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Pg. 433 #3, 5, 11, 15, 16, 17, 23, 29, 33, 35, 53, 60

3. $g(x) = \begin{cases} x^2 + 4, & \text{for} \quad x \leq 0, \\ 4 - x, & \text{for} \quad x > 0 \end{cases}$

$$\int_{-2}^{3} g(x) = \int_{-2}^{0} x^2 + 4 + \int_{0}^{3} 4 - x \tag{1}$$

$$= \frac{x^3}{3} + 4x\bigg|_{-2}^{0} + 4x - \frac{x^2}{2}\bigg|_{0}^{3} \tag{2}$$

$$= \left(\left(\frac{(0)^3}{3} + 4(0)\right) - \left(\frac{(-2)^3}{3} + 4(-2)\right)\right) + \left(\left(4(3) - \frac{(3)^2}{2}\right) - \left(4(0) - \frac{(0)^2}{2}\right)\right) \tag{3}$$

$$= 18\frac{1}{6} \tag{4}$$

5. $f(x) = \begin{cases} -x^2 - 6x + 7, & \text{for} \quad x < 1 \\ \frac{3}{2}x - 1, & \text{for} \quad x \geq 1 \end{cases}$

$$\int_{-6}^{4} f(x) = \int_{-6}^{1} -x^3 - 6x + 7 + \int_{1}^{4} \frac{3}{2}x - 1 \tag{5}$$

$$= -\frac{x^3}{3} - 3x^2 + 7x\bigg|_{-6}^{1} + \frac{3x^2}{4} - x\bigg|_{1}^{4} \tag{6}$$

$$= \left(\left(-\frac{(1)^3}{3} - 3(1)^2 + 7(1)\right) - \left(-\frac{(-6)^3}{3} - 3(-6)^2 + 7(-6)\right)\right) + \left(\left(\frac{3(4)^2}{4} - (4)\right) - \left(\frac{3(1)^2}{4} - (1)\right)\right) \tag{7}$$

$$= 89\frac{11}{12} \tag{8}$$

11. $f(x) = x^2 - x - 5, \quad g(x) = x + 10$

$$x^2 - x - 5 = x + 10 \tag{9}$$

$$x^2 - 2x - 15 = 0 \tag{10}$$

$$(x + 3)(x - 5) = 0 \tag{11}$$

$$x = -3, 5 \tag{12}$$

15. $f(x) = x^4 - 8x^3 + 18x^2, \quad g(x) = x + 28$

$$
\begin{align}
A &= \int_{-1}^{4} g(x) - \int_{-1}^{4} f(x) \tag{13}\\
&= \int_{-1}^{4} x + 28 - \int_{-1}^{4} x^4 - 8x^3 + 18x^2 \tag{14}\\
&= \frac{x^2}{2} + 28x \Big|_{-1}^{4} - \frac{x^5}{5} - 2x^4 + 6x^3 \Big|_{-1}^{4} \tag{15}\\
&= \left(\left(\frac{(4)^2}{2} + 28(4)\right) - \left(\frac{(-1)^2}{2} + 28(-1)\right)\right) \nonumber\\
&\quad - \left(\left(\frac{(4)^5}{5} - 2(4)^4 + 6(4)^3\right) - \left(\frac{(-1)^5}{5} - 2(-1)^4 + 6(-1)^3\right)\right) \tag{16}\\
&= 62.5 \tag{17}
\end{align}
$$

16. $f(x) = 4x - x^2, \quad g(x) = x^2 - 6x + 8$

$$
\begin{align}
A &= \int_{1}^{4} f(x) - \int_{1}^{4} g(x) \tag{18}\\
&= \int_{1}^{4} 4x - x^2 - \int_{1}^{4} x^2 - 6x + 8 \tag{19}\\
&= 2x^2 - \frac{x^3}{3} \Big|_{1}^{4} - \frac{x^3}{3} - 3x^2 + 8x \Big|_{1}^{4} \tag{20}\\
&= \left(\left(2(4)^2 - \frac{(4)^3}{3}\right) - \left(2(1)^2 - \frac{(1)^3}{3}\right)\right) \nonumber\\
&\quad - \left(\left(\frac{(4)^3}{3} - 3(4)^2 + 8(4)\right) - \left(\frac{(1)^3}{3} - 3(1)^2 + 8(1)\right)\right) \tag{21}\\
&= 9 \tag{22}
\end{align}
$$

17. $y = x, \quad y = x^3, \quad x = 0, \quad x = 1$

$$
\begin{align}
A &= \int_{0}^{1} x - \int_{0}^{1} x^3 \tag{23}\\
&= \frac{x^2}{2} \Big|_{0}^{1} - \frac{x^4}{4} \Big|_{0}^{1} \tag{24}\\
&= \left(\left(\frac{(1)^2}{2}\right) - \left(\frac{(0)^2}{2}\right)\right) - \left(\left(\frac{(1)^4}{4}\right) - \left(\frac{(0)^4}{4}\right)\right) \tag{25}\\
&= \frac{1}{4} \tag{26}
\end{align}
$$

23. $y = 2x - x^2, \quad y = -x$

Find where the graphs meet:

$$2x - x^2 = -x \tag{27}$$
$$3x - x^2 = 0 \tag{28}$$
$$x(3 - x) = 0 \tag{29}$$
$$x = 0, 3 \tag{30}$$

Find the graph that is higher between the interval:

$$2(1) - (1)^2 = 1 \tag{31}$$
$$-(1) = -1 \tag{32}$$

Find the area of the difference of the two graphs between the interval:

$$A = \int_0^3 2x - x^2 - \int_0^3 -x \tag{33}$$
$$= x^2 - \frac{x^3}{3}\bigg|_0^3 - -\frac{x^2}{2}\bigg|_0^3 \tag{34}$$
$$= \left(\left((3)^2 - \frac{(3)^3}{3}\right) - \left((0)^2 - \frac{(0)^3}{3}\right)\right) - \left(\left(-\frac{(3)^2}{2}\right) - \left(-\frac{(3)^2}{2}\right)\right) \tag{35}$$
$$= 4\frac{1}{2} \tag{36}$$

29. $y = 4 - x^2, \quad y = 4 - 4x$

Find where the graphs meet:

$$4 - x^2 = 4 - 4x \tag{37}$$
$$-x^2 + 4x = 0 \tag{38}$$
$$-x(x - 4) = 0 \tag{39}$$
$$x = 0, 4 \tag{40}$$

Find the graph that is higher between the interval:

$$4 - (1)^2 = 3 \tag{41}$$
$$4 - 4(1) = 0 \tag{42}$$

Find the area of the difference of the two graphs between the interval:

$$A = \int_0^4 4 - x^2 - \int_0^4 4 - 4x \tag{43}$$
$$= 4x - \frac{x^3}{3}\bigg|_0^4 - 4x - 2x^2\big|_0^4 \tag{44}$$
$$= \left(\left(4(4) - \frac{(4)^3}{3}\right) - \left(4(0) - \frac{(0)^3}{3}\right)\right) - \left(\left(4(4) - 2(4)^2\right) - \left(4(0) - 2(0)^2\right)\right) \tag{45}$$
$$= 10\frac{2}{3} \tag{46}$$

33. $y = 2x^2 - 6x + 5, \quad y = x^2 + 6x - 15$

Find where the graphs meet:

$$2x^2 - 6x + 5 = x^2 + 6x - 15 \tag{47}$$
$$x^2 - 12x + 20 = 0 \tag{48}$$
$$(x-10)(x-2) = 0 \tag{49}$$
$$x = 2, 10 \tag{50}$$

Find the graph that is higher between the interval:

$$2(3)^2 - 6(3) + 5 = 5 \tag{51}$$
$$(3)^2 + 6(3) - 15 = 12 \tag{52}$$

Find the area of the difference of the two graphs between the interval:

$$A = \int_2^{10} x^2 + 6x - 15 - \int_2^{10} 2x^2 - 6x + 5 \tag{53}$$
$$= \frac{x^3}{3} + 3x^2 - 15x\Big|_2^{10} - \frac{2x^3}{3} - 3x^2 + 5x\Big|_2^{10} \tag{54}$$
$$= \left(\left(\frac{(10)^3}{3} + 3(10)^2 - 15(10)\right) - \left(\frac{(2)^3}{3} + 3(2)^2 - 15(2)\right)\right)$$
$$- \left(\left(\frac{2(10)^3}{3} - 3(10)^2 + 5(10)\right) - \left(\frac{2(2)^3}{3} - 3(2)^2 + 5(2)\right)\right) \tag{55}$$
$$= 85\frac{1}{3} \tag{56}$$

35. $y = 4 - x^2; \quad [-2, 2]$

$$y_{\text{avg}} = \frac{1}{2+2}\int_{-2}^{2} 4 - x^2 \tag{57}$$
$$= \frac{1}{4} \cdot 4x - \frac{x^3}{3}\Big|_{-2}^{2} \tag{58}$$
$$= \frac{1}{4} \cdot \left(\left(4(2) - \frac{(2)^3}{3}\right) - \left(4(-2) - \frac{(-2)^3}{3}\right)\right) \tag{59}$$
$$= 2\frac{2}{3} \tag{60}$$

53. $W(t) = -6t^2 + 12t + 90, \quad t$ in $[0, 5]$

a) Find the speed at the beginning of the interval.

$$-(0)t^2 + 12(0) + 90 = 90 \text{ words per minute} \tag{61}$$

b) Find the maximum speed and when it occurs.

$$
\begin{align}
W'(t) &= -12t + 12 \tag{62}\\
-12t + 12 &= 0 \tag{63}\\
-12(t-1) &= 0 \tag{64}\\
t &= 1 \text{ minute} \tag{65}\\
W(1) &= 94 \text{ words per minute, at } t = 1 \text{ minute} \tag{66}
\end{align}
$$

c) Find the average speed over the 5-min interval

$$
\begin{align}
W_{\text{avg}} &= \frac{1}{5-0}\int_0^5 -6t^2 + 12t + 90 \tag{67}\\
&= \frac{1}{5} \cdot -2t^3 + 6t^2 + 90t\Big|_0^5 \tag{68}\\
&= \frac{1}{5} \cdot \big(\big(-2(5)^3 + 6(5)^2 + 90(5)\big) - \big(-2(0)^3 + 6(0)^2 + 90(0)\big)\big) \tag{69}\\
&= 70 \text{ words per minute} \tag{70}
\end{align}
$$

60. $y = e^x, \quad y = e^{-x}, \quad x = -2$

Find where the graphs meet:

$$
\begin{align}
e^x &= e^{-x} \tag{71}\\
e^x - e^{-x} &= 0 \tag{72}\\
x &= 0 \tag{73}
\end{align}
$$

Find the graph that is higher between the interval:

$$
\begin{align}
e^{(-1)} &= \frac{1}{e} \tag{74}\\
e^{-(-1)} &= e \tag{75}
\end{align}
$$

Find the area of the difference of the two graphs between the interval:

$$
\begin{align}
A &= \int_{-2}^0 e^{-x} - \int_{-2}^0 e^x \tag{76}\\
&= -e^{-x}\Big|_{-2}^0 - e^x\Big|_{-2}^0 \tag{77}\\
&= \big(\big(-e^{-(0)}\big) - \big(-e^{-(-2)}\big)\big) - \big(\big(e^{(0)}\big) - \big(e^{(-2)}\big)\big) \tag{78}\\
&\approx 5.524 \tag{79}
\end{align}
$$