**3.1** A squirrel has x- and y-coordinates (1.1 m, 3.4 m) at time  $t_1 = 0$  and coordinates (5.3 m, -0.5 m) at time  $t_2 = 3.0$  s. For this time interval, find (a) the components of the average velocity, and (b) the magnitude and direction of the average velocity

## **Solution:**

(a)

$$\vec{v}_{\text{av-}x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{(5.3 \,\text{m}) - (1.1 \,\text{m})}{3.0 \,\text{s} - 0} = \frac{4.2 \,\text{m}}{3.0 \,\text{s}} = \frac{7 \,\text{m}}{5 \,\text{s}} \approx 1.4 \,\text{m/s}$$
 (1)

$$\vec{v}_{\text{av-}y} = \frac{y_2 - y_1}{t_2 - t_1} = \frac{(-0.5 \,\text{m}) - (3.4 \,\text{m})}{3.0 \,\text{s} - 0} = \frac{-3.9 \,\text{m}}{3.0 \,\text{s}} = -\frac{13 \,\text{m}}{10 \,\text{s}} = -1.3 \,\text{m/s}$$
(2)

(b)

$$\|\vec{v}_{\text{av}}\| = \sqrt{\vec{v}_{\text{av-}x}^2 + \vec{v}_{\text{av-}y}^2} = \sqrt{(1.4 \,\text{m/s})^2 + (-1.3 \,\text{m/s})^2} \approx 1.9 \,\text{m/s}$$
 (3)

$$\tan \alpha = \frac{\vec{v}_{\text{av-}y}}{\vec{v}_{\text{av-}x}} = \frac{-1.3 \,\text{m/s}}{1.4 \,\text{m/s}}$$
 (4)

$$\alpha = 360^{\circ} - \text{rad2deg}\left(\arctan\left(\frac{-1.3 \,\text{m/s}}{1.4 \,\text{m/s}}\right)\right) \approx 317^{\circ}$$
 (5)

**3.3** A web page designer creates an animation in which a dot on a computer screen has position

$$\hat{r} = \left[4.0 \,\text{cm} + \left(2.5 \,\text{cm/s}^2\right) t^2\right] \hat{\imath} + (5.0 \,\text{cm/s}) t \hat{\jmath}.$$

(a) Find the magnitude and direction of the dot's average velocity between t=0 and t=2.0 s. (b) Find the magnitude and direction of the instantaneous velocity at t=0, t=1.0 s, and t=2.0 s. (c) Sketch the dot's trajectory from t=0 to t=2.0 s, and show the velocities calculated in part (b).

## **Solution:**

(a)

$$\vec{r}(2.0 \,\mathrm{s}) = \left[4.0 \,\mathrm{cm} + \left(2.5 \,\mathrm{cm/s^2}\right) \left(2.0 \,\mathrm{s}\right)^2\right] \hat{\imath} + \left(5.0 \,\mathrm{cm/s}\right) \left(2.0 \,\mathrm{s}\right) \hat{\jmath} \tag{6}$$

$$= (14 \,\mathrm{cm}) \,\hat{\imath} + (10 \,\mathrm{cm}) \,\hat{\jmath} \tag{7}$$

$$\vec{r}(0) = \left[4.0 \,\text{cm} + \left(2.5 \,\text{cm/s}^2\right) \left(0\right)^2\right] \hat{\imath} + \left(5.0 \,\text{cm/s}\right) \left(0\right) \hat{\jmath} \tag{8}$$

$$= (4.0 \,\mathrm{cm}) \,\hat{\imath} \tag{9}$$

$$\vec{v}_{\text{av}} = \frac{\vec{r}(2.0\,\text{s}) - \vec{r}(0)}{2.0\,\text{s} - 0} = \frac{(10\,\text{cm})\,\hat{\imath} + (10\,\text{cm})\,\hat{\jmath}}{2.0\,\text{s}}$$
(10)

$$= (5.0 \,\mathrm{cm/s}) \,\hat{\imath} + (5.0 \,\mathrm{cm/s}) \,\hat{\jmath} \tag{11}$$

$$\|\vec{v}_{\text{av}}\| = \sqrt{\vec{v}_{\text{av-}x}^2 + \vec{v}_{\text{av-}y}^2} = \sqrt{(5.0 \,\text{cm/s})^2 + (5.0 \,\text{cm/s})^2} \approx 7.1 \,\text{m}$$
 (12)

$$\tan \alpha = \frac{\vec{v}_{\text{av-}y}}{\vec{v}_{\text{ov},x}} \tag{13}$$

$$\alpha = \text{rad2deg}\left(\arctan\left(\frac{5.0 \text{ cm/s}}{5.0 \text{ cm/s}}\right)\right) = 45^{\circ}$$
 (14)

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$
(15)

$$= \frac{d}{dt} \left[ 4.0 \,\text{cm} + \left( 2.5 \,\text{cm/s}^2 \right) t^2 \right] \hat{i} + \frac{d}{dt} \left( 5.0 \,\text{cm/s} \right) t \hat{j} \tag{16}$$

$$= (5.0 \,\mathrm{cm/s}) \,t\hat{\imath} + (5.0 \,\mathrm{cm/s}) \,\hat{\jmath} \tag{17}$$

$$\|\vec{v}\| = \sqrt{\left[ (5.0 \,\text{cm/s}) \,t \right]^2 + (5.0 \,\text{cm/s})^2} = \sqrt{(25 \,\text{cm/s}) \,t^2 + 25 \,\text{cm/s}}$$
 (18)

$$\tan \alpha = \frac{\vec{v}_y}{\vec{v}_r} \tag{19}$$

$$\alpha = \operatorname{rad2deg}\left(\arctan\left(\frac{5.0\,\mathrm{cm/s}}{(5.0\,\mathrm{cm/s})\,t}\right)\right) \tag{20}$$

$$\vec{v}(0) = (5.0 \,\text{cm/s})(0)\,\hat{i} + (5.0 \,\text{cm/s})\,\hat{j} = (5.0 \,\text{cm/s})\,\hat{j}$$
 (21)

$$\|\vec{v}(0)\| = 5.0 \,\text{cm/s}, \quad \alpha_{\vec{v}(0)} = 90^{\circ}$$
 (22)

$$\vec{v}(1.0\,\mathrm{s}) = (5.0\,\mathrm{cm/s})(1.0\,\mathrm{s})\,\hat{\imath} + (5.0\,\mathrm{cm/s})\,\hat{\jmath} = (5.0\,\mathrm{cm/s})\,\hat{\imath} + (5.0\,\mathrm{cm/s})\,\hat{\jmath} \tag{23}$$

$$\|\vec{v}(1.0 \,\mathrm{s})\| \approx 7.1 \,\mathrm{cm/s}, \quad \alpha_{\vec{v}(1.0 \,\mathrm{s})} = 45^{\circ}$$
 (24)

$$\vec{v}(2.0\,\text{s}) = (5.0\,\text{cm/s})(2.0\,\text{s})\,\hat{\imath} + (5.0\,\text{cm/s})\,\hat{\jmath} = (10\,\text{cm/s})\,\hat{\imath} + (5.0\,\text{cm/s})\,\hat{\jmath} \tag{25}$$

$$\|\vec{v}(2.0 \,\mathrm{s})\| \approx 11 \,\mathrm{cm/s}, \quad \alpha_{\vec{v}(2.0 \,\mathrm{s})} \approx 27^{\circ}$$
 (26)

(c)

