**Exercise**

1.1 **Analysis:** since the prize money and ticket price is fixed, the variables is the number of people buying a ticket and the winning probability, which is represented by the symbol  and respectively.

**Results:** The demand/performance equilibrium of a raffle is shown as figure 1.1.1. the winning probability decreases with the number of people buying a ticket, which is so called performance function. in contrast, the number of people  buying a ticket increases with winning probability, which is so called demand function. the point where the two curves intersect is the equilibrium point , when the number of people buying a ticket is such that it creates a winning probability that corresponds to the same number of people buying a ticket.

N



winning prob





number of people

demand

performance

figure 1.1.1 the demand/performance equilibrium of a raffle

if the number of people is less thanas shown in figure 1.1.2, the winning probability is high, which attracts more people to buy a ticket. as the increase of the number of people who buy tickets, the winning probability decreases until reaching the equilibrium point.



winning prob







N

number of people

demand

performance

figure 1.1.2 the number of people buying a ticket is too low in a raffle

if the number of people buying a ticket is more than as shown in figure1.1.3, the winning probability is very low. in this case, more and more people will forgo this raffle gradually . as the number of people buying a ticket decreases, the winning probability increases until reaching the equilibrium point.





winning prob





number of people

demand

performance

figure 1.1.3 the number of people buying a ticket is too high in a raffle

1.2 firstly, whether the two paths is all used or only one of them is used should be demonstrated.

the free-flow travel time of link 1 is 2, and the link 2 is 1. if travel time of the link 2 is 2, the corresponding flow is 0.33, which is less than the trip rate 4. therefore ,both the two links are used simultaneously.

according to the network, the flow on link 3 is the sum of link 1 and 2. therefore:



therefore,



according to user equilibrium theory, at user equilibrium, all the travel time on all paths is equal. that is



namely,



the formula is as follows:



therefore,, and 

in summary

 

1.3(a) at equilibrium, all the travel time on each links is equal, and the sum of all flow on each links is the trip rate, the expression is as follows:



(b)the first step is to determine which paths is used. the method is as follows:

set all the flow on each path being zero to calculate the free-flow travel time . rank these links in ascending order of free-flow travel time. the ranked link is recorded as ,where represents the link number, the represents the number ranked by free-flow travel time. the is the free-flow travel time of link  after the rank based on the free-flow travel time. set the performance function of being equal to the free-flow travel time of , that is , the corresponding flow is perceived. therefore, this method can calculate all the flow of corresponding with free-flow travel time of except the link with the most free-flow travel time. sum these  in the ascending order of  until the sum is just larger or equal to the trip rate . the rest of links that is not participate the sum is unused links, and the link participated the sum is used.

the second step is to determine the flow on each link at equilibrium. after the first step, all the links is grouped to two groups. the first group concludes links that is used and the second group is unused links. for the unused links, the flow is zero. but for the used links, use the user equilibrium to calculate the flow on each link as the first question.