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ME C231B Assignment: Kalman Filtering (part 2)

```
close all
```

1.c.ii

```
Amat = repmat(1,[1 1 20]);
nX = size(Amat,1);
Emat = repmat(0,[1 1 20]);
Cmat = repmat(1,[1 1 20]);
Fmat = repmat(1,[1 1 20]);
sX = 1000; sW = 1; m0=2;
for k=1:6
    [LkBatch,VkBatch,eVar] = ...
        batchKF(Amat,Emat,Cmat,Fmat,sX,sW,k);
```

```
    fprintf('Horizon number is %d',k)
    LkBatch(end-2*nX+1:end,:)
    VkBatch(end-2*nX+1:end)
end
```

```
Horizon number is 1
ans =

    0.9990
    0.9990


ans =

   1.0e-03 *

    0.9990
    0.9990

Horizon number is 2
ans =

    0.4998    0.4998
    0.4998    0.4998


ans =

   1.0e-03 *

    0.4998
    0.4998

Horizon number is 3
ans =

    0.3332    0.3332    0.3332
    0.3332    0.3332    0.3332


ans =

   1.0e-03 *

    0.3332
    0.3332

Horizon number is 4
ans =

    0.2499    0.2499    0.2499    0.2499
    0.2499    0.2499    0.2499    0.2499
```

```
ans =

   1.0e-03 *

    0.2499
    0.2499

Horizon number is 5
ans =

    0.2000    0.2000    0.2000    0.2000    0.2000
    0.2000    0.2000    0.2000    0.2000    0.2000


ans =

   1.0e-03 *

    0.2000
    0.2000

Horizon number is 6
ans =

    0.1666    0.1666    0.1666    0.1666    0.1666    0.1666
    0.1666    0.1666    0.1666    0.1666    0.1666    0.1666


ans =

   1.0e-03 *

    0.1666
    0.1666
```

1.c.iv

```
Amat = repmat(1,[1 1 20]);
nX = size(Amat,1);
Emat = repmat(0,[1 1 20]);
Cmat = repmat(1,[1 1 20]);
Fmat = repmat(1,[1 1 20]);
sX = 0.1; sW = 5; m0=4;
for k=1:6
    [LkBatch,VkBatch,eVar] = ...
        batchKF(Amat,Emat,Cmat,Fmat,sX,sW,k);
    fprintf('Horizon number is %d',k)
    LkBatch(end-2*nX+1:end,:)
    VkBatch(end-2*nX+1:end)
end
```

```
Horizon number is 1
```

```
ans =

    0.0196
    0.0196


ans =

    0.9804
    0.9804

Horizon number is 2
ans =

    0.0192    0.0192
    0.0192    0.0192


ans =

    0.9615
    0.9615

Horizon number is 3
ans =

    0.0189    0.0189    0.0189
    0.0189    0.0189    0.0189


ans =

    0.9434
    0.9434

Horizon number is 4
ans =

    0.0185    0.0185    0.0185    0.0185
    0.0185    0.0185    0.0185    0.0185


ans =

    0.9259
    0.9259

Horizon number is 5
ans =

    0.0182    0.0182    0.0182    0.0182    0.0182
    0.0182    0.0182    0.0182    0.0182    0.0182
```

```
ans =

    0.9091
    0.9091

Horizon number is 6
ans =

    0.0179    0.0179    0.0179    0.0179    0.0179    0.0179
    0.0179    0.0179    0.0179    0.0179    0.0179    0.0179


ans =

    0.8929
    0.8929
```

2

Please review code in KF231B.m

3. Steady-State Behavior

```
% We assume those following are the process state-space matrices.
A = [2,-1;-3,4];
B = [1;1];
E = [2;1];
C = [2,1];
F = 3;
N = 10; % Horizontal Number
% Initial Conditions
xkk1 = 0; % Average values
Sxkk1 = 1; % Variance of the initial conditions
Swk = 1; % Variance of the time-invariant disturbence
uk = 1;
yk = 1;
for k = 1:10
    [xk1k,Sxk1k,xkk,Sxkk,Sykk1,Lk,Hk,Gk,wkk] =
 KF231B(xkk1,Sxkk1,A,B,C,E,F,Swk,uk,yk);
    xkk1 = xk1k;
    Sxkk1 = Sxk1k;
    fprintf('Step %d',k);
    disp(Lk);
    disp(Hk);
end
```

```
Step 1    0.6429
    0.0714

    0.1429
    0.0714
```

```
Step 2    0.0539
    1.5809

    0.0456
    0.3859

Step 3   42.8388
 -110.0164

    7.9868
  -22.1086

Step 4   -7.6380
   22.2456

   -1.6554
    4.3206

Step 5   -6.8907
   20.2265

   -1.4850
    3.9405

Step 6   -6.9478
   20.3908

   -1.5026
    3.9676

Step 7   -6.9649
   20.4382

   -1.5071
    3.9761

Step 8   -6.9766
   20.4709

   -1.5102
    3.9819

Step 9   -6.9822
   20.4867

   -1.5117
    3.9846

Step 10   -6.9852
   20.4950

   -1.5125
    3.9861
```

4. Separating signal from noise

4.b

Separating Signal from Noise

ME C231B, UC Berkeley, Spring 2018

Sample Time

For this discrete-time example, set TS=-1. In Matlab, this just means an unspecified sampling time, totally within the context of pure discrete-time systems.

```
TS = -1;
```

Create high-pass filter

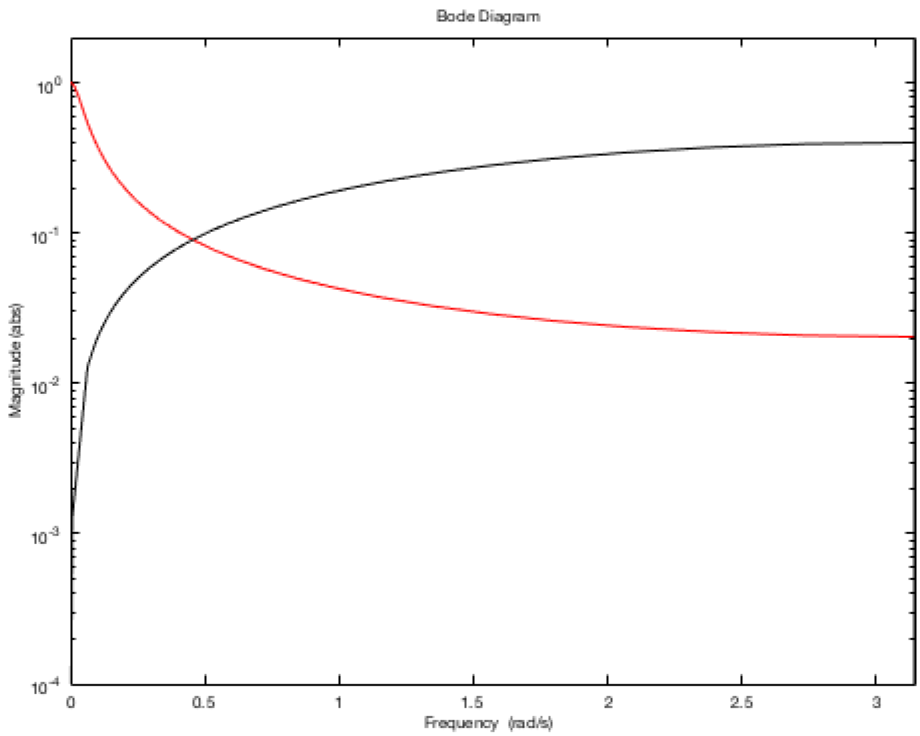
```
P1 = 0.4*tf([.5 -.5],[1 0],TS);
[A1,E1,C1,F1] = ssdata(P1);
nX1 = size(A1,1); % will be 1
nW1 = size(E1,2); % will be 1
```

Create low-pass filter

```
P2 = tf(.04,[1 -.96],TS);
[A2,E2,C2,F2] = ssdata(P2);
nX2 = size(A2,1); % will be 1
nW2 = size(E2,2); % will be 1
```

Bode plot of both

```
bOpt = bodeoptions;
bOpt.PhaseVisible = 'off';
bOpt.MagUnits = 'abs';
bOpt.MagScale = 'log';
bOpt.FreqScale = 'linear';
bOpt.Xlim = [0 pi];
bOpt.Ylim = [1e-4 2];
bodeplot(P2,'r',P1,'k',bOpt)
```



Form overall system which adds the outputs

```
A = blkdiag(A1,A2);
E = blkdiag(E1,E2);
C = [C1 C2];
F = [F1 F2];
nX = size(A,1);
nY = size(C,1);
nW = size(E,2);
```

Noise variance and initial condition variance

Keep it simple, and make everything Identity (appropriate dimension)

```
SigW = eye(nW);
Sxkk1 = eye(nX);
```

Run several iterations to get the steady-state Kalman Gains

```
nIter = 40;
for i=1:nIter
Swk = SigW;
```

```
[~,Sxk1k,~,Sxkk,Sykk1,Lk,Hk,Gk,~] = ...
KF231B([],Sxkk1,A,[],C,E,F,Swk,[],[]);
Sxkk1 = Sxk1k;
disp(Lk)
end
```

```
    0.4433
    0.6809

    0.9603
    1.5483

    0.9959
    1.5457

    1.0636
    1.2801

    1.1575
    1.0569

    1.2405
    0.9206

    1.2995
    0.8505

    1.3353
    0.8204

    1.3538
    0.8108

    1.3618
    0.8094

    1.3645
    0.8101

    1.3651
    0.8109

    1.3652
    0.8111

    1.3652
    0.8109

    1.3653
    0.8107

    1.3654
    0.8106
```

1.3655
0.8105

1.3656
0.8104

1.3656
0.8104

1.3656
0.8104

1.3656
0.8104

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1.3656
0.8104

```
    1.3656
    0.8104

    1.3656
    0.8104

    1.3656
    0.8104

    1.3656
    0.8104

    1.3656
    0.8104

    1.3656
    0.8104
```

Form Kalman filter with 3 outputs

```
AKF = A-Lk*C;
BKF = Lk;
CKF = [eye(nX);eye(nX)-Hk*C;-Gk*C];
DKF = [zeros(nX,nY);Hk;Gk];
SSKF = ss(AKF,BKF,CKF,DKF,TS);
```

Form matrix to extract estimate of y2_{k|k}

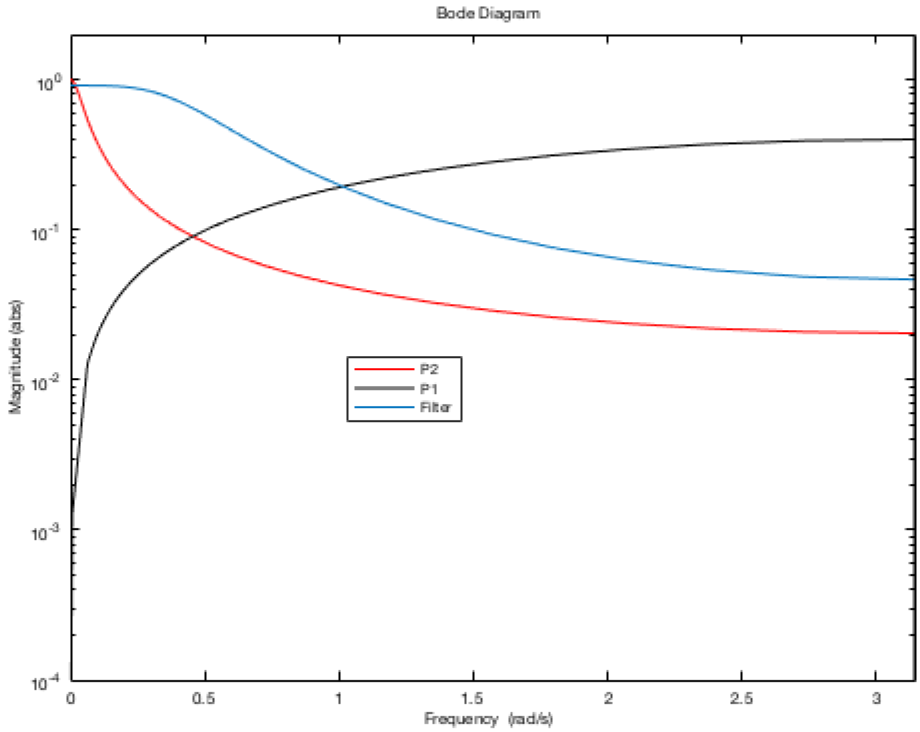
We need [0 C2]*xhat_{k|k} + [0 F2]*what_{k|k}. Everything is scalar dimension, but we can form this matrix properly so that the example would work on other systems too.

```
M = [zeros(nY,nX) zeros(nY,nX1) C2 zeros(nY,nW1) F2];
```

Bode plot of filter

It makes sense that the filter will try to "pass" some low frequencies, to preserve y2, but will cutoff high-frequencies to reject y1. The "pass" region should extend over the region where P2 has modest gain. The Bode magnitude plot confirms this

```
bodeplot(P2,'r',P1,'k',M*SSKF,bOpt)
legend('P2','P1','Filter');
```



Single Simulation

Create a w sequence consistent with variance assumption

```
wSeq = randn(100,2);
```

Get y1 and y2 (separate simulations) for later comparison

```
y1 = lsim(P1,wSeq(:,1));
y2 = lsim(P2,wSeq(:,2));
y = y1 + y2;
```

Form the cascade (system output goes directly to Kalman Filter), and simulate, obtaining the outputs of Kalman Filter

```
Est = lsim(SSKF*ss(A,E,C,F,TS),wSeq);
```

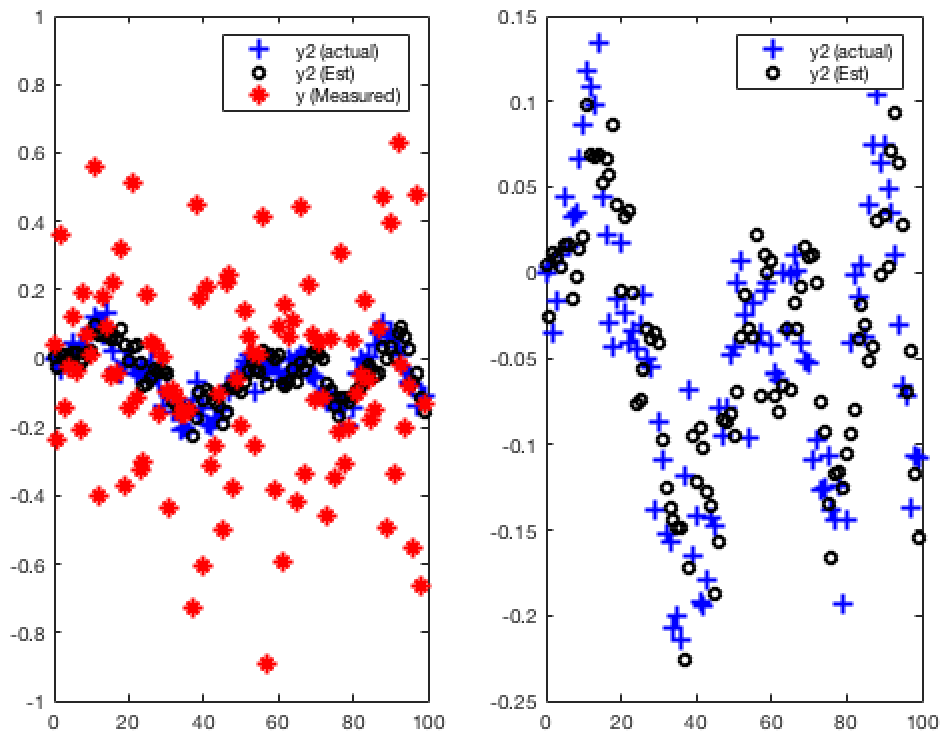
Form Estimate of y2

Est matrix is 100-by-6, so use transpose correctly to do reconstruction as a matrix multiply

```
y2Est = (M*Est')';
```

Plot

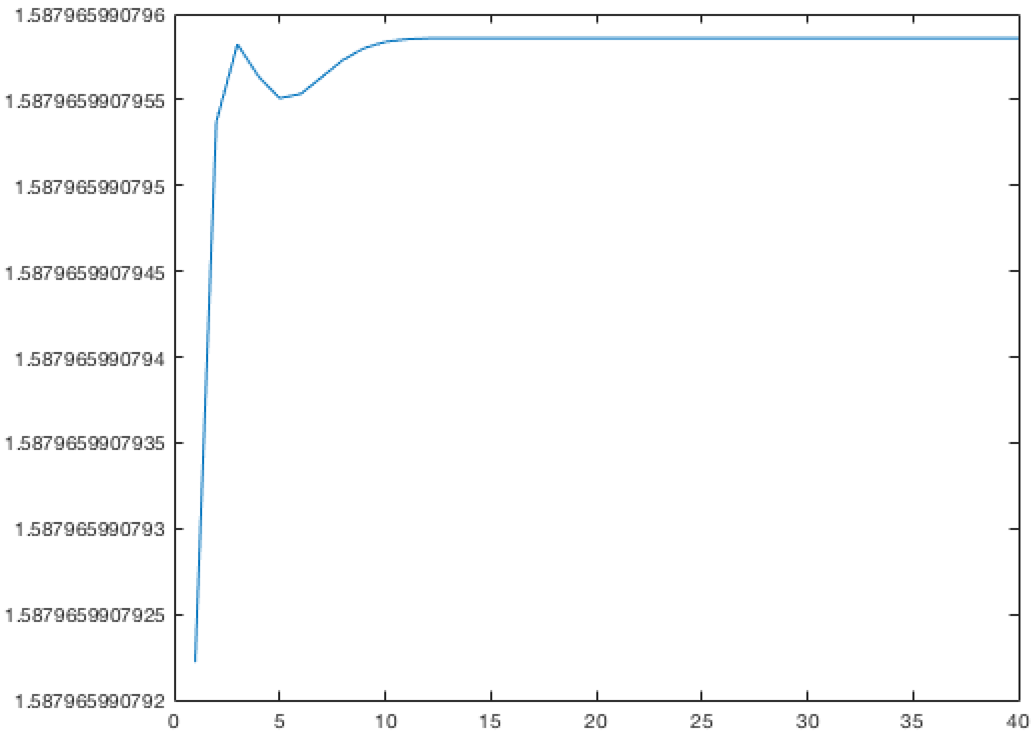
```
subplot(1,2,1);
plot(0:99,y2,'b+',0:99,y2Est,'ko',0:99,y,'r*');
legend('y2 (actual)','y2 (Est)','y (Measured)');
subplot(1,2,2);
plot(0:99,y2,'b+',0:99,y2Est,'ko');
legend('y2 (actual)','y2 (Est)');
```



4.c

It takes about 10 steps to converge in most cases.

```
nIter = 40;
Lk_list = zeros(2,40);
for i=1:nIter
Swk = SigW;
[~,Sxk1k,~,Sxkk,Sykk1,Lk,Hk,Gk,~] = ...
KF231B([],Sxkk1,A,[],C,E,F,Swk,[],[]);
Sxkk1 = Sxk1k;
Lk_list(:,i) = Lk;
end
figure
plot(1:40,sqrt(Lk_list(1,:).^2+Lk_list(2,:).^2))
```



4.d

Create plant

```
T = 100;  % sequence length, data runs from 0:(T-1)
nX = 2;
nW = 2;
nY = 1;
arrayA = repmat([0,0;0,0.96],[1 1 T]);
arrayE = repmat([0.5,0;0,0.25],[1 1 T]);
arrayC = repmat([-0.4,0.16],[1 1 T]);
arrayF = repmat([0.2,0],[1 1 T]);
```

Create/Declare variance of disturbance and initial condition

```
arraySW = repmat([1,0;0,1], [1,1,T]);
Sx0 = eye(nX);
m0 = 2;
```

Initialize KF states with appropriate values

```
Sxii1 = Sx0;
```

```
xii1 = m0;
```

Create a specific initial condition and noise sequence

Under ideal circumstances, this should be consistent with the statistical assumptions made in the previous code cell. When studying robustness, namely how the filter performance degrades as assumptions are not met, it may be useful to create an initial condition and noise sequence which is not consistent with the assumptions wSeq = randn(2,100);

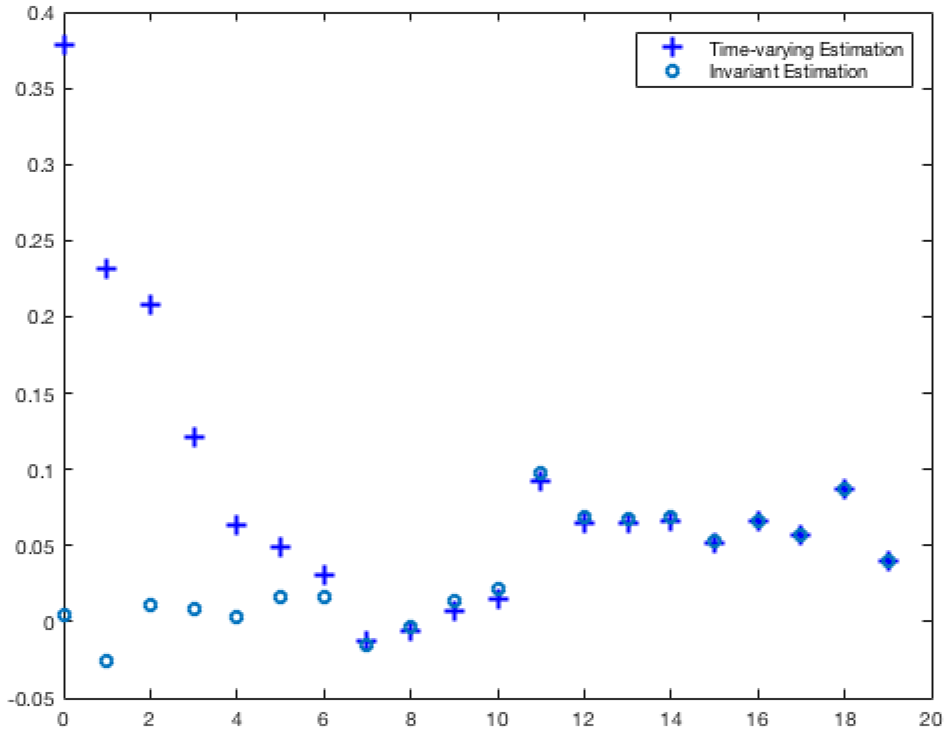
```
wSeq = wSeq';
x0 = 0;
```

Simulate the system/KF one step at a time

```
emptyB = [];  % this template is for no control signal
emptyu = [];  % this template is for no control signal
y = zeros(nY,T);
xSeq = zeros(nX,T);
xSeq(:,1) = x0;
xEii1 = zeros(nX,T); xEii1(:,1) = m0;
xEii = zeros(nX,T); yEii = zeros(nY,T); y2Eii = zeros(1,T);
for i=0:T-1
   iMatlab = i+1;
   Ai = arrayA(:,:,iMatlab);
   Ei = arrayE(:,:,iMatlab);
   Ci = arrayC(:,:,iMatlab);
   Fi = arrayF(:,:,iMatlab);
   Swi = arraySW(:,:,iMatlab);
   wi = wSeq(:,iMatlab);
   xi = xSeq(:,iMatlab);

   % Get y(i) from system model, using x(i) and w(i)
   y(:,iMatlab) = Ci*xi + Fi*wi;
   % Get estimates of x(i+1|i) using y(i) from KF
   [xi1i,Sxi1i,xii,Sxii,Syii1,Li,Hi,Gi,wii] =
 KF231B(xEii1(:,iMatlab),Sxii1,...
      Ai,emptyB,Ci,Ei,Fi,Swi,emptyu,y(:,iMatlab));
   xEii(:,iMatlab) = xii;
   xEii1(:,iMatlab+1) = xi1i;
   % Get x(i+1) from system model, using x(i) and w(i)
   xSeq(:,iMatlab+1) = Ai*xi + Ei*wi;
   % Shift the error-variance estimate so that when loop-index i
 advances,
   % the initial condition for this variance is correct.
   Sxii1 = Sxi1i;
   yEii(:,iMatlab) = Ci*xii;
   y2Eii(iMatlab) = C2*xii(2) + F2*wii(2);
end
y2 = lsim(P2,wSeq(2,:));
figure
plot(0:19,y2Eii(:,1:20),'b+',0:19,y2Est(1:20,:),'o');
```

```
legend('Time-varying Estimation', 'Invariant Estimation');
```



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