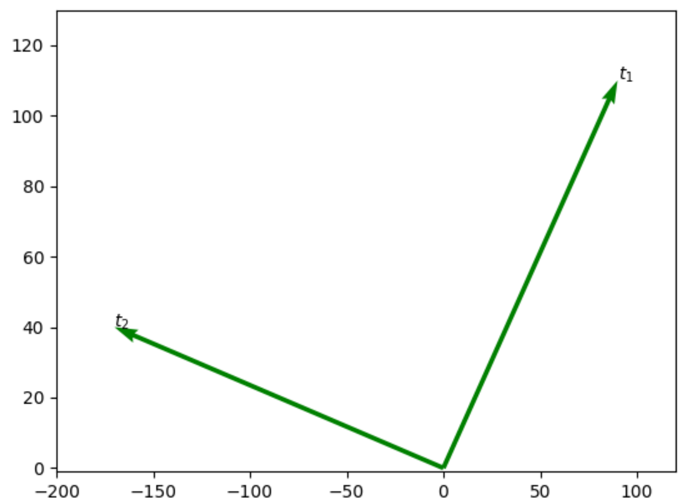
3.5 A jet plane is flying at a constant altitude. At time $t_1 = 0$, it has components of velocity $v_x = 90$ m/s, $v_y = 110$ m/s. At time $t_2 = 30.0$ s, the components are $v_x = -170$ m/s, $v_y = 40$ m/s. (a) Sketch the velocity vectors at t_1 and t_2. How do these two vectors differ? For this time interval calculate (b) the components of the average acceleration, and (c) the magnitude and direction of the average acceleration.

Solution:



The two vectors differ in their direction on the x-axis.

$$\vec{a}_{\text{av}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{(-170\,\text{m/s} - 90\,\text{m/s})\,\hat{\imath} + (40\,\text{m/s} - 110\,\text{m/s})\,\hat{\jmath}}{30.0\,\text{s} - 0} \tag{1}$$

$$= \frac{(-260\,\text{m/s})\,\hat{\imath} + (70\,\text{m/s})\,\hat{\jmath}}{30.0\,\text{s}} = \left(-\frac{26}{3}\,\text{m/s}^2\right)\hat{\imath} + \left(-\frac{7}{3}\,\text{m/s}^2\right)\hat{\jmath} \tag{2}$$

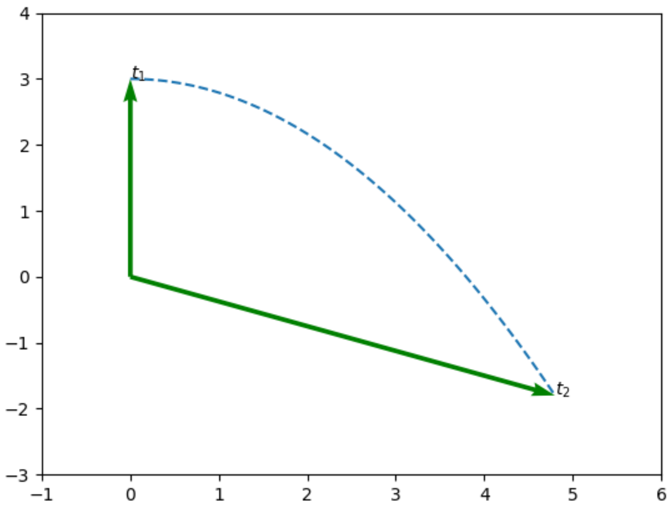
$$\|\vec{a}_{\text{av}}\| = \sqrt{\vec{a}_{\text{av-}x}{}^2 + \vec{a}_{\text{av-}y}{}^2} = \sqrt{\left(-\frac{26}{3}\,\text{m/s}^2\right)^2 + \left(-\frac{7}{3}\,\text{m/s}^2\right)^2} \approx 8.98\,\text{m/s}^2 \tag{3}$$

$$\tan\beta = \frac{\vec{a}_{\text{av-}y}}{\vec{a}_{\text{av-}x}} \Rightarrow \beta = 180^\circ + \text{rad2deg}\left(\arctan\left(\frac{-\frac{7}{3}\,\text{m/s}^2}{-\frac{26}{3}\,\text{m/s}^2}\right)\,\text{m/s}^2\right) \approx 195^\circ \tag{4}$$

3.7 The coordinates of a bird flying in the xy-plane are given by $x(t) = \alpha t$ and $y(t) = 3.0\,\mathrm{m} - \beta t^2$, where $\alpha = 2.4\,\mathrm{m/s}$ and $\beta = 1.2\,\mathrm{m/s^2}$. (a) Sketch the path of the bird between $t = 0$ and $t = 2.0\,\mathrm{s}$. (b) Calculate the velocity and acceleration vectors of the bird as functions of time. (c) Calculate the magnitude and direction of the bird's velocity and acceleration at $t = 2.0\,\mathrm{s}$. (d) Sketch the velocity and acceleration vectors at $t = 2.0\,\mathrm{s}$. At this instant, is the bird's speed increasing, decreasing, or not changing? Is the bird turning? If so, in what direction?

Solution:

(a)



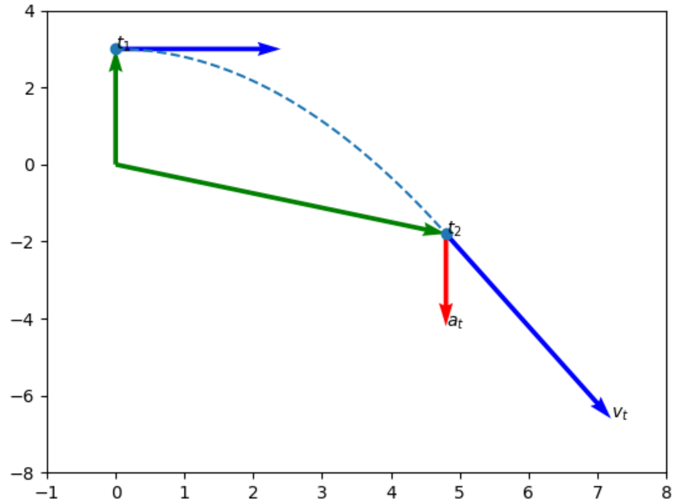
(b)

$$\vec{r} = (x(t))\,\hat{\imath} + (y(t))\,\hat{\jmath} \tag{5}$$
$$\vec{v} = (x'(t))\,\hat{\imath} + (y'(t))\,\hat{\jmath} \tag{6}$$
$$= (\alpha)\,\hat{\imath} + (-2\beta t)\,\hat{\jmath} \tag{7}$$
$$\vec{a} = (x''(t))\,\hat{\imath} + (y''(t))\,\hat{\jmath} \tag{8}$$
$$= (0)\,\hat{\imath} + (-2\beta)\,\hat{\jmath} \tag{9}$$

(c)

$$\|\vec{v}\| = \sqrt{(2.4\,\mathrm{m/s})^2 + \left(-2\left(1.2\,\mathrm{m/s^2}\right)(2.0\,\mathrm{s})\right)^2} \approx 5.4\,\mathrm{m/s} \tag{10}$$
$$\alpha = \mathrm{rad2deg}\left(\arctan\left(\frac{-2\left(1.2\,\mathrm{m/s^2}\right)(2.0\,\mathrm{s})}{(2.4\,\mathrm{m/s})}\right)\right) + 360^\circ \approx 297^\circ \tag{11}$$
$$\|\vec{a}\| = -2\left(1.2\,\mathrm{m/s^2}\right) \approx -2.4\,\mathrm{m/s^2} \tag{12}$$
$$\beta = \mathrm{rad2deg}\left(\arctan\left(\frac{\left(-2.4\,\mathrm{m/s^2}\right)}{0}\right)\right) + 180^\circ = 270^\circ \tag{13}$$

(d)



$$\text{At this instant, where } t = 2.0\,\text{s, the bird's speed is increasing and is turning right.} \tag{14}$$