*March 28th, 2018*

**Exercise**

1. The precondition is that the first derivate of  is continuously differentiable.

The function is strictly decreasing to the left of the minimum, which indicates the first derivative of  is less than zero. In contrast, the function  is strictly increasing to the right of the minimum, which indicates the first derivative of  is larger than zero. Of course, the first derivative of  is zero at a local minimum.

2. Firstly, prove the necessity. That is, if condition [2.4a] holds, then [2.4b] is true.

If  is convex, for any  ,

 [2.4a]

That is



Let , the left limit of the formula above is



That is



Because  and  are equivalent, so

 [2.4b]

Next, prove the sufficiency. That is, if condition [2.4b] holds, then [2.4a] is true.

If  [2.4b]

Take ,. Let ,

Let 1 and replace in [2.4b], and replace . Then





The first formula above multiply , and the second formula above multiply . Then both ends of the two equations add to each other,



That is:

 [2.4a]

3. Firstly, prove the necessity. That is, if condition [2.4a] holds, then [2.4c] is true.

If  is strictly convex, for any 

 [2.4a]

That is (from problem 2):

[2.4b]

According to Taylor's formula:  


The following formula can be deduced from the two formulas above:



Where,





Let , the following formula (namely[2.4c])is obtained:

[2.4c]

Next, prove the sufficiency. That is, if condition [2.4c] holds, then [2.4a] is true.

If the following condition holds,

[2.4c]

According to Taylor's formula:  


Because the following formula is reasonable for any 



Thus

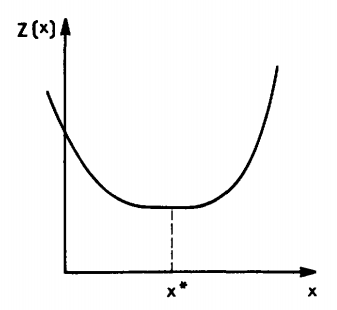


That is

 [2.4a]

4.(a)NO

Because it can't guarantee the objective is strictly convex in the vicinity of the stationary point. For example, the following function has more than one minimum, and it is convex and the constraint set is convex.



(b)YES

If a function is ditonic, it is strictly decreasing to the left of the minimum and strictly increasing to the right. The ditonic function has a single minimum. The above two conditions can guarantee the ditonic and unconstrained function  has a unique minimum.

(c)NO

For a monotonically increasing and unconstrained function, it has no stationary point. That is, it's first derivative can't equals zero.

(d)NO

If the concave function is as the first figure 1-a, its feasible region is the whole domain. Then it only has maximum. However, if the concave function is as figure 1-b, its feasible region is from a to b. Then it has a unique minimum.

Figure 1-a Figure 1-b

(e)NO

For example ,the function shown as the following figure 2 is piecewise continuous over a convex constraint set, but it has more than one minimum point. Because it can't guarantee the objective is strictly convex in the vicinity of the stationary point.

Figure 2

5.(a) the standard form of this program is as follows:  
 

Subject to

(b)the point that minimizes  is 

Therefore, the second constraint is binding constraint.



For the first constraint, because it is not binding constraint, let 

For the second constraint, because it is binding constraint, .



Because of , therefore



Thus, the first-order conditions [2.7] hold at the point that minimizes  in the feasible region.

6.Let a=2,b=3

(a) Let 



Because is positive define everywhere, the function is strictly convex. The contours is as follows.



(b)Let 



Because the leading minor of in the upper left corner is larger than and equals to zero in the feasible region constrained by . Therefore, the function is convex, but has multiple equal minima as the following figure shown.



(3) Let , the contours is as follows.



(d) I don't know how to how to make figure of constrained planning model.

7.Because the direction of the gradient is the normal direction(法线方向) of isosurface（等值面）or isopleth (等值线) of objective function. In this direction, the direction derivative gets the miximum. The direction derivative indicates the rate of change of the function value along a certain direction at a certain point. Therefore the gradient always points in the direction of the steepest increase in . In contrast, the opposite direction of the gradient is the direction in which the value of the function falls fastest.

8. 





9.Define  are convex functions which feasible region is . 

Because are convex functions on , thus





...



Sum all the above inequalities



Therefore, is a convex function.

In summary, the sum of the convex functions is a convex function.

10. The value of the dual variable used in solving Eqs. [2.15] is , and the minimum value is  at the point .

Let the relaxing amount , then .

The problem is



Subject to 

The Kuhn-Tucker conditions for this problem are as follows:  


Assuming that the value of the dual variable does note change, that is, . The minimum of the new problem is  at the point .

The minimum value of  decreased 0.002, which just equals to .

Therefore , measures the sensitivity of to the binding constraint.

11.(a) For the nonnegative programs, the Kuhn-Tucker conditions are as follows:



These conditions are discussed according to or.

If , the Kuhn-Tucker conditions are written as:



If , the Kuhn-Tucker conditions are written as:



Thus, for , the Kuhn-Tucker conditions are can be written as follows:  


The shown above, for , Eq.[2.19] hold.

I don't know how to vanish the conditions in Kuhn-Tucker .

(b) For programs with nonnegative and equality constraints, let denotes the Lagrangians multiplier of nonnegative constraint, let denotes the Lagrangians multiplier of equality constraint, the Kuhn-Tucker conditions are as follows:



If , the Kuhn-Tucker conditions are written as:



If , the Kuhn-Tucker conditions are written as:



Thus, for , the Kuhn-Tucker conditions are can be written as follows:



Specially, there are no restrictions for the sign of .

12. At the solution point, the Lagrangian of  can be written as :

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At the minimum point. If the th constrained is relaxed by a small amount, ,and is replaced by , the new minimum value of will approximately equal the old value minus . Thus a relaxation of the th constraint by improves the optimal value of the objective function by, approximately, .

13. The Kuhn-Tucker conditions for this problem are as follows:



The minimum value is  at the point , and .

The  of is



Because  is positive definite, the objective function is strictly convex everywhere.

Besides, the feasible region is convex.

Thus, the solution is unique.

14.Fistly, the Lagrangian with respect to the equality constraints should be formed as:



Subject to 

The original program can be written as an program with nonnegative constraints. Therefore, the conditions for the above program to find stationary point as follows:  


The stationary point of  is ,. Because the function is convex, is the minimum point. Thus, the optimal solution of original program is  at point .

Plot the feasible region.

15. The program belongs to linear program.

This problem was solve by GAMS. The minimum value of is 1 at the point .

Plot the feasible region and the contours of the objective function.

16. The gradient of is



Let ,that is



The stationary point is .

The Hessian is



Where, the leading minor of Hessian matrix in the left upper corner are:







Thus, the Hessian matrix is positive definite. The objective function is strictly convex everywhere. The stationary point is the global minimum point.

17.

(a) For the minimization program with standard inequality constraints, the gradient of the objective function exists within the cone formed by the gradients of the constraints, and thus can be expressed as the nonnegative linear combination of the gradients of the constraints. That is 

Thus, .

For the minimization program with standard equality constraints, the Lagrangian of the program is

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Because ****** always equals to zero, thus the sign of can have any sign.

18. The Kuhn-Tucker conditionsfor a stationary point of this program are as follows:



Because the equality can be written as



For the inequality constraints, the sign of is nonnegative, thus for the equality, the sign of is nonnegative unstricted.