Eric Martin
ME 472
Project 2

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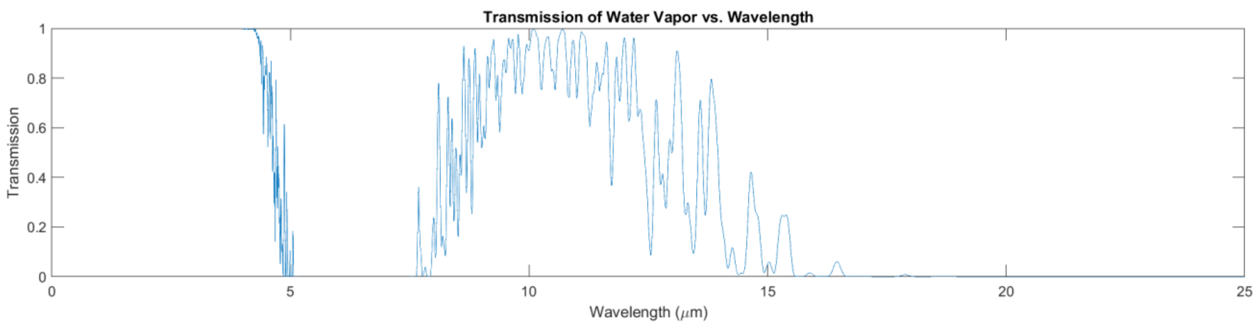
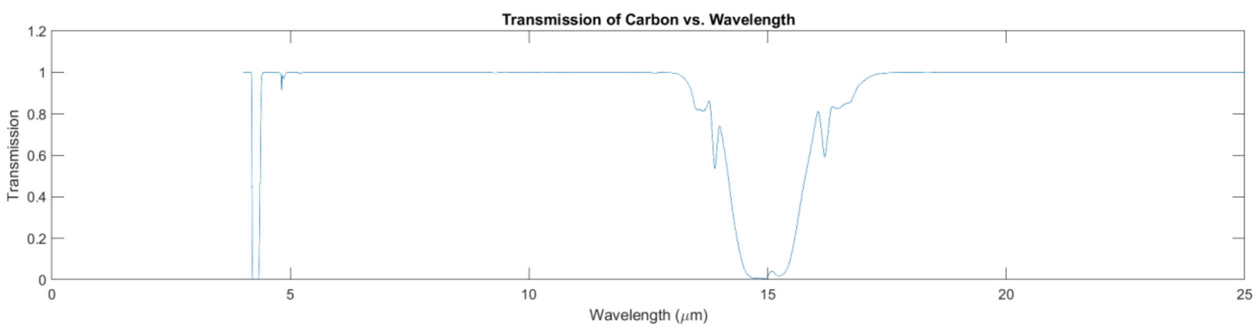
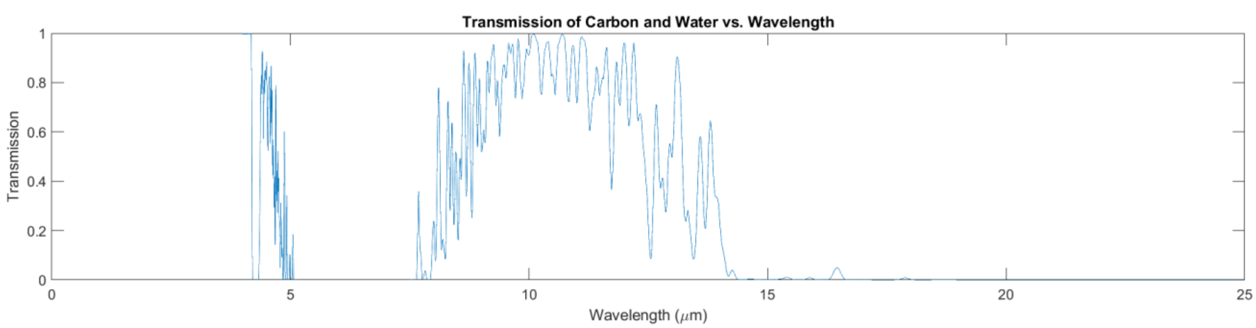
1. As a function of wavelength ($4 < \lambda[\mu m] < 25$) plot the transmissivity of the atmosphere for water vapor, carbon dioxide, and the combined transmissivity on separate graphs.

The three plots corresponding to water, carbon, and combined, are shown below.

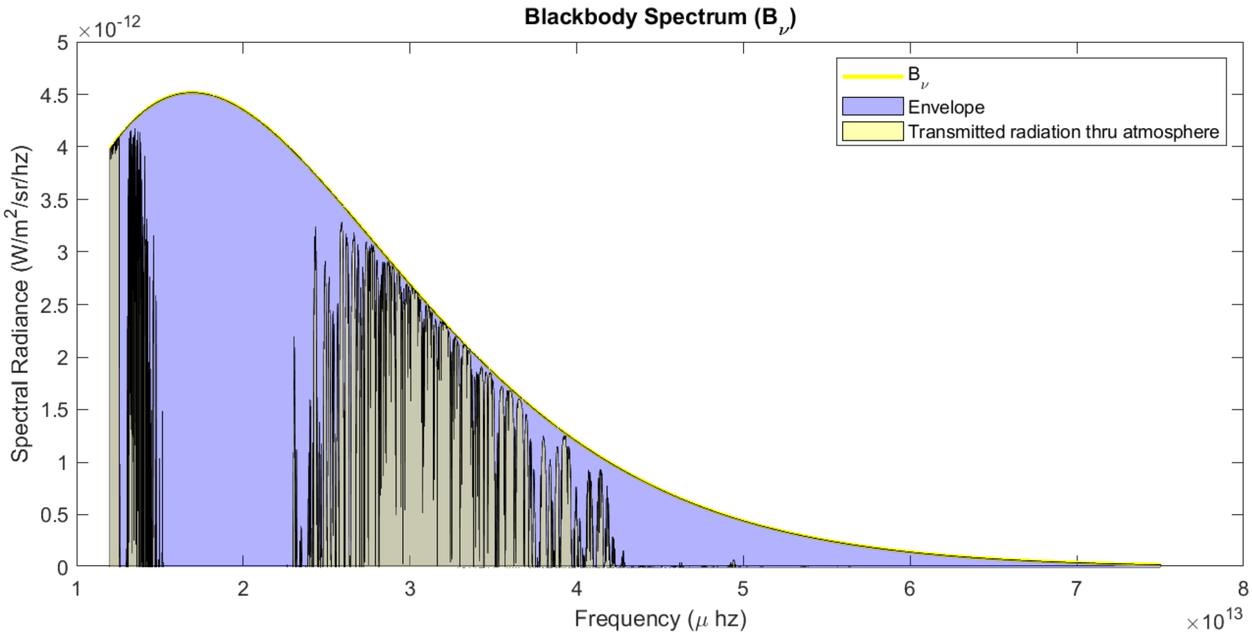


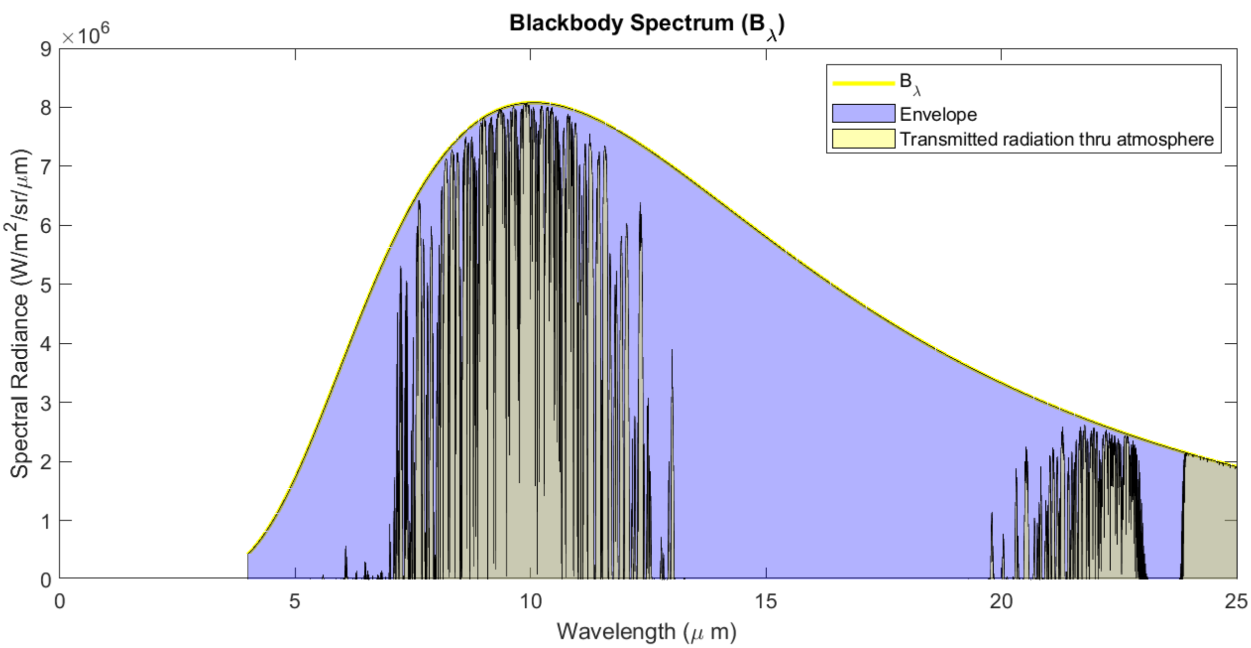




2. Compute the blackbody spectrum (both B_ν and B_λ) for terrestrial radiation assuming an average earth temperature of 288 K. Make two plots that show the blackbody envelope and the transmitted radiation through the atmosphere, *i.e.,* the combined effect from the previous problem.

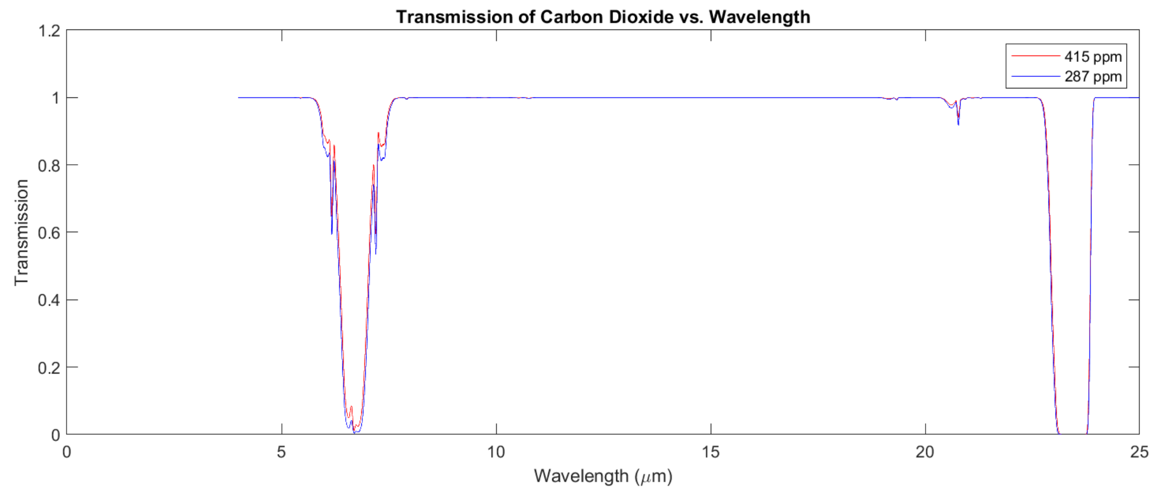
The two plots are shown below.

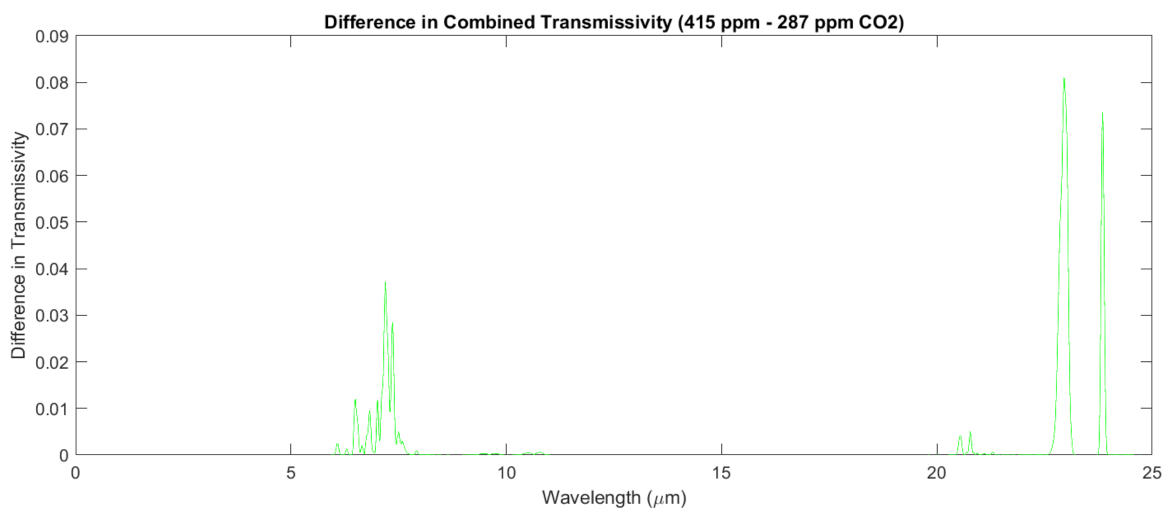




3. Compute the carbon dioxide transmissivity for a pre-industrial concentration of 287 ppm. Plot the two spectra on the same axes as above. On a separate plot using the same abscissa, plot the difference in the combined transmissivity between 415 and 287 ppm of carbon dioxide.

The two plots are shown below.



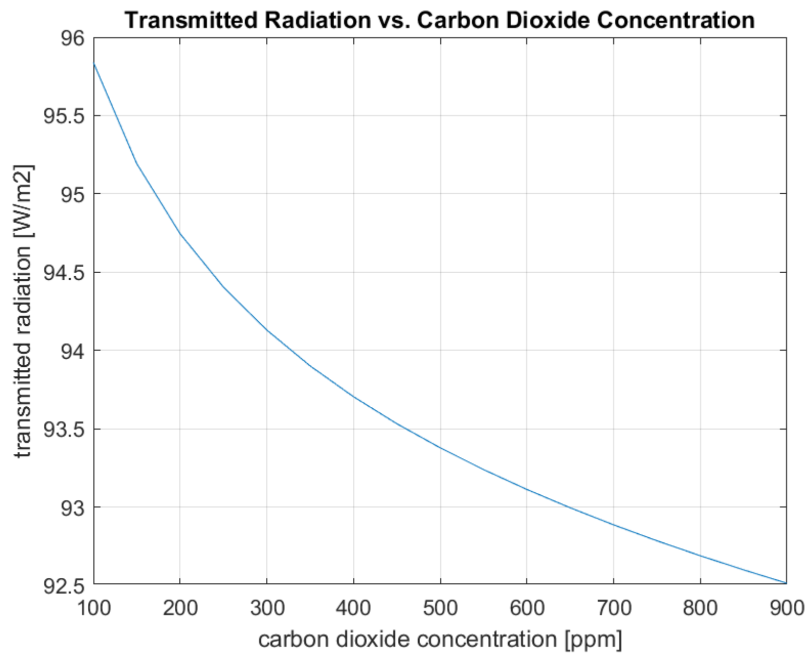


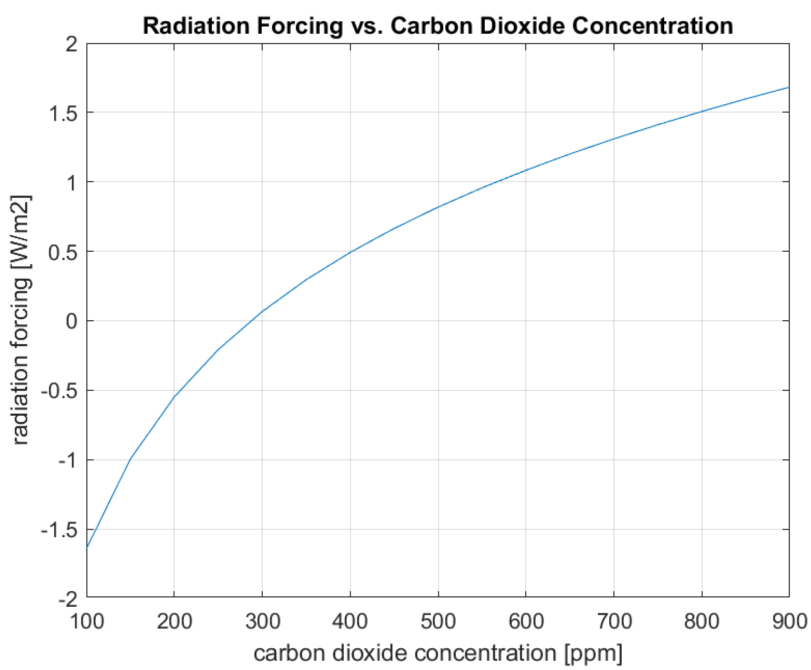
4. Integrate the result from part 2 over wavelength (or frequency) to find the total emission from the surface and the total transmission through the atmosphere [W/m^2]. Calculate an average absorptivity of the outgoing radiation through the troposphere.

```
>> proj2_q4
Total emisson from surface [W/m^2]: 317.8064
Total transmission thru atmosphere [W/m^2]: 93.6504
Absorbance value: 0.70532
```

5. For a range of carbon dioxide concentration ranging from 100 to 900 ppm, plot the transmitted radiation [W/m2] vs. carbon dioxide concentration. On a different plot with the same abscissa and using 287 ppm (the pre-industrial level) as the baseline, plot the radiation forcing.

The two plots are shown below.





The MATLAB code for each question is pasted below. All functions are at the very end.

QUESTION 1:

```
clear all
close all
clc
%% organizing data
load("HitranSpectra_f23 (1).mat")
H2O = H2O * 1e-4;
CO2 = CO2 * 1e-4;
lam = nu.^(-1)*10^4;     % convert wave number to wavelength [μm]
k_nu_table = [lam;H2O;CO2]; % matrix w/ k_nu for H2O and CO2 for given lambda

%% isolating desired wavelengths in data
lower_lam = 4;
upper_lam = 25;
condition = (lam >= lower_lam) & (lam <= upper_lam);
[row_indices, col_indices] = find(condition);
col_start = col_indices(1);
col_end = col_indices(21000);
% k_nu_table column numbers we're interested in: 2421 -> 23420
wavelength_indices = 2421:23420;
lambda_4_25 = lam(wavelength_indices);

% create arrays to hold new wavelength data
chi_water = zeros(size(wavelength_indices));
transmission_water = zeros(size(wavelength_indices));
chi_carbon = zeros(size(wavelength_indices));
transmission_carbon = zeros(size(wavelength_indices));

%% loop through chi integration and transmission conversion for each wavelength
totalTimeStart = tic;
for i = 1:length(wavelength_indices)
    k_nu_value_water = k_nu_table(2, wavelength_indices(i));
    k_nu_value_carbon = k_nu_table(3, wavelength_indices(i));

    integrand_water = @(z) k_nu_value_water * density_water(z);
    integrand_carbon = @(z) k_nu_value_carbon * density_carbon(z);

    chi_water(i) = integral(integrand_water, 0, 20, 'ArrayValued', true);
    chi_carbon(i) = integral(integrand_carbon, 0, 20, 'ArrayValued', true);

    transmission_water(i) = exp(-chi_water(i));
    transmission_carbon(i) = exp(-chi_carbon(i));
    transmission_total(i) = transmission_water(i) * transmission_carbon(i);
%#ok<SAGROW>
end

totalTimeElapsed = toc(totalTimeStart);
fprintf('Total elapsed time: %.2f seconds\n', totalTimeElapsed);

figure;
transmission_water_filt = TransFilter(1e4./lam,transmission_water,5);
plot(lam(col_start:col_end), transmission_water_filt);
xlabel('Wavelength (\mum)');
ylabel('Transmission');
title('Transmission of Water Vapor vs. Wavelength');
```

```
figure;
transmission_carbon_filt = TransFilter(1e4./lam,transmission_carbon,5);
plot(lam(col_start:col_end), transmission_carbon_filt);
xlabel('Wavelength (\mum)');
ylabel('Transmission');
title('Transmission of Carbon vs. Wavelength');

figure;
transmission_total_filt = TransFilter(1e4./lam,transmission_total,5);
plot(lam(col_start:col_end), transmission_total_filt);
xlabel('Wavelength (\mum)');
ylabel('Transmission');
title('Transmission of Carbon and Water vs. Wavelength');
```

QUESTION 2:

```
h = 6.626e-34; % joule's constant [J*s]
c = 3.0e8;     % [m/s]
k = 1.38e-23;  % boltzmann constant [J/K]
T = 288;       % temp [K]

% wavelength range in meters
lambda = linspace(4e-6, 25e-6, length(wavelength_indices));
lambda_um = lambda * 1e6;
nu = c ./ lambda;

% BB spectra
B_nu = 2 * h * nu.^3 ./ (c^2.*(exp((h .* nu) ./ ( k * T)) - 1));
B_lam = (2 * h * c^2) ./ (lambda.^5 .* (exp((h * c) ./ (lambda * k .* T)) - 1));
transmission_B_nu = B_nu .* transmission_total;
transmission_B_lam = B_lam .* transmission_total;

% plot1:
figure;
plot(nu, B_nu, 'y','LineWidth', 2);
hold on
x_shade_1 = [nu, fliplr(nu)];
y_shade_1 = [B_nu, zeros(size(B_nu))];
y_shade_2 = [transmission_B_nu, zeros(size(transmission_B_nu))];
fill(x_shade_1, y_shade_1, 'b', 'FaceAlpha', 0.3);
fill(x_shade_1, y_shade_2, 'y', 'FaceAlpha', 0.3);
xlabel('Frequency (\mu hz)');
ylabel('Spectral Radiance (W/m^2/sr/hz)');
title('Blackbody Spectrum (B_{\nu})');
legend('B_{\nu}', 'Envelope','Transmitted radiation thru atmosphere');
hold off;

% plot2:
figure;
plot(lambda_um, B_lam, 'y','LineWidth', 2);
hold on
x_shade_2 = [lambda_um, fliplr(lambda_um)];
```

```
y_shade2_1 = [B_lam, zeros(size(B_lam))];
y_shade2_2 = [transmission_B_lam, zeros(size(transmission_B_lam))];
fill(x_shade_2, y_shade2_1, 'b', 'FaceAlpha', 0.3);
fill(x_shade_2, y_shade2_2, 'y', 'FaceAlpha', 0.3);
xlabel('Wavelength (\mu m)');
ylabel('Spectral Radiance (W/m^2/sr/\mum)');
title('Blackbody Spectrum (B_{\lambda})');
legend('B_{\lambda}', 'Envelope','Transmitted radiation thru atmosphere');
hold off;
```

QUESTION 3:

```
R = 8.314;
carbon_concentration_PI = 287;
chi_carbon_PI = zeros(size(wavelength_indices));
transmission_carbon_PI = zeros(size(wavelength_indices));
transmission_total_PI = zeros(size(wavelength_indices));
transmission_difference = zeros(size(wavelength_indices));

for i = 1:length(wavelength_indices)
    k_nu_value_carbon = k_nu_table(3, wavelength_indices(i));
    carbon_function_287 = @(z) k_nu_value_carbon * density_carbon(z,
carbon_concentration_PI);
    chi_carbon_PI(i) = integral(carbon_function_287, 0, 20, 'ArrayValued', true);
    transmission_carbon_PI(i) = exp(-chi_carbon_PI(i));
    transmission_total_PI(i) = transmission_water(i) * transmission_carbon_PI(i);
    transmission_difference(i) = abs(transmission_total(i) -
transmission_total_PI(i));
end

figure;
d = TransFilter(1e4./lam,transmission_carbon_PI,5)
plot(lambda_um, d, 'r', 'DisplayName', '415 ppm');
hold on;
plot(lambda_um, transmission_carbon_filt, 'b', 'DisplayName', '287 ppm');
xlabel('Wavelength (\mum)');
ylabel('Transmission');
title('Transmission of Carbon Dioxide vs. Wavelength');
legend;

figure;
e = TransFilter(1e4./lam,transmission_difference,5);
plot(lambda_um, e, 'g');
xlabel('Wavelength (\mum)');
ylabel('Difference in Transmissivity');
title('Difference in Combined Transmissivity (415 ppm - 287 ppm CO2)');
```

QUESTION 4:

```
B_lam = @(lambda) (2 * h * (c*10^6)^2) ./ (lambda.^5 .* (exp((h * (c*10^6)) ./
(lambda * k .* T)) - 1));
E_b = pi .* integral(B_lam,4,25)/(10^-6)^2;
E_b_atmosphere = E_b * mean(transmission_total);
```

```
absorptivity = 1 - mean(transmission_total);

disp(['Total emisson from surface [W/m^2]: ', num2str(E_b)]);
disp(['Total transmission thru atmosphere [W/m^2]: ', num2str(E_b_atmosphere)]);
disp(['Absorbance value: ', num2str(absorptivity)]);
```

QUESTION 5:

```
ppm_range = 100:50:900;

chi_carbon_range = zeros(length(ppm_range),length(wavelength_indices));
transmission_carbon_range = zeros(length(ppm_range),length(wavelength_indices));
transmission_total_range = zeros(length(ppm_range),length(wavelength_indices));

for i=1:length(ppm_range)
    for j = 1:length(wavelength_indices)
        k_nu_carbon_value = k_nu_table(3, wavelength_indices(j));
        chi_carbon_range(i, j) = integral(@(z) k_nu_carbon_value * density_carbon(z,
ppm_range(i)), 0, 20, 'ArrayValued', true);
        transmission_carbon_range(i, j) = exp(-chi_carbon_range(i, j));
        transmission_total_range(i, j) = transmission_water(j) *
transmission_carbon_range(i, j);
    end
end

E_b_atmosphere_287 = E_b * mean(transmission_total_PI);
mean_transmission_total = mean(transmission_total_range,2);
E_b_atmosphere = E_b * mean_transmission_total;
forcing = E_b_atmosphere_287 - E_b_atmosphere;
figure;
plot(ppm_range, E_b_atmosphere);
xlabel('carbon dioxide concentration [ppm]');
ylabel('transmitted radiation [W/m2]');
title('Transmitted Radiation vs. Carbon Dioxide Concentration');
grid on;

figure;
plot(ppm_range, forcing);
xlabel('carbon dioxide concentration [ppm]');
ylabel('radiation forcing [W/m2]');
title('Radiation Forcing vs. Carbon Dioxide Concentration');
grid on;
```

ALL FUNCTIONS USED:

```matlab
%% temperature as function of z
function T = temp(z)
    if z<=11
        T = (292 - 6.5 * z);
    elseif (11<z) & (z<=20) %#ok<AND2>
        T = 216.5;
    elseif z>20
        T = (216.5 + 1) * (z - 20);
    end
end


%% carbon density as function of z and ppm
function rho_c_ppm = density_carbon(z, ppm)
    R = 8.314;
    T = temp(z);
    rho_c_ppm = (ppm * 6.022e23) / (R * T);
end


%% water number density as function of z
function rho_w = density_water(z)
    T = temp(z);
    RH = 0.5;
    R = 8.314;
    if T >= 0
        P_sat = 0.61121 * exp((18.678 - (T/234.5))*(T / (257.14 + T)));
    else
        P_sat = 0.61115 * exp((23.036 - (T/333.7))*(T / (279.82 + T)));
    end
    P_vap = RH * P_sat;
    rho_w_mol = P_vap / (R * (T+273.15));
    rho_w = rho_w_mol * 6.022e26;
end


%% carbon number density as  function of z
function rho_c = density_carbon(z)
    carbon_concentration = 415;
    R = 8.314;
    T = temp(z);
    rho_c = (carbon_concentration * 6.022e23) / (R * T);
end
```