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Pg. 524 #1-31 eoo, 42-43, 45

1. $y' = 5x^4$

$$=x^5+C\tag{1}$$

$$=x^5, \quad x^5-1, \quad x^5+1$$
 (2)

 $5. \quad y' = \frac{8}{x} - x^2 + x^5$

$$= 8\ln|x| - \frac{x^3}{3} + \frac{x^6}{6} + C \tag{3}$$

$$= 8\ln|x| - \frac{x^3}{3} + \frac{x^6}{6}, \quad 8\ln|x| - \frac{x^3}{3} + \frac{x^6}{6} - 1, \quad 8\ln|x| - \frac{x^3}{3} + \frac{x^6}{6} + 1 \tag{4}$$

9. $f'(x) = x^{2/3} - x$; f(1) = -6

$$=\frac{3}{5}x^{5/3} - \frac{x^2}{2} + C \tag{5}$$

$$-6 = \frac{3}{5} \left(1\right)^{5/3} - \frac{(1)^2}{2} + C \tag{6}$$

$$C = -6 - \frac{3}{5} + \frac{1}{2} = -\frac{61}{10} \tag{7}$$

$$f(x) = \frac{3}{5}x^{5/3} - \frac{x^2}{2} - \frac{61}{10} \tag{8}$$

13. Show that $y = e^x + 3xe^x$ is a solution of y'' - 2y' + y = 0.

$$y' = e^x + 3(xe^x - e^x)$$
 (9)

$$y'' = e^x + 3(xe^x - e^x - e^x)$$
 (10)

$$(e^{x} + 3(xe^{x} - e^{x} - e^{x})) - 2(e^{x} + 3(xe^{x} - e^{x})) + (e^{x} + 3xe^{x}) = 0$$
(11)

$$e^{x} + 3xe^{x} - 3e^{x} - 3e^{x} - 2e^{x} - 6xe^{x} + 6e^{x} + e^{x} + 3xe^{x} = 0 \quad \checkmark$$
 (12)

17. $3y^2 \frac{dy}{dx} = 8x$

$$3\int y^2 dy = 8\int x dx \tag{13}$$

$$y^3 = 4x^2 + C \tag{14}$$

$$y = \sqrt[3]{4x^2 + C} \tag{15}$$

21. $\frac{dy}{dx} = \frac{6}{y}$

$$\int y \, dy = 6 \int dx \tag{16}$$

$$\frac{y^2}{2} = 6x + C (17)$$

$$y = \sqrt{12x + C} \tag{18}$$

25. $y' = 5y^{-2}$; y = 3 when x = 2

$$\frac{dy}{dx} = 5y^{-2} \tag{19}$$

$$\frac{1}{5} \int y^2 \, dy = \int dx \tag{20}$$

$$\frac{1}{15}y^3 = x + C \tag{21}$$

$$y = \sqrt[3]{15x + C} \tag{22}$$

$$3 = \sqrt[3]{15(2) + C} \tag{23}$$

$$27 = 30 + C (24)$$

$$C = -3 \tag{25}$$

$$y = \sqrt[3]{15x - 3} \tag{26}$$

29. $\frac{dP}{dt} = 2P$

$$\frac{1}{2} \int \frac{dP}{P} = \int dt \tag{27}$$

$$\frac{1}{2}\ln|P| = t + C\tag{28}$$

$$P = C_1 e^{2t}$$
, where $C_1 = \pm e^C$ (29)

42. $\frac{dP}{dt} = kP$

(a)

$$\frac{1}{k} \int \frac{dP}{P} = \int dt \tag{30}$$

$$\frac{1}{k}\ln|P| = t + C\tag{31}$$

$$P = C_1 e^{kt} (32)$$

(b)

$$P_0 = 0 (33)$$

$$P = P_0 e^{kt} (34)$$

43. $\frac{dR}{dS} = k \cdot \frac{R}{S}$

$$S \cdot dR = k \cdot R \cdot dS \tag{35}$$

$$\int \frac{dR}{R} = k \int \frac{dS}{S} \tag{36}$$

$$ln |R| = k ln |S| + C$$
(37)

$$R = C_1 e^{k \ln S} \tag{38}$$

$$= C_1 S^k, \quad \text{where } C_1 = e^C \tag{39}$$

45. $e^{-1/x} \cdot \frac{dy}{dx} = x^{-2} \cdot y^2$

$$\int y^{-2} dy = \int x^{-2} e^{x^{-1}} dx \tag{40}$$

$$\int x^{-2}e^{x^{-1}} dx = \begin{bmatrix} u = x^{-1} \\ du = -x^{-2} dx \end{bmatrix} = -\int e^{u} du$$
 (41)

$$-y^{-1} = -e^{x^{-1}} + C (42)$$

$$y = \frac{1}{e^{1/x} - C} \tag{43}$$