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**Pg. 463 #17, 26-39, 41-43, 45-53 odd**

**17.** Give an interpretation of the shaded region.

$$\text{Total words in } t \text{ minutes.} \tag{1}$$

**26.** Find the area of the region bounded by $y = 3x^2$ and $y = 9x$

Find where the graphs meet:

$$3x^2 = 9x \tag{2}$$
$$3x^2 - 9x = 0 \tag{3}$$
$$x^2 - 3x = 0 \tag{4}$$
$$x(x-3) = 0 \tag{5}$$
$$x = 0 \ \ \& \ \ x = 3 \tag{6}$$

Find the graph that is higher between the interval:

$$3(1)^2 < 9(1) \tag{7}$$

Find the area of the difference of the two graphs between the interval:

$$\int_0^3 9x \ dx - \int_0^3 3x^2 \ dx = \frac{9}{2}x^2 \ \Big|_0^3 - x^3 \ \Big|_0^3 \tag{8}$$
$$= \frac{9}{2}(3)^2 - (3)^3 \tag{9}$$
$$= \frac{81}{2} - \frac{54}{2} \tag{10}$$
$$= 13\frac{1}{2} \tag{11}$$

Evaluate using substitution.

**27.** $\int x^3 e^{x^4} \ dx$

$$= \begin{bmatrix} u = x^4 \\ du = 4x^3 \ dx \end{bmatrix} = \frac{1}{4}\int e^u \ du \tag{12}$$
$$= \frac{1}{4}e^{x^4} + C \tag{13}$$

**28.** $\int \frac{24t^5}{4t^6 + 3} \ dt$

$$= \begin{bmatrix} u = 4t^6 \\ du = 24t^5 \ dt \end{bmatrix} = \int \frac{du}{u+3} \tag{14}$$
$$= \ln\left(4t^6 + 3\right) + C \tag{15}$$

**29.** $\displaystyle\int \frac{\ln(4x)}{2x}\,dx$

$$= \begin{bmatrix} u = \ln(4x) \\ du = \dfrac{1}{x}\,dx \end{bmatrix} = 2\int u\,du \tag{16}$$

$$= 2\ln(4x) + C \tag{17}$$

**30.** $\displaystyle\int 2e^{-3x}\,dx$

$$= -\frac{2}{3}e^{-3x} + C \tag{18}$$

Evaluate using integration by parts.

**31.** $\displaystyle\int 3xe^{3x}\,dx$

$$= 3\int xe^{3x}\,dx \tag{19}$$

$$\int xe^{3x}\,dx = \begin{bmatrix} u = x & v = \dfrac{e^{3x}}{3} \\ du = dx & dv = e^{3x}\,dx \end{bmatrix} = uv - \int v\,du \tag{20}$$

$$= (x)\left(\frac{e^{3x}}{3}\right) - \int \left(\frac{e^{3x}}{3}\right)(dx) \tag{21}$$

$$\int 3xe^{3x}\,dx = 3\left(\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x}\right) \tag{22}$$

$$= xe^{3x} - \frac{1}{3}e^{3x} + C \tag{23}$$

**32.** $\displaystyle\int \ln\sqrt[3]{x^2}\,dx$

$$= \frac{2}{3}\int \ln x\,dx \tag{24}$$

$$\int \ln x\,dx = \begin{bmatrix} u = \ln x & v = x \\ du = \dfrac{1}{x}\,dx & dv = dx \end{bmatrix} = uv - \int v\,du \tag{25}$$

$$= (\ln x)(x) - \int (x)\left(\frac{1}{x}\,dx\right) \tag{26}$$

$$= x\ln(x) - x \tag{27}$$

$$\int \ln\sqrt[3]{x^2}\,dx = \frac{2}{3}(x\ln(x) - x) + C \tag{28}$$

**33.** $\int 3x^2 \ln x \; dx$

$$= 3 \int x^2 \ln x \; dx \tag{29}$$

$$\int x^2 \ln x \; dx = \begin{bmatrix} u = \ln x & v = \dfrac{x^3}{3} \\ du = \dfrac{1}{x} \; dx & dv = x^2 \; dx \end{bmatrix} = uv - \int v \; du \tag{30}$$

$$= (\ln x) \left( \frac{x^3}{3} \right) - \int \left( \frac{x^3}{3} \right) \left( \frac{1}{x} \; dx \right) \tag{31}$$

$$= \frac{1}{3} x^3 \ln (x) - \frac{x^3}{9} \tag{32}$$

$$\int 3x^2 \ln x \; dx = x^3 \ln (x) - \frac{x^3}{3} + C \tag{33}$$

Evaluate using tables of integration.

**34.** $\int \dfrac{1}{49 - x^2} \; dx$

$$= \frac{1}{14} \ln \left| \frac{7 + x}{7 - x} \right| + C \tag{34}$$

**35.** $\int x^2 e^{5x} \; dx$

$$= \frac{x^2 e^{5x}}{5} - \frac{2}{5} \int x e^{5x} \; dx \tag{35}$$

$$\int x e^{5x} \; dx = \frac{1}{25} \cdot e^{5x} (5x - 1) \tag{36}$$

$$= \frac{1}{5} x e^{5x} - \frac{1}{25} e^{5x} \tag{37}$$

$$\int x^2 e^{5x} \; dx = \frac{1}{5} x^2 e^{5x} - \frac{2}{5} \left( \frac{1}{5} x e^{5x} - \frac{1}{25} e^{5x} \right) \tag{38}$$

$$= \frac{1}{5} x^2 e^{5x} - \frac{2}{25} e^{5x} + \frac{2}{125} e^{5x} + C \tag{39}$$

**36.** $\int \dfrac{x}{7x + 1} \; dx$

$$= \frac{1}{49} + \frac{x}{7} - \frac{1}{49} \ln |1 + 7x| + C \tag{40}$$

**37.** $\int \dfrac{dx}{\sqrt{x^2 - 36}}$

$$= \ln \left| x + \sqrt{x^2 + 6} \right| + C \tag{41}$$

**38.** $\int x^6 \ln x \; dx$

$$= x^7 \left[ \frac{\ln x}{7} - \frac{1}{49} \right] + C \tag{42}$$

**39.** $\int x e^{8x} \; dx$

$$= \frac{1}{64} e^{8x} (8x - 1) + C \tag{43}$$

Word problems.

**41.** Find the average value of $y = xe^{-x}$ over $[0, 2]$

$$y_{\text{av}} = \frac{1}{2} \int_0^2 x e^{-x} \; dx \tag{44}$$

$$\int_0^2 x e^{-x} \; dx = \begin{array}{cc} D & I \\ \hline x & e^{-x} \\ \searrow^{+} & \\ 1 & -e^{-x} \\ \searrow^{-} & \\ 0 & e^{-x} \end{array} = -xe^{-x} - e^{-x} \tag{45}$$

$$y_{av} = \frac{1}{2} \left( -xe^{-x} - e^{-x} \Big|_0^2 \right) \tag{46}$$

$$= \frac{1}{2} \left( \left( -2e^{-2} - e^{-2} \right) - (-1) \right) \tag{47}$$

$$= \frac{1}{2} \left( 1 - 3e^{-2} \right) \approx 0.297 \tag{48}$$

**42.** $\int_0^4 3t^2 + 2t \; dt$

$$= 3 \int_0^4 t^2 \; dt + 2 \int_0^4 t \; dt \tag{49}$$

$$= t^3 + t^2 \Big|_0^4 \tag{50}$$

$$= 80 \text{ mi} \tag{51}$$

**43.** $\int_0^4 3e^{3t} \; dt$

$$= e^{3t} \Big|_0^4 \tag{52}$$

$$= e^{12} - 1 \approx \$162,753.79 \tag{53}$$

Integrate using any method.

**45.** $\displaystyle\int \frac{12t^2}{4t^3+7}\ dt$

$$= \begin{bmatrix} u = 4t^3 \\ du = 12t^3\ dt \end{bmatrix} = \int \frac{du}{u+7} \tag{54}$$

$$= \ln\left|4t^3+7\right| + C \tag{55}$$

$$\tag{56}$$

**47.** $\displaystyle\int 5x^4 e^{x^5}\ dx$

$$= \begin{bmatrix} u = x^5 \\ du = 5x^4\ dx \end{bmatrix} = \int e^u\ du \tag{57}$$

$$= e^{x^5} + C \tag{58}$$

**49.** $\displaystyle\int t^7\left(t^8+3\right)^{11}\ dt$

$$= \begin{bmatrix} u = t^8+3 \\ du = 8t^7\ dt \end{bmatrix} = \frac{1}{7}\int u^{11}\ du \tag{59}$$

$$= \frac{\left(t^8+3\right)^{12}}{84} + C \tag{60}$$

**51.** $\displaystyle\int x\ln(8x)\ dx$

$$= -\frac{1}{4}x^2 + \frac{1}{2}x^2\ln(8x) + C \tag{61}$$

**53.** $\displaystyle\int \frac{dx}{e^x+2}$

$$= \begin{bmatrix} u = e^x+2 \\ du = e^x dx \end{bmatrix} = \int \frac{1}{u\,(u-2)} du \tag{62}$$

$$= -\frac{1}{2}\ln\left|\frac{u}{-2+u}\right| \tag{63}$$

$$= -\frac{1}{2}\ln\left|\frac{e^x+2}{-2+e^x+2}\right| \tag{64}$$

$$= -\frac{1}{2}\ln\left|\frac{e^x}{e^x}+\frac{2}{e^x}\right| \tag{65}$$

$$= -\frac{1}{2}\ln\left(1+2e^{-x}\right) + C \tag{66}$$