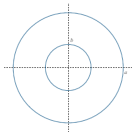
An ellipse has the equation:

$$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1} \quad (1)$$

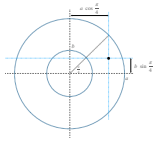
And can be drawn by:

$x = a\cos(t) \quad y = b\sin(t) \quad 0 \le t \le 2\pi$

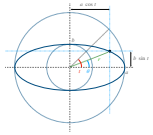
So let's draw. Start with two circles with radius a and b.



Now let's draw a point at $t = 45°$



This actually draws the ellipse point at a different angle



So the ellipse angle is then:

$$\tan\theta = \frac{b}{a}\frac{\sin t}{\cos t}$$

$$\tan\theta = \frac{b}{a}\tan t$$

$$\boxed{\theta = \arctan\left(\frac{b}{a}\tan t\right)} \quad (2)$$

$$\boxed{t = \arctan\left(\frac{a}{b}\tan\theta\right)} \quad (3)$$

So how to find r at θ?

Substitute x and y coordinates in terms of r into (1):

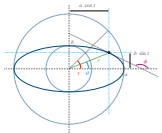
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{(r\cos\theta)^2}{a^2}+\frac{(r\sin\theta)^2}{b^2}=1$$

$$r^2b^2\cos^2\theta + r^2a^2\sin^2\theta = a^2b^2$$

$$\boxed{r(\theta) = \frac{a^2b^2}{\sqrt{(b\cos\theta)^2+(a\sin\theta)^2}}} \quad (4)$$

How to find angle θ from a tangent line with angle σ?



Gradient of the tangent is:

$$m = \frac{\Delta y}{\Delta x}$$

Where:

$$\Delta y = \frac{dy}{dx} = \frac{d}{dx}b\sin t = b\cos t$$

$$\Delta x = \frac{dx}{dy} = \frac{d}{dy}a\cos t = -a\sin t$$

Therefore:

$$m = \frac{\Delta y}{\Delta x} = -\frac{b}{a}\cot t = -\frac{b}{a\tan t} = \frac{b\cos t}{-a\sin t}$$

The gradient is related to ϕ with the tan function:

$$\tan\phi = m = -\frac{b}{a\tan t}$$

$$\boxed{\phi = \arctan\left(-\frac{b}{a\tan t}\right) = \arctan\left(\frac{b\cos t}{-a\sin t}\right)} \quad (5)$$

$$\boxed{t = \arctan\left(-\frac{b}{a\tan\phi}\right)} \quad (6)$$

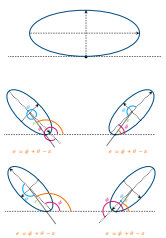
Substituting (6) into (2)

$$\theta = \arctan\left(\frac{b}{a}\tan t\right)$$

$$\theta = \arctan\left(-\frac{b}{a}\frac{b}{a\tan\phi}\right)$$

$$\boxed{\theta = \arctan\left(-\frac{b^2}{a^2\tan\phi}\right)} \quad (7)$$

So we have a point, and an ellipse. We want to rotate the ellipse and then position it so that its horizontal tangent is touching the point



1. Find θ from σ with (7)
2. Find r from θ with (4)
3. Find σ from $\sigma = \phi + \theta - \pi$
4. x offset: $r\cos\sigma$
5. y offset: $r\sin\sigma$