3.1 A squirrel has x- and y-coordinates (1.1 m, 3.4 m) at time $t_1 = 0$ and coordinates (5.3 m, -0.5 m) at time $t_2 = 3.0$ s. For this time interval, find (a) the components of the average velocity, and (b) the magnitude and direction of the average velocity

Solution:

(a)

$$\vec{v}_{\text{av-}x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{(5.3\,\text{m}) - (1.1\,\text{m})}{3.0\,\text{s} - 0} = \frac{4.2\,\text{m}}{3.0\,\text{s}} = \frac{7\,\text{m}}{5\,\text{s}} \approx 1.4\,\text{m/s} \tag{1}$$

$$\vec{v}_{\text{av-}y} = \frac{y_2 - y_1}{t_2 - t_1} = \frac{(-0.5\,\text{m}) - (3.4\,\text{m})}{3.0\,\text{s} - 0} = \frac{-3.9\,\text{m}}{3.0\,\text{s}} = -\frac{13\,\text{m}}{10\,\text{s}} = -1.3\,\text{m/s} \tag{2}$$

(b)

$$\|\vec{v}_{\text{av}}\| = \sqrt{\vec{v}_{\text{av-}x}{}^2 + \vec{v}_{\text{av-}y}{}^2} = \sqrt{(1.4\,\text{m/s})^2 + (-1.3\,\text{m/s})^2} \approx 1.9\,\text{m/s} \tag{3}$$

$$\tan\alpha = \frac{\vec{v}_{\text{av-}y}}{\vec{v}_{\text{av-}x}} = \frac{-1.3\,\text{m/s}}{1.4\,\text{m/s}} \tag{4}$$

$$\alpha = 360^\circ - \text{rad2deg}\left(\arctan\left(\frac{-1.3\,\text{m/s}}{1.4\,\text{m/s}}\right)\right) \approx 317^\circ \tag{5}$$

3.3 A web page designer creates an animation in which a dot on a computer screen has position

$$\hat{r} = \left[4.0\,\text{cm} + \left(2.5\,\text{cm/s}^2\right) t^2\right] \hat{\imath} + (5.0\,\text{cm/s})\, t\hat{\jmath}.$$

(a) Find the magnitude and direction of the dot's average velocity between $t = 0$ and $t = 2.0$ s. (b) Find the magnitude and direction of the instantaneous velocity at $t = 0$, $t = 1.0$ s, and $t = 2.0$ s. (c) Sketch the dot's trajectory from $t = 0$ to $t = 2.0$ s, and show the velocities calculated in part (b).

Solution:

(a)

$$\vec{r}\,(2.0\,\text{s}) = \left[4.0\,\text{cm} + \left(2.5\,\text{cm/s}^2\right)(2.0\,\text{s})^2\right] \hat{\imath} + (5.0\,\text{cm/s})\,(2.0\,\text{s})\,\hat{\jmath} \tag{6}$$

$$= (14\,\text{cm})\,\hat{\imath} + (10\,\text{cm})\,\hat{\jmath} \tag{7}$$

$$\vec{r}\,(0) = \left[4.0\,\text{cm} + \left(2.5\,\text{cm/s}^2\right)(0)^2\right] \hat{\imath} + (5.0\,\text{cm/s})\,(0)\,\hat{\jmath} \tag{8}$$

$$= (4.0\,\text{cm})\,\hat{\imath} \tag{9}$$

$$\vec{v}_{\text{av}} = \frac{\vec{r}\,(2.0\,\text{s}) - \vec{r}\,(0)}{2.0\,\text{s} - 0} = \frac{(10\,\text{cm})\,\hat{\imath} + (10\,\text{cm})\,\hat{\jmath}}{2.0\,\text{s}} \tag{10}$$

$$= (5.0\,\text{cm/s})\,\hat{\imath} + (5.0\,\text{cm/s})\,\hat{\jmath} \tag{11}$$

$$\|\vec{v}_{\text{av}}\| = \sqrt{\vec{v}_{\text{av-}x}{}^2 + \vec{v}_{\text{av-}y}{}^2} = \sqrt{(5.0\,\text{cm/s})^2 + (5.0\,\text{cm/s})^2} \approx 7.1\,\text{m/s} \tag{12}$$

$$\tan\alpha = \frac{\vec{v}_{\text{av-}y}}{\vec{v}_{\text{av-}x}} \tag{13}$$

$$\alpha = \text{rad2deg}\left(\arctan\left(\frac{5.0\,\text{cm/s}}{5.0\,\text{cm/s}}\right)\right) = 45^\circ \tag{14}$$

(b)

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k} \tag{15}$$

$$= \frac{d}{dt}\left[4.0\,\mathrm{cm} + \left(2.5\,\mathrm{cm/s^2}\right)t^2\right]\hat{\imath} + \frac{d}{dt}\left(5.0\,\mathrm{cm/s}\right)t\hat{\jmath} \tag{16}$$

$$= \left(5.0\,\mathrm{cm/s}\right)t\hat{\imath} + \left(5.0\,\mathrm{cm/s}\right)\hat{\jmath} \tag{17}$$

$$\|\vec{v}\| = \sqrt{\left[\left(5.0\,\mathrm{cm/s}\right)t\right]^2 + \left(5.0\,\mathrm{cm/s}\right)^2} = \sqrt{\left(25\,\mathrm{cm/s}\right)t^2 + 25\,\mathrm{cm/s}} \tag{18}$$

$$\tan\alpha = \frac{\vec{v}_y}{\vec{v}_x} \tag{19}$$

$$\alpha = \mathrm{rad2deg}\left(\arctan\left(\frac{5.0\,\mathrm{cm/s}}{\left(5.0\,\mathrm{cm/s}\right)t}\right)\right) \tag{20}$$

$$\vec{v}\left(0\right) = \left(5.0\,\mathrm{cm/s}\right)\left(0\right)\hat{\imath} + \left(5.0\,\mathrm{cm/s}\right)\hat{\jmath} = \left(5.0\,\mathrm{cm/s}\right)\hat{\jmath} \tag{21}$$

$$\|\vec{v}\left(0\right)\| = 5.0\,\mathrm{cm/s}, \quad \alpha_{\vec{v}(0)} = 90^\circ \tag{22}$$

$$\vec{v}\left(1.0\,\mathrm{s}\right) = \left(5.0\,\mathrm{cm/s}\right)\left(1.0\,\mathrm{s}\right)\hat{\imath} + \left(5.0\,\mathrm{cm/s}\right)\hat{\jmath} = \left(5.0\,\mathrm{cm/s}\right)\hat{\imath} + \left(5.0\,\mathrm{cm/s}\right)\hat{\jmath} \tag{23}$$

$$\|\vec{v}\left(1.0\,\mathrm{s}\right)\| \approx 7.1\,\mathrm{cm/s}, \quad \alpha_{\vec{v}(1.0\,\mathrm{s})} = 45^\circ \tag{24}$$

$$\vec{v}\left(2.0\,\mathrm{s}\right) = \left(5.0\,\mathrm{cm/s}\right)\left(2.0\,\mathrm{s}\right)\hat{\imath} + \left(5.0\,\mathrm{cm/s}\right)\hat{\jmath} = \left(10\,\mathrm{cm/s}\right)\hat{\imath} + \left(5.0\,\mathrm{cm/s}\right)\hat{\jmath} \tag{25}$$

$$\|\vec{v}\left(2.0\,\mathrm{s}\right)\| \approx 11\,\mathrm{cm/s}, \quad \alpha_{\vec{v}(2.0\,\mathrm{s})} \approx 27^\circ \tag{26}$$

(c)