1.27 Compute the x- and y-components of the vectors $\vec{A}, \vec{B}, \vec{C}$, and $\vec{D}$ in Fig. E1.24.

$$\vec{A}_x = 0\,\mathrm{m}, \quad \vec{A}_y = -8.0\,\mathrm{m} \tag{1}$$
$$\vec{B}_x = \|\vec{B}\| \cos\theta = (15.0\,\mathrm{m})\left(\cos\left(90.0^\circ - 30.0^\circ\right)\right) \approx 7.50\,\mathrm{m} \tag{2}$$
$$\vec{B}_y = \|\vec{B}\| \sin\theta = (15.0\,\mathrm{m})\left(\sin\left(90.0^\circ - 30.0^\circ\right)\right) \approx 13.0\,\mathrm{m} \tag{3}$$
$$\vec{C}_x = \|\vec{C}\| \cos\theta = (12.0\,\mathrm{m})\left(\cos\left(25.0^\circ + 180.^\circ\right)\right) \approx -10.9\,\mathrm{m} \tag{4}$$
$$\vec{C}_y = \|\vec{C}\| \sin\theta = (12.0\,\mathrm{m})\left(\sin\left(25.0^\circ + 180.^\circ\right)\right) \approx -5.07\,\mathrm{m} \tag{5}$$
$$\vec{D}_x = \|\vec{D}\| \cos\theta = (10.0\,\mathrm{m})\left(\cos\left(53.0^\circ + 90.0^\circ\right)\right) \approx -7.99\,\mathrm{m} \tag{6}$$
$$\vec{D}_y = \|\vec{D}\| \sin\theta = (10.0\,\mathrm{m})\left(\sin\left(53.0^\circ + 90.0^\circ\right)\right) \approx 6.02\,\mathrm{m} \tag{7}$$

1.29 Vector $\vec{A}$ has y-component $A_y = +9.60\,\mathrm{m}$. $\vec{A}$ makes an angle of 32.0° counterclockwise from the $+y$-axis. (a) What is the x-component of $\vec{A}$? (b) What is the magnitude of $\vec{A}$?

(a)

$$\tan\left(32.0^\circ + 90.0^\circ\right) = \frac{9.60\,\mathrm{m}}{\vec{A}_x} \tag{8}$$
$$\vec{A}_x = \frac{9.60\,\mathrm{m}}{\tan 122^\circ} \approx -6.00\,\mathrm{m} \tag{9}$$

(b)

$$9.60\,\mathrm{m} = A \sin\left(32.0^\circ + 90.0^\circ\right) \tag{10}$$
$$A = \frac{9.60}{\sin\left(32.0^\circ + 90.0^\circ\right)} \approx 11.3\,\mathrm{m} \tag{11}$$

1.31 For the vectors $\vec{A}$ and $\vec{B}$ in Fig. E1.24, use the method of components to find the magnitude and direction of (a) the vector sum $\vec{A} + \vec{B}$; (b) the vector sum $\vec{B} + \vec{A}$; (c) the vector difference $\vec{A} - \vec{B}$; (d) the vector difference $\vec{B} - \vec{A}$.

(a)

$$\|\vec{A} + \vec{B}\| = \sqrt{(0\,\mathrm{m} + 7.50\,\mathrm{m})^2 + (-8.0\,\mathrm{m} + 13.0\,\mathrm{m})^2} \approx 9.01\,\mathrm{m} \tag{12}$$
$$\angle(\vec{A} + \vec{B}) = \arctan\left(\frac{-8.0\,\mathrm{m} + 13.0\,\mathrm{m}}{0\,\mathrm{m} + 7.5\,\mathrm{m}}\right) \times \frac{180}{\pi} \approx 33.7^\circ \tag{13}$$

(b)

$$\vec{B} + \vec{A} = \langle 9.01\,\mathrm{m}, 33.7^\circ \rangle; \quad \text{Commutative Property of Addition} \tag{14}$$

(c)

$$\|\vec{A} - \vec{B}\| = \sqrt{(0\,\mathrm{m} - 7.50\,\mathrm{m})^2 + (-8.0\,\mathrm{m} - 13.0\,\mathrm{m})^2} \approx 19.6\,\mathrm{m} \tag{15}$$
$$\angle(\vec{A} - \vec{B}) = \arctan\left(\frac{-8.0\,\mathrm{m} - 13.0\,\mathrm{m}}{0\,\mathrm{m} - 7.5\,\mathrm{m}}\right) \times \frac{180}{\pi} \approx 70.3^\circ + 180^\circ \approx 250^\circ \tag{16}$$

(d)

$$\|\vec{B} - \vec{A}\| = \sqrt{(7.50\,\mathrm{m} - 0\,\mathrm{m})^2 + (13.0\,\mathrm{m} + 8.0\,\mathrm{m})^2} \approx 22.3\,\mathrm{m} \tag{17}$$
$$\angle(\vec{B} - \vec{A}) = \arctan\left(\frac{13.0\,\mathrm{m} + 8.0\,\mathrm{m}}{7.5\,\mathrm{m} - 0\,\mathrm{m}}\right) \times \frac{180}{\pi} \approx 70.3^\circ \tag{18}$$

1.33 A disoriented physics professor drives 3.25 km north, then 2.20 km west, and then 1.50 km south. Find the magnitude and direction of the resultant displacement, using the method of components. In a vector-addition diagram (roughly to scale), show that the resultant displacement found from your diagram is in qualitative agreement with the result you obtained by using the method of components.

$$\|\vec{R}\| = \sqrt{(2.20\,\text{km})^2 + (3.25\,\text{km} - 1.50\,\text{km})^2} \approx 2.81\,\text{km} \tag{19}$$

$$\angle\vec{R} = \arctan\left(\frac{3.25\,\text{km} - 1.50\,\text{km}}{2.20\,\text{km}}\right) \times \frac{180}{\pi} \approx 38.5^\circ \text{ north of west} \tag{20}$$

1.35 Vector $\vec{A}$ is 2.80 cm long and is 60.0° above the x-axis in the first quadrant. Vector $\vec{B}$ is 1.90 cm long and is 60.0° below the x-axis in the fourth quadrant (Fig. E1.35). Use components to find the magnitude and direction of (a) $\vec{A} + \vec{B}$; (b) $\vec{A} - \vec{B}$; (c) $\vec{B} - \vec{A}$. In each case, sketch the vector addition or subtraction and show that your numerical answers are in qualitative agreement with your sketch.

Find the components of $\vec{A}$ and $\vec{B}$:

$$\vec{A}_x = \|\vec{A}\| \cos\theta = (2.80\,\text{cm})\,(\cos(60.0^\circ)) \approx 1.40\,\text{cm} \tag{21}$$

$$\vec{A}_y = \|\vec{A}\| \sin\theta = (2.80\,\text{cm})\,(\sin(60.0^\circ)) \approx 2.42\,\text{cm} \tag{22}$$

$$\vec{B}_x = \|\vec{B}\| \cos\theta = (1.90\,\text{cm})\,(\cos(360^\circ - 60.0^\circ)) \approx 0.95\,\text{cm} \tag{23}$$

$$\vec{B}_y = \|\vec{B}\| \sin\theta = (1.90\,\text{cm})\,(\sin(360^\circ - 60.0^\circ)) \approx -1.65\,\text{cm} \tag{24}$$

(a)

$$\|\vec{A} + \vec{B}\| = \sqrt{\left(\vec{A}_x + \vec{B}_x\right)^2 + \left(\vec{A}_y + \vec{B}_y\right)^2} \approx 2.48\,\text{km} \tag{25}$$

$$\angle(\vec{A} + \vec{B}) = \arctan\left(\frac{\vec{A}_y + \vec{B}_y}{\vec{A}_x + \vec{B}_x}\right) \times \frac{180}{\pi} \approx 18.3^\circ \tag{26}$$

(b)

$$\|\vec{A} - \vec{B}\| = \sqrt{\left(\vec{A}_x - \vec{B}_x\right)^2 + \left(\vec{A}_y - \vec{B}_y\right)^2} \approx 4.10\,\text{km} \tag{27}$$

$$\angle(\vec{A} - \vec{B}) = \arctan\left(\frac{\vec{A}_y - \vec{B}_y}{\vec{A}_x - \vec{B}_x}\right) \times \frac{180}{\pi} \approx 83.7^\circ \tag{28}$$

(c)

$$\|\vec{B} - \vec{A}\| = \sqrt{\left(\vec{B}_x - \vec{A}_x\right)^2 + \left(\vec{B}_y - \vec{A}_y\right)^2} \approx 4.10\,\text{km} \tag{29}$$

$$\angle(\vec{B} - \vec{A}) = \arctan\left(\frac{\vec{B}_y - \vec{A}_y}{\vec{B}_x - \vec{A}_x}\right) \times \frac{180}{\pi} \approx 83.7^\circ + 180^\circ \approx 264^\circ \tag{30}$$