln xy + x + y = 2$

   2.2. $x = \ln (x + y + 1)$

   2.3. $x + \ln x^2 y + 3y^2 = 2x^2 - 1$

3. Find the $\frac{dy}{dx}$ of the given equation by logarithmic differentiation.

   3.1. $y = (5x - 4)(x^2 + 3)(3x^3 - 5)$

   3.2. $y = \frac{x^2 (x-1)^2 (x+2)^3}{(x-4)^5}$

   3.3. $y = \frac{x^3 + 2x}{\sqrt[5]{x^7+1}}$

   3.4. $y = x^{\sqrt{x}}$

   3.5. $y = (\sin x)^{\tan x}$

   3.6. $y = x^{\ln x}$

Answer Key

1. Find the differential of the given function.

1.1. $h(x) = \ln \sqrt{4 + 5x}$

$$\begin{aligned} &= \ln (4 + 5x)^{1/2} \\ &= \frac{1}{2} \ln (4 + 5x) \\ &= \frac{1}{2} \cdot \frac{5}{(4 + 5x)} \\ &= \frac{5}{(8 + 10x)} \end{aligned}$$

1.2. $f(x) = \ln (3x + 1)^2$

$$\begin{aligned} &= 2 \ln (3x + 1) \\ &= 2 \cdot \frac{3}{(3x + 1)} \\ &= \frac{6}{(3x + 1)} \end{aligned}$$

1.3. $g(x) = \ln^2 (3x + 1)$

1.4. $F(x) = \ln (\sin 5x)$

$$\begin{aligned} &= \frac{5 \cos 5x}{\sin 5x} \\ &= 5 \cot 5x \end{aligned}$$

1.5. $f(x) = \cos (\ln x)$

$$= -\frac{\sin (\ln x)}{x}$$

1.6. $f(x) = x \ln x$

$$\begin{aligned} &= x \cdot \frac{1}{x} + 1 \cdot \ln x \\ &= 1 + \ln x \end{aligned}$$

1.7. $f(x) = \ln\left[(5x-3)^4(2x^2+7)^3\right]$

$$\begin{aligned}
&= \ln(5x-3)^4 + \ln(2x^2+7)^3 \\
&= 4\ln(5x-3) + 3\ln(2x^2+7) \\
&= 4 \cdot \frac{5}{5x-3} + 3 \cdot \frac{4x}{2x^2+7} \\
&= \frac{20}{5x-3} + \frac{12x}{2x^2+7}
\end{aligned}$$

1.8. $h(x) = \frac{x}{\ln x}$

1.9. $g(x) = \ln \sqrt[3]{\frac{x+1}{x^2+1}}$

1.10. $h(x) = \frac{\log x}{x}$

1.11. $f(x) = \log\left(\frac{x}{x+1}\right)$

1.12. $f(x) = \sqrt{\log_a x}$

2. Find the $\frac{dy}{dx}$ of the given equations by implicit differentiation.

2.1. $\ln xy + x + y = 2$

$$\begin{aligned}
\frac{1}{xy} \cdot \frac{dy}{dx}[xy] + 1 + \frac{dy}{dx} &= 0 \\
\frac{x \cdot \frac{dy}{dx} + 1 \cdot y}{xy} + 1 + \frac{dy}{dx} &= 0 \\
\frac{x\frac{dy}{dx} + y}{xy} + 1 + \frac{dy}{dx} &= 0 \\
x\frac{dy}{dx} + y + xy + xy\frac{dy}{dx} &= 0 \\
x\frac{dy}{dx} + xy\frac{dy}{dx} &= -y - xy \\
\frac{dy}{dx}[x + xy] &= -y - xy \\
\frac{dy}{dx} &= \frac{-y-xy}{x+xy}
\end{aligned}$$

2.2. $x = \ln(x+y+1)$

$$\begin{aligned}
1 &= \frac{1}{x+y+1} \cdot \frac{dy}{dx}[x+y+1] \\
1 &= \frac{1}{x+y+1}\left[1 + \frac{dy}{dx}\right] \\
x+y+1 &= 1 + \frac{dy}{dx} \\
\frac{dy}{dx} &= x+y
\end{aligned}$$

2.3. $x + \ln x^2 y + 3y^2 = 2x^2 - 1$

3. Find the $\frac{dy}{dx}$ of the given equation by logarithmic differentiation.

3.1. $y = (5x-4)(x^2+3)(3x^3-5)$

$$\begin{aligned}
\ln(y) &= \ln(5x-4) + \ln(x^2+3) + \ln(3x^3-5) \\
\frac{1}{y}\frac{dy}{dx} &= \frac{5}{5x-4} + \frac{2x}{x^2+3} + \frac{9x^2}{3x^3-5} \\
\frac{dy}{dx} &= y\left[\frac{5}{5x-4} + \frac{2x}{x^2+3} + \frac{9x^2}{3x^3-5}\right] \\
\frac{dy}{dx} &= (5x-4)(x^2+3)(3x^3-5)\left[\frac{5}{5x-4} + \frac{2x}{x^2+3} + \frac{9x^2}{3x^3-5}\right]
\end{aligned}$$

3.2. $y = \frac{x^2(x-1)^2(x+2)^3}{(x-4)^5}$

$$\begin{aligned}
y &= (x^2) + (x-1)^2 + (x+2)^3 - (x-4)^5 \\
\ln(y) &= 2\ln(x) + 2\ln(x-1) + 3\ln(x+2) - 5\ln(x-4) \\
\frac{1}{y}\frac{dy}{dx} &= \frac{2}{x} + \frac{2}{x-1} + \frac{3}{x+2} - \frac{5}{x-4} \\
\frac{dy}{dx} &= y\left[\frac{2}{x} + \frac{2}{x-1} + \frac{3}{x+2} - \frac{5}{x-4}\right] \\
\frac{dy}{dx} &= \frac{x^2(x-1)^2(x+2)^3}{(x-4)^5}\left[\frac{2}{x} + \frac{2}{x-1} + \frac{3}{x+2} - \frac{5}{x-4}\right]
\end{aligned}$$

3.3. $y = \frac{x^3+2x}{\sqrt[5]{x^7+1}}$

$$\begin{aligned}
y &= (x^3+2x) - (x^7+1)^{\frac{1}{5}} \\
\ln(y) &= \ln(x^3+2x) - \frac{1}{5}\ln(x^7+1) \\
\frac{1}{y}\frac{dy}{dx} &= \frac{3x^2+2}{x^3+2x} - \frac{7x^6}{5(x^7+1)} \\
\frac{dy}{dx} &= y\left[\frac{3x^2+2}{x^3+2x} - \frac{7x^6}{5(x^7+1)}\right] \\
\frac{dy}{dx} &= \frac{x^3+2x}{\sqrt[5]{x^7+1}}\left[\frac{3x^2+2}{x^3+2x} - \frac{7x^6}{5(x^7+1)}\right]
\end{aligned}$$

3.4. $y = x^{\sqrt{x}}$

$$\begin{aligned}
\ln(y) &= \sqrt{x}\ln(x) \\
\frac{1}{y}\frac{dy}{dx} &= \sqrt{x}\cdot\frac{1}{x} + \frac{1}{2}x^{-\frac{1}{2}}\cdot\ln(x) \\
\frac{1}{y}\frac{dy}{dx} &= \frac{\sqrt{x}}{x} + \frac{\ln(x)}{2x^{\frac{1}{2}}} \\
\frac{1}{y}\frac{dy}{dx} &= \frac{1}{\sqrt{x}} + \frac{\ln(x)}{2\sqrt{x}} \\
\frac{1}{y}\frac{dy}{dx} &= \frac{2+\ln(x)}{2\sqrt{x}} \\
\frac{dy}{dx} &= y\left[\frac{2+\ln(x)}{2\sqrt{x}}\right] \\
\frac{dy}{dx} &= x^{\sqrt{x}}\left[\frac{2+\ln(x)}{2\sqrt{x}}\right]
\end{aligned}$$

3.5. $y = (\sin x)^{\tan x}$

3.6. $y = x^{\ln x}$

2 Natural Logs and Exponentials

1. Find the differentiation of each function with respect to x.

   1.1. $y = \ln x^3$

   1.2. $y = e^{2x^3}$

   1.3. $y = \ln \ln 2x^4$

   1.4. $y = \ln \ln 3x^3$

   1.5. $y = \cos \ln 4x^3$

   1.6. $y = e^{e^{3x^2}}$

   1.7. $y = e^{(4x^3+5)^2}$

   1.8. $y = \ln 4x^2 \cdot (-x^3 - 4)$

   1.9. $y = \ln \left(-\frac{4x^4}{x^3-3} \right)^5$

   1.10. $y = \frac{e^{5x^4}}{e^{4x^2+3}}$

Answer Key

1. Find the differentiation of each function with respect to x.

   1.1. $y = \ln x^3$

   $$y = 3\ln(x)$$
   $$\frac{dy}{dx} = \frac{3}{x}$$

   1.2. $y = e^{2x^3}$

   1.3. $y = \ln\ln 2x^4$

   1.4. $y = \ln\ln 3x^3$

   1.5. $y = \cos\ln 4x^3$

   1.6. $y = e^{e^{3x^2}}$

   1.7. $y = e^{(4x^3+5)^2}$

   1.8. $y = \ln 4x^2 \cdot (-x^3 - 4)$

   1.9. $y = \ln\left(-\frac{4x^4}{x^3-3}\right)^5$

   1.10. $y = \frac{e^{5x^4}}{e^{4x^2+3}}$

3 Logs and Exponentials

1. Find the differentiation of each function with respect to x.

   1.1. $y = 4^{4x^4}$

   1.2. $y = 4^{-5x^3}$

   1.3. $y = \log_3 3x^2$

   1.4. $y = \log_2 4x^2$

   1.5. $y = \log_3 (3x^5 + 5)^5$

   1.6. $y = \log_5 (-5x^3 - 2)^3$

   1.7. $y = (4^{x^3} + 2)^3$

   1.8. $y = 3^{(x^4+1)^3}$

   1.9. $y = 3^{\cos 3x^4}$

   1.10. $y = \log_5 \tan 4x^4$