1.43 For the vectors, $\vec{A}, \vec{B}$, and $\vec{C}$ in Fig. E1.24, find the scalar products (a) $\vec{A} \cdot \vec{B}$; (b) $\vec{B} \cdot \vec{C}$; (c) $\vec{A} \cdot \vec{C}$.

(a)

$$\vec{A} \cdot \vec{B} = \|A\|\|B\| \cos\phi = (8.00\,\mathrm{m})\,(15.0\,\mathrm{m}) \cos\left(270^\circ - 60.0^\circ\right) \approx -104\,\mathrm{m}^2 \tag{1}$$

(b)

$$\vec{B} \cdot \vec{C} = \|B\|\|C\| \cos\phi = (15.0\,\mathrm{m})\,(12.0\,\mathrm{m}) \cos\left((180^\circ + 25.0^\circ) - 60.0^\circ\right) \approx -147\,\mathrm{m}^2 \tag{2}$$

(c)

$$\vec{A} \cdot \vec{C} = \|A\|\|C\| \cos\phi = (8.00\,\mathrm{m})\,(12.0\,\mathrm{m}) \cos\left(270^\circ - (180^\circ + 25.0^\circ)^\circ\right) \approx 40.6\,\mathrm{m}^2 \tag{3}$$

1.45 Find the angle between each of these pairs of vectors:

(a) $\vec{A} = -2.00\hat{\imath} + 6.00\hat{\jmath}$ and $\vec{B} = 2.00\hat{\imath} - 3.00\hat{\jmath}$
(b) $\vec{A} = 3.00\hat{\imath} + 5.00\hat{\jmath}$ and $\vec{B} = 10.00\hat{\imath} + 6.00\hat{\jmath}$
(c) $\vec{A} = -4.00\hat{\imath} + 2.00\hat{\jmath}$ and $\vec{B} = 7.00\hat{\imath} + 14.00\hat{\jmath}$

(a)

$$\cos\phi = \frac{\vec{A} \cdot \vec{B}}{\|\vec{A}\|\|\vec{B}\|} = \frac{\vec{A}_x \vec{B}_x + \vec{A}_y \vec{B}_y}{\|\vec{A}\|\|\vec{B}\|} \tag{4}$$

$$= \frac{(-2.00)\,(2.00) + (6.00)\,(-3.00)}{\sqrt{(-2.00)^2 + (6.00)^2}\sqrt{(2.00)^2 + (-3.00)^2}} \approx -0.965 \tag{5}$$

$$\phi = \arccos\left(-0.965\right) \approx 2.88 \times \frac{180}{\pi} \approx 165^\circ \tag{6}$$

(b)

$$\cos\phi = \frac{\vec{A} \cdot \vec{B}}{\|\vec{A}\|\|\vec{B}\|} = \frac{\vec{A}_x \vec{B}_x + \vec{A}_y \vec{B}_y}{\|\vec{A}\|\|\vec{B}\|} \tag{7}$$

$$= \frac{(3.00)\,(10.00) + (5.00)\,(6.00)}{\sqrt{(3.00)^2 + (5.00()^2}\sqrt{(10.00)^2 + (6.00)^2}} \approx 0.882 \tag{8}$$

$$\phi = \arccos\left(0.882\right) \approx 0.490 \times \frac{180}{\pi} \approx 28.1^\circ \tag{9}$$

(c)

$$\cos\phi = \frac{\vec{A} \cdot \vec{B}}{\|\vec{A}\|\|\vec{B}\|} = \frac{\vec{A}_x \vec{B}_x + \vec{A}_y \vec{B}_y}{\|\vec{A}\|\|\vec{B}\|} \tag{10}$$

$$= \frac{(-4.00)\,(7.00) + (2.00)\,(14.00)}{\sqrt{(-4.00)^2 + (2.00)^2}\sqrt{(7.00)^2 + (14.00)^2}} \approx 0.00 \tag{11}$$

$$\phi = \arccos\left(0.00\right) \approx 1.57 \times \frac{180}{\pi} \approx 90.0^\circ \tag{12}$$

1.47 For the two vectors $\vec{A}$ and $\vec{D}$ in Fig. E1.24, find the magnitude and direction of (a) the vector product $\vec{A} \times \vec{D}$; (b) the vector product $\vec{D} \times \vec{A}$.

(a)

$$\vec{A} \times \vec{D} = \|\vec{A}\|\|\vec{D}\| \sin\phi \tag{13}$$

$$= (-8.00\,\mathrm{m})\,(10.0\,\mathrm{m}) \sin\left(270^\circ - (90.0^\circ + 53.0^\circ)\right) \approx \left(-63.9\,\mathrm{m}^2\right)\hat{k} \tag{14}$$

(b)

$$\vec{D} \times \vec{A} = \|\vec{D}\|\|\vec{A}\| \sin\phi \tag{15}$$

$$= (10.0\,\mathrm{m})\,(-8.00\,\mathrm{m}) \sin\left((90.0^\circ + 53.0^\circ) - 270^\circ\right) \approx \left(63.9\,\mathrm{m}^2\right)\hat{k} \tag{16}$$