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Pg. 463 #17, 26-39, 41-43, 45-53 odd

17. Give an interpretation of the shaded region.

$$\text{Total words in } t \text{ minutes.} \quad (1)$$

26. Find the area of the region bounded by $y = 3x^2$ and $y = 9x$

Find where the graphs meet:

$$3x^2 = 9x \quad (2)$$

$$3x^2 - 9x = 0 \quad (3)$$

$$x^2 - 3x = 0 \quad (4)$$

$$x(x - 3) = 0 \quad (5)$$

$$x = 0 \text{ \& } x = 3 \quad (6)$$

Find the graph that is higher between the interval:

$$3(1)^2 < 9(1) \quad (7)$$

Find the area of the difference of the two graphs between the interval:

$$\int_0^3 9x \, dx - \int_0^3 3x^2 \, dx = \left. \frac{9}{2}x^2 \right|_0^3 - \left. x^3 \right|_0^3 \quad (8)$$

$$= \frac{9}{2}(3)^2 - (3)^3 \quad (9)$$

$$= \frac{81}{2} - \frac{54}{2} \quad (10)$$

$$= 13\frac{1}{2} \quad (11)$$

Evaluate using substitution.

$$27. \int x^3 e^{x^4} \, dx$$

$$= \left[\begin{array}{l} u = x^4 \\ du = 4x^3 \, dx \end{array} \right] = \frac{1}{4} \int e^u \, du \quad (12)$$

$$= \frac{1}{4} e^{x^4} + C \quad (13)$$

$$28. \int \frac{24t^5}{4t^6 + 3} \, dt$$

$$= \left[\begin{array}{l} u = 4t^6 \\ du = 24t^5 \, dt \end{array} \right] = \int \frac{du}{u + 3} \quad (14)$$

$$= \ln(4t^6 + 3) + C \quad (15)$$

$$29. \int \frac{\ln(4x)}{2x} dx$$

$$= \left[\begin{array}{l} u = \ln(4x) \\ du = \frac{1}{x} dx \end{array} \right] = 2 \int u du \quad (16)$$

$$= 2 \ln(4x) + C \quad (17)$$

$$30. \int 2e^{-3x} dx$$

$$= -\frac{2}{3}e^{-3x} + C \quad (18)$$

Evaluate using integration by parts.

$$31. \int 3xe^{3x} dx$$

$$= 3 \int xe^{3x} dx \quad (19)$$

$$\int xe^{3x} dx = \left[\begin{array}{ll} u = x & v = \frac{e^{3x}}{3} \\ du = dx & dv = e^{3x} dx \end{array} \right] = uv - \int v du \quad (20)$$

$$= (x) \left(\frac{e^{3x}}{3} \right) - \int \left(\frac{e^{3x}}{3} \right) (dx) \quad (21)$$

$$\int 3xe^{3x} dx = 3 \left(\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} \right) \quad (22)$$

$$= xe^{3x} - \frac{1}{3}e^{3x} + C \quad (23)$$

$$32. \int \ln \sqrt[3]{x^2} dx$$

$$= \frac{2}{3} \int \ln x dx \quad (24)$$

$$\int \ln x dx = \left[\begin{array}{ll} u = \ln x & v = x \\ du = \frac{1}{x} dx & dv = dx \end{array} \right] = uv - \int v du \quad (25)$$

$$= (\ln x)(x) - \int (x) \left(\frac{1}{x} dx \right) \quad (26)$$

$$= x \ln(x) - x \quad (27)$$

$$\int \ln \sqrt[3]{x^2} dx = \frac{2}{3} (x \ln(x) - x) + C \quad (28)$$

$$\begin{aligned}
33. \quad \int 3x^2 \ln x \, dx &= 3 \int x^2 \ln x \, dx \quad (29)
\end{aligned}$$

$$\int x^2 \ln x \, dx = \left[\begin{array}{ll} u = \ln x & v = \frac{x^3}{3} \\ du = \frac{1}{x} dx & dv = x^2 dx \end{array} \right] = uv - \int v \, du \quad (30)$$

$$= (\ln x) \left(\frac{x^3}{3} \right) - \int \left(\frac{x^3}{3} \right) \left(\frac{1}{x} dx \right) \quad (31)$$

$$= \frac{1}{3} x^3 \ln(x) - \frac{x^3}{9} \quad (32)$$

$$\int 3x^2 \ln x \, dx = x^3 \ln(x) - \frac{x^3}{3} + C \quad (33)$$

Evaluate using tables of integration.

$$\begin{aligned}
34. \quad \int \frac{1}{49 - x^2} \, dx &= \frac{1}{14} \ln \left| \frac{7 + x}{7 - x} \right| + C \quad (34)
\end{aligned}$$

$$\begin{aligned}
35. \quad \int x^2 e^{5x} \, dx &= \frac{x^2 e^{5x}}{5} - \frac{2}{5} \int x e^{5x} \, dx \quad (35)
\end{aligned}$$

$$\int x e^{5x} \, dx = \frac{1}{25} \cdot e^{5x} (5x - 1) \quad (36)$$

$$= \frac{1}{5} x e^{5x} - \frac{1}{25} e^{5x} \quad (37)$$

$$\int x^2 e^{5x} \, dx = \frac{1}{5} x^2 e^{5x} - \frac{2}{5} \left(\frac{1}{5} x e^{5x} - \frac{1}{25} e^{5x} \right) \quad (38)$$

$$= \frac{1}{5} x^2 e^{5x} - \frac{2}{25} e^{5x} + \frac{2}{125} e^{5x} + C \quad (39)$$

$$\begin{aligned}
36. \quad \int \frac{x}{7x + 1} \, dx &= \frac{1}{49} + \frac{x}{7} - \frac{1}{49} \ln |1 + 7x| + C \quad (40)
\end{aligned}$$

$$\begin{aligned}
37. \quad \int \frac{dx}{\sqrt{x^2 - 36}} &= \ln \left| x + \sqrt{x^2 + 6} \right| + C \quad (41)
\end{aligned}$$

$$\begin{aligned}
 38. \quad & \int x^6 \ln x \, dx \\
 &= x^7 \left[\frac{\ln x}{7} - \frac{1}{49} \right] + C
 \end{aligned} \tag{42}$$

$$\begin{aligned}
 39. \quad & \int x e^{8x} \, dx \\
 &= \frac{1}{64} e^{8x} (8x - 1) + C
 \end{aligned} \tag{43}$$

Word problems.

41. Find the average value of $y = xe^{-x}$ over $[0, 2]$

$$y_{\text{av}} = \frac{1}{2} \int_0^2 x e^{-x} \, dx \tag{44}$$

$$\begin{array}{c}
 D \qquad \qquad I \\
 \hline
 \int_0^2 x e^{-x} \, dx = \begin{array}{c} x \swarrow + e^{-x} \\ 1 \quad \quad \searrow - e^{-x} \\ 0 \quad \quad \searrow e^{-x} \end{array} = -x e^{-x} - e^{-x}
 \end{array} \tag{45}$$

$$y_{\text{av}} = \frac{1}{2} \left(-x e^{-x} - e^{-x} \Big|_0^2 \right) \tag{46}$$

$$= \frac{1}{2} \left((-2e^{-2} - e^{-2}) - (-1) \right) \tag{47}$$

$$= \frac{1}{2} (1 - 3e^{-2}) \approx 0.297 \tag{48}$$

$$\begin{aligned}
 42. \quad & \int_0^4 3t^2 + 2t \, dt \\
 &= 3 \int_0^4 t^2 \, dt + 2 \int_0^4 t \, dt
 \end{aligned} \tag{49}$$

$$= t^3 + t^2 \Big|_0^4 \tag{50}$$

$$= 80 \text{ mi} \tag{51}$$

$$\begin{aligned}
 43. \quad & \int_0^4 3e^{3t} \, dt \\
 &= e^{3t} \Big|_0^4
 \end{aligned} \tag{52}$$

$$= e^{12} - 1 \approx \$162,753.79 \tag{53}$$

Integrate using any method.

$$\begin{aligned}
 45. \quad \int \frac{12t^2}{4t^3 + 7} dt &= \left[\begin{array}{l} u = 4t^3 \\ du = 12t^2 dt \end{array} \right] = \int \frac{du}{u + 7} & (54) \\
 &= \ln |4t^3 + 7| + C & (55) \\
 & & (56)
 \end{aligned}$$

$$\begin{aligned}
 47. \quad \int 5x^4 e^{x^5} dx &= \left[\begin{array}{l} u = x^5 \\ du = 5x^4 dx \end{array} \right] = \int e^u du & (57) \\
 &= e^{x^5} + C & (58)
 \end{aligned}$$

$$\begin{aligned}
 49. \quad \int t^7 (t^8 + 3)^{11} dt &= \left[\begin{array}{l} u = t^8 + 3 \\ du = 8t^7 dt \end{array} \right] = \frac{1}{8} \int u^{11} du & (59) \\
 &= \frac{(t^8 + 3)^{12}}{84} + C & (60)
 \end{aligned}$$

$$\begin{aligned}
 51. \quad \int x \ln(8x) dx &= -\frac{1}{4}x^2 + \frac{1}{2}x^2 \ln(8x) + C & (61)
 \end{aligned}$$

$$\begin{aligned}
 53. \quad \int \frac{dx}{e^x + 2} &= \int \frac{1 + e^x}{e^x + 2} - \frac{e^x}{e^x + 2} dx & (62) \\
 &= \int \frac{1 + e^x}{e^x + 2} dx - \int \frac{e^x}{e^x + 2} dx & (63) \\
 \int \frac{1 + e^x}{e^x + 2} dx &= \int \frac{e^x}{e^x + 2} dx + \int \frac{1}{e^x + 2} dx & (64) \\
 \int \frac{e^x}{e^x + 2} dx &= \left[\begin{array}{l} u = e^x + 2 \\ du = e^x \end{array} \right] = \frac{du}{u} & (65) \\
 &= \ln(e^x + 2) & (66) \\
 \int \frac{dx}{e^x + 2} &= \int \frac{dx}{e^x + 2} & (67)
 \end{aligned}$$