

**1.43** For the vectors,  $\vec{A}$ ,  $\vec{B}$ , and  $\vec{C}$  in Fig. E1.24, find the scalar products (a)  $\vec{A} \cdot \vec{B}$ ; (b)  $\vec{B} \cdot \vec{C}$ ; (c)  $\vec{A} \cdot \vec{C}$ .

(a)

$$\vec{A} \cdot \vec{B} = \|A\| \|B\| \cos \phi = (8.00 \text{ m}) (15.0 \text{ m}) \cos (270^\circ - 60.0^\circ) \approx -104 \text{ m}^2 \quad (1)$$

(b)

$$\vec{B} \cdot \vec{C} = \|B\| \|C\| \cos \phi = (15.0 \text{ m}) (12.0 \text{ m}) \cos ((180^\circ + 25.0^\circ) - 60.0^\circ) \approx -147 \text{ m}^2 \quad (2)$$

(c)

$$\vec{A} \cdot \vec{C} = \|A\| \|C\| \cos \phi = (8.00 \text{ m}) (12.0 \text{ m}) \cos (270^\circ - (180^\circ + 25.0^\circ)) \approx 40.6 \text{ m}^2 \quad (3)$$

**1.45** Find the angle between each of these pairs of vectors:

(a)  $\vec{A} = -2.00\hat{i} + 6.00\hat{j}$  and  $\vec{B} = 2.00\hat{i} - 3.00\hat{j}$

(b)  $\vec{A} = 3.00\hat{i} + 5.00\hat{j}$  and  $\vec{B} = 10.00\hat{i} + 6.00\hat{j}$

(c)  $\vec{A} = -4.00\hat{i} + 2.00\hat{j}$  and  $\vec{B} = 7.00\hat{i} + 14.00\hat{j}$

(a)

$$\cos \phi = \frac{\vec{A} \cdot \vec{B}}{\|\vec{A}\| \|\vec{B}\|} = \frac{\vec{A}_x \vec{B}_x + \vec{A}_y \vec{B}_y}{\|\vec{A}\| \|\vec{B}\|} \quad (4)$$

$$= \frac{(-2.00)(2.00) + (6.00)(-3.00)}{\sqrt{(-2.00)^2 + (6.00)^2} \sqrt{(2.00)^2 + (-3.00)^2}} \approx -0.965 \quad (5)$$

$$\phi = \arccos(-0.965) \approx 2.88 \times \frac{180}{\pi} \approx 165^\circ \quad (6)$$

(b)

$$\cos \phi = \frac{\vec{A} \cdot \vec{B}}{\|\vec{A}\| \|\vec{B}\|} = \frac{\vec{A}_x \vec{B}_x + \vec{A}_y \vec{B}_y}{\|\vec{A}\| \|\vec{B}\|} \quad (7)$$

$$= \frac{(3.00)(10.00) + (5.00)(6.00)}{\sqrt{(3.00)^2 + (5.00)^2} \sqrt{(10.00)^2 + (6.00)^2}} \approx 0.882 \quad (8)$$

$$\phi = \arccos(0.882) \approx 0.490 \times \frac{180}{\pi} \approx 28.1^\circ \quad (9)$$

(c)

$$\cos \phi = \frac{\vec{A} \cdot \vec{B}}{\|\vec{A}\| \|\vec{B}\|} = \frac{\vec{A}_x \vec{B}_x + \vec{A}_y \vec{B}_y}{\|\vec{A}\| \|\vec{B}\|} \quad (10)$$

$$= \frac{(-4.00)(7.00) + (2.00)(14.00)}{\sqrt{(-4.00)^2 + (2.00)^2} \sqrt{(7.00)^2 + (14.00)^2}} \approx 0.00 \quad (11)$$

$$\phi = \arccos(0.00) \approx 1.57 \times \frac{180}{\pi} \approx 90.0^\circ \quad (12)$$

**1.47** For the two vectors  $\vec{A}$  and  $\vec{D}$  in Fig. E1.24, find the magnitude and direction of (a) the vector product  $\vec{A} \times \vec{D}$ ; (b) the vector product  $\vec{D} \times \vec{A}$ .

(a)

$$\vec{A} \times \vec{D} = \|\vec{A}\| \|\vec{D}\| \sin \phi \quad (13)$$

$$= (-8.00 \text{ m}) (10.0 \text{ m}) \sin (270^\circ - (90.0^\circ + 53.0^\circ)) \approx (-63.9 \text{ m}^2) \hat{k} \quad (14)$$

(b)

$$\vec{D} \times \vec{A} = \|\vec{D}\| \|\vec{A}\| \sin \phi \quad (15)$$

$$= (10.0 \text{ m}) (-8.00 \text{ m}) \sin ((90.0^\circ + 53.0^\circ) - 270^\circ) \approx (63.9 \text{ m}^2) \hat{k} \quad (16)$$