PINNs for Navier Stokes equations

CASML-2024 : Preconference Workshop (Day 2)

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Outline

Introduction	• Governing equations
PINNs Architecture for NSE	
Demonstrative example	• Falkner-Skan Boundary Layer
FastVPINNs for NSE	• Fluid flow problems • Forward & Inverse problems on complex geometries

Navier Stokes equation

- Incompressible General Form:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} + f_i$$

$$\frac{\partial u_i}{\partial x_i} = 0$$

- 2D Steady Incompressible Navier-Stokes eqn. reduces to:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

- **Nonlinearity:** The equations involve nonlinear terms $(u\frac{\partial u}{\partial x}, v\frac{\partial u}{\partial y})$, making them challenging to solve analytically or numerically.
- **Coupled equations** (Momentum-Continuity Coupling)

PINNs Architecture for NSE

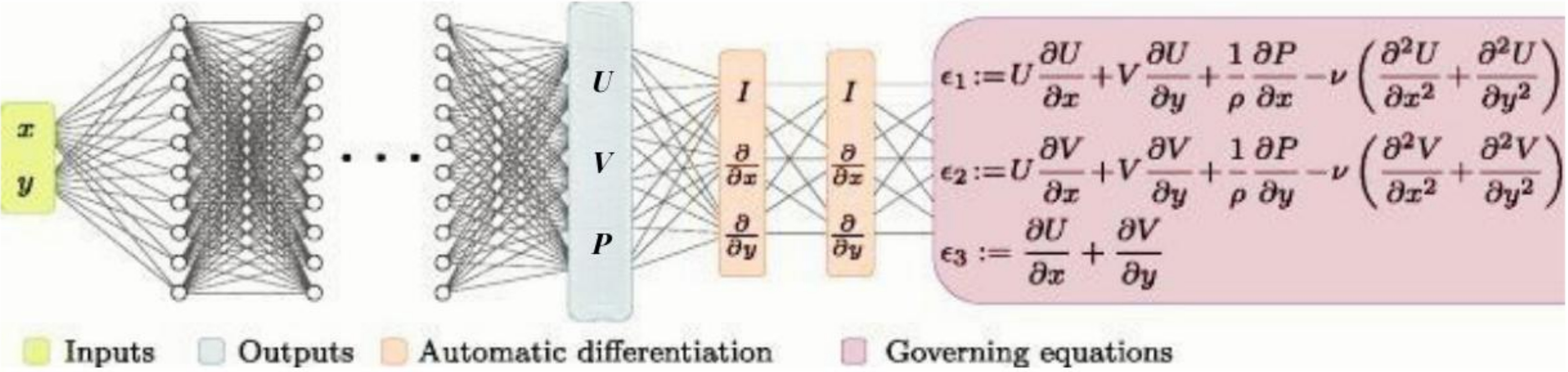


Figure: PINNs for NSE2d

Let, $\mathbf{U}_b^n(\theta) = [u, v, p]_{pred}$, where '$\theta$' represents the parameters of the network and $\tilde{\mathbf{U}}_b^n = [u, v, p]$ be the ground truth.

$$L_b = \frac{1}{N_b}\sum_{n=1}^{N_b}\left|\mathbf{U}_b^n(\theta) - \tilde{\mathbf{U}}_b^n\right|^2$$

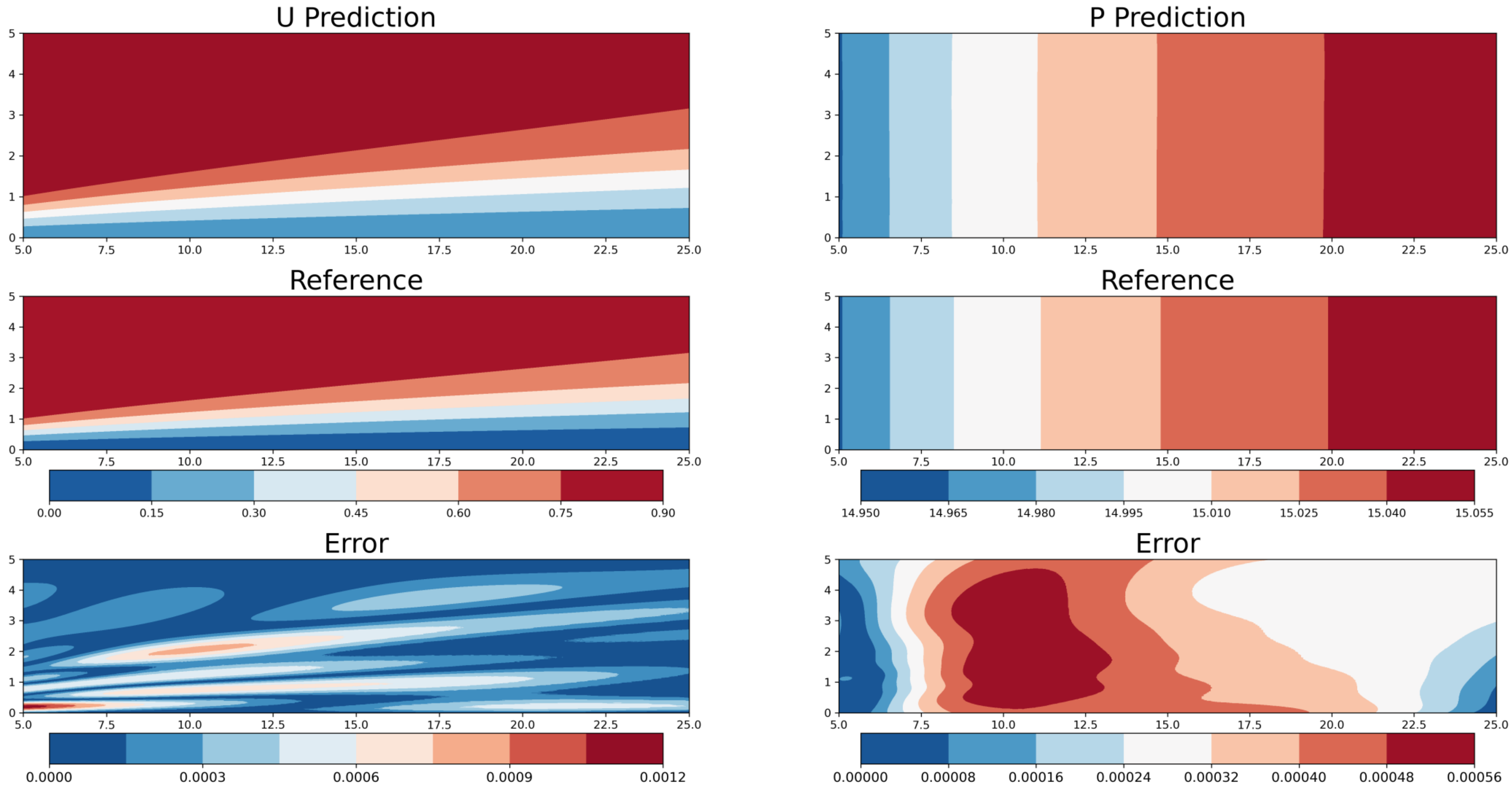
$$L_r = \frac{1}{N_r}\sum_{i=1}^{3}\sum_{j=1}^{N_r} w_i \epsilon_{i,j}^2$$

PINNs Architecture for NSE

N_b and N_r represents the number of boundary and residual points, respectively. w_i represents the residual weighting.

$$\text{Total Loss}, L = L_r + \beta L_b$$

Demonstrative case: Falkner-Skan BL



Thank you..!

Questions ?

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