Optimization Tutorial Assignment 2

1. Maximize $Z = 12x_1 + 15x_2 + 14x_3$

Subject to $-x_1 + x_2 \leq 0$

$-x_1 + 2x_3 \leq 0$

$x_1 + x_2 + x_3 \leq 100$

$x_1, x_2, x_3 \geq 0$

Standard form:-

Max. $Z = 12x_1 + 15x_2 + 14x_3 + 0 \cdot S_1 + 0 \cdot S_2 + 0 \cdot S_3$

Subject to $-x_1 + x_2 + S_1 = 0$

$-x_1 + 2x_3 + S_2 = 0$

$x_1 + x_2 + x_3 + S_3 = 100$

$x_1, x_2, x_3 \geq 0$, $S_1, S_2, S_3 \geq 0$

Basis	C	x_1	x_2	x_3	S_1	S_2	S_3	RHS	θ_i
S_1	0	-1	1	0	1	0	0	0	$0/1 = 0$ ←
S_2	0	-1	0	2	0	1	0	0	$0/0$
S_3	0	1	1	1	0	0	1	100	$100/1 = 100$
	C_j	12	15	14	0	0	0		
	Z_j	0	0	0	0	0	0		
	Δ_j	12	15	14	0	0	0		

↑

Basis	C	x_1	x_2	x_3	S_1	S_2	S_3	RHS	θ
x_2	15	-1	1	0	1	0	0	0	0
S_2	0	-1	0	2	0	1	0	0	0
S_3	0	2	0	1	-1	0	1	100	50 ←
	C_j	12	15	14	0	0	0		
	Z_j	-15	15	0	15	0	0		
	Δ_j	27 ↑	0	14	-15	0	0		

x_2	15	0	1	1/2	1/2	0	1/2	50	100
S_2	0	0	0	5/2	-1/2	1	1/2	50	20 ←
x_1	12	1	0	1/2	-1/2	0	1/2	50	25
	C_j	12	15	14	0	0	0		
	Z_j	12	15	23/2	3/2	0	27/2		
	Δ_j	0	0	1/2 ↑	-3/2	0	-27/2		

x_2	15	0	1	0	3/5	-1/5	2/5	40
x_3	14	0	0	1	-1/5	2/5	1/5	20
x_1	12	1	0	0	-2/5	-1/5	2/5	40
	C_j	12	15	14	0	0	0	
	Z_j	12	15	14	7/5	1/5	68/5	
	Δ_j	0	0	0	-7/5	-1/5	-68/5	

$\therefore x_2 = 40,\ x_3 = 20\ \&\ x_1 = 40 \Rightarrow \underline{Z = 1360}$

2. Minimize $Z = x_1 + x_2 + x_3$

Subject to $x_1 + 3x_2 + 4x_3 = 5$

$x_1 + 2x_2 \leq 3$

$2x_2 + x_3 \leq -4.$

$x_1 \geq 0,\ x_2 \leq 0,\ x_3$ is unrestricted in sign.

<u>Standard form</u> :-

Minimize $Z = x_1 - x_4 + x_5 - x_6 + 0 \cdot S_1 + 0 S_2$ (Let $x_2 = -x_4,\ x_3 = x_5 - x_6$)

Subject to $x_1 + 3x_4 + 4x_5 - 4x_6 = 5$

$x_1 - 2x_4 + S_1 = 3$

$2x_4 - x_5 + x_6 - S_2 = 4$

$x_1 \geq 0,\ x_4 \geq 0,\ x_5 \geq 0,\ x_6 \geq 0,\ S_1 \geq 0,\ S_2 \geq 0$

<u>Canonical form</u> :-

Minimize $Z = x_1 - x_4 + x_5 - x_6 + 0 \cdot S_1 + 0 \cdot S_2 + MA_1 + MA_2$

Subject to $x_1 - 3x_4 + 4x_5 - 4x_6 + A_1 = 5$

$x_1 - 2x_4 + S_1 = 3$

$2x_4 - x_5 + x_6 - S_2 + A_2 = 4.$

$x_1, x_4, x_5, x_6, S_1, S_2, A_1, A_2 \geq 0.$

Basis	C	x_1	x_4	x_5	x_6	S_1	S_2	A_1	A_2	RHS	θ_j
A_1	M	1	-3	4	-4	0	0	1	0	5	5/4 ←
S_1	0	1	-2	0	0	1	0	0	0	3	3/0
A_2	M	0	2	-1	1	0	-1	0	1	4	-4
	C_j	1	-1	1	-1	0	0	M	M		
	Z_j	M	-M	3M	-3M	0	-M	M	M		
	Δ_j	1-M	-1+M	1-3M ↑	3M-1	0	M	0	0		

$[1\ -3\ \ 4\ -4\ \ 0\ \ 0\ \ 1\ \ 0]\ 5$

$[0\ \ 6\ -3\ \ 3\ \ 0\ -3\ \ 0\ \ 3]\ 12$

Basis	C	x_1	x_4	x_5	x_6	S_1	S_2	A_1	A_2	RHS	θ
x_5	1	1/4	-3/4	1	-1	0	0	1/4	0	5/4	
S_1	0	1	-2	0	0	1	0	0	0	3	
A_2	M	1/4	5/4	0	0	0	-1	1/4	1	21/4	21/5 ←
	C_j	1	-1	1	-1	0	0	M	M		
	Z_j	$1/4+\frac{M}{4}$	$\frac{5}{4}M-3/4$	1	-1	0	-M	$\frac{M+1}{4}$	M		
	Δ_j	$\frac{-M}{4}+\frac{3}{4}$	$\frac{-5}{4}M-3/4$	0	0	0	M	$\frac{3M}{4}-\frac{1}{4}$	0		
			↑								

Basis	C	x_1	x_4	x_5	x_6	S_1	S_2	A_1	A_2	RHS	θ
x_5	1	8/20	0	1	-1	0	-3/5	8/20	3/5	88/20	-88/12
S_1	0	7/5	0	0	0	1	-8/5	2/5	8/5	57/5	-57/8
x_4	-1	1/5	1	0	0	0	-4/5	-1/5	4/5	21/5	-21/4
	C_j	1	-1	1	-1	0	0	M	M		
	Z_j	1/5	-1	1	-1	0	1/5	1/5	-1/5		
	Δ_j	4/5	0	0	0	0	-1/5	M-1/5	M+1/5		
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The problem is unbounded, since the coeffecients in S_2 column are negative.

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3. Maximize $Z = 4x_1 + 10x_2$

Subject to $2x_1 + x_2 \leq 10$

$2x_1 + 5x_2 \leq 20$

$2x_1 + 3x_2 \leq 18$

$x_1; x_2 \geq 0$.

Standard form :-

Maximize $Z = 4x_1 + 10x_2 + 0 \cdot S_1 + 0 \cdot S_2 + 0 \cdot S_3$

Subject to $2x_1 + x_2 + S_1 = 10$

$2x_1 + 5x_2 + S_2 = 20$

$2x_1 + 3x_2 + S_3 = 18$

$x_1, x_2, S_1, S_2, S_3 \geq 0$

Basis	C	x_1	x_2	S_1	S_2	S_3	RHS	θ_j
S_1	0	2	1	1	0	0	10	10
S_2	0	2	5	0	1	0	20	4 ←
S_3	0	2	3	0	0	1	18	6
	C_j	4	10	0	0	0		
	Z_j	0	0	0	0	0		
	Δ_j	4	10 ↑	0	0	0		

S_1	0	$8/5$	0	1	$-1/5$	0	6
x_2	10	$2/5$	1	0	$1/5$	0	4
S_3	0	$-2/5$	0	0	$-3/5$	1	6
	C_j	4	10	0	0	0	
	Z_j	4	10	0	2	0	
	Δ_j	$\boxed{0}$	0	0	-2	0	

$\because$ $\Delta_j = 0$ for x_1 and there is no net improvement, the problem has multiple sol.

4. Minimize $Z = x_1 + 2x_2 + x_3$

Subject to $\frac{3}{2}x_1 + 2x_2 + x_3 \geq 8$

$x_1 + x_2/2 + x_3/2 \leq 1$

$x_1, x_2, x_3 \geq 0$

Canonical version :-

Minimize $Z = x_1 + 2x_2 + x_3 + 0 \cdot S_1 + 0 \cdot S_2 + MA_1$

Subject to $3x_1 + 4x_2 + 2x_3 - S_1 + A_1 = 16$

$2x_1 + x_2 + x_3 + S_2 = 2$

$$\Rightarrow C = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 & M \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 & x_2 & x_3 & S_1 & S_2 & A_1 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 4 & 2 & -1 & 0 & 1 \\ 2 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \quad b = \begin{bmatrix} 16 \\ 2 \end{bmatrix}$$

Iteration 1

Basic variables :- $A_1, S_2 \Rightarrow B_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\bar{C}_1 = C_1 - \begin{bmatrix} M & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = 1 - 3M$$

$$\bar{C}_2 = C_2 - \begin{bmatrix} M & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} = -4M + 2 \leftarrow \qquad \theta = \begin{bmatrix} 16/4 \\ 2/1 \end{bmatrix} \leftarrow$$

$$\bar{C}_3 = C_3 - \begin{bmatrix} M & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 1 - 2M$$

$$\bar{C}_4 = C_4 - \begin{bmatrix} M & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix} = 0 + M$$

Iteration 2 :-

Entering variable : x_2, leaving variable : S_2

$$B_1 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \Rightarrow B_1^{-1} = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$$

$$\bar{P}_1 = B_1^{-1} P_1 = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \end{bmatrix}$$

$$\bar{P}_3 = B_1^{-1} P_3 = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$\bar{P}_4 = B_1^{-1} P_4 = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\bar{P}_5 = B^{-1} P_5 = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$$

$$\bar{c}_1 = c_1 - \begin{bmatrix} M & 2 \end{bmatrix} \begin{bmatrix} -5 \\ 2 \end{bmatrix} = 1 + 5M - 4 = 5M - 3$$

$$\bar{c}_3 = c_3 - \begin{bmatrix} M & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} = 1 + 2M - 2 = 2M - 1$$

$$\bar{c}_4 = c_4 - \begin{bmatrix} M & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix} = 0 + M = M$$

$$\bar{c}_5 = c_5 - \begin{bmatrix} M & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} = 0 + 4M - 2$$

$$\bar{b} = B_1^{-1} B = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 16 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \end{bmatrix} = \begin{bmatrix} A_1 \\ x_2 \end{bmatrix}$$

Since an artificial variable is in the solution, the solution is infeasible.

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5. Maximize $Z = 2x_1 + x_2$

Subject to $4x_1 + 3x_2 \leq 12$

$4x_1 + x_2 \leq 8$

$4x_1 - x_2 \leq 8$

$x_1, x_2 \geq 0$

Standard form :-

Maximize $Z = 2x_1 + x_2 + 0 \cdot s_1 + 0 \cdot s_2 + 0 s_3$

Subject to $4x_1 + 3x_2 + s_1 = 12$

$4x_1 + x_2 + s_2 = 8$

$4x_1 - x_2 + s_3 = 8$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

$$C = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 & x_2 & s_1 & s_2 & s_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 3 & 1 & 0 & 0 \\ 4 & 1 & 0 & 1 & 0 \\ 4 & -1 & 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 12 \\ 8 \\ 8 \end{bmatrix}$$

Iteration 1

→ Basis variable :- s_1, s_2, s_3 $\Rightarrow B_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\bar{c}_1 = c_1 - \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix} = 2 \leftarrow$$

$$\bar{c}_2 = c_2 - \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} = 1$$

$$\Rightarrow \theta = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix} \begin{matrix} \\ \leftarrow \\ \\ \end{matrix}$$

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Iteration 2:- Entering variable :- x_1
Leaving variable :- s_2

$$\mathbb{B}_1 = \begin{bmatrix} 1 & 4 & 0 \\ 0 & 4 & 0 \\ 0 & 4 & 1 \end{bmatrix} \rightarrow \mathbb{B}_1^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1/4 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\overline{P}_2 = \mathbb{B}_1^{-1} P_2 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1/4 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1/4 \\ -2 \end{bmatrix}$$

$$\overline{P}_4 = \mathbb{B}_1^{-1} P_4 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1/4 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ +1/4 \\ -1 \end{bmatrix}$$

$$\overline{b} = \mathbb{B}_1^{-1} b = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1/4 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 12 \\ 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$$

$$\overline{C}_2 = C_2 - \begin{bmatrix} 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1/4 \\ 0 \end{bmatrix} = 1 - 1/2 = 1/2 \leftarrow$$

$$\overline{C}_4 = C_4 - \begin{bmatrix} 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1/4 \\ -1 \end{bmatrix} = 0 - 1/2 = -1/2$$

$$\theta = \begin{bmatrix} 2 \\ 8 \\ 0/0 \end{bmatrix} \leftarrow$$

Iteration 3:- Entering variable : x_2 , leaving variable : s_1

$$B_2^{-1} = \begin{bmatrix} 1/2 & 0 & 0 \\ -1/8 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} , \quad B_2 = \begin{bmatrix} 2 & 0 & 0 \\ 1/4 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\bar{P}_3 = B_2^{-1} P_3 = \begin{bmatrix} 1/2 & 0 & 0 \\ -1/8 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1/8 \\ 1 \end{bmatrix}$$

$$\bar{P}_4 = B_2^{-1} P_4 = \begin{bmatrix} 1/2 & 0 & 0 \\ -1/8 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1/4 \\ -1 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 3/8 \\ -2 \end{bmatrix}$$

$$\bar{b} = B_2^{-1} b = \begin{bmatrix} 1/2 & 0 & 0 \\ -1/8 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3/2 \\ 4 \end{bmatrix}$$

$$\bar{C}_3 = C_3 - \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1/2 \\ -1/8 \\ 1 \end{bmatrix} = 0 - (1/2 - 1/8) = -3/8$$

$$\bar{C}_4 = C_4 - \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} -1/2 \\ 3/8 \\ -2 \end{bmatrix} = 0 - (-1/2 + 3/4) = -1/2$$

$$\therefore \begin{bmatrix} x_2 \\ x_1 \\ S_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3/2 \\ 4 \end{bmatrix} \Rightarrow \boxed{Z = 5}$$

(Unique Solution)

6. Consider the following integer programming problem :-

Minimize $Z = 3x_1 + 2.5x_2$

Subject to $x_1 + 2x_2 \geq 20$

$3x_1 + 2x_2 \geq 50$

$x_1, x_2 \geq 0$ & integers

Canonical form :-

Minimize $Z = 3x_1 + 2.5x_2 + 0.S_1 + 0.S_2 + MA_1 + MA_2$

Subject to $x_1 + 2x_2 - S_1 + A_1 = 20$

$3x_1 + 2x_2 - S_2 + A_2 = 50$

~~$x_1, x_2 \geq 0$ & integers~~ $x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$ & integers.

Iteration 1:- $B_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = B_0^{-1}$, $b = \begin{bmatrix} 20 \\ 50 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2 & -1 & 0 & 1 & 0 \\ 3 & 2 & -0 & -1 & 0 & 1 \end{bmatrix}$

$x = [x_1 \quad x_2 \quad S_1 \quad S_2 \quad A_1 \quad A_2]$

$c = [3 \quad 2.5 \quad 0 \quad 0 \quad M \quad M]$

$\overline{C}_1 = 3 - [M \quad M]\begin{bmatrix} 1 \\ 3 \end{bmatrix} = 3 - 4M$ $\qquad \theta = \begin{bmatrix} 10 \\ 25 \end{bmatrix}$

$\overline{C}_2 = 2.5 - [M \quad M]\begin{bmatrix} 2 \\ 2 \end{bmatrix} = 2.5 - 4M$

$\overline{C}_3 = 0 - [M \quad M]\begin{bmatrix} -1 \\ 0 \end{bmatrix} = M$

$\overline{C}_4 = 0 - [M \quad M]\begin{bmatrix} 0 \\ -1 \end{bmatrix} = M$

Iteration 2 Entering variable : x_2 , leaving variable : A_1.

$$\mathbb{B}_1 = \begin{bmatrix} 2 & 0 \\ 2 & 1 \end{bmatrix} \Rightarrow \mathbb{B}_1^{-1} = \begin{bmatrix} 1/2 & 0 \\ -1 & 1 \end{bmatrix}$$

$$\overline{P}_1 = \mathbb{B}_1^{-1} P_1 = \begin{bmatrix} 1/2 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 2 \end{bmatrix}, \quad \overline{b} = \begin{bmatrix} 1/2 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 50 \end{bmatrix} = \begin{bmatrix} 10 \\ 30 \end{bmatrix}$$

$$\overline{P}_3 = \mathbb{B}_1^{-1} P_3 = \begin{bmatrix} 1/2 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$$

$$\overline{P}_4 = \mathbb{B}_1^{-1} P_4 = \begin{bmatrix} 1/2 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\overline{P}_5 = \mathbb{B}_1^{-1} P_5 = \begin{bmatrix} 1/2 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1 \end{bmatrix}$$

$$\overline{C}_1 = 3 - \begin{bmatrix} 2.5 & M \end{bmatrix} \begin{bmatrix} 1/2 \\ 2 \end{bmatrix} = 3 - (5 + 2M) = -2 - 2M$$

$$\overline{C}_3 = 0 - \begin{bmatrix} 2.5 & M \end{bmatrix} \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} = -(-5 + M) = 5 - M$$

$$\overline{C}_4 = 0 - \begin{bmatrix} 2.5 & M \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = M \qquad \theta = \begin{bmatrix} 20 \\ 15 \end{bmatrix}$$

$$\overline{C}_5 = M - \begin{bmatrix} 2.5 & M \end{bmatrix} \begin{bmatrix} 1/2 \\ -1 \end{bmatrix} = 2M - 5$$

Iteration 3 Entering variable : x_1, leaving variable : A_2.

$$\Rightarrow \mathbb{B}_2 = \begin{bmatrix} 1 & 1/2 \\ 0 & 2 \end{bmatrix} \Rightarrow \mathbb{B}_2^{-1} = \begin{bmatrix} 1 & -1/4 \\ 0 & 1/2 \end{bmatrix}$$

$$\overline{P}_3 = \begin{bmatrix} 1 & -1/4 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} = \begin{bmatrix} -3/4 \\ 1/2 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \overline{P}_2, \; \overline{P}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\overline{P}_4 = \begin{bmatrix} 1 & -1/4 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} +1/4 \\ -1/2 \end{bmatrix}$$

$$\overline{P}_5 = \begin{bmatrix} 1 & -1/4 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1/2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3/4 \\ -1/2 \end{bmatrix}$$

$$\overline{P}_6 = \begin{bmatrix} 1 & -1/4 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/4 \\ 1/2 \end{bmatrix}$$

$$\overline{C}_3 = -\begin{bmatrix} 2.5 & 3 \end{bmatrix} \begin{bmatrix} -3/4 \\ 1/2 \end{bmatrix} = 3/8$$

$$\overline{C}_4 = -\begin{bmatrix} 2.5 & 3 \end{bmatrix} \begin{bmatrix} 1/4 \\ -1/2 \end{bmatrix} = 7/8$$

$$\overline{C}_5 = M - \begin{bmatrix} 2.5 & 3 \end{bmatrix} \begin{bmatrix} 3/4 \\ -1/2 \end{bmatrix} > 0$$

$$\overline{C}_6 = M - \begin{bmatrix} 2.5 & 3 \end{bmatrix} \begin{bmatrix} 3/4 \\ -1/2 \end{bmatrix} > 0$$

$$\Rightarrow \overline{b} = \begin{bmatrix} 1 & -1/4 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 10 \\ 30 \end{bmatrix} = \begin{bmatrix} 5/2 \\ 15 \end{bmatrix} \Rightarrow \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} 5/2 \\ 15 \end{bmatrix} = \begin{bmatrix} 2\frac{1}{2} \\ 15 \end{bmatrix}$$

Using Gomory's cutting plane method,

$$0 \cdot x_1 + x_2 - 3/4 S_1 + 1/4 S_2 + 3/4 A_1 - 1/4 A_2 = 2\tfrac{1}{2}$$

$$\Rightarrow x_2 + S_1 + 1/4 S_1 + 1/4 S_2 + 3/4 A_1 - A_2 + 3/4 A_2 = 2 + 1/2$$

$$\Rightarrow x_2 - S_1 - A_2 - 2 = -1/4 S_1 - 1/4 S_2 - 3/4 A_1 - 3/4 A_2 + 1/2$$

$$\Rightarrow -1/4 S_1 - 1/4 S_2 - 3/4 A_1 - 3/4 A_2 + 1/2 \leq 0$$

$$\Rightarrow -1/4 S_1 - 1/4 S_2 - 3/4 A_1 - 3/4 A_2 \leq -1/2$$

$$\Rightarrow S_1 + S_2 + 3A_1 + 3A_2 \geq 2.$$

Modified problem:—

Minimize : $Z = 3x_1 + 2.5x_2 + 0 \cdot s_1 + 0 \cdot s_2 + M \cdot A_1 + M \cdot A_2$

Subjected to : $\left[\begin{array}{l} x_2 - \frac{3}{4}s_1 + \frac{1}{4}s_2 + \frac{3}{4}A_1 - \frac{1}{4}A_2 = 2.5 \\ x_1 + \frac{1}{2}s_1 - \frac{1}{2}s_2 - \frac{1}{2}A_1 + \frac{1}{2}A_2 = 15 \\ s_1 + s_2 + 3A_1 + 3A_2 \geq 2 \end{array}\right.$

Canonical form:—

Minimize: $Z = 3x_1 + 2.5x_2 + 0 \cdot S_1 + 0 \cdot S_2 + 0 \cdot S_3 + M \cdot A_1 + M \cdot A_2 + M \cdot A_3$

Subject to :

$$4x_2 - 3S_1 + S_2 + 3A_1 - A_2 = 10$$
$$2x_1 + S_1 - S_2 - A_1 + A_2 = 30$$
$$S_1 + S_2 + 3A_1 + 3A_2 - S_3 + A_3 = 2$$

$$\Rightarrow B_0 = \begin{array}{c} \begin{array}{ccc} x_2 & x_1 & A_3 \end{array} \\ \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \qquad \Rightarrow A = \begin{array}{c} \begin{array}{cccccccc} x_2 & x_1 & S_1 & S_2 & S_3 & A_1 & A_2 & A_3 \end{array} \\ \begin{bmatrix} 4 & 0 & -3 & 1 & 0 & 3 & -1 & 0 \\ 0 & 2 & 1 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 3 & 3 & 1 \end{bmatrix} \end{array}$$

$$b = \begin{bmatrix} 10 \\ 30 \\ 2 \end{bmatrix} \qquad \Rightarrow B_0^{-1} = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 1 & 0 & -3/4 & 1/4 & 0 & 3/4 & -1/4 & 0 \\ 0 & 1 & 1/2 & -1/2 & 0 & -1/2 & 1/2 & 0 \\ 0 & 0 & 1 & 1 & -1 & 3 & 3 & 1 \end{bmatrix} \qquad \Rightarrow b = \begin{bmatrix} 2.5 \\ 15 \\ 2 \end{bmatrix}$$

$$C_3 = -\begin{bmatrix} 2.5 & 3 & M \end{bmatrix} \begin{bmatrix} -3/4 \\ 1/2 \\ 1 \end{bmatrix} = (3/8 - M) \qquad C_5 = -\begin{bmatrix} 2.5 & 3M \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = M$$

$$C_4 = -\begin{bmatrix} 2.5 & 3 & M \end{bmatrix} \begin{bmatrix} 1/4 \\ -1/2 \\ 1 \end{bmatrix} = 7/8 - M \qquad C_6 = M - \begin{bmatrix} 2.5 & 3 & M \end{bmatrix} \begin{bmatrix} 3/4 \\ -1/2 \\ 3 \end{bmatrix} = \underline{\underline{-3/8 - 2M}}$$

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$$\bar{C}_7 = M - [2.5 \;\; 3 \;\; M]\begin{bmatrix}-1/4\\1/2\\3\end{bmatrix} = -7/8 - 2M \leftarrow \quad , \quad \theta = \begin{bmatrix}-10\\30\\6\end{bmatrix} \leftarrow$$

Iteration 2:- Entering variable : A_2 , leaving variable : A_3

$$B_1 = \begin{bmatrix}1 & 0 & -1/4\\0 & 1 & 1/2\\0 & 0 & 3\end{bmatrix} \Rightarrow B_1^{-1} = \begin{bmatrix}1 & 0 & 1/12\\0 & 1 & -1/6\\0 & 0 & 1/3\end{bmatrix}$$

$$\bar{P}_3 = \begin{bmatrix}1 & 0 & 1/12\\0 & 1 & -1/6\\0 & 0 & 1/3\end{bmatrix}\begin{bmatrix}-3/4\\1/2\\1\end{bmatrix} = \begin{bmatrix}-2/3\\1/3\\1/3\end{bmatrix} ; \quad \bar{P}_4 = \begin{bmatrix}1 & 0 & 1/12\\0 & 1 & -1/6\\0 & 0 & 1/3\end{bmatrix}\begin{bmatrix}1/4\\-1/2\\1\end{bmatrix} = \begin{bmatrix}1/6\\-2/3\\1/3\end{bmatrix}$$

$$\bar{P}_5 = \begin{bmatrix}1 & 0 & 1/12\\0 & 1 & -1/6\\0 & 0 & 1/3\end{bmatrix}\begin{bmatrix}0\\0\\-1\end{bmatrix} = \begin{bmatrix}-1/12\\1/6\\-1/3\end{bmatrix} ; \quad \bar{P}_6 = \begin{bmatrix}1 & 0 & 1/12\\0 & 1 & -1/6\\0 & 0 & 1/3\end{bmatrix}\begin{bmatrix}3/4\\-1/2\\3\end{bmatrix} = \begin{bmatrix}1\\-1\\1\end{bmatrix}$$

$$\bar{P}_8 = \begin{bmatrix}1 & 0 & 1/12\\0 & 1 & -1/6\\0 & 0 & 1/3\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}1/12\\-1/6\\1/3\end{bmatrix} ; \quad \bar{b} = \begin{bmatrix}1 & 0 & 1/12\\0 & 1 & -1/6\\0 & 0 & 1/3\end{bmatrix}\begin{bmatrix}2.5\\15\\2\end{bmatrix} = \begin{bmatrix}8/3\\44/3\\2/3\end{bmatrix}$$

$$\bar{C}_3 = -[2.5 \;\; 3 \;\; M]\begin{bmatrix}-2/3\\1/3\\1/3\end{bmatrix} = 2/3 - M/3 ; \quad \bar{C}_4 = -[2.5 \;\; 3 \;\; M]\begin{bmatrix}1/6\\-2/3\\1/3\end{bmatrix} = 19/12 - M/3$$

$$\bar{C}_5 = M - [2.5 \;\; 3 \;\; M]\begin{bmatrix}-1/12\\1/6\\-1/3\end{bmatrix} = -7/24 + M/3 ; \quad \bar{C}_6 = M - [2.5 \;\; 3 \;\; M]\begin{bmatrix}3/4\\-1\\1\end{bmatrix} = 1/2$$

$$\bar{C}_8 = M - [2.5 \;\; 3 \;\; M]\begin{bmatrix}1/12\\-1/6\\1/3\end{bmatrix} = 7/24 + 2M/3 \quad \Rightarrow \theta = \begin{bmatrix}-4\\44\\2\end{bmatrix} \leftarrow$$

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<u>Iteration 3:-</u> Entering variable :- S_1 ; leaving variable :- A_2

$$B_2 = \begin{bmatrix} 1 & 0 & -2/3 \\ 0 & 1 & 1/3 \\ 0 & 0 & 1/3 \end{bmatrix} \Rightarrow B_2^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\overline{P}_4 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1/6 \\ -2/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 5/6 \\ -1 \\ 1 \end{bmatrix} \qquad \overline{P}_5 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} -1/12 \\ -1/6 \\ -1/12 \end{bmatrix} = \begin{bmatrix} -3/4 \\ 1/2 \\ -1 \end{bmatrix}$$

$$\overline{P}_6 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 3 \end{bmatrix} \qquad \overline{P}_7 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$\overline{P}_8 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1/12 \\ -1/6 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 3/4 \\ -1/2 \\ 1 \end{bmatrix} \qquad \overline{b} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 8/3 \\ 44/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \\ 2 \end{bmatrix}$$

$$\overline{C}_4 = -\begin{bmatrix} 2.5 & 3 & 0 \end{bmatrix} \begin{bmatrix} 5/6 \\ -1 \\ 1 \end{bmatrix} = 11/12 \qquad \overline{C}_8 = M - \begin{bmatrix} 2.5 & 3 & 0 \end{bmatrix} \begin{bmatrix} \\ \\ \\ \end{bmatrix} = M - (\ ) > 0$$

$$\overline{C}_5 = -\begin{bmatrix} 2.5 & 3 & 0 \end{bmatrix} \begin{bmatrix} -3/4 \\ 1/2 \\ -1 \end{bmatrix} = -3/8 \Rightarrow \boxed{\begin{bmatrix} x_2 \\ x_1 \\ S_1 \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \\ 2 \end{bmatrix}}$$

$$\overline{C}_6 = M - \begin{bmatrix} 2.5 & 3 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 3 \end{bmatrix} = M - (\ ) > 0$$

$\rightarrow$ optimized solution

$$\boxed{Z = 52}$$

$$\overline{C}_7 = M - \begin{bmatrix} 2.5 & 3 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = M - (\cdot) > 0$$