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Pg. 488 #17, 19, 21, 25, 27, 39, 43-47 odd, 49-51

17. $\displaystyle\int_0^\infty xe^x\,dx$

$$= \lim_{b\to\infty}\int_0^b xe^x\,dx = \begin{array}{ccc} D & & I \\ \hline x & + & e^x \\ 1 & - & e^x \\ 0 & & e^x \end{array} = \lim_{b\to\infty}\left[xe^x - e^x\right]_0^b = \text{Divergent} \tag{1}$$

19. $\displaystyle\int_0^\infty me^{-mx}\,dx,\ m > 0$

$$= \lim_{b\to\infty}\int_0^b me^{-mx}\,dx = \lim_{b\to\infty}\left[-e^{-mx}\right]_0^b = \text{Convergent}; 1 \tag{2}$$

21. $\displaystyle\int_\pi^\infty \frac{dt}{t^{1.001}}$

$$= \lim_{b\to\infty}\int_\pi^b t^{-1.001}\,dt = \lim_{b\to\infty}\left[-\frac{1000}{t^{0.001}}\right]_\pi^b = \text{Convergent}; \frac{1000}{\pi^{0.001}} \tag{3}$$

25. $\displaystyle\int_2^\infty 1/x^2\,dx$

$$= \lim_{b\to\infty}\int_2^b x^{-2}\,dx = \lim_{b\to\infty}\left[-\frac{1}{x}\right]_2^b = \frac{1}{2} \tag{4}$$

27. $\displaystyle\int_0^\infty 2xe^{-x^2}\,dx$

$$= \lim_{b\to\infty}\left[2\int_0^b xe^{-x^2}\,dx\right] = \left[\begin{array}{c} u = -x^2 \\ du = -2x\,dx \end{array}\right] = \lim_{b\to\infty}\left[-\frac{1}{e^{x^2}}\right]_0^b = 1 \tag{5}$$

39. $\displaystyle\lim_{T\to\infty}\int_0^T e^{0.00003t}\,dt$

$$= \lim_{T\to\infty}\left[\frac{1}{0.00003}\left(1 - e^{-0.00003T}\right)\right] = \frac{1}{0.00003} = 33,333\frac{1}{3}\text{ lb} \tag{6}$$

43. $\displaystyle\int_0^\infty \frac{dx}{x^{2/3}}$

$$= \lim_{b\to\infty} \int_0^b x^{-2/3}\ dx = \lim_{b\to\infty} \left[-3x^{1/3}\right]_0^b = \text{Divergent} \tag{7}$$

45. $\displaystyle\int_0^\infty \frac{dx}{(x+1)^{3/2}}$

$$= \lim_{b\to\infty} \int_0^b (x+1)^{-3/2}\ dx = \begin{bmatrix} u = x+1 \\ du = dx \end{bmatrix} = \lim_{b\to\infty} \left[-\frac{2}{\sqrt{x+1}}\right]_0^b = \text{Convergent; } 2 \tag{8}$$

47. $\displaystyle\int_0^\infty xe^{-x^2}\ dx$

$$= \lim_{b\to\infty} \int_0^b xe^{-x^2}\ dx = \begin{bmatrix} u = -x^2 \\ du = -2x\ dx \end{bmatrix} = \lim_{b\to\infty} \left[-\frac{1}{2e^{x^2}}\right]_0^b = \text{Convergent; } \frac{1}{2} \tag{9}$$

49. $\displaystyle\int_0^\infty te^{-kt}\ dt, \quad k > 0$

$$= \lim_{b\to\infty} \int_0^b te^{-kt}\ dt = \begin{array}{ccc} D & & I \\ \hline t & + & e^{-kt} \\ 1 & - & -\frac{1}{k}e^{-kt} \\ 0 & & \frac{1}{k^2}e^{-kt} \end{array} = \lim_{b\to\infty} \left[-\frac{t}{ke^{-kt}} - \frac{1}{k^2e^{-kt}}\right]_0^b = \frac{1}{k^2} \tag{10}$$

The integral represents the total drug dosage. (11)

50. A physician prescribes a dosage of 100 mg. Find k.

$$100 = \frac{1}{k^2} \tag{12}$$

$$k = \sqrt{\frac{1}{100}} \tag{13}$$

51. Consider the functions $y = \frac{1}{x^2}$ and $y = \frac{1}{x}$. Suppose that you go to a paint store to buy paint to cover the region under each graph over the interval $[1, \infty)$. Discuss whether you could be successful and why or why not.

I could be successful as both integrals $\displaystyle\int_1^\infty \frac{1}{x^2}\ dx$ and $\displaystyle\int_1^\infty \frac{1}{x}\ dx$ are convergent. (14)