2.7 A car is stopped at a traffic light. It then travels along a straight road such that its distance from the light is given by $x(t) = bt^2 - ct^3$, where $b = 2.40\,\mathrm{m/s^2}$ and $c = 0.120\,\mathrm{m/s^3}$. (a) Calculate the average velocity of the car for the time interval $t = 0$ to $t = 10.0\,\mathrm{s}$. (b) Calculate the instantaneous velocity of the car at $t = 0, t = 5.0\,\mathrm{s}$ and $t = 10.0\,\mathrm{s}$ (c) How long after starting from rest is the car again at rest?

Solution:

(a)

$$x_1 = x\left(0\,\mathrm{s}\right) = \left(2.40\,\mathrm{m/s^2}\right)\left(0\,\mathrm{s}\right)^2 - \left(0.120\,\mathrm{m/s^3}\right)\left(0\,\mathrm{s}\right)^3 = 0\,\mathrm{m} \tag{1}$$

$$x_2 = x\left(10.0\,\mathrm{s}\right) = \left(2.40\,\mathrm{m/s^2}\right)\left(10.0\,\mathrm{s}\right)^2 - \left(0.120\,\mathrm{m/s^3}\right)\left(10.0\,\mathrm{s}\right)^3 = 120\,\mathrm{m} \tag{2}$$

$$v_{\text{av-}x} = \frac{120\,\mathrm{m} - 0\,\mathrm{m}}{10.0\,\mathrm{s} - 0\,\mathrm{s}} = 12.0\,\mathrm{m/s} \tag{3}$$

(b)

$$v_x = \lim_{t\to 0}\frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{x\left(0.001\,\mathrm{s}\right) - x\left(0\,\mathrm{s}\right)}{0.001\,\mathrm{s} - 0\,\mathrm{s}} \approx 0.002\,\mathrm{m/s}, \quad \text{where } t = 0\,\mathrm{s} \tag{4}$$

$$= \lim_{t\to 0}\frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{x\left(1\times 10^{-9}\,\mathrm{s}\right) - x\left(0\,\mathrm{s}\right)}{1\times 10^{-9}\,\mathrm{s} - 0\,\mathrm{s}} \approx 0\,\mathrm{m/s}, \quad \text{where } t = 0\,\mathrm{s} \tag{5}$$

$$v_x = \lim_{t\to 0}\frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{x\left(5.001\,\mathrm{s}\right) - x\left(5.0\,\mathrm{s}\right)}{5.001\,\mathrm{s} - 5.0\,\mathrm{s}} \approx 15.0\,\mathrm{m/s}, \quad \text{where } t = 5\,\mathrm{s} \tag{6}$$

$$v_x = \lim_{t\to 0}\frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{x\left(10.001\,\mathrm{s}\right) - x\left(10.0\,\mathrm{s}\right)}{10.001\,\mathrm{s} - 10.0\,\mathrm{s}} \approx 12.0\,\mathrm{m/s}, \quad \text{where } t = 10.0\,\mathrm{s} \tag{7}$$

(c)

$$v_x = \frac{dx}{dt} = \frac{d}{dt}\left[\left(2.40\,\mathrm{m/s^2}\right)t^2 - \left(0.120\,\mathrm{m/s^3}\right)t^3\right] \tag{8}$$

$$0\,\mathrm{m/s} = \left(4.80\,\mathrm{m/s^2}\right)t - \left(0.360\,\mathrm{m/s^3}\right)t^2 \tag{9}$$

$$0\,\mathrm{m/s} = t\left(4.80\,\mathrm{m/s^2} - \left(0.360\,\mathrm{m/s^3}\right)t\right) \tag{10}$$

$$t = \frac{-4.80\,\mathrm{m/s^2}}{-0.360\,\mathrm{m/s^3}} \approx 13.3\,\mathrm{s} \tag{11}$$

2.9 A ball moves in a straight line (the x-axis). The graph in **Fig. E2.9** shows this ball's velocity as a function of time. (a) What are the ball's average speed and average velocity during the first 3.0 s? (b) Suppose that the ball moved in such a way that the graph segment after 2.0 s was -3.0 m/s instead of +3.0 m/s. Find the ball's average speed and average velocity in this case.

Solution:

(a)

$$s_{\text{av-}x} = \frac{|2| + |2| + |3|}{3} = \frac{7}{3} \approx 2.33\,\mathrm{m/s} \tag{12}$$

$$v_{\text{av-}x} = \frac{2 + 2 + 3}{3} = \frac{7}{3} \approx 2.33\,\mathrm{m/s} \tag{13}$$

(b)

$$s_{\text{av-}x} = \frac{|2| + |2| + |-3|}{3} = \frac{7}{3} \approx 2.33\,\text{m/s} \tag{14}$$

$$v_{\text{av-}x} = \frac{2 + 2 - 3}{3} = \frac{1}{3} \approx 0.33\,\text{m/s} \tag{15}$$

2.11 A test car travels in a straight line along the x-axis. The graph in **Fig. E2.11** shows the car's position x as a function of time. Find its instantaneous velocity at points A through G.

Solution:

$$A_{v_x} = \frac{B_y - A_y}{B_x - A_x} \approx \frac{35\,\text{m} - 24\,\text{m}}{2.1\,\text{s} - 0.4\,\text{s}} \approx 6.5\,\text{m/s} \tag{16}$$

$$B_{v_x} = A_{v_x} \approx 6.5\,\text{m/s} \tag{17}$$

$$C_{v_x} = 0\,\text{m/s} \tag{18}$$

$$D_{v_x} = \frac{E_y - D_y}{E_x - D_x} \approx \frac{0\,\text{m} - 30\,\text{m}}{6\,\text{s} - 5.2\,\text{s}} \approx -37.5\,\text{m/s} \tag{19}$$

$$E_{v_x} = D_{v_x} \approx -37.5\,\text{m/s} \tag{20}$$

$$F_{v_x} = E_{v_x} \approx -37.5\,\text{m/s} \tag{21}$$

$$G_{v_x} = 0\,\text{m/s} \tag{22}$$