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Pg. 596 #37-44, 47, 52, 54-55, 58

Evaluate.

37. $\int 3x^5 dx$

$$= 3 \cdot \frac{x^6}{6} + C \quad (1)$$

$$= \frac{x^6}{2} + C \quad (2)$$

38. $\int_{-1}^0 (2e^x + 1) dx$

$$= 2 \int_{-1}^0 e^x dx + \int_{-1}^0 dx \quad (3)$$

$$= 2 [e^x]_{-1}^0 + [x]_{-1}^0 \quad (4)$$

$$= 2 [e^0 - e^{-1}] + [0 - (-1)] \quad (5)$$

$$= 2 \left[1 - \frac{1}{e} \right] + 1 \quad (6)$$

$$= 2 - \frac{2}{e} + 1 \quad (7)$$

$$= 3 - \frac{2}{e} \quad (8)$$

39. $\int \frac{x}{(7-3x)^2} dx$ (Use Table 1.)

$$= \frac{7}{(-3)^2(7-3x)} + \frac{1}{(-3)^2} \ln |7-3x| + C \quad (9)$$

$$= \frac{7}{9(7-3x)} + \frac{1}{9} \ln |7-3x| + C \quad (10)$$

40. $\int x^3 e^{x^4} dx$ (Do not use Table 1.)

$$= \left[\begin{array}{l} u = x^4 \\ du = 4x^3 dx \end{array} \right] = \frac{1}{4} \int e^u du \quad (11)$$

$$= \frac{1}{4} e^{x^4} + C \quad (12)$$

$$41. \int (x+3) \ln x \, dx$$

$$= \left[\begin{array}{ll} u = \ln x & v = \frac{x^2}{2} + 3x \\ du = \frac{1}{x} dx & dv = x + 3 dx \end{array} \right] = uv - \int v \, du \quad (13)$$

$$= (\ln x) \left(\frac{x^2}{2} + 3x \right) - \int \left(\frac{x^2}{2} + 3x \right) \left(\frac{1}{x} dx \right) \quad (14)$$

$$= \left(\frac{x^2}{2} + 3x \right) \ln x - \int \frac{x}{2} dx - \int 3 dx \quad (15)$$

$$= \left(\frac{x^2}{2} + 3x \right) \ln x - \frac{x^2}{4} - 3x + C \quad (16)$$

$$42. \int \frac{75}{x} dx$$

$$= 75 \ln |x| + C \quad (17)$$

$$43. \int_0^1 3\sqrt{x} \, dx$$

$$= 3 \left[x^{3/2} \right]_0^1 \quad (18)$$

$$= 3 (1^{3/2} - 0^{3/2}) \quad (19)$$

$$= 3 \quad (20)$$

44. Find the area under the graph of $y = x^2 + 3x$ over the interval $[1, 5]$.

$$= \int_1^5 x^2 \, dx + \int_1^5 3x \, dx \quad (21)$$

$$= \left[\frac{x^3}{3} \right]_1^5 + \left[\frac{3x^2}{2} \right]_1^5 \quad (22)$$

$$= \left[\frac{(5)^3}{3} - \frac{(1)^3}{3} \right] + \left[\frac{3(5)^2}{2} - \frac{3(1)^2}{2} \right] \quad (23)$$

$$= \frac{124}{3} + \frac{72}{2} \quad (24)$$

$$= \frac{248}{6} + \frac{216}{6} \quad (25)$$

$$= \frac{464}{6} \quad (26)$$

$$= \frac{232}{3} \quad (27)$$

47. Determine whether the following improper integral is convergent or divergent, and calculate its value if it is convergent:

$$\int_3^{\infty} \frac{1}{x^7} dx.$$

$$= \lim_{b \rightarrow \infty} \int_3^b x^{-7} dx \quad (28)$$

$$= \lim_{b \rightarrow \infty} \left[\frac{1}{-6x^6} \right]_3^b \quad (29)$$

$$= 0 - \frac{1}{-6(3)^6} \quad (30)$$

$$= \frac{1}{4374}; \quad \text{Convergent.} \quad (31)$$

52. Solve the differential equation $dy/dx = xy$.

$$\int \frac{dy}{y} = \int x dx \quad (32)$$

$$\ln y = \frac{x^2}{2} + C \quad (33)$$

$$y = C_1 e^{x^2/2}, \quad \text{where } C_1 = e^C \quad (34)$$

Given $f(x, y) = e^y + 4x^2y^3 + 3x$, find each of the following.

54. f_x

$$= 8xy^3 + 3 \quad (35)$$

55. f_{yy}

$$= \frac{\partial}{\partial y} (e^y + 12x^2y^2) \quad (36)$$

$$= e^y + 24x^2y \quad (37)$$

58. Evaluate

$$\int_0^3 \int_{-1}^2 e^x dy dx.$$

$$= \int_0^3 [ye^x]_{-1}^2 dx \quad (38)$$

$$= \int_0^3 [2e^x + e^x] dx \quad (39)$$

$$= 3 \int_0^3 e^x dx \quad (40)$$

$$= 3 [e^x]_0^3 dx \quad (41)$$

$$= 3 (e^3 - e^0) \quad (42)$$

$$= 3 (e^3 - 1) \quad (43)$$