1 Integrals Yielding the Natural Logarithmic Function

DEFINITION. Since the natural logarithm is undefined for negative numbers, you will often encounter expressions of the form $\ln|u|$. Hence, for derivatives involving absolute values, if u is a differentiable function of x such that $u \neq 0$, then

$$\frac{d}{dx}[\ln|u|] = \frac{u'}{u}$$

EXAMPLE 1.0.0. Evaluate the equation $\int \frac{1}{u} du$.

$$= \ln|u| + C$$

DEFINITION. For any rational number $n \neq -1$,

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + C$$

However, for any rational number $n = -1$,

$$\int u^n \, du = \ln|u| + C$$

EXAMPLE 1.0.1. Evaluate the equation $\int \frac{2}{x} \, dx$.

EXAMPLE 1.0.2. Evaluate the equation $\int \frac{dx}{4x-1}$.

EXAMPLE 1.0.3. Find the area of the region on the x-axis and the line $x = 3$, bounded by the graph of

$$y = \frac{x}{x^2+1}$$

EXAMPLE 1.0.4. Evaluate the equation $\int \frac{x+1}{x^2+2x} \, dx$.

EXAMPLE 1.0.5. Evaluate the equation $\int \frac{\sec^2 x}{\tan x} \, dx$.

EXAMPLE 1.0.6. Evaluate the equation $\int \frac{\ln x}{x} \, dx$.

NOTE. The integrals to which this formula or rules can be applied may appear in disguised form. For instance, when a rational function has a numerator of degree greater than or equal to that of the denominator, division may reveal a form to which you can apply the rule.

EXAMPLE 1.0.7. Evaluate the equation $\int \frac{x^2+x+1}{x^2+1}\,dx$.

EXAMPLE 1.0.8. Evaluate the equation $\int_0^2 \frac{x^2+2}{x+1}\,dx$.

2 Integrals Involving Logarithmic Functions

EXAMPLE 2.0.1. Evaluate the equation $\int \frac{\log_{10} x}{x}$.

EXAMPLE 2.0.2. Evaluate the equation $\int \frac{(\log_3 x)^2}{x}$.

3 Integral of Trigonometric Functions

EXAMPLE 3.0.1. Evaluate the equation $\int \tan x\,dx$.

$$\begin{aligned}
&= -\int \frac{-\sin x}{\cos x}\,dx && u = \cos x \\
&= -\int \frac{du}{u} && du = -\sin x\,dx \\
&= -\ln|u| + C \\
&= -\ln|\cos x| + C \\
&= \ln|(\cos x)^{-1}| + C \\
&= \ln|\sec x| + C
\end{aligned}$$

EXAMPLE 3.0.2. Evaluate the equation $\int \cot x\,dx$.

$$\begin{aligned}
&= \int \frac{\cos x}{\sin x}\,dx && u = \sin x \\
&= \int \frac{du}{u} && du = \cos x\,dx \\
&= \ln|u| + C \\
&= \ln|\sin x| + C
\end{aligned}$$