1 AE332: Modelling and Analysis Lab

1.1 Session 2: Simulating a Cannonball

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SC-Code: SC21B026

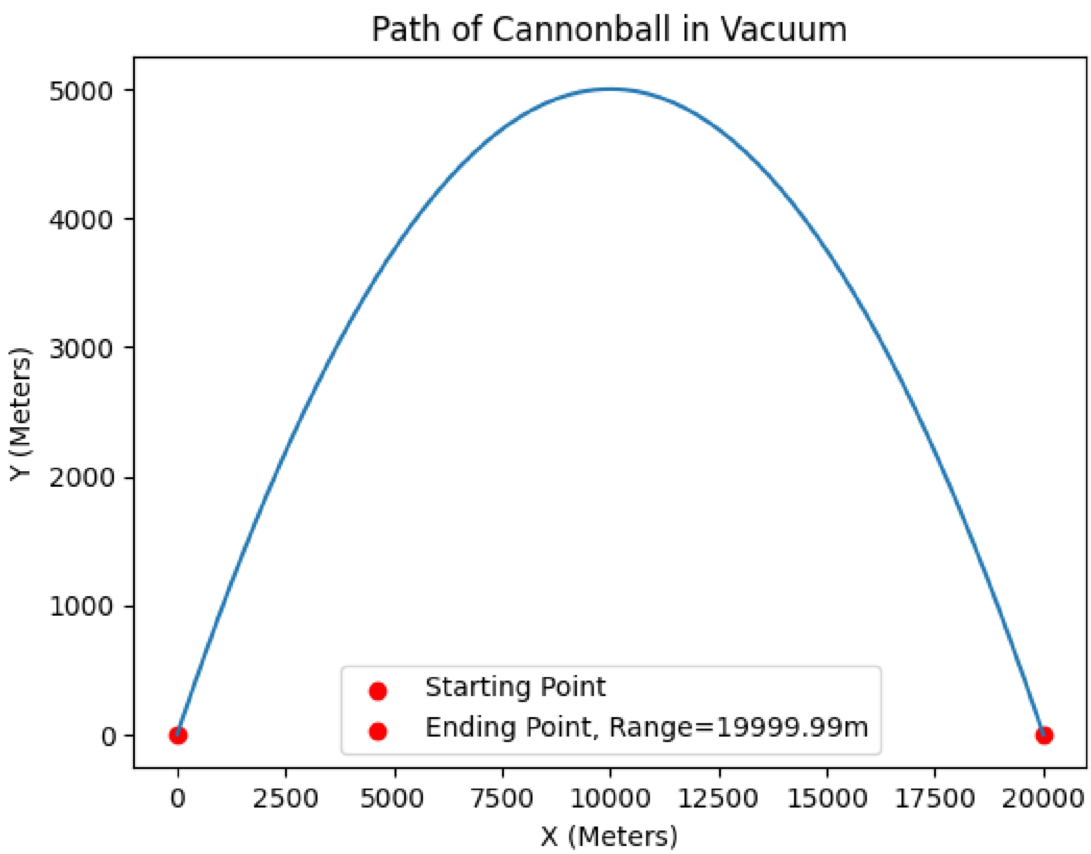
[1]:
```
import numpy as np
import matplotlib.pyplot as plt
import Atmosphere as atm
import scipy.integrate as sci
```

1.2 Simulation of Cannonball in Vacuum

[42]:
```
#Initital Conditions
u = 442.71888
theta = np.radians(45)
m = 40
g = 9.8
sol0 = np.array([0, u*np.cos(theta),0, u*np.sin(theta)])
def derive(t,u):
    return [u[1], 0, u[3],  -g]
t0, tf = 0, 100
t_eval = np.linspace(0,100, 100000)

def hit_ground(t,y):
    return y[2]
hit_ground.terminal = True
hit_ground.direction = -1
```

[43]:
```
sol = sci.solve_ivp(derive, (t0, tf), sol0, t_eval = t_eval, dense_output=True,␣
 ↪events=hit_ground)
```

[44]:
```
plt.plot(sol.y[0], sol.y[2])
plt.xlabel('X (Meters)')
plt.ylabel('Y (Meters)')
plt.title('Path of Cannonball in Vacuum')
plt.scatter(sol.y[0][0], sol.y[2][0], color='r', label='Starting Point')
plt.scatter(sol.y[0][-1], sol.y[2][-1], color = 'r', label = 'Ending Point,␣
 ↪Range={}m'.format(np.round(sol.y[0][-1], 2)))
plt.legend()
solVacuum = sol
```



[45]:
```
v = np.hypot(sol.y[1], sol.y[3])
E = m*g*sol.y[2] + 0.5*m*v*v
print("Error: ", np.abs(np.max(E-(0.5*m*u*u))))
```

```
Error:  5.587935447692871e-09
```

[46]:
```
#Reducing the rtol and atol
sol = sci.solve_ivp(derive, (t0, tf), sol0, t_eval = t_eval, dense_output=True,
 ↪events=hit_ground, rtol=1e-12, atol=1e-12)
v = np.hypot(sol.y[1], sol.y[3])
E = m*g*sol.y[2] + 0.5*m*v*v
print("Error: ", np.abs(np.max(E-(0.5*m*u*u))))
```

```
Error:  1.4901161193847656e-08
```

1.3 Cannonball in Atmosphere

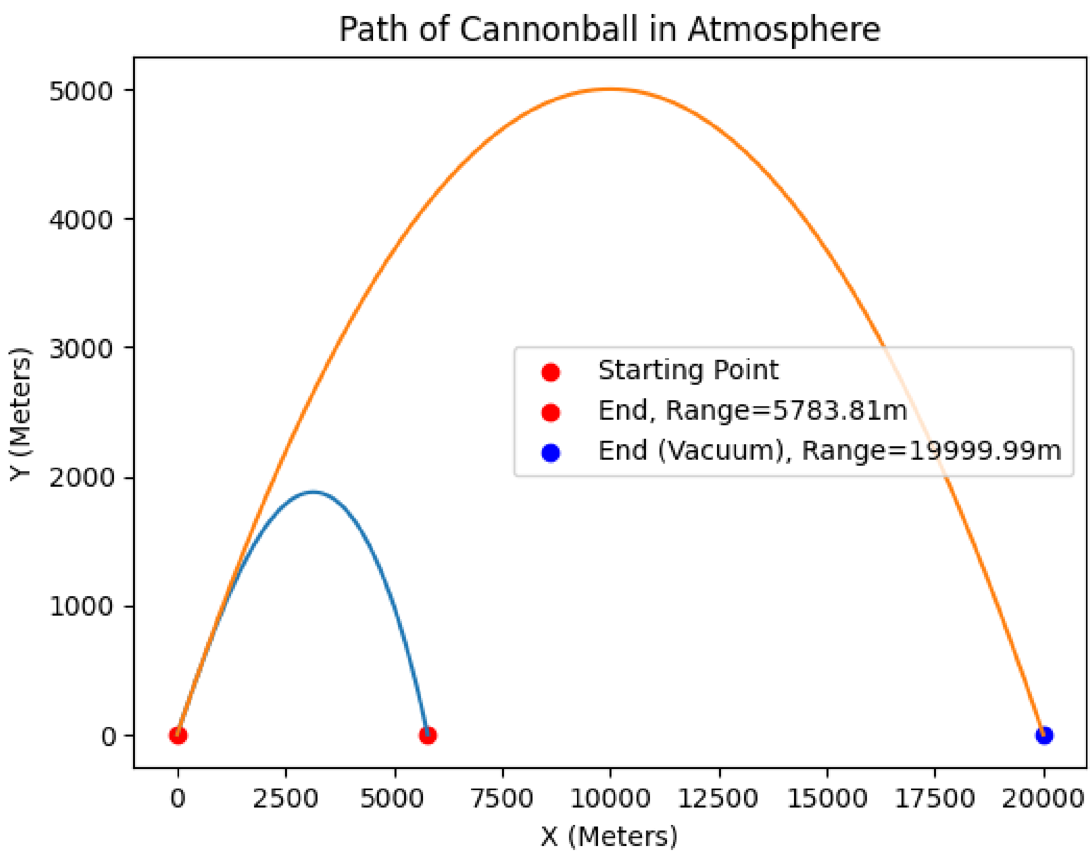
1.3.1 Problem 1

[47]:
```
#Initital Conditions
u = 442.71888
theta = np.radians(45)
m = 40
g = 9.8
rho_s = 8000
d = np.cbrt(6*m/np.pi/rho_s)

s = np.pi*d*d/4
sol0 = np.array([0, u*np.cos(theta),0, u*np.sin(theta), 0.5*m*u*u])
def derive(t,u):
    v = np.hypot(u[1],u[3])
    prm = atm.atmParam(u[2],v) #Temp(h), Density(h) , Viscosity(h), Mach(h,v),
 ↪logRe(h,v), Cd(h,v)
    fd = 0.5*prm[1]*v*s*prm[5]
    return [u[1], -fd*u[1]/m, u[3],  -g - (fd*u[3]/m), fd * v *v]
t0, tf = 0, 100
t_eval = np.linspace(0,100, 100000)


def hit_ground(t,y):
    return y[2]
hit_ground.terminal = True
hit_ground.direction = -1
```

[48]:
```
sol = sci.solve_ivp(derive, (t0, tf), sol0, t_eval = t_eval, dense_output=True,
 ↪events=hit_ground)
```

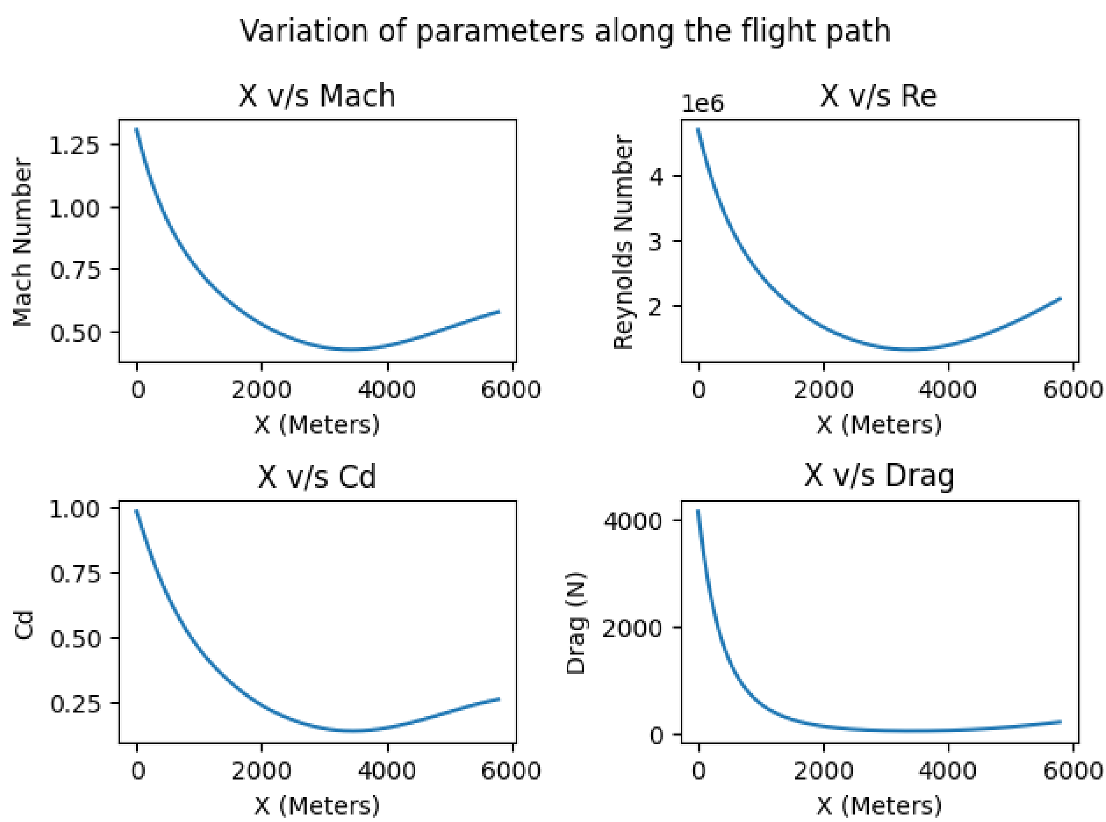
[49]:
```
plt.plot(sol.y[0], sol.y[2])
plt.plot(solVacuum.y[0], solVacuum.y[2])
plt.xlabel('X (Meters)')
plt.ylabel('Y (Meters)')
plt.title('Path of Cannonball in Atmosphere')
plt.scatter(sol.y[0][0], sol.y[2][0], color='r', label='Starting Point')
plt.scatter(sol.y[0][-1], sol.y[2][-1], color = 'r', label = 'End, Range={}m'.
 ↪format(np.round(sol.y[0][-1], 2)))
plt.scatter(solVacuum.y[0][-1], solVacuum.y[2][-1], color = 'b', label = 'End
 ↪(Vacuum), Range={}m'.format(np.round(solVacuum.y[0][-1], 2)))
plt.legend(loc='center right')
```

[49]:
```
<matplotlib.legend.Legend at 0x21184806310>
```



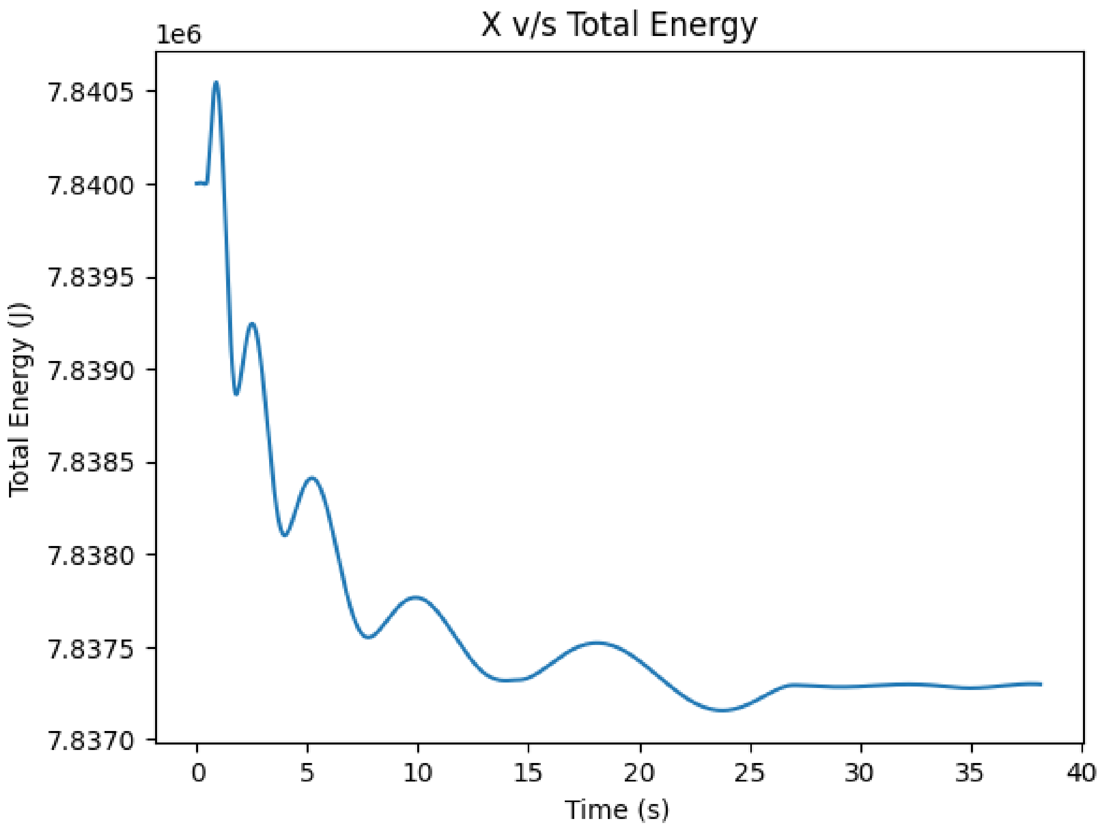
[50]:
```
v = np.hypot(sol.y[1],sol.y[3])
prm = atm.atmParam(sol.y[2],v) #Temp(h), Density(h) , Viscosity(h), Mach(h,v),␣
↪logRe(h,v), Cd(h,v)
plt.subplot(2,2,1)
plt.plot(sol.y[0], prm[3], label='Mach Number')
plt.xlabel('X (Meters)')
plt.ylabel('Mach Number')
plt.title("X v/s Mach")
plt.subplot(2,2,2)
plt.plot(sol.y[0], np.exp(prm[4]), label='Reynolds Number')
plt.xlabel('X (Meters)')
plt.ylabel('Reynolds Number')
plt.title("X v/s Re")
plt.subplot(2,2,3)
plt.plot(sol.y[0], prm[5], label='Cd')
plt.xlabel('X (Meters)')
plt.ylabel('Cd')
plt.title("X v/s Cd")
plt.subplot(2,2,4)
```

```
fd = 0.5*prm[1]*v*v*s*prm[5]
plt.plot(sol.y[0], fd, label='Cd')
plt.xlabel('X (Meters)')
plt.ylabel('Drag (N)')
plt.title("X v/s Drag")
plt.suptitle("Variation of parameters along the flight path")
plt.tight_layout(pad=1)
```




```
[51]: E = m*g*sol.y[2] + 0.5*m*v*v + sol.y[4]
plt.plot(sol.t, E)
plt.xlabel('Time (s)')
plt.ylabel('Total Energy (J)')
plt.title("X v/s Total Energy")
print("Error: ",np.abs(np.max(E) - np.min(E)))
```

```
Error:  3389.597687105648
```



[52]:

```
#Reducing atol and vtol
sol = sci.solve_ivp(derive, (t0, tf), sol0, t_eval = t_eval, dense_output=True,␣
 ↪events=hit_ground, atol=1e-13, rtol=1e-13)
v = np.hypot(sol.y[1],sol.y[3])
E = m*g*sol.y[2] + 0.5*m*v*v + sol.y[4]
print("Error: ", np.abs(np.max(E) - np.min(E)))
```

```
Error:  6.882473826408386e-07
```

1.4 Problem 2

[93]:

```
#intitial conditions
u = 710
theta = np.radians(45)
m = 20000
g = 9.8
rho_s = 8000
d = np.cbrt((6*m)/(np.pi*rho_s))
print(m, rho_s, d)
s = np.pi*d*d/4
sol0 = np.array([0, u*np.cos(theta),0, u*np.sin(theta), 0.5*m*u*u])
```

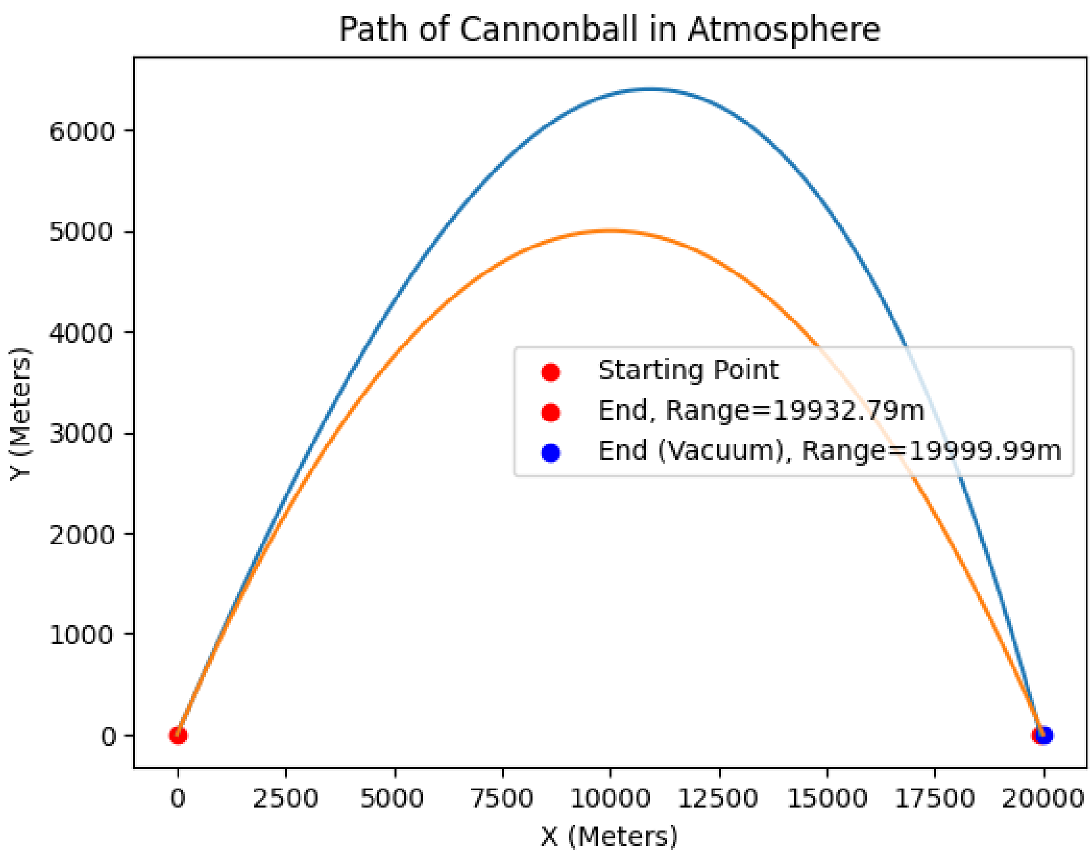
```python
def derive(t,u):
    v = np.hypot(u[1],u[3])
    prm = atm.atmParam(u[2],v) #Temp(h), Density(h) , Viscosity(h), Mach(h,v), logRe(h,v), Cd(h,v)
    fd = 0.5*prm[1]*v*s*prm[5]
    return [u[1], -fd*u[1]/m, u[3],  -g - (fd*u[3]/m), fd * v *v]
t0, tf = 0, 100
t_eval = np.linspace(0,100, 100000)

def hit_ground(t,y):
    return y[2]
hit_ground.terminal = True
hit_ground.direction = -1
```

```
20000 8000 1.6838903009606296
```

[94]:
```python
sol = sci.solve_ivp(derive, (t0, tf), sol0, t_eval = t_eval, dense_output=True, events=hit_ground)
```

[95]:
```python
plt.plot(sol.y[0], sol.y[2])
plt.plot(solVacuum.y[0], solVacuum.y[2])
plt.xlabel('X (Meters)')
plt.ylabel('Y (Meters)')
plt.title('Path of Cannonball in Atmosphere')
plt.scatter(sol.y[0][0], sol.y[2][0], color='r', label='Starting Point')
plt.scatter(sol.y[0][-1], sol.y[2][-1], color = 'r', label = 'End, Range={}m'.format(np.round(sol.y[0][-1], 2)))
plt.scatter(solVacuum.y[0][-1], solVacuum.y[2][-1], color = 'b', label = 'End (Vacuum), Range={}m'.format(np.round(solVacuum.y[0][-1], 2)))
plt.legend(loc='center right')
```

[95]:
```
<matplotlib.legend.Legend at 0x21184d36ad0>
```



The feasibility of acheiving of the 20km range is very low since providing the initial velocity of 710 m/s is not practical. As we increase the mass of the cannon ball, the diameter of the ball also increases which in turn increases the drag.

1.4.1 Sensitivity of Elevation Angle with Range

[39]:

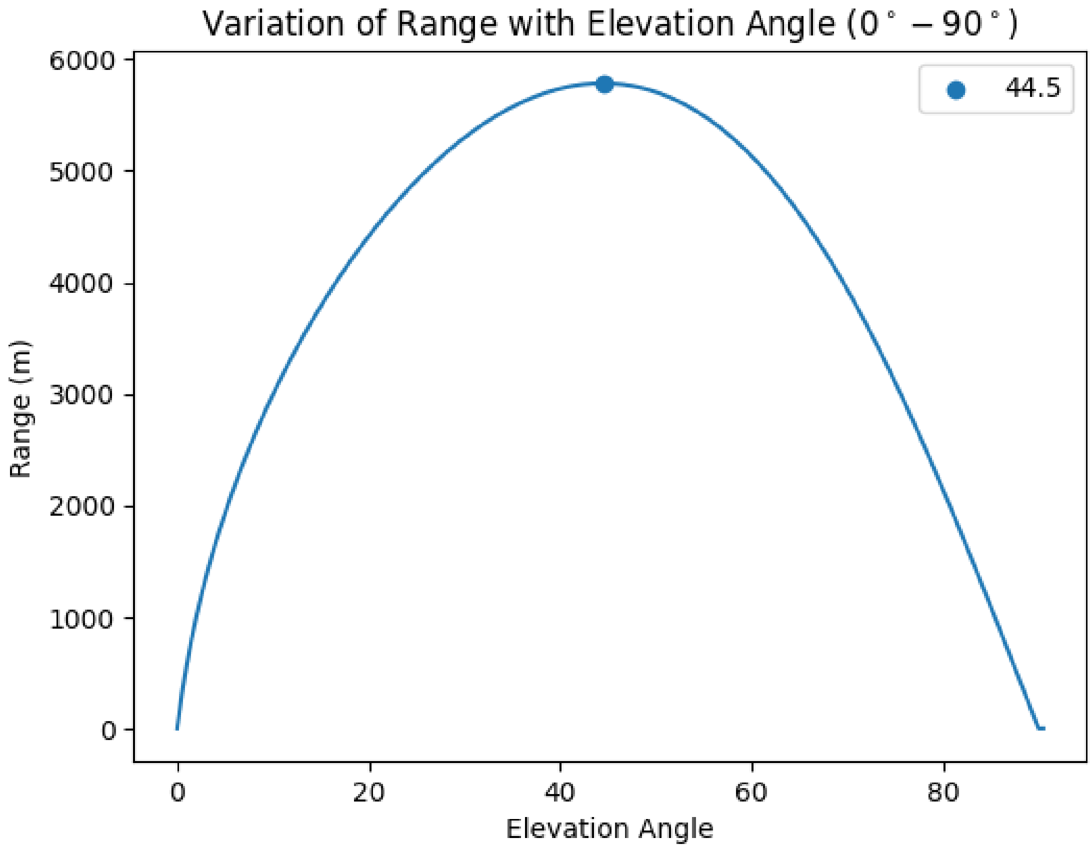
```
#Inititial Conditions
u = 442.71888
m = 40
g = 9.8
rho_s = 8000
d = np.cbrt(6*m/np.pi/rho_s)
s = np.pi*d*d/4

def derive(t,u):
    v = np.hypot(u[1],u[3])
    prm = atm.atmParam(u[2],v) #Temp(h), Density(h) , Viscosity(h), Mach(h,v),␣
 ↪logRe(h,v), Cd(h,v)
    fd = 0.5*prm[1]*v*s*prm[5]
    return [u[1], -fd*u[1]/m, u[3],  -g - (fd*u[3]/m), fd * v *v]
```

```
t0, tf = 0, 100
t_eval = np.linspace(0,100, 100000)


def hit_ground(t,y):
    return y[2]
hit_ground.terminal = True
hit_ground.direction = -1
```

[40]:
```
thetaL = np.arange(0,91,0.5)
Range = np.ones_like(thetaL)
for x in range(thetaL.size):
    theta = thetaL[x]*np.pi/180
    sol0 = np.array([0, u*np.cos(theta),0, u*np.sin(theta), 0.5*m*u*u])
    sol = sci.solve_ivp(derive, (t0, tf), sol0, t_eval = t_eval,␣
 ↪dense_output=True, events=hit_ground)
    Range[x] = np.max(sol.y[0])
```

[41]:
```
plt.plot(thetaL, Range)
thetamax = thetaL[np.where(Range==np.max(Range))]
plt.scatter(thetamax, np.max(Range), label=str(thetamax[0]))
plt.xlabel('Elevation Angle')
plt.ylabel('Range (m)')
plt.legend()
plt.title(r"Variation of Range with Elevation Angle $(0^\circ-90^\circ)$ ")
```

[41]:
```
Text(0.5, 1.0, 'Variation of Range with Elevation Angle $(0^\\circ-90^\\circ)$
')
```



The derivative of the above curve gives the sensitivity of range with respect to elevation angle.