2.51 A rocket starts from rest and moves upward from the surface of the earth. For the first 10.0 s of its motion, the vertical acceleration of the rocket is given by $a_y = \left(2.80\,\mathrm{m/s^3}\right) t$, where the $+y$-direction is upward. (a) What is height of the rocket above the surface of the earth at $t = 10.0$ s? (b) What is the speed of the rocket when it is 325 m above the surface of the earth?

Solution:

(a)

$$v_y = \int_0^t \left[\left(2.80\,\mathrm{m/s^3}\right) t\right] dt \tag{1}$$

$$= 2.80\,\mathrm{m/s^3} \int_0^t t\,dt \tag{2}$$

$$= 2.80 \left[\frac{t^2}{2}\right] \tag{3}$$

$$= 1.40t^2 \tag{4}$$

$$y = 1.40 \int_0^{10} t^2\,dt \tag{5}$$

$$= \frac{7}{5}\left[\frac{t^3}{3}\right]_0^{10} \tag{6}$$

$$= \frac{7(10)^3}{15} \tag{7}$$

$$= \frac{7000}{15} \approx 467\,\mathrm{m} \tag{8}$$

(b)

$$325\,\mathrm{m} = \frac{7t^3}{15} \tag{9}$$

$$t^3 = \frac{4875}{7}\,\mathrm{m} \tag{10}$$

$$t = \sqrt[3]{\frac{4875}{7}} \tag{11}$$

$$v_y = 1.40 \left(\sqrt[3]{\frac{4875}{7}}\right)^2 \tag{12}$$

$$\approx 110\,\mathrm{m/s} \tag{13}$$

2.53 The acceleration of a motorcycle is given by $a_x(t) = At - Bt^2$, where $A = 1.50\,\mathrm{m/s^3}$ and $B = 0.120\,\mathrm{m/s^4}$. The motorcycle is at rest the origin at time $t = 0$. (a) Find its position and velocity as functions of time. (b) Calculate the maximum velocity it attains.

(a)

$$v_x = \int_0^t \left(1.50\,\mathrm{m/s^3}\right) t - \left(0.120\,\mathrm{m/s^4}\right) t^2\,dt \tag{14}$$

$$= \frac{\left(1.50\,\mathrm{m/s^3}\right) t^2}{2} - \frac{\left(0.120\,\mathrm{m/s^4}\right) t^3}{3} \tag{15}$$

$$= \left(0.75\,\mathrm{m/s^3}\right) t^2 - \left(0.040\,\mathrm{m/s^4}\right) t^3 \tag{16}$$

$$x = \int_0^t \left(0.75\,\mathrm{m/s^3}\right) t^2 - \left(0.040\,\mathrm{m/s^4}\right) t^3\,dt \tag{17}$$

$$= \frac{\left(0.75\,\mathrm{m/s^3}\right) t^3}{3} - \frac{\left(0.040\,\mathrm{m/s^4}\right) t^4}{4} \tag{18}$$

$$= \left(0.25\,\mathrm{m/s^3}\right) t^3 - \left(0.010\,\mathrm{m/s^4}\right) t^4 \tag{19}$$

(b)

$$0 = \left(1.50\,\mathrm{m/s^3}\right) t - \left(0.120\,\mathrm{m/s^4}\right) t^2 \tag{20}$$

$$t = \frac{-\left(1.50\,\mathrm{m/s^3}\right) \pm \sqrt{\left(1.50\,\mathrm{m/s^3}\right)^3 - 4\left(0.120\,\mathrm{m/s^4}\right)(0)}}{2\left(0.120\,\mathrm{m/s^4}\right)} \tag{21}$$

$$t = 0\,\mathrm{s} \text{ or } t = 12.5\,\mathrm{s} \tag{22}$$

$$v_x = \left(0.75\,\mathrm{m/s^3}\right)(12.5\,\mathrm{s})^2 - \left(0.040\,\mathrm{m/s^4}\right)(12.5\,\mathrm{s})^3 \tag{23}$$

$$\approx 39.1\,\mathrm{m/s} \tag{24}$$