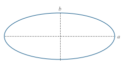
An ellipse has the equation:

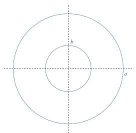
$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1} \qquad (1)$$

And can be drawn by:

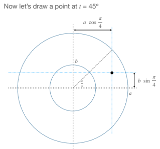
$x = a\ \cos(t) \quad y = b\ \sin(t) \quad 0 \le t \le 2\pi$



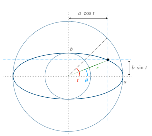
So let's draw. Start with two circles with radius a and b.



Now let's draw a point at t = 45°



This actually draws the ellipse point at a different angle



So the ellipse angle is then:

$$\tan\theta = \frac{b\ \sin t}{a\ \cos t}$$

$$\tan\theta = \frac{b}{a}\tan t$$

$$\boxed{\theta = \arctan\left(\frac{b}{a}\tan t\right)} \qquad (2)$$

$$\boxed{t = \arctan\left(\frac{a}{b}\tan\theta\right)} \qquad (3)$$

So how to find r at θ?

Substitute x and y coordinates in terms of r into (1):

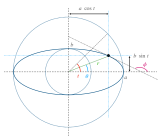
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{(r\ \cos\theta)^2}{a^2} + \frac{(r\ \sin\theta)^2}{b^2} = 1$$

$$r^2b^2\cos^2\theta + r^2a^2\sin^2\theta = a^2b^2$$

$$\boxed{r(\theta) = \frac{a^2b^2}{\sqrt{(b\ \cos\theta)^2 + (a\ \sin\theta)^2}}} \qquad (4)$$

How to find angle θ from a tangent line with angle $ø$?



Gradient of the tangent is:

$$m = \frac{\Delta y}{\Delta x}$$

Where:

$$\Delta y = \frac{d\,y}{d\,x} = \frac{d}{d\,x} b\ \sin t = b\ \cos t$$

$$\Delta x = \frac{d\,x}{d\,y} = \frac{d}{d\,y} a\ \cos t = -\,a\ \sin t$$

Therefore:

$$m = \frac{\Delta y}{\Delta x} = -\frac{b}{a}\cot t = -\frac{b}{a\ \tan t} = \frac{b\ \cos t}{-a\ \sin t}$$

The gradient is related to ø with the tan function:

$$\tan\phi = m = -\frac{b}{a\ \tan t}$$

$$\boxed{\phi = \arctan\left(-\frac{b}{a\ \tan t}\right) = \arctan\left(\frac{b\ \cos t}{-a\ \sin t}\right)} \qquad (5)$$

$$\boxed{t = \arctan\left(-\frac{b}{a\ \tan\phi}\right)} \qquad (6)$$

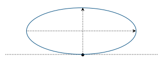
Substituting (6) into (2)

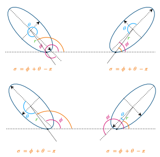
$$\theta = \arctan\left(\frac{b}{a}\tan t\right)$$

$$\theta = \arctan\left(-\frac{b}{a}\frac{b}{a\ \tan\phi}\right)$$

$$\boxed{\theta = \arctan\left(-\frac{b^2}{a^2\tan\phi}\right)} \qquad (7)$$

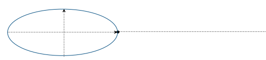
So we have a point, and an ellipse. We want to rotate the ellipse and then position it so that its horizontal tangent is touching the point

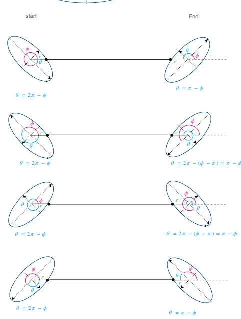




1. Find θ from $ø$ with (7)
2. Find r from θ with (4)
3. Find σ from $\quad \sigma = \phi + \theta - \pi$
4. x offset: $\quad r\ \cos\sigma$
5. y offset: $\quad r\ \sin\sigma$

If the label is on the end, then it is slightly different





$$\boxed{\theta = 2\pi - \phi} \qquad (8)$$

$$\boxed{\theta = \pi - \phi} \qquad (9)$$

1. Find θ from $ø$ with (8) for start of line and (9) for end of line
2. Find r from θ with (4)
3. x offset: -r for start of line, +r for end of line
4. y offset: 0