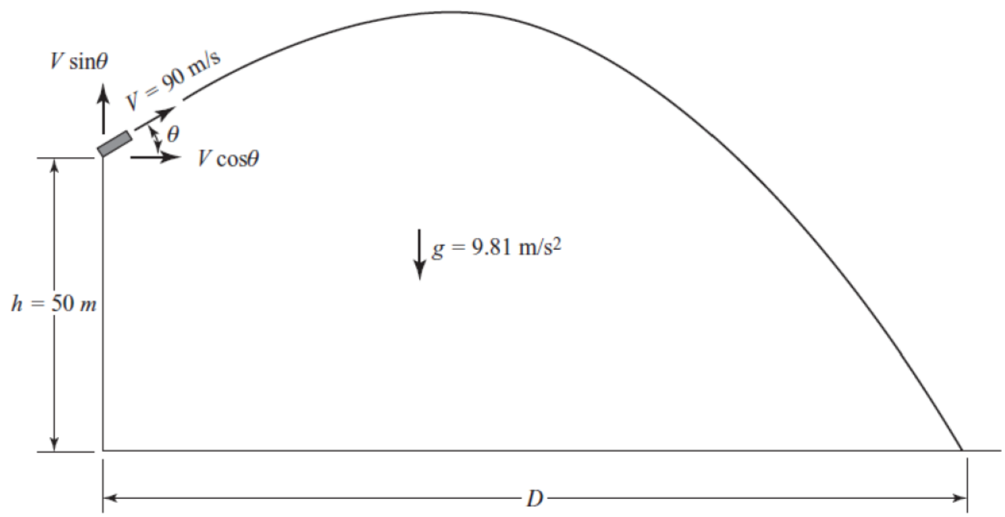
Home Assignment-1

To be submitted on or before 17-09-2024 on Moodle

1. A projectile released from a height h at an angle θ with respect to the horizontal in a gravitational field g, shown in figure below, travels a distance D when it hits the ground. If h=50m and v=90 m/s, determine the angle θ ($0 \leq \theta \leq 80$) for which D is maximum.



Hint: Derive the objective function

$$D = \left(\frac{V \sin\theta}{g} + \sqrt{\frac{2h}{g} + \left(\frac{V \sin\theta}{g} \right)^2} \right) V \cos\theta$$

and write down the constraints.

2. Consider that there are m number of manufacturing facilities (F_1, F_2, …F_m) to be established so as to cater to the demands of n number of retailers (R_1, R_2, ..R_n). The quantities of the item each manufacturing facility must supply to each of the retailers is given by Q_{ij}, where i=1,2,..m is the index for manufacturing facilities and j=1,2,..n is the index for retailers. Let there are p number of possible locations (L_1, L_2,..L_p) for establishing these manufacturing facilities and $p>m$. The cost per unit quantity of transporting the item from each location k (k=1,2,..p) to each retailer j (j=1,2,..n) is given by C_{kj}. Write a mathematical formulation to allocate all the manufacturing facilities to one of the available locations such that the total cost of transportation to the retailers is minimized. Note: A location cannot accommodate more than one manufacturing facility.

3. A canning company operates two canning plants. There are three growers (S1, S2 and S3) who are willing to supply fresh fruits in the following amounts:

- S1: 200 tonnes at Rs.1100/tonne
- S2: 310 tonnes at Rs.1000/tonne
- S3: 420 tonnes at Rs.900/tonne

Shipping costs from growers to the canning plants in Rs. per tonne are:

	To:	Plant A	Plant B
From:	S1	3000	3500
	S2	2000	2500
	S3	6000	4000

Canning plant capacities and the labour costs are:

	Plant A	Plant B
Capacity	460 tonnes	560 tonnes
Labour cost	Rs.26000/tonne	Rs.21000/tonne

The canned fruits are sold at Rs 50000/tonne to the distributors. The company can sell at this price all they can produce. The objective is to find the best mixture of the quantities supplied by the three growers to the two plants so that the company maximises its profits. Formulate the problem.

4. Acid company, a chemical engineering company manufactures two types of acids A and B. Both of these involve two chemical operations for each. Each unit of Acid A requires 3 hours on operation 1 and 4 hours on operation 2. Each unit of Acid B requires 3 hours on Operation and 2 hours on operation 2. Available time for operation 1 is 20 hours and for operation 2, 18 hours. The production of B also results in a by-product C at no extra cost. A part of this by-product can be sold at a profit, but the remainder has to be destroyed. Acid A sells for P_1 rupees profit per unit while acid B sells for P_2 rupees profit per unit. The by-product C can be sold at a unit profit of P_3 units, but remainder has to be destroyed. The destruction cost is P_4 rupees per unit. Forecasts by sales department show that a maximum of K units of by-product C can be sold; n units of C results for every unit of B produced. Formulate a linear programming problem to determine the production quantities of A, B and C, so that the total profits will be maximized.

5. Consider that there are m number of bins (B_1, B_2,…B_m) each of volume capacity V and weight capacity W. Let there n number of items (T_1,T_2,…,T_n) with volumes, v_1, v_2,..,v_n, and weights, w_1, w_2,..w_n, respectively. The items are to be packed in the bins such that the volume and weight capacity constraints must be satisfied and the total volume of items packed within all the bins is maximized. Write an integer programming formulation for the above problem.

6. Three students want to go from their hostel to their department. They have only one bicycle and only one person can ride the bicycle at a time. The three students can walk at speeds a_1, a_2 and a_3, respectively and can ride the bicycle at speeds b_1, b_2 and b_3, respectively. Assume that the slowest cycling speed is more than the fastest walking speed. A person riding the cycle can park it at any point and walk the rest of the journey. He does not wait near the cycle till the next person arrives (the cycle is safe even if it is unlocked and left unattended). A person rides the cycle only once during the travel. Write the mathematical formulation (decision variables and its bounds, objective function and constraints) that finds out when earliest all three can reach the department. That is the time taken by the last student to reach the department has to be minimized. Assume the distance between the hostel and the department to be d.

7. A coffee shop provides three types of coffee (strong, medium and light) to its customers in various outlets. They use three varieties of coffee beans to make the three types of coffee blends from which the types of coffee are provided. Although the recipes for the three types are not fully precise, certain restrictions must be satisfied when combining the three varieties of beans:
Component 1 should constitute 10% to 20% of final Blend 1 by weight
Component 2 should constitute 30% to 35% of Blend 3 by weight.
Components 2 and 3 combined should constitute at least 70% of Blend 2 by weight

   In addition to these restrictions, there is a limited availability of the three varieties. The maximum weekly availability is 8000, 10000 and 9000 kg, respectively Weekly capacity of the shop to make all three blends is 25000 kg. The weekly production of Blend 1 should be atleast 5000 kg. Given that the three beans varieties cost the manufacturer Rs. 120, Rs. 130 and Rs. 110 per kg, respectively, and the coffee sold using the blends yields Rs. 300, Rs. 320 and Rs. 280 per kg, respectively, write a mathematical formulation (decision variables, objective function and all constraints) to find the number of kg of each variety to be used in each type to maximize the final profit.

8. A farming company owns 50 plots of land, each of 1 acre that can produce 10 different agricultural products (wheat, rice, vegetables, etc). A plot of land can be allotted only one product that produces the product during the entire year, and the product is changed in the next year as part of crop rotation. The company expects to gain Rs. P_{ik} per kg for a product i produced in each year k. The annual yield of product i on plot j is known and is given by Y_{ik} kg, which is the same for all the periods. The company has a policy to cultivate atleast five different products in the 50 plots of land in a year, and a product is not cultivated in more than 10 plots in a year. Write down the mathematical formulation (objective function, constraints and decision variables) to allot products to plots in each year to maximize the profit earned by the company in the next 5 years.

9. A farmer has 20 acres of land. He produces three products, X, Y and Z. Average yield (harvest/produce) per acre for X, Y and Z is 400, 600 and 200 kg, respectively. The selling

price per kg of X, Y and Z is Rs. 20, Rs 15 and Rs. 25, respectively. Each product needs fertilizers. Per acre need for fertilizer for X, Y, and Z is 200, 300 and 100 kg, respectively. The cost of fertilizer is Rs. 10 per kg. Labour requirements for X, Y and Z are 10, 12 and 8 man-hours per acre, respectively. The cost of labour is Rs. 40 per man-hour. The maximum availability of labour is 2000 man-hours. Formulate a mathematical programming model (objective function, constraints and decision variables) to allocate land for each product to maximize profit.

10. Determine the dimensions of an open box of maximum volume that can be constructed from an A4 sheet (210 mm x 297 mm) by cutting four squares of side *x* from the corners and folding and gluing the edges as shown in fig.