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Pg. 554 #13, 17, 19, 21, 25, 27, 31, 33, 37, 39, 45, 67

Find f_x and f_y.

13. $f(x, y) = y \ln(x + 2y)$

$$f_x = \frac{y}{x+2y} \tag{1}$$
$$f_y = y \cdot \frac{2}{x+2y} + \ln(x+2y) \tag{2}$$
$$= \frac{2y}{x+2y} + \ln(x+2y) \tag{3}$$

17. $f(x, y) = \dfrac{x}{y} - \dfrac{y}{3x}$

$$f_x = \frac{1}{y} + \frac{y}{3x^2} \tag{4}$$
$$f_y = -\frac{x}{y^2} - \frac{1}{3x} \tag{5}$$

19. $f(x, y) = 3(2x + y - 5)^2$

$$f_x = 2 \cdot 3\,(2x + y - 5) \cdot 2 \tag{6}$$
$$= 12\,(2x + y - 5) \tag{7}$$
$$f_y = 2 \cdot 3\,(2x + y - 5) \cdot 1 \tag{8}$$
$$= 6\,(2x + y - 5) \tag{9}$$

Find $\frac{\partial f}{\partial b}$ and $\frac{\partial f}{\partial m}$.

21. $f(b, m) = m^3 + 4m^2 b - b^2 + (2m + b - 5)^2 + (3m + b - 6)^2$

$$\frac{\partial f}{\partial b} = 4m^2 - 2b + 2\,(2m + b - 5) + 2\,(3m + b - 6) \tag{10}$$
$$= 4m^2 - 2b + 4m + 2b - 10 + 6m + 2b - 12 \tag{11}$$
$$= 4m^2 + 10m + 2b - 22 \tag{12}$$
$$\frac{\partial f}{\partial m} = 3m^2 + 8mb + 4\,(2m + b - 5) + 6\,(3m + b - 6) \tag{13}$$
$$= 3m^2 + 8mb + 8m + 4b - 20 + 18m + 6b - 36 \tag{14}$$
$$= 3m^2 + 8mb + 10b + 26m - 56 \tag{15}$$

Find f_x, f_y, and f_λ.

25. $f(x, y, \lambda) = x^2 + y^2 - \lambda(10x + 2y - 4)$

$$f_x = 2x - 10\lambda \tag{16}$$
$$f_y = 2y - 2\lambda \tag{17}$$
$$f_\lambda = -(10x + 2y - 4) \tag{18}$$

Find the four second-order partial derivatives.

27. $f(x, y) = 5xy$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}(5y) = 0 \tag{19}$$
$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}(5y) = 5 \tag{20}$$
$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}(5x) = 5 \tag{21}$$
$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}(5x) = 0 \tag{22}$$

31. $f(x, y) = x^5 y^4 + x^3 y^2$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(5x^4y^4 + 3x^2y^2\right) = 20x^3y^4 + 6xy^2 \tag{23}$$
$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(5x^4y^4 + 3x^2y^2\right) = 20x^4y^3 + 6x^2y \tag{24}$$
$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(4x^5y^3 + 2x^3y\right) = 20x^4y^3 + 6x^2y \tag{25}$$
$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(4x^5y^3 + 2x^3y\right) = 12x^5y^2 + 2x^3 \tag{26}$$

Find f_{xx}, f_{xy}, f_{yx}, and f_{yy}.

33. $f(x, y) = 2x - 3y$

$$f_{xx} = \frac{\partial}{\partial x}(2) = 0 \tag{27}$$
$$f_{xy} = \frac{\partial}{\partial y}(2) = 0 \tag{28}$$
$$f_{yx} = \frac{\partial}{\partial x}(-3) = 0 \tag{29}$$
$$f_{yy} = \frac{\partial}{\partial y}(-3) = 0 \tag{30}$$

37. $f(x,y) = x + e^y$

$$f_{xx} = \frac{\partial}{\partial x}(1) = 0 \tag{31}$$

$$f_{xy} = \frac{\partial}{\partial y}(1) = 0 \tag{32}$$

$$f_{yx} = \frac{\partial}{\partial x}(e^y) = 0 \tag{33}$$

$$f_{yy} = \frac{\partial}{\partial y}(e^y) = e^y \tag{34}$$

39. $f(x,y) = y \ln x$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{y}{x}\right) = -\frac{y}{x^2} \tag{35}$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{y}{x}\right) = \frac{1}{x} \tag{36}$$

$$f_{yx} = \frac{\partial}{\partial x}(\ln x) = \frac{1}{x} \tag{37}$$

$$f_{yy} = \frac{\partial}{\partial y}(\ln x) = 0 \tag{38}$$

Temperature-humidity heat index. In the summer, humidity interacts with the outdoor temperature, making a person feel hotter due to a reduced heat loss from the skin caused by higher humidity. The temperature-humidity index, T_h, is what the temperature would have to be with no humidity in order to give the same heat effect. One index often used is given by

$$T_h = 1.98T - 1.09(1 - H)(T - 58) - 56.9,$$

where T is the air temperature, in degrees Fahrenheit, and H is the relative humidity, expressed as a decimal. Find the temperature-humidity index in each case. Round to the nearest tenth of a degree.

45. $T = 85°$ and $H = 60\%$

$$T_h = 1.98(85°) - 1.09(1 - 0.60)(85° - 58) - 56.9 \approx 99.6° \tag{39}$$

67. Consider $f(x,y) = \ln(x^2 + y^2)$. Show that f is a solution to the partial differential equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{2x}{x^2 + y^2}\right) = \frac{2(x^2 + y^2) - 4x^2}{(x^2 + y^2)^2} = \frac{-2x^2 + 2y^2}{(x^2 + y^2)^2} \tag{40}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{2y}{x^2 + y^2}\right) = \frac{2(x^2 + y^2) - 4y^2}{(x^2 + y^2)^2} = \frac{2x^2 - 2y^2}{(x^2 + y^2)^2} \tag{41}$$

$$\frac{-2x^2 + 2y^2}{(x^2 + y^2)^2} + \frac{2x^2 - 2y^2}{(x^2 + y^2)^2} = 0 \tag{42}$$