Lab 5: Vehicle Dynamics and Extended Kalman Filter

In this lab you will model and simulate a nonlinear vehicle dynamics model. You will then implement a standard and an extended Kalman filter to estimate the system states.

Vehicle Dynamics Model

In this lab we will model the vehicle using a bicycle model similar to that presented in class and shown in Figure 1:

$$\begin{aligned}
\dot{v}_x &= v_y\omega_z - \frac{2}{m}F_f(\alpha_f)\sin(\delta) + \frac{F_x}{m} \\
\dot{v}_y &= -v_x\omega_z + \frac{2}{m}\left[F_f(\alpha_f)\cos(\delta) + F_r(\alpha_r)\right] \\
\dot{\omega}_z &= \frac{2}{J_z}\left[aF_f(\alpha_f)\cos(\delta) - bF_r(\alpha_r)\right] \\
\dot{X} &= v_x\cos(\psi) - v_y\sin(\psi) \\
\dot{Y} &= v_x\sin(\psi) + v_y\cos(\psi) \\
\dot{\psi} &= \omega_z
\end{aligned}$$

where the longitudinal force F_x and the steering angle δ are inputs. We will refer to the dynamics as $\dot{x} = f(x, u)$ where $x = \begin{bmatrix} v_x & v_y & \omega_z & X & Y & \psi \end{bmatrix}^\top$ is the state vector and $u = \begin{bmatrix} F_x & \delta \end{bmatrix}^\top$ is the input vector.

The tire forces are given by

$$\begin{aligned}
F_f(\alpha_f) &= m\frac{a}{a+b}A\sin(C\arctan(B\alpha_f)) \\
F_r(\alpha_r) &= m\frac{b}{a+b}A\sin(C\arctan(B\alpha_r))
\end{aligned}$$

and the front and rear slip angles are

$$\begin{aligned}
\alpha_f &= \arctan\left(\frac{v_y + a\omega_z}{v_x}\right) - \delta \\
\alpha_r &= \arctan\left(\frac{v_y - b\omega_z}{v_x}\right)
\end{aligned}$$

Note that if $v_x = 0$ the slip angles are undefined. In this case set the slip angles to 0.

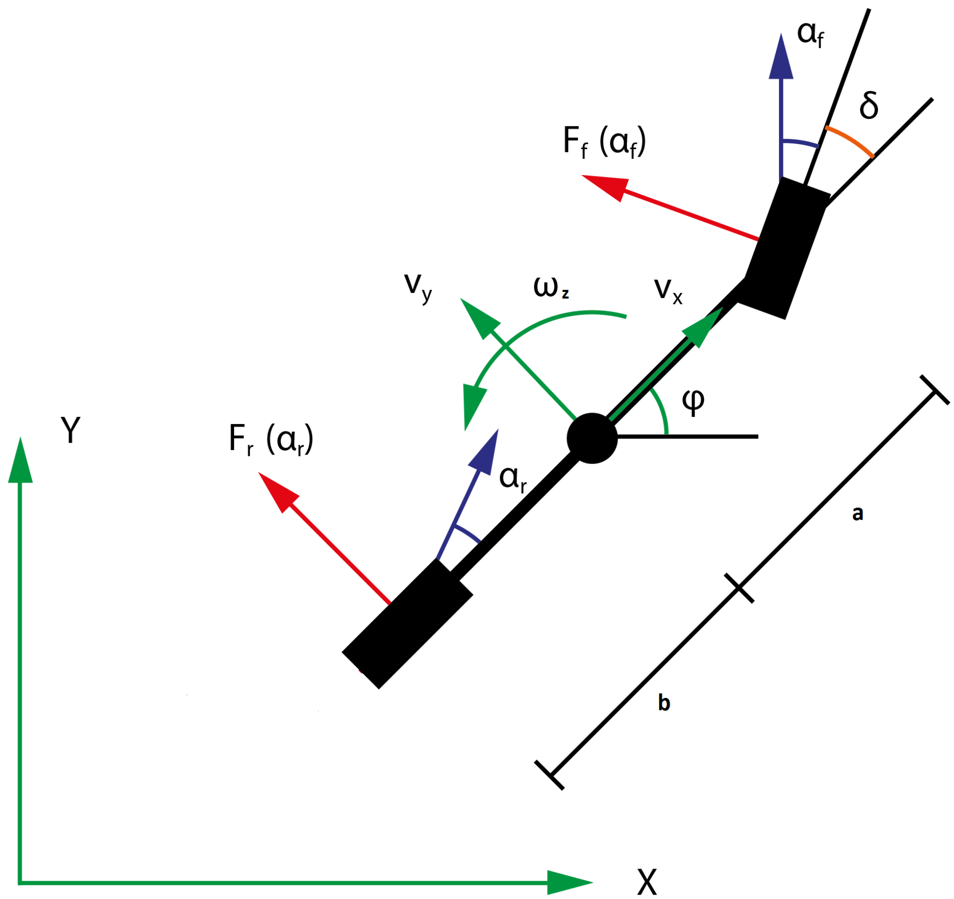

Figure 1: Bicycle Model.

The vehicle parameters are

$$m = 2237\ kg$$
$$J_z = 5112\ kg - m^2$$
$$a = 1.46\ m$$
$$b = 1.55\ m$$

and the tire model parameters are

$$A = -6.8357$$
$$B = 0.0325$$
$$C = 238.9874$$

We assume that we measure longitudinal velocity v_x, lateral acceleration a_y, and yaw rate ω_z. Therefore, the measurement equation is

$$y = h(x,u) = \begin{bmatrix} v_x \\ \frac{2}{m}\left[F_f(\alpha_f)\cos(\delta) + F_r(\alpha_r)\right] \\ \omega_z \end{bmatrix}.$$

Using the Kalman filter we will attempt to estimate the sideslip angle β using these measurements and the system dynamics. The sideslip angle is defined as

$$\beta := \arctan\left(\frac{v_y}{v_x}\right)$$

and is the direction of travel of the center of mass of the vehicle.

1. Simulate the nonlinear continuous time system with the following sets of different inputs for 20 seconds:

   - **Constant Velocity Skidpad:** Let the initial longitudinal velocity be $20\ m/s$, the longitudinal force $F_x = 2500N$, and the steering angle $\delta = 10$ degrees.
   - **Constant Acceleration Skidpad:** Let the initial longitudinal velocity be $5\ m/s$, the longitudinal force $F_x = 8000\ N$, and the steering angle $\delta = 10$ degrees.
   - **Lane Change:** With $F_x = 0\ N$ and an initial velocity of $20\ m/s$ apply zero steering input for 5 seconds, then apply a step steering input of $\delta = 5$ degrees for two seconds, apply zero steering for 5 seconds, and then apply a step steering input of $\delta = -5$ degrees for two seconds.

   For each test plot the input signals F_x and δ, the resulting position X vs. Y, and the lateral and longitudinal velocities v_y and v_x.

2. Discretize the nonlinear dynamics by a forward Euler approximation with sampling time $\Delta t = 0.01$:

   $$\begin{aligned} x_{k+1} &= x_k + \Delta_t f(x_k, u_k) \\ y_k &= h(x_k, u_k) \end{aligned}$$

   Simulate the **Lane Change** scenario with the discretized dynamics and plot the resulting position X vs. Y, and the lateral and longitudinal velocities v_y and v_x to make sure that the discretized system is a good approximation of the continuous time system.

3. Perform a symbolic linearization of the discretized system dynamics and measurement equations. You will use these results to implement an extended Kalman Filter in the following problems so you will need to get expressions for the (A, B, C, D) matrices as a function of the states and inputs. Report the (A, B, C, D) matrices of the linearized system at the operating point:

   $$\begin{aligned} v_x &= 10\ m/s \\ v_y = \omega_z = X = Y = \psi &= 0 \\ F_x = \delta &= 0 \end{aligned}$$

4. **Note that the value of the tire model parameter A was originally incorrect and has been changed to -6.8357. Make sure you update this value so the simulation and Kalman filter estimates will be reflective of the measured data.**

Download the `VehicleData.mat` file from bCourses. This file contains real data from a vehicle test. It includes the inputs:

- `Fx_in` - longitudinal force
- `delta_in` - steering angle

and measurements:

- `ay_meas` - lateral acceleration
- `vx_meas` - longitudinal velocity
- `yawRate_meas` - yaw rate
- `beta_meas` - sideslip angle

Implement a standard Kalman filter on the discrete time linearized system that uses the measurements of lateral acceleration, longitudinal velocity, and yaw rate to estimate the sideslip angle. Plot the estimates of the lateral acceleration, yaw rate, and sideslip angle compared to the actual measurements.

To do this the discrete time dynamics should be linearized about the operating point:

$$\begin{aligned} v_x &= 10\ m/s \\ v_y = \omega_z = X = Y = \psi &= 0 \\ F_x = \delta &= 0 \end{aligned}$$

The initial state mean and error covariance are

$$m_0 = \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \Sigma_0 = \begin{bmatrix} 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 \end{bmatrix}.$$

We assume the process and measurement noise w effect the system through the matrices

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and has covariance

$$W = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.05 \end{bmatrix}.$$

5. Using the same data and noise characteristics implement an extended Kalman filter to estimate the sideslip angle of the vehicle. Plot the estimates of the lateral acceleration, yaw rate, and sideslip angle compared to the actual measurements and the estimates from the standard Kalman filter.

   How does the extended Kalman filter estimates compare to the standard Kalman filter? Are there specific times where the extended Kalman filter performs significantly better? Explain why this would happen.