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Pg. 463 #17, 26-39, 41-43, 45-53 odd

17. Give an interpretation of the shaded region.

$$\text{Total words in } t \text{ minutes.} \tag{1}$$

26. Find the area of the region bounded by $y = 3x^2$ and $y = 9x$

Find where the graphs meet:

$$\begin{aligned} 3x^2 &= 9x && (2) \\ 3x^2 - 9x &= 0 && (3) \\ x^2 - 3x &= 0 && (4) \\ x(x-3) &= 0 && (5) \\ x = 0 \ \ \& \ \ x &= 3 && (6) \end{aligned}$$

Find the graph that is higher between the interval:

$$3(1)^2 < 9(1) \tag{7}$$

Find the area of the difference of the two graphs between the interval:

$$\begin{aligned} \int_0^3 9x\ dx - \int_0^3 3x^2\ dx &= \frac{9}{2}x^2 \Big|_0^3 - x^3 \Big|_0^3 && (8) \\ &= \frac{9}{2}(3)^2 - (3)^3 && (9) \\ &= \frac{81}{2} - \frac{54}{2} && (10) \\ &= 13\frac{1}{2} && (11) \end{aligned}$$

Evaluate using substitution.

27. $\displaystyle\int x^3 e^{x^4}\ dx$

$$\begin{aligned} &= \begin{bmatrix} u = x^4 \\ du = 4x^3\ dx \end{bmatrix} = \frac{1}{4}\int e^u\ du && (12) \\ &= \frac{1}{4}e^{x^4} + C && (13) \end{aligned}$$

28. $\displaystyle\int \frac{24t^5}{4t^6+3}\ dt$

$$\begin{aligned} &= \begin{bmatrix} u = 4t^6 \\ du = 24t^5\ dt \end{bmatrix} = \int \frac{du}{u+3} && (14) \\ &= \ln\left(4t^6 + 3\right) + C && (15) \end{aligned}$$

29. $\displaystyle\int \frac{\ln(4x)}{2x}\ dx$

$$= \begin{bmatrix} u = \ln(4x) \\ du = \dfrac{1}{x}\ dx \end{bmatrix} = 2\int u\ du \tag{16}$$

$$= 2\ln(4x) + C \tag{17}$$

30. $\displaystyle\int 2e^{-3x}\ dx$

$$= -\frac{2}{3}e^{-3x} + C \tag{18}$$

Evaluate using integration by parts.

31. $\displaystyle\int 3xe^{3x}\ dx$

$$= 3\int xe^{3x}\ dx \tag{19}$$

$$\int xe^{3x}\ dx = \begin{bmatrix} u = x & v = \dfrac{e^{3x}}{3} \\ du = dx & dv = e^{3x}\ dx \end{bmatrix} = uv - \int v\ du \tag{20}$$

$$= (x)\left(\frac{e^{3x}}{3}\right) - \int \left(\frac{e^{3x}}{3}\right)(dx) \tag{21}$$

$$\int 3xe^{3x}\ dx = 3\left(\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x}\right) \tag{22}$$

$$= xe^{3x} - \frac{1}{3}e^{3x} + C \tag{23}$$

32. $\displaystyle\int \ln\sqrt[3]{x^2}\ dx$

$$= \frac{2}{3}\int \ln x\ dx \tag{24}$$

$$\int \ln x\ dx = \begin{bmatrix} u = \ln x & v = x \\ du = \dfrac{1}{x}\ dx & dv = dx \end{bmatrix} = uv - \int v\ du \tag{25}$$

$$= (\ln x)(x) - \int (x)\left(\frac{1}{x}\ dx\right) \tag{26}$$

$$= x\ln(x) - x \tag{27}$$

$$\int \ln\sqrt[3]{x^2}\ dx = \frac{2}{3}(x\ln(x) - x) + C \tag{28}$$

33. $\int 3x^2 \ln x \; dx$

$$= 3 \int x^2 \ln x \; dx \tag{29}$$

$$\int x^2 \ln x \; dx = \begin{bmatrix} u = \ln x & v = \dfrac{x^3}{3} \\ du = \dfrac{1}{x} \; dx & dv = x^2 \; dx \end{bmatrix} = uv - \int v \; du \tag{30}$$

$$= (\ln x) \left(\frac{x^3}{3}\right) - \int \left(\frac{x^3}{3}\right) \left(\frac{1}{x} \; dx\right) \tag{31}$$

$$= \frac{1}{3} x^3 \ln(x) - \frac{x^3}{9} \tag{32}$$

$$\int 3x^2 \ln x \; dx = x^3 \ln(x) - \frac{x^3}{3} + C \tag{33}$$

Evaluate using tables of integration.

34. $\int \frac{1}{49 - x^2} \; dx$

$$= \frac{1}{14} \ln \left| \frac{7 + x}{7 - x} \right| + C \tag{34}$$

35. $\int x^2 e^{5x} \; dx$

$$= \frac{x^2 e^{5x}}{5} - \frac{2}{5} \int x e^{5x} \; dx \tag{35}$$

$$\int x e^{5x} \; dx = \frac{1}{25} \cdot e^{5x} (5x - 1) \tag{36}$$

$$= \frac{1}{5} x e^{5x} - \frac{1}{25} e^{5x} \tag{37}$$

$$\int x^2 e^{5x} \; dx = \frac{1}{5} x^2 e^{5x} - \frac{2}{5} \left(\frac{1}{5} x e^{5x} - \frac{1}{25} e^{5x} \right) \tag{38}$$

$$= \frac{1}{5} x^2 e^{5x} - \frac{2}{25} e^{5x} + \frac{2}{125} e^{5x} + C \tag{39}$$

36. $\int \frac{x}{7x + 1} \; dx$

$$= \frac{1}{49} + \frac{x}{7} - \frac{1}{49} \ln |1 + 7x| + C \tag{40}$$

37. $\int \frac{dx}{\sqrt{x^2 - 36}}$

$$= \ln \left| x + \sqrt{x^2 + 6} \right| + C \tag{41}$$

38. $\int x^6 \ln x \; dx$

$$= x^7 \left[\frac{\ln x}{7} - \frac{1}{49} \right] + C \tag{42}$$

39. $\int xe^{8x} \; dx$

$$= \frac{1}{64} e^{8x} \left(8x - 1 \right) + C \tag{43}$$

Word problems.

41. Find the average value of $y = xe^{-x}$ over [0, 2]

$$y_{\text{av}} = \frac{1}{2} \int_0^2 xe^{-x} \; dx \tag{44}$$

$$\int_0^2 xe^{-x} \; dx = \begin{array}{ccc} D & & I \\ \hline x & + & e^{-x} \\ 1 & - & -\,e^{-x} \\ 0 & & e^{-x} \end{array} = -xe^{-x} - e^{-x} \tag{45}$$

$$y_{av} = \frac{1}{2} \left(-xe^{-x} - e^{-x} \Big|_0^2 \right) \tag{46}$$

$$= \frac{1}{2} \left(\left(-2e^{-2} - e^{-2} \right) - (-1) \right) \tag{47}$$

$$= \frac{1}{2} \left(1 - 3e^{-2} \right) \approx 0.297 \tag{48}$$

42. $\int_0^4 3t^2 + 2t \; dt$

$$= 3 \int_0^4 t^2 \; dt + 2 \int_0^4 t \; dt \tag{49}$$

$$= t^3 + t^2 \Big|_0^4 \tag{50}$$

$$= 80 \text{ mi} \tag{51}$$

43. $\int_0^4 3e^{3t} \; dt$

$$= e^{3t} \Big|_0^4 \tag{52}$$

$$= e^{12} - 1 \approx \$162,753.79 \tag{53}$$

Integrate using any method.

45. $\displaystyle\int \frac{12t^2}{4t^3+7}\ dt$

$$\begin{aligned}
&= \begin{bmatrix} u = 4t^3 \\ du = 12t^3\ dt \end{bmatrix} = \int \frac{du}{u+7} && (54)\\
&= \ln\left|4t^3+7\right| + C && (55)\\
& && (56)
\end{aligned}$$

47. $\displaystyle\int 5x^4 e^{x^5}\ dx$

$$\begin{aligned}
&= \begin{bmatrix} u = x^5 \\ du = 5x^4\ dx \end{bmatrix} = \int e^u\ du && (57)\\
&= e^{x^5} + C && (58)
\end{aligned}$$

49. $\displaystyle\int t^7\left(t^8+3\right)^{11}\ dt$

$$\begin{aligned}
&= \begin{bmatrix} u = t^8+3 \\ du = 8t^7\ dt \end{bmatrix} = \frac{1}{7}\int u^{11}\ du && (59)\\
&= \frac{\left(t^8+3\right)^{12}}{84} + C && (60)
\end{aligned}$$

51. $\displaystyle\int x\ln(8x)\ dx$

$$= -\frac{1}{4}x^2 + \frac{1}{2}x^2\ln(8x) + C \qquad (61)$$

53. $\displaystyle\int \frac{dx}{e^x+2}$

$$\begin{aligned}
&= \begin{bmatrix} u = e^x+2 \\ du = e^x dx \end{bmatrix} = \int \frac{1}{u(u-2)}du && (62)\\
&= -\frac{1}{2}\ln\left|\frac{u}{-2+u}\right| && (63)\\
&= -\frac{1}{2}\ln\left|\frac{e^x+2}{-2+e^x+2}\right| && (64)\\
&= -\frac{1}{2}\ln\left|\frac{e^x}{e^x}+\frac{2}{e^x}\right| && (65)\\
&= -\frac{1}{2}\ln\left(1+2e^{-x}\right) + C && (66)
\end{aligned}$$