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Pg. 451 #1-35 eoo, 39, 41, 49

1. $\int 4xe^{4x}\ dx$

$$\begin{aligned}
&= \begin{bmatrix} u = 4x & v = \dfrac{e^{4x}}{4} \\ du = 4\ dx & dv = e^{4x}\ dx \end{bmatrix} = uv - \int v\ du && (1)\\
&= (4x)\left(\frac{e^{4x}}{4}\right) - \int \left(\frac{e^{4x}}{4}\right)(4\ dx) && (2)\\
&= xe^{4x} - \frac{1}{4}e^{4x} + C && (3)
\end{aligned}$$

5. $\int xe^{5x}\ dx$

$$\begin{aligned}
&= \begin{bmatrix} u = x & v = \dfrac{1}{5}e^{5x} \\ du = dx & dv = e^{5x}\ dx \end{bmatrix} = uv - \int v\ du && (4)\\
&= (x)\left(\frac{1}{5}e^{5x}\right) - \int \left(\frac{1}{5}e^{5x}\right)(dx) && (5)\\
&= \frac{1}{5}xe^{5x} - \frac{1}{25}e^{5x} + C && (6)\\
& && (7)
\end{aligned}$$

9. $\int x^2 \ln(x+5)\, dx$

$$\begin{aligned}
&= \begin{bmatrix} u = \ln(x+5) & v = \dfrac{x^3}{3} \\ du = \dfrac{1}{x}\ dx & dv = x^2\ dx \end{bmatrix} = uv - \int v\ du && (8)\\
&= (\ln(x+5))\left(\frac{x^3}{3}\right) - \int \left(\frac{x^3}{3}\right)\left(\frac{1}{x}\ dx\right) && (9)\\
&= \frac{x^3 \ln(x+5)}{3} - \frac{1}{3}\int x^2\ dx && (10)\\
&= \frac{x^3 \ln(x+5)}{3} - \frac{x^3}{9} + C && (11)
\end{aligned}$$

13. $\int \ln(x+5)\,dx$

$$
\begin{aligned}
&= \begin{bmatrix} u = x+5 \\ du = dx \end{bmatrix} = \int \ln(u)\,du && (12)\\
&= u\ln(u) - u + C && (13)\\
&= (x+5)\ln(x+5) - (x+5) + C && (14)\\
&= (x+5)\ln(x+5) - x + C && (15)
\end{aligned}
$$

17. $\int (x-1)\ln x \; dx$

$$
\begin{aligned}
&= \begin{bmatrix} u = \ln x & v = \dfrac{x^2}{2} - x \\ du = \dfrac{1}{x}\; dx & dv = (x-1)\,dx \end{bmatrix} = uv - \int v \; du && (16)\\
&= (\ln x)\left(\frac{x^2}{2} - x\right) - \int \left(\frac{x^2}{2} - x\right)\left(\frac{1}{x}\; dx\right) && (17)\\
&= \frac{x^2 \ln x}{2} - x\ln x - \int \frac{x}{2} - 1 \; dx && (18)\\
&= \frac{x^2 \ln x}{2} - x\ln x - \left(\frac{1}{2}\int x \; dx - \int 1 \; dx\right) && (19)\\
&= \left(\frac{x^2}{2} - x\right)\ln x - \frac{x^2}{4} + x + C && (20)\\
& && (21)
\end{aligned}
$$

21. $\int x^3 \ln(2x)\,dx$

$$
\begin{aligned}
&= \begin{bmatrix} u = \ln(2x) & v = \dfrac{x^4}{4} \\ du = \dfrac{1}{x}\; dx & dv = x^3 \; dx \end{bmatrix} = uv - \int v \; du && (22)\\
&= (\ln(2x))\left(\frac{x^4}{4}\right) - \int \left(\frac{x^4}{4}\right)\left(\frac{1}{x}\; dx\right) && (23)\\
&= \frac{x^4 \ln(2x)}{4} - \frac{1}{4}\int x^3 \; dx && (24)\\
&= \frac{x^4}{4}\ln(2x) - \frac{x^4}{16} + C && (25)
\end{aligned}
$$

25. $\int x^2 e^{2x}\ dx$

$$= \begin{bmatrix} u = x^2 & v = \dfrac{e^{2x}}{2} \\ du = 2x\ dx & dv = e^{2x}\ dx \end{bmatrix} = uv - \int v\ du \tag{26}$$

$$= \left(x^2\right)\left(\frac{e^{2x}}{2}\right) - \int \left(\frac{e^{2x}}{2}\right)(2x\ dx) \tag{27}$$

$$= \frac{1}{2}x^2 e^{2x} - \int x e^{2x}\ dx \tag{28}$$

$$\int x e^{2x}\ dx = \begin{bmatrix} u = x & v = \dfrac{e^{2x}}{2} \\ du = dx & dv = e^2 x\ dx \end{bmatrix} = uv - \int v\ du \tag{29}$$

$$= (x)\left(\frac{e^{2x}}{2}\right) - \int \left(\frac{e^{2x}}{2}\right)(dx) \tag{30}$$

$$= \frac{1}{2}x e^{2x} - \frac{1}{2}\int e^{2x}\ dx \tag{31}$$

$$= \frac{1}{2}x e^{2x} - \frac{1}{4}e^2 x + C \tag{32}$$

$$= \frac{1}{2}x^2 e^{2x} - \left(\frac{1}{2}x e^{2x} - \frac{1}{4}e^2 x\right) + C \tag{33}$$

$$= \frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{4}e^2 x + C \tag{34}$$

29. $\int \left(x^4+4\right) e^{3x}\ dx$

$$= \begin{bmatrix} u = x^4+4 & v = \dfrac{e^3 x}{3} \\ du = 4x^3\ dx & dv = e^{3x}\ dx \end{bmatrix} = uv - \int v\ du \tag{35}$$

$$= \left(x^4+4\right)\left(\frac{e^3 x}{3}\right) - \int \left(\frac{e^3 x}{3}\right)\left(4x^3\ dx\right) \tag{36}$$

$$= \frac{1}{3}x^4 e^3 x + \frac{4}{3}xe^3 - \frac{4}{3}\int x^4 e^3\ dx \tag{37}$$

$$\int x^4 e^3\ dx = \begin{bmatrix} u = x^4 & v = \dfrac{e^3}{3} \\ du = 4x^3\ dx & dv = e^3\ dx \end{bmatrix} = uv - \int v\ du \tag{38}$$

$$= \left(x^4\right)\left(\frac{e^3}{3}\right) - \int \left(\frac{e^3}{3}\right)\left(4x^3\ dx\right) \tag{39}$$

$$= \frac{1}{3}x^4 e^3 - \frac{4}{3}\int x^3 e^3\ dx \tag{40}$$

$$\int x^3 e^3\ dx = \begin{bmatrix} u = x^3 & v = \dfrac{e^3}{3} \\ du = 3x^2\ dx & dv = e^3\ dx \end{bmatrix} = uv - \int v\ du \tag{41}$$

$$= \left(x^3\right)\left(\frac{e^3}{3}\right) - \int \left(\frac{e^3}{3}\right)\left(3x^2\ dx\right) \tag{42}$$

$$= \frac{1}{3}x^3 e^3 - \int x^2 e^3\ dx \tag{43}$$

$$\int x^2 e^3\ dx = \begin{bmatrix} u = x^2 & v = 2x \\ du = \dfrac{e^3}{3} & dv = e^3\ dx \end{bmatrix} = uv - \int v\ du \tag{44}$$

$$= \left(x^2\right)(2x) - \int (2x)\left(\frac{e^3}{3}\right) \tag{45}$$

$$= 2x^3 - \frac{2}{3}\int xe^3\ dx \tag{46}$$

$$\int xe^3\ dx = \begin{bmatrix} u = x & v = \dfrac{e^3}{3} \\ du = dx & dv = e^3\ dx \end{bmatrix} = uv - \int v\ du \tag{47}$$

$$= (x)\left(\frac{e^3}{3}\right) - \int \left(\frac{e^3}{3}\right)(dx) \tag{48}$$

$$= \frac{1}{3} - \frac{e^3}{9} + C \tag{49}$$

$$\int \left(x^4+4\right) e^{3x}\ dx = \frac{1}{3}x^4 e^3 x + \frac{4}{3}xe^3 - \frac{4}{3}\left(\frac{1}{3}x^4 e^3 - \frac{4}{3}\left(\frac{1}{3}x^3 e^3 - \left(2x^3 - \frac{2}{3}\left(\frac{1}{3} - \frac{e^3}{9}\right)\right)\right)\right) + C \tag{50}$$

33. $\displaystyle\int_2^6 \ln(x+8)\,dx$

$$= \begin{bmatrix} u = \ln(x+8) & v = x \\ du = \dfrac{1}{x+8}\ dx & dv = dx \end{bmatrix} = uv - \int v\ du \tag{51}$$

$$= (\ln(x+8))(x) - \int (x)\left(\frac{1}{x+8}\ dx\right) \tag{52}$$

$$= x\ln(x+8) - \int \frac{x}{x+8}\ dx \tag{53}$$

$$\int \frac{x}{x+8}\ dx = \begin{bmatrix} u = \dfrac{1}{x+8} & v = \dfrac{x^2}{2} \\ du = -\dfrac{1}{x^2}\ dx & dv = x\ dx \end{bmatrix} = uv - \int v\ du \tag{54}$$

$$= \left(\frac{1}{x+8}\right)\left(\frac{x^2}{2}\right) - \int \left(\frac{x^2}{2}\right)\left(-\frac{1}{x^2}\ dx\right) \tag{55}$$

$$= \frac{x^2}{2(x+8)} + \frac{1}{2}\int 1dx \tag{56}$$

$$= \frac{x^2}{2(x+8)} + \frac{x}{2} \tag{57}$$

$$\int_2^6 \ln(x+8)\,dx = x\ln(x+8) - \left(\frac{x^2}{2(x+8)} + \frac{x}{2}\right)\Bigg|_2^6 \tag{58}$$

39. $K(t) = 10te^{-t}$

a)

$$\int_0^T K(t) = \begin{bmatrix} u = 10t & v = -e^{-t} \\ du = 10\ dt & dv = e^{-t}\ dt \end{bmatrix} = uv - \int v\ du \tag{59}$$

$$= (10t)\left(-e^{-t}\right) - \int \left(-e^{-t}\right)(10\ dt) \tag{60}$$

$$= -10te^{-t} - 10e^{-t}\ \Bigg|_0^T \tag{61}$$

$$= -10(T)e^{-(T)} - 10e^{-(T)} - \left(-10(0)e^{-(0)} - 10e^{-(0)}\right) \tag{62}$$

$$= -10Te^{-T} - 10e^{-T} + 10 \tag{63}$$

b)

$$-10(4)e^{-(4)} - 10e^{-(4)} + 10 \approx 9.084 \text{ kW-h} \tag{64}$$

41. $\displaystyle\int \sqrt{x}\ln x\ dx$

$$= \begin{bmatrix} u = \ln x & v = \dfrac{2}{3}x^{3/2} \\ du = \dfrac{1}{x}\ dx & dv = \sqrt{x}\ dx \end{bmatrix} = uv - \int v\ du \tag{65}$$

$$= (\ln x)\left(\frac{2}{3}x^{3/2}\right) - \int \left(\frac{2}{3}x^{3/2}\right)\left(\frac{1}{x}\ dx\right) \tag{66}$$

$$= \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int \sqrt{x}\ dx \tag{67}$$

$$= \frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + C \tag{68}$$

49. Verify that for any positive integer n, $\displaystyle\int x^n e^x\ dx = x^n e^x - n\int x^{n-1}e^x\ dx$

$$\int x^n e^x\ dx = \begin{bmatrix} u = x^n & v = e^x \\ du = nx^{n-1} & dv = e^x\ dx \end{bmatrix} = uv - \int v\ du \tag{69}$$

$$= (x^n)(e^x) - \int (e^x)\left(nx^{n-1}\right) \tag{70}$$

$$= x^n e^x - n\int x^{n-1}e^x\ dx \tag{71}$$