1 AE332: Modelling and Analysis Lab

1.1 Session 3: Simulating Shells and Rockets

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SC-Code: SC21B026

[1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import scipy.integrate as sci
import scipy.interpolate as inp
import Atmosphere as atm
from matplotlib.patches import Rectangle
plt.rcParams['text.usetex'] = True
```

1.1.1 Importing the data from excel

[2]:
```
df = pd.read_excel('data-shell.xlsx')
print(df.head())
CAjoff = np.array([[0.4309, 0.4331, 0.434, 0.4084, 0.3876, 0.4582, 0.461, 0.
 ↪4688, 0.4953, 0.6134],
          [0.4683, 0.4709, 0.4729, 0.4599, 0.4171, 0.4716, 0.578, 0.5858, 0.
 ↪6139, 0.7416],
          [0.6796, 0.6774, 0.6797, 0.67, 0.6197, 0.6678, 0.7619, 0.7686, 0.7932,␣
 ↪0.9058],
          [0.7946, 0.7922, 0.7945, 0.7867, 0.7334, 0.7771, 0.8632, 0.8694, 0.
 ↪8922, 0.9964],
          [0.8887, 0.886,  0.8883, 0.8826, 0.8263, 0.8663, 0.9456, 0.9513, 0.
 ↪9724, 1.069],
          [0.9595, 0.9575, 0.9602, 0.9578, 0.8943, 0.9361, 1.0177, 1.023,  1.
 ↪0426, 1.1323],
          [1.0027, 1.0004, 1.0031, 1.0036, 0.9379, 0.9756, 1.0529, 1.0578, 1.
 ↪0761, 1.1601],
          [0.842,  0.8389, 0.8413, 0.8445, 0.8003, 0.8048, 0.8664, 0.9805, 0.
 ↪9964, 1.0696],
          [0.7338, 0.7298, 0.7317, 0.7356, 0.7049, 0.6887, 0.7387, 0.832,  0.
 ↪8455, 0.9076],
          [0.6602, 0.6552, 0.6567, 0.6605, 0.645,  0.611,  0.6512, 0.7246, 0.
 ↪7357, 0.7873],
          [0.6441, 0.6389, 0.6402, 0.644,  0.6295, 0.5943, 0.6328, 0.7026, 0.
 ↪7134, 0.7632],
          [0.62,   0.6142, 0.6153, 0.6186, 0.607,  0.5696, 0.6045, 0.6664, 0.
 ↪6764, 0.7226],
          [0.5946, 0.5951, 0.5959, 0.5983, 0.5896, 0.5507, 0.5829, 0.6385, 0.
 ↪6479, 0.6915],
```

```
        [0.5499, 0.5502, 0.5508, 0.5526, 0.5454, 0.5062, 0.5363, 0.5867, 0.
↪5958, 0.638],
        [0.5347, 0.5349, 0.5352, 0.5366, 0.5311, 0.4915, 0.5199, 0.5659, 0.
↪5747, 0.6154]
        ])
```

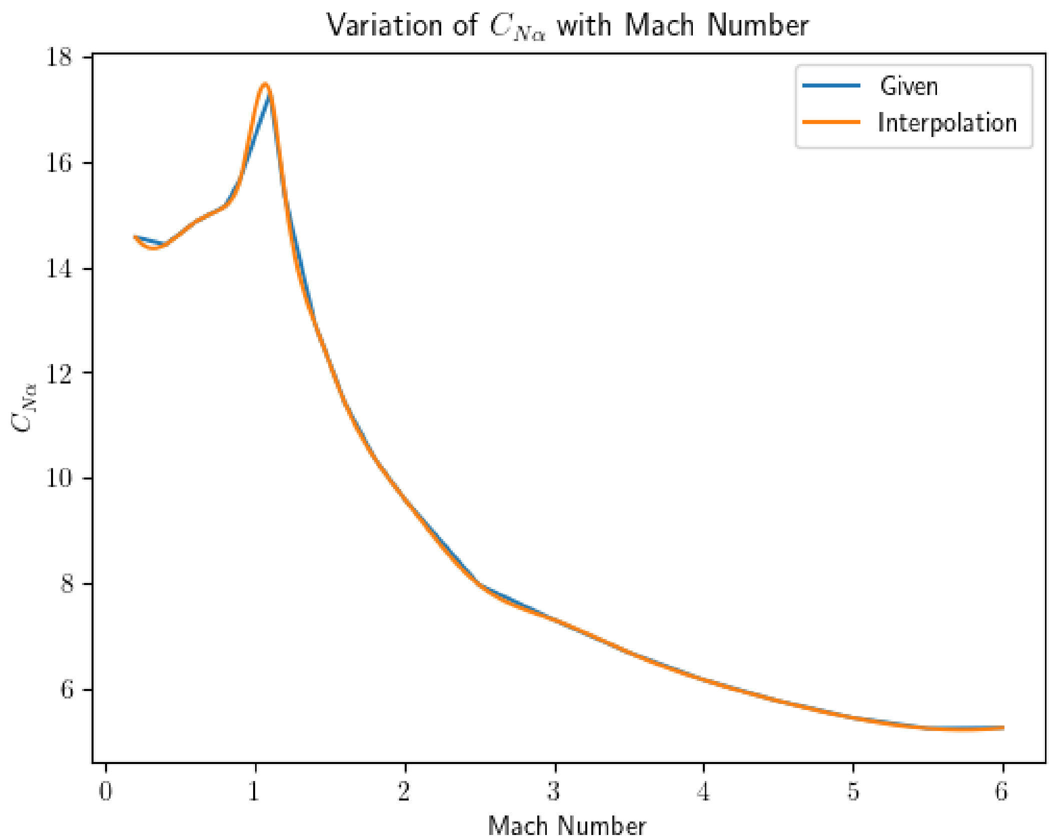
```
   Mach  CNAlpha    XCPLen  MachJoff    Hjoff
0   0.2    14.57  0.842868       0.4      0.0
1   0.4    14.43  0.851050       0.6   5000.0
2   0.6    14.86  0.850584       0.8  10000.0
3   0.8    15.16  0.848138       0.9  20000.0
4   0.9    15.67  0.851823       1.0  30000.0
```

1.1.2 Interpolation Functions

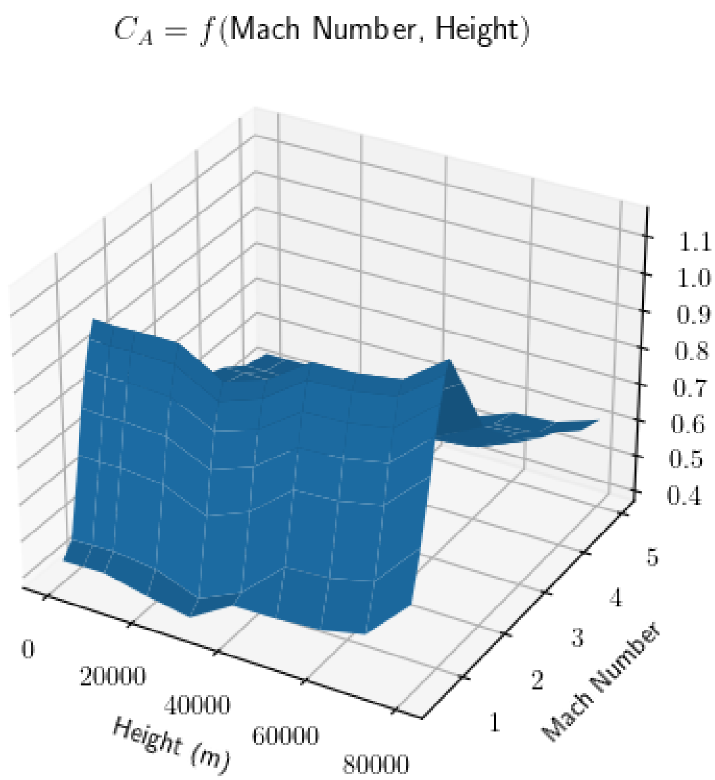
[3]:
```
CNAlphaInterp = inp.interp1d(df.Mach, df.CNAlpha, kind='cubic')
CPInterp = inp.interp1d(df.Mach, df.CNAlpha, kind='cubic')
CAInterp = inp.RectBivariateSpline(df.MachJoff[:15], df.Hjoff[:10], CAjoff)
```

[4]:
```
mach = np.linspace(0.2, 6, 10000)
plt.plot(df.Mach, df.CNAlpha, label='Given')
plt.plot(mach, CNAlphaInterp(mach),label='Interpolation')
ax = plt.gca()
ax.set_xlabel('Mach Number')
ax.set_ylabel(r'$C_{N\alpha}$')
plt.legend()
ax.set_title(r'Variation of $C_{N\alpha}$ with Mach Number')
```

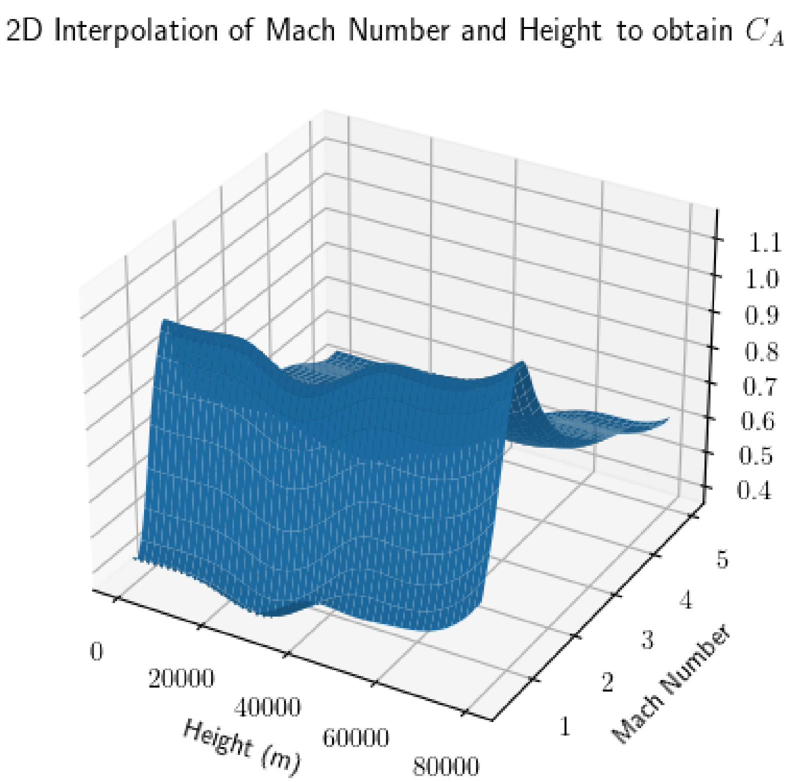
[4]:
```
Text(0.5, 1.0, 'Variation of $C_{N\\alpha}$ with Mach Number')
```



[5]:
```
fig, ax = plt.subplots(subplot_kw={"projection": "3d"})
xx, yy = np.meshgrid(df.Hjoff[:10], df.MachJoff[:15])
surf = ax.plot_surface(xx,yy, CAjoff)
plt.xlabel('Height (m)')
plt.ylabel('Mach Number')
plt.title(r'$C_A$ = $f($Mach Number, Height$)$')
plt.show()
```



[6]:
```
fig, ax = plt.subplots(subplot_kw={"projection": "3d"})
hh = np.linspace(0, 80000, 100000)
mm = np.linspace(0.4, 5, 1000)
xx, yy = np.meshgrid(hh,mm)
surf = ax.plot_surface(xx, yy, CAInterp(mm,hh))
plt.xlabel('Height (m)')
plt.ylabel('Mach Number')
plt.title('2D Interpolation of Mach Number and Height to obtain $C_A$')
plt.show()
```



1.1.3 Simulation of Shell

[38]:
```
l = 1.115
m = 40
I = 4.29
S = 80.12e-4
com = 0.70769
U = 867
g = 9.8
theta0 = np.radians(45)
sol0 = np.array([0, U*np.cos(theta0),0, U*np.sin(theta0), theta0, 0, 0.5*m*U*U])
def deriv(t,x):
    theta = x[4]
    alpha = theta - np.arctan2(x[3],x[1])
    v = np.hypot(x[1],x[3])
    prm = atm.atmParam(x[2],v) #Temp(h), Density(h) , Viscosity(h), Mach(h,v),
 ↪logRe(h,v), Cd(h,v)
    fN = 0.5*prm[1]*v*v*S*CNAlphaInterp(prm[3])*alpha
    fA = 0.5*prm[1]*v*v*S*CAInterp(prm[3], x[2])[0][0]
    s = np.sin(theta)
    c = np.cos(theta)
    xddot = (-fN*s - fA*c)/m
```

```
    yddot = (fN*c - fA*s - m*g)/m
    thetaddot = -fN * (CPInterp(prm[3])*l - com) / I
    if (alpha > np.radians(2)):
        raise ValueError("Alpha more than 2 Degrees")
    return [x[1], xddot, x[3], yddot, x[5], thetaddot, fA*v]

t0, tf = 0, 100
t_eval = np.linspace(0,100, 100000)

def hit_ground(t,y):
    return y[2]
hit_ground.terminal = True
hit_ground.direction = -1
```

[39]:
```
sol = sci.solve_ivp(deriv, (t0, tf), sol0, t_eval = t_eval, dense_output=True,␣
↪events=hit_ground, atol=1e-12, rtol=1e-12)
```

[40]:
```
v = np.hypot(sol.y[1], sol.y[3])
E = m*g*sol.y[2] + 0.5*m*v*v + sol.y[6]
print("Error: ",np.max(E) - np.min(E))
```

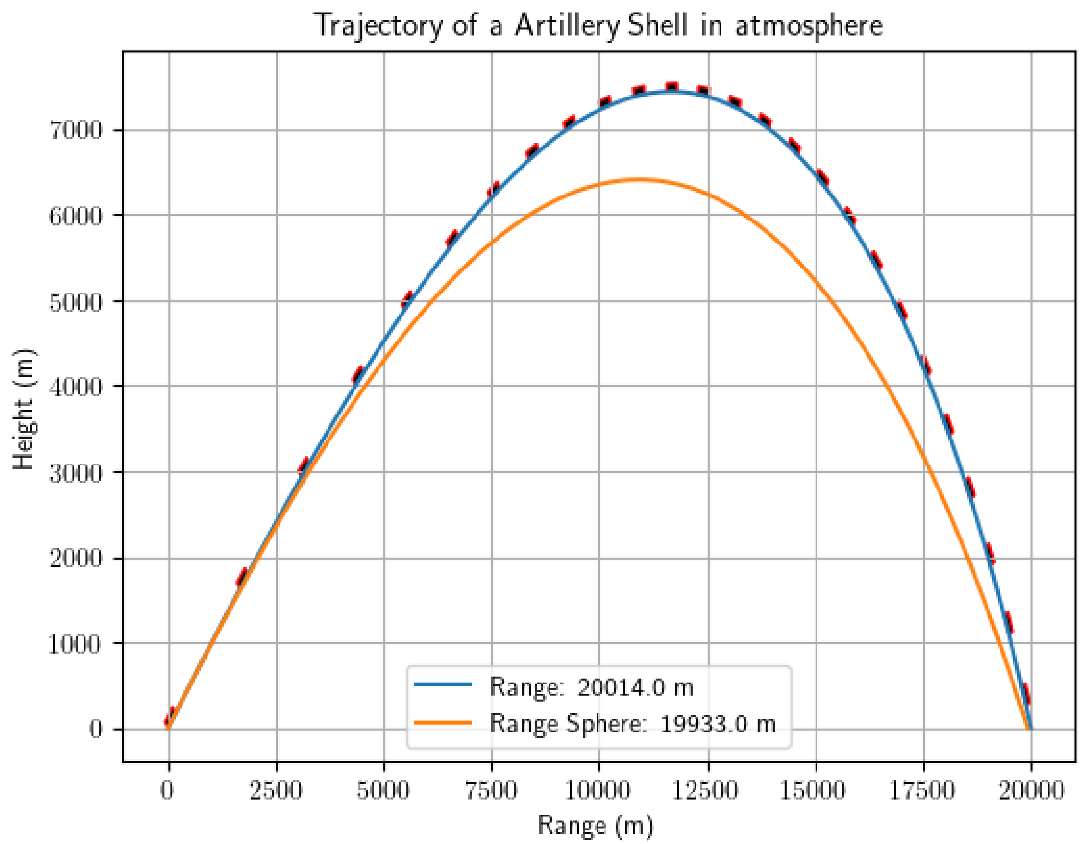
```
Error:  0.016091778874397278
```

[41]:
```
u = 710
theta = np.radians(45)
m = 20000
rho_s = 8000
d = np.cbrt((6*m)/(np.pi*rho_s))
s = np.pi*d*d/4
sol_sphere0 = np.array([0, u*np.cos(theta),0, u*np.sin(theta), 0.5*m*u*u])
def derive(t,u):
    v = np.hypot(u[1],u[3])
    prm = atm.atmParam(u[2],v) #Temp(h), Density(h) , Viscosity(h), Mach(h,v),␣
↪logRe(h,v), Cd(h,v)
    fd = 0.5*prm[1]*v*s*prm[5]
    return [u[1], -fd*u[1]/m, u[3],  -g - (fd*u[3]/m), fd * v *v]
```

[42]:
```
sol_sphere = sci.solve_ivp(derive, (t0, tf), sol_sphere0, t_eval = t_eval,␣
↪dense_output=True, events=hit_ground)
```

[43]:
```
plt.plot(sol.y[0], sol.y[2], label='Range: {} m'.format(np.round(np.max(sol.
↪y[0])),5))
plt.plot(sol_sphere.y[0], sol_sphere.y[2], label='Range Sphere: {} m'.format(np.
↪round(np.max(sol_sphere.y[0])),5))
plt.xlabel('Range (m)')
plt.ylabel('Height (m)')
plt.grid()
```

```
plt.legend(loc='lower center')
plt.title('Trajectory of a Artillery Shell in atmosphere')
for x in range(0,sol.y[0].size,3000):
    plt.gca().add_patch(Rectangle((sol.y[0][x],sol.y[2][x]),250,100,
                    angle=np.degrees(sol.y[4][x]),
                    edgecolor='red',
                    facecolor='black'))
```



[44]:
```
U_L = np.arange(200,1000,10)
l = 1.115
m = 40
I = 4.29
S = 80.12e-4
com = 0.70769
g = 9.8
RangeL = np.empty_like(U_L)

for i in range(U_L.size):
    U = U_L[i]
    print(U)
```

```
    sol0 = np.array([0, U*np.cos(theta0),0, U*np.sin(theta0), theta0, 0, 0.
↪5*m*U*U])
    try:
        sol = sci.solve_ivp(deriv, (t0, tf), sol0, t_eval = t_eval,␣
↪dense_output=True, events=hit_ground)
        RangeL[i] = np.max(sol.y[0])
    except ValueError as ve:
        print(ve)
        RangeL[i] = 0
```

```
200
210
220
230
240
250
260
270
280
290
300
310
320
330
340
350
360
370
380
390
400
410
Alpha more than 2 Degrees
420
430
440
450
460
470
480
490
500
510
520
530
540
```

```
550
560
570
580
590
600
610
620
630
640
650
660
670
680
690
700
710
720
730
740
750
760
770
780
790
800
810
820
830
840
850
860
870
880
890
900
910
920
930
940
950
Alpha more than 2 Degrees
960
970
980
Alpha more than 2 Degrees
990
```

[25]:
```
plt.plot(U_L, RangeL)
plt.xlabel('Initial Velocity $(m/s)$')
plt.ylabel('Range $(m)$')
plt.title('Initial Velocity v/s Range of the artilery shell')
```

[25]:
```
Text(0.5, 1.0, 'Initial Velocity v/s Range of the artilery shell')
```

The range of the artilery shell increases almost linearly with the initial velocity of the shell.

1.2 Simulation of rocket

[26]:
```
dfR = pd.read_excel('data-rocket.xlsx')
print(dfR.head())
CAjon=np.array([[0.3835,        0.3857,        0.3866,        0.361,        0.
↪3401,        0.4108,        0.4123,        0.4162,        0.428,        0.
↪4786],
[0.4158,        0.4184,        0.4204,        0.4074,        0.3646,        0.
↪4191,        0.5255,        0.5307,        0.549,        0.6322],
[0.6169,        0.6147,        0.617,        0.6073,        0.557,        0.
↪6051,        0.6991,        0.7039,        0.7214,        0.801],
```

```
[0.7267,       0.7244,       0.7267,       0.7189,       0.6655,       0.
 ↪7093,       0.7954,       0.7999,       0.8163,       0.8914],
[0.8158,       0.8131,       0.8154,       0.8096,       0.7534,       0.
 ↪7933,       0.8726,       0.8768,       0.8922,       0.9629],
[0.8902,       0.8882,       0.8909,       0.8885,       0.825,       0.
 ↪8668,       0.9484,       0.9524,       0.9668,       1.0329],
[0.9371,       0.9348,       0.9375,       0.938,       0.8722,       0.
 ↪91,       0.9872,       0.9909,       1.0045,       1.067],
[0.7873,       0.7842,       0.7866,       0.7898,       0.7456,       0.
 ↪7501,       0.8117,       0.9258,       0.9379,       0.9939],
[0.6864,       0.6824,       0.6843,       0.6882,       0.6575,       0.
 ↪6412,       0.6913,       0.7846,       0.7953,       0.8446],
[0.6208,       0.6158,       0.6173,       0.6212,       0.6056,       0.
 ↪5716,       0.6118,       0.6852,       0.6944,       0.7369],
[0.6077,       0.6024,       0.6038,       0.6075,       0.593,       0.
 ↪5578,       0.5964,       0.6661,       0.6751,       0.7164],
[0.5872,       0.5814,       0.5825,       0.5858,       0.5742,       0.
 ↪5367,       0.5716,       0.6336,       0.642,       0.681],
[0.5654,       0.5659,       0.5667,       0.5691,       0.5605,       0.
 ↪5216,       0.5537,       0.6093,       0.6174,       0.6547],
[0.5316,       0.532,       0.5325,       0.5344,       0.5272,       0.
 ↪488,       0.5181,       0.5685,       0.5763,       0.6128],
[0.5193,       0.5194,       0.5198,       0.5211,       0.5156,       0.
 ↪4761,       0.5045,       0.5504,       0.5581,       0.5936]])
```

```
   Mach  C_N_Alpha      x_CP  Majoff   hAjoff  Majon     hAon     tm  \
0   0.2      14.57  0.842868     0.4      0.0    0.4      0.0  0.000
1   0.4      14.43  0.851050     0.6   5000.0    0.6   5000.0  0.040
2   0.6      14.86  0.850584     0.8  10000.0    0.8  10000.0  0.041
3   0.8      15.16  0.848138     0.9  20000.0    0.9  20000.0  0.042
4   0.9      15.67  0.851823     1.0  30000.0    1.0  30000.0  0.043

   thrustKN  massExpelled
0     0.000         0.000
1     5.921         0.000
2     6.335         0.002
3     7.164         0.006
4     7.933         0.011
```

1.2.1 Interpolation Functions

[27]:
```
CNAlphaInterpR = inp.interp1d(dfR.Mach[:19], dfR.C_N_Alpha[:19], kind='cubic')
CPInterpR = inp.interp1d(dfR.Mach[:19], dfR.x_CP[:19], kind='cubic')
CA_joffInterpR = inp.RectBivariateSpline(dfR.Majoff[:15], dfR.hAjoff[:10],␣
 ↪CAjoff)
CA_jonInterpR = inp.RectBivariateSpline(dfR.Majon[:15], dfR.hAon[:10], CAjon)
ThrustKNInterp = inp.interp1d(dfR.tm, dfR.thrustKN, kind='cubic')
```

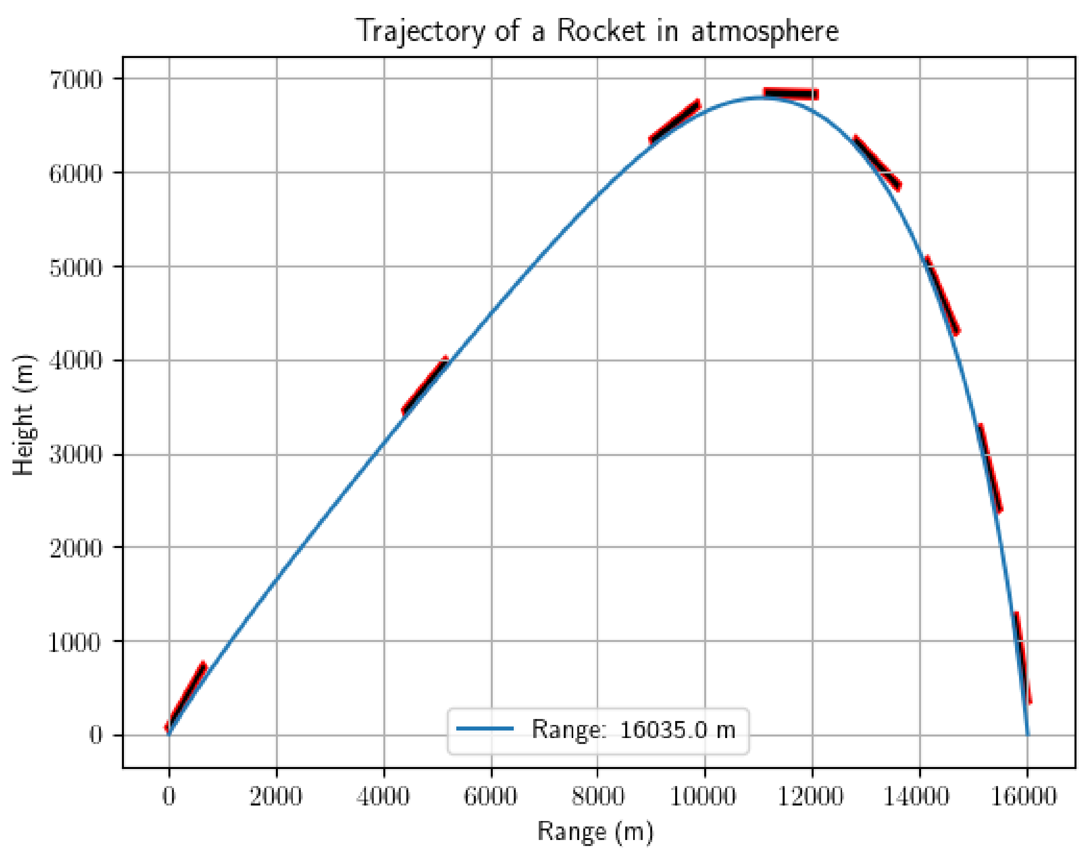
```
massExpl_Interp = inp.interp1d(dfR.tm, dfR.massExpelled, kind='cubic')
```

[134]:
```
l = 2.277
m0 = 88
R = 287
S = np.pi*0.207*0.207/4 #
AE = np.pi*0.125*0.125/4
com = 0.70769
U = 100
g = 9.8
mp0 = 48.771
mS = 39.229
t_eval = np.linspace(0,1000, 100000)
tthrust = 11.778
theta0 = np.radians(45)
sol0 = np.array([0, U*np.cos(theta0),0, U*np.sin(theta0), theta0, 0, 0.5*m0*U*U])
def deriv(t,x):
    theta = x[4]
    alpha = theta - np.arctan2(x[3],x[1])
    v = np.hypot(x[1],x[3])
    prm = atm.atmParam(x[2],v) #Temp(h), Density(h) , Viscosity(h), Mach(h,v),␣
 ↪logRe(h,v), Cd(h,v)
    fN = 0.5*prm[1]*v*v*S*CNAlphaInterpR(prm[3])*alpha
    if t <= tthrust:
        fA = 0.5*prm[1]*v*v*S*CA_jonInterpR(prm[3], x[2])[0][0]
        T = ThrustKNInterp(t)*1000 - prm[0]*prm[1]*R*AE
        mp_expl = massExpl_Interp(t)
        X = (43.681 + 1.4735*mp_expl)/(39.229 + mp_expl)
        I = 16.318 + 39.229*(1.1135-X)**2 + (0.0979 + (1.4735 - X)**2)*mp_expl
    else:
        fA = 0.5*prm[1]*v*v*S*CA_joffInterpR(prm[3], x[2])[0][0]
        T = 0
        mp_expl = 48.771
        X = 1.1135
        I = 16.318
    m = mp0 - mp_expl + mS
    s = np.sin(theta)
    c = np.cos(theta)
    xddot = (T*c -fN*s - fA*c)/m
    yddot = (fN*c + T*s - fA*s - m*g)/m
    thetaddot = -fN * (CPInterpR(prm[3])*l - X) / I
    if (alpha > np.radians(2)):
        raise ValueError("Alpha more than 2 Degrees")
    return [x[1], xddot, x[3], yddot, x[5], thetaddot, fA*v]
t0, tf = 0, 1000
def hit_ground(t,y):
    return y[2]
```

```
hit_ground.terminal = True
hit_ground.direction = -1
```

[135]:
```
sol = sci.solve_ivp(deriv, (t0, tf), sol0, t_eval = t_eval, dense_output=True,␣
↪events=hit_ground)
```

[136]:
```
plt.plot(sol.y[0], sol.y[2], label='Range: {} m'.format(np.round(np.max(sol.
↪y[0])),5))
plt.xlabel('Range (m)')
plt.ylabel('Height (m)')
plt.grid()
plt.legend(loc='lower center')
plt.title('Trajectory of a Rocket in atmosphere')
for x in range(0,sol.y[0].size,1000):
    plt.gca().add_patch(Rectangle((sol.y[0][x],sol.y[2][x]),1000,100,
                    angle=np.degrees(sol.y[4][x]),
                    edgecolor='red',
                    facecolor='black'))
```



[137]:
```
angle = np.arange(0,91,1)
Range_L  = np.empty_like(angle)
Height_L = np.empty_like(angle)
for i in range(angle.size):
    print(angle[i])
    theta0 = np.radians(angle[i])
    sol0 = np.array([0, U*np.cos(theta0),0, U*np.sin(theta0), theta0, 0, 0.
 ↪5*m0*U*U])
    try:
        sol = sci.solve_ivp(deriv, (t0, tf), sol0, t_eval = t_eval,␣
 ↪dense_output=True, events=hit_ground)
        Range_L[i] = np.max(sol.y[0])
        Height_L[i] = np.max(sol.y[2])
    except ValueError as ve:
        print(ve)
        Range_L[i] = 0
        Height_L[i] = 0
```

```
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
```

```
78
79
80
81
82
Alpha more than 2 Degrees
83
A value (0.1992947543409542) in x_new is below the interpolation range's minimum
value (0.2).
84
A value (0.19852833995424451) in x_new is below the interpolation range's
minimum value (0.2).
85
A value (0.19983626107992858) in x_new is below the interpolation range's
minimum value (0.2).
86
A value (0.19218277921761642) in x_new is below the interpolation range's
minimum value (0.2).
87
A value (0.19726823119067527) in x_new is below the interpolation range's
minimum value (0.2).
88
A value (0.19177844846277248) in x_new is below the interpolation range's
minimum value (0.2).
89
A value (0.1879083369270822) in x_new is below the interpolation range's minimum
value (0.2).
90
Alpha more than 2 Degrees
```

[138]:

```
print("Angle with Maximum Range: ", angle[np.where(Range_L==np.max(Range_L))][0])
print("Angle with Maximum Height: ", angle[np.where(Height_L==np.
↪max(Height_L))][0])
```

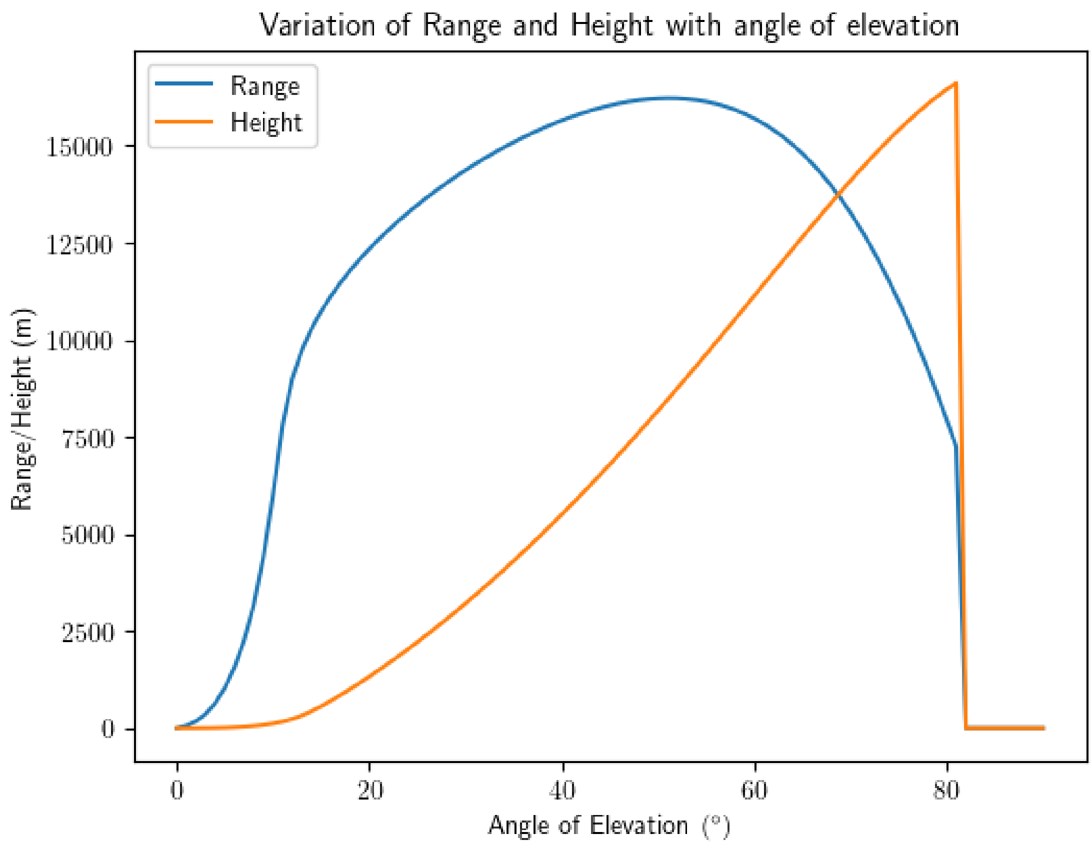
```
Angle with Maximum Range:  51
Angle with Maximum Height:  81
```

[139]:

```
plt.plot(angle, Range_L, label='Range')
plt.plot(angle, Height_L, label='Height')
plt.title("Variation of Range and Height with angle of elevation")
plt.xlabel("Angle of Elevation $(^\circ)$")
plt.ylabel("Range/Height (m)")
plt.legend()
```

[139]:

```
<matplotlib.legend.Legend at 0x2c2397d8210>
```



Therefore, we get the maximum range at 49 Degrees and maximum height at 83 Degrees for the intitial velocity of 100 m/s.