

```
In [4]: import matplotlib
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
from sklearn import linear_model
from sklearn.preprocessing import StandardScaler
from sklearn.metrics import mean_squared_error
from math import sqrt
import tensorflow as tf
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import Ridge
from sklearn.linear_model import Lasso
from sklearn.model_selection import train_test_split
from sklearn.utils import resample
scaler = StandardScaler()
```

## 1-1. Fit the data using linear (1st order) regression model (matrix form, gradient descent method).

**Plot the data with the fitted line. Print optimized weights. Print Root Mean Squared Error (RMSE).**

```
In [198]: data1 = pd.read_csv("ex2data1.csv")
X_1 = data1.as_matrix(columns=data1.columns[0:1])
Y_1 = data1.as_matrix(columns=data1.columns[1:])
X_1_coloumn_added = np.c_[np.ones((data1.shape[0], 1)), X_1]
```

```
In [10]: def weight_optimizer(data, degree, Label, learn_rate = .01, epochs = 100000,
regularisation_alpha=0):
    theta_1 = np.random.randn(degree+1, 1)
    for epoch in range(epochs):
        gradients = (2/data.shape[0]) * (data.T.dot(data.dot(theta_1) - La
bel) + regularisation_alpha * theta_1)
        theta_1 = theta_1 - learn_rate * gradients
    return theta_1
```

```
In [11]: def rmse_calculator(data, weights, actual_label):
    predict_y = data.dot(weights)
    rms = sqrt(mean_squared_error(actual_label, predict_y))
    return rms
```

```
In [12]: # Please input sorted data
def graph_plotter(X_1_poly_dn_scaled_coloumn_added,weights,actual_label,
graph_label=None):
    X_1_poly_dn_scaled_coloumn_added_sorted = X_1_poly_dn_scaled_coloumn
_added [X_1_poly_dn_scaled_coloumn_added[:,1].argsort()]
    plt.plot(X_1_poly_dn_scaled_coloumn_added_sorted[:,1:2], actual_labe
l, 'bo')
    y_1_dn_plot = X_1_poly_dn_scaled_coloumn_added_sorted.dot(weights)
    plt.plot(X_1_poly_dn_scaled_coloumn_added_sorted[:,1:2], y_1_dn_plot
,label=graph_label)
    legend = plt.legend(loc='upper right', shadow=True)
```

```
In [13]: def polynomial_adder(data,degree):
    poly_features = PolynomialFeatures(degree=degree, include_bias=False
)
    X_1_poly_dn = poly_features.fit_transform(data)
    X_1_poly_dn_scaled = scaler.fit_transform(X_1_poly_dn)
    X_1_poly_dn_scaled_coloumn_added = np.c_[np.ones((data.shape[0], 1
)), X_1_poly_dn_scaled]
    ## Sorting for graph plotting
    #X_1_poly_dn_scaled_coloumn_added = X_1_poly_dn_scaled_coloumn_added
[X_1_poly_dn_scaled_coloumn_added[:,1].argsort()]
    return X_1_poly_dn_scaled_coloumn_added
```

## Optimized Weight - Deg1

```
In [204]: theta_1 = weight_optimizer(X_1_coloumn_added,1,Y_1)
print (theta_1)

[[ 1.07581132]
 [-1.1887203 ]]
```

## RMSE - Deg1

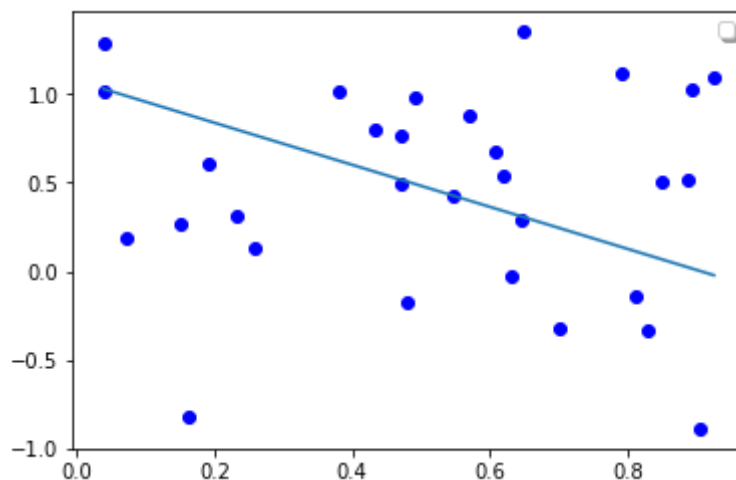
```
In [205]: rms_1 = rmse_calculator(X_1_coloumn_added,theta_1,Y_1)
print (rms_1)

0.47624021947640577
```

## Plot - Deg1

```
In [206]: graph_plotter(X_1_coloumn_added,theta_1,Y_1)
```

No handles with labels found to put in legend.



## 1-2. Fit the data using 2nd order polynomial regression model (matrix form, gradient descent method).

```
In [207]: X_1_poly_d2_scaled_coloumn_added = polynomial_adder(X_1,2)
```

### Optimized Weight - deg2

```
In [208]: theta_1_d2 = weight_optimizer(X_1_poly_d2_scaled_coloumn_added,2,Y_1,epo
chs=1000000)
print (theta_1_d2)
```

```
[[ 0.45269151]
 [ 1.13735736]
 [-1.51236155]]
```

### RMSE

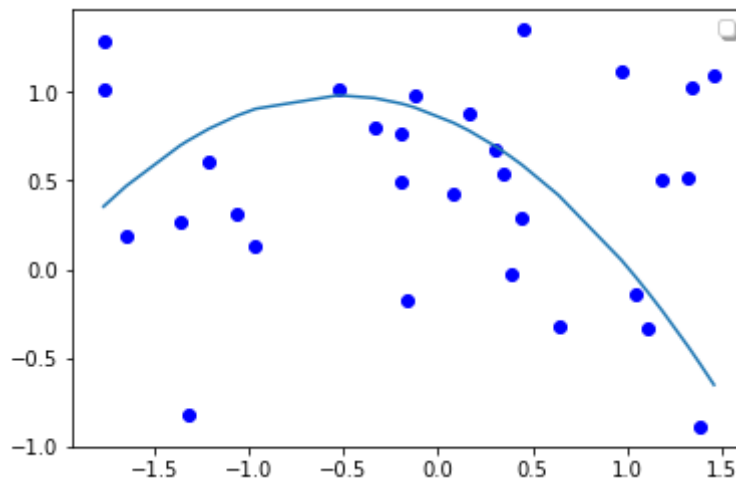
```
In [209]: rms_1_d2 = rmse_calculator(X_1_poly_d2_scaled_coloumn_added,theta_1_d2,Y
_1)
print (rms_1_d2)
```

```
0.2856992946970416
```

### Plot - deg2

```
In [210]: graph_plotter(X_1_poly_d2_scaled_coloumn_added,theta_1_d2,Y_1)
```

No handles with labels found to put in legend.



## 1-3. Fit the data using 4th order polynomial regression model (matrix form, gradient descent method).

```
In [211]: X_1_poly_d4_scaled_coloumn_added = polynomial_adder(X_1,4)
```

### Optimized Weight - deg4

```
In [212]: theta_1_d4 = weight_optimizer(X_1_poly_d4_scaled_coloumn_added,4,Y_1,epo
chs=1000000)
print (theta_1_d4)
```

```
[[ 0.45269151]
 [ 1.29328167]
 [-1.97124121]
 [ 0.40758024]
 [-0.09772327]]
```

### RMSE Deg-4

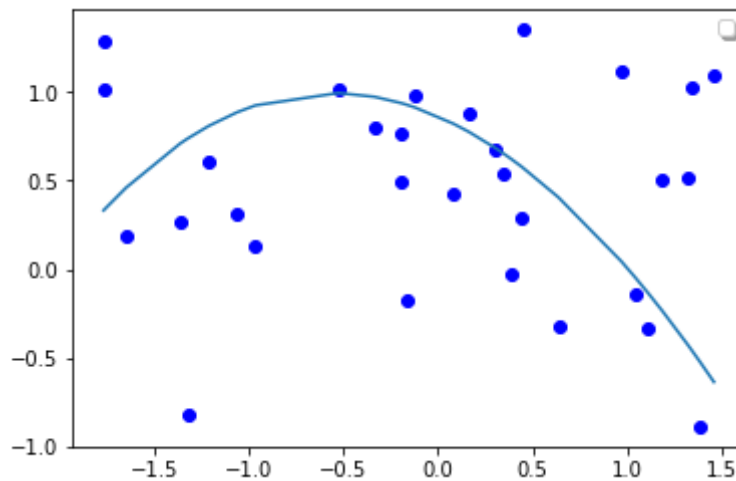
```
In [213]: rms_1_d4 = rmse_calculator(X_1_poly_d4_scaled_coloumn_added,theta_1_d4,Y
_1)
print (rms_1_d4)
```

```
0.2853539310798953
```

### Graph\_Deg 4

```
In [214]: graph_plotter(X_1_poly_d4_scaled_coloumn_added,theta_1_d4,Y_1)
```

No handles with labels found to put in legend.



## 1-4. Fit the data using 16th order polynomial regression model (matrix form, gradient descent method).

```
In [215]: X_1_poly_d16_scaled_coloumn_added = polynomial_adder(X_1,16)
```

### Optimized Weight - deg16

```
In [216]: theta_1_d16 = weight_optimizer(X_1_poly_d16_scaled_coloumn_added,16,Y_1,
epochs=1000000)
print (theta_1_d16)
```

```
[[ 0.45269151]
 [ 0.90569215]
 [-0.63905604]
 [ 0.58174172]
 [-2.18981068]
 [-0.36381962]
 [-0.48679136]
 [ 0.03242304]
 [ 1.22399681]
 [ 1.3783914 ]
 [ 0.51790883]
 [ 0.54350124]
 [-1.12891568]
 [ 0.26915691]
 [ 0.5333662 ]
 [-0.19490954]
 [-1.39357019]]
```

## RMSE - Deg4

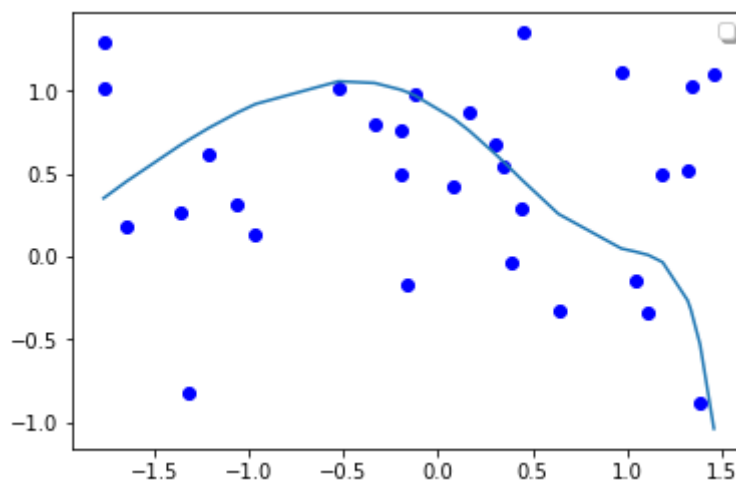
```
In [217]: rms_1_d16 = rmse_calculator(X_1_poly_d16_scaled_coloumn_added,theta_1_d16,Y_1)
          print (rms_1_d16)
```

0.2625062559634639

## Graph\_Deg 16

```
In [218]: graph_plotter(X_1_poly_d16_scaled_coloumn_added,theta_1_d16,Y_1)
```

No handles with labels found to put in legend.



**1-5. Fit the data using 16th order polynomial regression model with ridge (L2 penalty) regularization (matrix form, gradient descent method).**

**You need to try at least 3 different L2 penalty (for example,  $\lambda = 0.1$ , 1, 10).**

**$\lambda = 0.1$**

**Optimized weights  $\lambda = 0.1$**

```
In [219]: theta_1_d16_regularised_1 = weight_optimizer(X_1_poly_d16_scaled_coloumn_
_added,16,Y_1,regularisation_alpha=0.1)
print (theta_1_d16_regularised_1)

[[ 0.45118755]
 [ 0.78496938]
 [-0.41986364]
 [-0.64770631]
 [-0.48516438]
 [-0.22691241]
 [ 0.00677979]
 [ 0.17744631]
 [ 0.27520327]
 [ 0.30862102]
 [ 0.28991242]
 [ 0.22673522]
 [ 0.12928467]
 [ 0.01020639]
 [-0.12850398]
 [-0.27605485]
 [-0.43514748]]
```

### RMSE - $\lambda = 0.1$

```
In [220]: rms_1_d16_regularised_1 = rmse_calculator(X_1_poly_d16_scaled_coloumn_ad
ded,theta_1_d16_regularised_1,Y_1)
print (rms_1_d16_regularised_1)

0.2693872247572115
```

### $\lambda = 1$

### Optimized weights $\lambda = 1$

```
In [221]: theta_1_d16_regularised_2 = weight_optimizer(X_1_poly_d16_scaled_coloumn_
          _added,16,Y_1,regularisation_alpha=1)
          print (theta_1_d16_regularised_2)

[[ 0.43808856]
 [ 0.30532714]
 [-0.11714469]
 [-0.21555533]
 [-0.18959658]
 [-0.12931817]
 [-0.06945018]
 [-0.02187918]
 [ 0.01069068]
 [ 0.02911397]
 [ 0.03545038]
 [ 0.03198174]
 [ 0.02082246]
 [ 0.00379041]
 [-0.01760999]
 [-0.04215991]
 [-0.06888419]]
```

### RMSE - $\lambda = 1$

```
In [222]: rms_1_d16_regularised_2 = rmse_calculator(X_1_poly_d16_scaled_coloumn_ad
          ded,theta_1_d16_regularised_2,Y_1)
          print (rms_1_d16_regularised_2)

0.30190806311237905
```

### $\lambda = 10$

### Optimized weights $\lambda = 10$



```
In [223]: theta_1_d16_regularised_3 = weight_optimizer(X_1_poly_d16_scaled_coloumn_
          _added,16,Y_1,regularisation_alpha=10)
          print (theta_1_d16_regularised_3)

[[ 0.33951863]
 [ 0.05560105]
 [-0.02707079]
 [-0.0564417 ]
 [-0.06173571]
 [-0.05764355]
 [-0.05057775]
 [-0.04323712]
 [-0.03668487]
 [-0.03126116]
 [-0.02699188]
 [-0.02377544]
 [-0.02146823]
 [-0.01992273]
 [-0.01900323]
 [-0.01859114]
 [-0.01858551]]
```

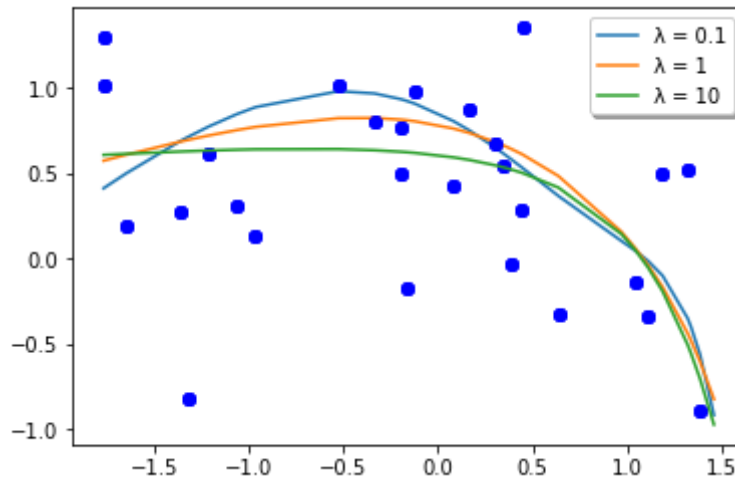
### RMSE - $\lambda = 10$

```
In [224]: rms_1_d16_regularised_3 = rmse_calculator(X_1_poly_d16_scaled_coloumn_ad
          ded,theta_1_d16_regularised_3,Y_1)
          print (rms_1_d16_regularised_3)

0.34575123097106086
```

### Graph Plot $\lambda = 0.1,1,10$

```
In [225]: graph_plotter(X_1_poly_d16_scaled_coloumn_added,theta_1_d16_regularised_
1,Y_1,graph_label="λ = 0.1")
graph_plotter(X_1_poly_d16_scaled_coloumn_added,theta_1_d16_regularised_
2,Y_1,graph_label= "λ = 1")
graph_plotter(X_1_poly_d16_scaled_coloumn_added,theta_1_d16_regularised_
3,Y_1,graph_label= "λ = 10")
```



## 1-6. Fit the data using 16th order polynomial regression model with scikit-learn Ridge model.

You need to try at least 3 different L2 penalty (for example,  $\lambda = 0.1$ , 1, 10).

```
In [226]: alpha = [0.1, 1, 10]
```

```
In [14]: def ridge_optimized_weight(data,label,alpha=0.1):
ridge_reg = Ridge(alpha=alpha, solver="cholesky")
ridge_model = ridge_reg.fit(data[:,1:], label)
print ("Model-Intercept",ridge_model.intercept_)
print ("Model-Co-ef",ridge_model.coef_.T)
return ridge_model
```

```
In [15]: def ridge_lasso_graph_plotter(data,actual_label,ridge_model,graph_label=
None):
X_1_poly_dn_scaled_coloumn_added_sorted = data[data[:,1].argsort()]
Y_ridge_model_1_predict = ridge_model.predict(X_1_poly_dn_scaled_col
oumn_added_sorted[:,1:])
plt.plot(X_1_poly_dn_scaled_coloumn_added_sorted[:,1:2], actual_labe
l, 'bo')
plt.plot(X_1_poly_dn_scaled_coloumn_added_sorted[:,1:2], Y_ridge_mod
el_1_predict,label=graph_label)
legend = plt.legend(loc='upper right', shadow=True)
```

```
In [16]: def ridge_lasso_rmse(data,actual_label,ridge_model):
          Y_ridge_model_1_predict = ridge_model.predict(data[:,1:])
          rms = sqrt(mean_squared_error(actual_label, Y_ridge_model_1_predict
          ))
          return rms
```

### Optimized\_Weights ridge $\lambda = 0.1$

```
In [230]: ridge_model_1 = ridge_optimized_weight(X_1_poly_d16_scaled_coloumn_added
          ,Y_1,0.1)
```

```
Model-Intercept [0.45269151]
Model-Co-ef [[ 0.78485691]
              [-0.41980483]
              [-0.6474652 ]
              [-0.48475686]
              [-0.22708056]
              [ 0.00646183]
              [ 0.17607267]
              [ 0.27503508]
              [ 0.30951311]
              [ 0.28986803]
              [ 0.22699526]
              [ 0.13087521]
              [ 0.01010497]
              [-0.12814542]
              [-0.27800671]
              [-0.43473061]]
```

### RMSE ridge $\lambda = 0.1$

```
In [231]: rms_ridge_1 = ridge_lasso_rmse(X_1_poly_d16_scaled_coloumn_added,Y_1,ridge_model_1)
          print(rms_ridge_1)
```

```
0.2693814634185468
```

### Optimized Weights Ridge $\lambda = 1$

```
In [232]: ridge_model_2 = ridge_optimized_weight(X_1_poly_d16_scaled_coloumn_added
, Y_1, 1)

Model-Intercept [0.45269151]
Model-Co-ef [[ 0.30532714]
 [-0.11714469]
 [-0.21555533]
 [-0.18959658]
 [-0.12931817]
 [-0.06945018]
 [-0.02187918]
 [ 0.01069068]
 [ 0.02911397]
 [ 0.03545038]
 [ 0.03198174]
 [ 0.02082246]
 [ 0.00379041]
 [-0.01760999]
 [-0.04215991]
 [-0.06888419]]
```

### RMSE ridge $\lambda = 1$

```
In [233]: rms_ridge_2 = ridge_lasso_rmse(X_1_poly_d16_scaled_coloumn_added, Y_1, ridge_model_2)
print(rms_ridge_2)

0.3015546921663988
```

### Optimized Weights Ridge $\lambda = 10$

```
In [234]: ridge_model_3 = ridge_optimized_weight(X_1_poly_d16_scaled_coloumn_added
, Y_1, 10)

Model-Intercept [0.45269151]
Model-Co-ef [[ 0.05560105]
 [-0.02707079]
 [-0.0564417 ]
 [-0.06173571]
 [-0.05764355]
 [-0.05057775]
 [-0.04323712]
 [-0.03668487]
 [-0.03126116]
 [-0.02699188]
 [-0.02377544]
 [-0.02146823]
 [-0.01992273]
 [-0.01900323]
 [-0.01859114]
 [-0.01858551]]
```

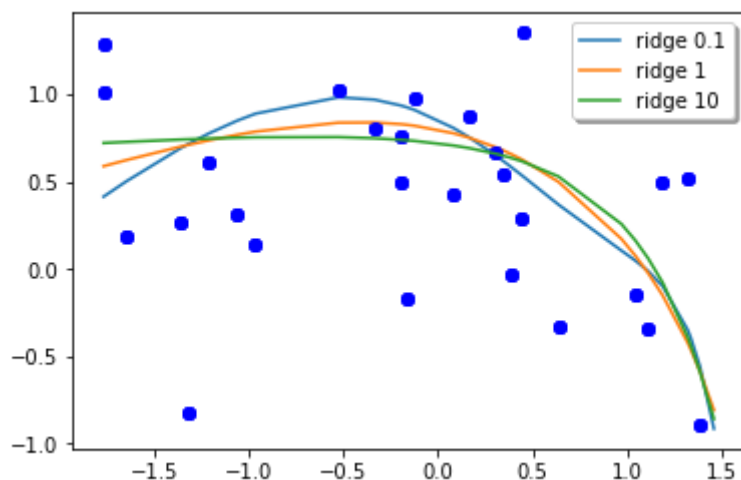
**RMSE ridge  $\lambda = 10$** 

```
In [235]: rms_ridge_3 = ridge_lasso_rmse(X_1_poly_d16_scaled_coloumn_added,Y_1,ridge_model_3)
          print(rms_ridge_3)
```

0.3267044743318329

**Graph Plot Ridge  $\lambda = 0.1, 1, 10$** 

```
In [236]: ridge_lasso_graph_plotter(X_1_poly_d16_scaled_coloumn_added,Y_1,ridge_model_1,graph_label="ridge 0.1")
          ridge_lasso_graph_plotter(X_1_poly_d16_scaled_coloumn_added,Y_1,ridge_model_2,graph_label="ridge 1")
          ridge_lasso_graph_plotter(X_1_poly_d16_scaled_coloumn_added,Y_1,ridge_model_3,graph_label="ridge 10")
```

**1-7. Fit the data using 16th order polynomial regression model with scikit-learn Lasso model.**

```
In [17]: def lasso_optimized_weight(data,label,alpha=0.1):
          lasso_reg = Lasso(alpha)
          lasso_model = lasso_reg.fit(data[:,1:], label)
          print ("Model-Intercept",lasso_model.intercept_)
          print ("Model-Co-ef",lasso_model.coef_.T)
          return lasso_model
```

**Optimized\_Weights lasso  $\lambda = 0.1$**

```
In [238]: lasso_model_1 = lasso_optimized_weight(X_1_poly_d16_scaled_coloumn_added
, Y_1, 0.1)

Model-Intercept [0.45269151]
Model-Co-ef [-0.          -0.          -0.          -0.          -0.
-0.37373874
-0.          -0.          -0.          -0.          -0.          -0.
-0.          -0.          -0.          -0.          ]
```

### RMSE lasso $\lambda = 0.1$

```
In [239]: rms_lasso_1 = ridge_lasso_rmse(X_1_poly_d16_scaled_coloumn_added, Y_1, las
so_model_1)
print(rms_lasso_1)

0.34466712593964577
```

### Optimized\_Weights lasso $\lambda = 1$

```
In [240]: lasso_model_2 = lasso_optimized_weight(X_1_poly_d16_scaled_coloumn_added
, Y_1, 1)

Model-Intercept [0.45269151]
Model-Co-ef [-0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -
0. -0.]
```

### RMSE lasso $\lambda = 1$

```
In [241]: rms_lasso_2 = ridge_lasso_rmse(X_1_poly_d16_scaled_coloumn_added, Y_1, las
so_model_2)
print(rms_lasso_2)

0.5772554182087681
```

### Optimized\_Weights lasso $\lambda = 10$

```
In [242]: lasso_model_3 = lasso_optimized_weight(X_1_poly_d16_scaled_coloumn_added
, Y_1, 10)

Model-Intercept [0.45269151]
Model-Co-ef [-0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -
0. -0.]
```

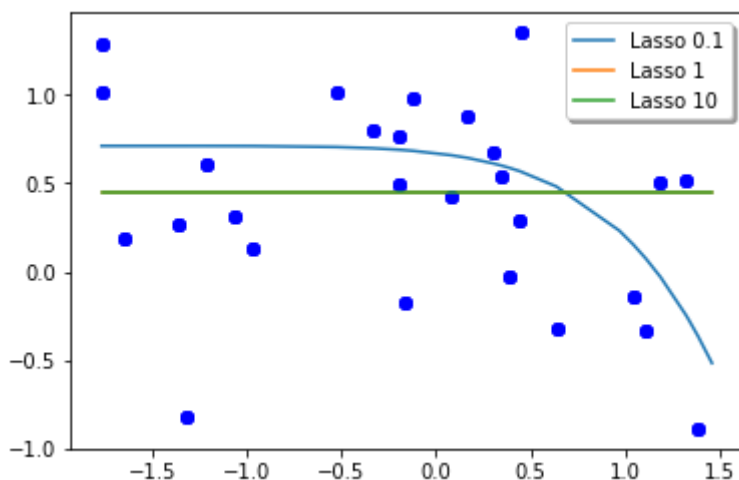
### RMSE lasso $\lambda = 10$

```
In [243]: rms_lasso_3 = ridge_lasso_rmse(X_1_poly_d16_scaled_coloumn_added,Y_1,lasso_model_3)
print(rms_lasso_3)
```

0.5772554182087681

### Graph Plot Lasso $\lambda = 0.1, 1, 10$

```
In [244]: ridge_lasso_graph_plotter(X_1_poly_d16_scaled_coloumn_added,Y_1,lasso_model_1,graph_label="Lasso 0.1")
ridge_lasso_graph_plotter(X_1_poly_d16_scaled_coloumn_added,Y_1,lasso_model_2,graph_label="Lasso 1")
ridge_lasso_graph_plotter(X_1_poly_d16_scaled_coloumn_added,Y_1,lasso_model_3,graph_label="Lasso 10")
```



## 2. Polynomial regression with train/validation/test

You need to split the data into training/validation/testing data set as 60% / 20% / 20%.

Please use `np.random.seed(1)` to have consistent data for evaluation

**2-1. Fit the data using linear (1st order) regression model (matrix form, gradient descent method).**

```

In [245]: # def plot_learning_curves(model, X, y):
#         X_train, X_val, y_train, y_val = train_test_split(X, y, test_size=
0.2)
#         train_errors, val_errors = [], []
#         for m in range(1, len(X_train)):
#             model.fit(X_train[:m], y_train[:m])
#             y_train_predict = model.predict(X_train[:m])
#             y_val_predict = model.predict(X_val)
#             train_errors.append(mean_squared_error(y_train_predict, y_train[:m]))
#             val_errors.append(mean_squared_error(y_val_predict, y_val))
#         plt.plot(np.sqrt(train_errors), "r-+", linewidth=2, label="train")
#         plt.plot(np.sqrt(val_errors), "b-", linewidth=3, label="val")

# lin_reg = LinearRegression()
# plot_learning_curves(lin_reg, X, y)

# from sklearn.pipeline import Pipeline

# polynomial_regression = Pipeline([
#     ("poly_features", PolynomialFeatures(degree=10, include_bias=False)),
#     ("lin_reg", LinearRegression()),
# ])

# plot_learning_curves(polynomial_regression, X, y)

```

```

In [6]: data2 = pd.read_csv("ex2data2.csv")
X_2 = data2.as_matrix(columns=data2.columns[0:1])
Y_2 = data2.as_matrix(columns=data2.columns[1:])
X_2_poly_d1_scaled = scaler.fit_transform(X_2)
X_2_column_added = np.c_[np.ones((data2.shape[0], 1)), X_2]
X_2_poly_d1_scaled_column_added = np.c_[np.ones((data2.shape[0], 1)), X_2_poly_d1_scaled]

/anaconda2/envs/carnd-term1/lib/python3.5/site-packages/sklearn/utils/validation.py:475: DataConversionWarning: Data with input dtype int64 was converted to float64 by StandardScaler.
warnings.warn(msg, DataConversionWarning)

```



```
In [7]: data2.head
```

```

Out[7]: <bound method NDFrame.head of
0      430    80000.0
1      460   247000.0
2      470   192500.0
3      490   150000.0
4      500   125000.0
5      520    82500.0
6      520   275000.0
7      520   330000.0
8      550   353000.0
9      560   299000.0
10     570    89950.0
11     580   220000.0
12     580   330600.0
13     590   156000.0
14     600   229000.0
15     620   175000.0
16     620   244900.0
17     630   148000.0
18     630   315000.0
19     630   430000.0
20     650   295000.0
21     660   100000.0
22     660   175000.0
23     660   225000.0
24     660   227450.0
25     670   240000.0
26     670   245000.0
27     670   279000.0
28     670   348000.0
29     680   110700.0
...
5374    5840  2200000.0
5375    5850  1530000.0
5376    5860  2400000.0
5377    6050  1600000.0
5378    6070  1550000.0
5379    6070  1570000.0
5380    6200  3300000.0
5381    6210  3200000.0
5382    6240  2950000.0
5383    6510  1900000.0
5384    6530  1600000.0
5385    6550  1500000.0
5386    6640  4500000.0
5387    6810  2480000.0
5388    6900  1140000.0
5389    7000  3200000.0
5390    7050  3800000.0
5391    7100  3200000.0
5392    7120   900000.0
5393    7320  1140000.0
5394    7420  1950000.0
5395    7620  1680000.0
5396    7710  3300000.0
5397    7850  2700000.0
5398    7880  2420000.0

```

```

5399      8000  5350000.0
5400      8010  5110000.0
5401      8670  2890000.0
5402     12050  7700000.0
5403     13540  2280000.0

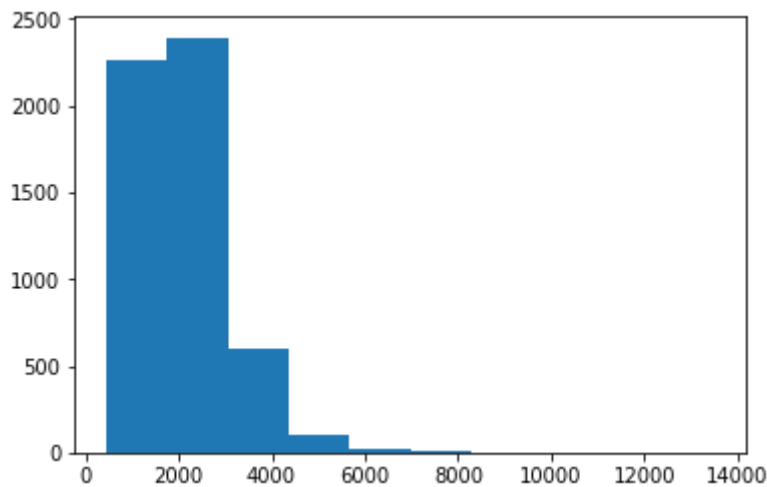
```

```
[5404 rows x 2 columns]>
```

```
In [247]: plt.hist(X_2)
```

```

Out[247]: (array([2.266e+03, 2.393e+03, 6.040e+02, 1.050e+02, 2.100e+01, 1.200e+0
1,
                1.000e+00, 0.000e+00, 1.000e+00, 1.000e+00]),
array([ 430., 1741., 3052., 4363., 5674., 6985., 8296., 9607.,
        10918., 12229., 13540.]),
<a list of 10 Patch objects>)
```



```

In [248]: X_2_train_val, X_2_test, y_2_train_val, y_2_test = train_test_split(X_2_
poly_d1_scaled_coloumn_added , Y_2, test_size=0.2,random_state=1)
X_2_train, X_2_val, y_2_train, y_2_val = train_test_split(X_2_train_val,
y_2_train_val, test_size=0.25,random_state=1)
print (X_2_coloumn_added.shape)
print (X_2_train_val.shape)
print (X_2_train.shape)

```

```
(5404, 2)
```

```
(4323, 2)
```

```
(3242, 2)
```

```

In [18]: def Mean_Square_Error_calculator(data,weights,actual_label):
        predict_y = data.dot(weights)
        rms = mean_squared_error(actual_label, predict_y)
        return rms

```

```

In [19]: def train_val_best_model(X_train, X_val, y_train, y_val,degree):
    epochs = [1000,10000,100000]
    learning_rate = [.001,.01,.1]
    train_errors, val_errors = [], []
    counter = 1
    combination = {}
    for epoch in epochs:
        for rate in learning_rate:
            weights = weight_optimizer(X_train,degree,y_train,learn_rate
= rate,epochs = epoch,regularisation_alpha=0)
            rmse_train = rmse_calculator(X_train,weights,y_train)
            rmse_val = rmse_calculator(X_val,weights,y_val)
            train_errors.append(rmse_train)
            val_errors.append(rmse_val)
            combination[counter] = (epoch,rate)
            counter += 1
    plt.plot(train_errors, "r-+", linewidth=2, label="train")
    plt.plot(val_errors, "b-", linewidth=3, label="val")
    legend = plt.legend(loc='upper right', shadow=True)
    ## Find the lowest validation error
    counter_val_error = list(enumerate(val_errors, 1))
    counter_val_error_sorted = sorted(counter_val_error, key=lambda x:x[
1])
    best_model = combination[(counter_val_error_sorted[0][0])]
    print(counter_val_error)
    print("Epoch and Learning Rate Combination",counter_val_error_sorted
[0][0],best_model)
    return best_model

```

```

In [251]: best_model_2 = train_val_best_model(X_2_train, X_2_val, y_2_train, y_2_v
al,1)
weights_best_model_2 = weight_optimizer(X_2_train,1,y_2_train,learn_rate
= best_model_2 [1],epochs = best_model_2 [0],regularisation_alpha=0)

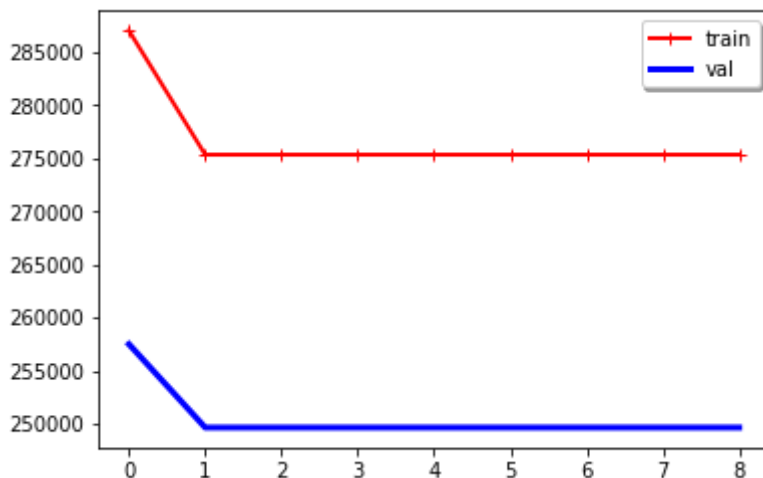
```

```

[(1, 257488.04094283126), (2, 249622.42769314421), (3, 249622.427705436
3), (4, 249622.4276900577), (5, 249622.42770543642), (6, 249622.4277054
363), (7, 249622.42770543747), (8, 249622.42770543642), (9, 249622.4277
054363)]

```

Epoch and Learning Rate Combination 4 (10000, 0.001)



## RMSE - Train,test Deg -1

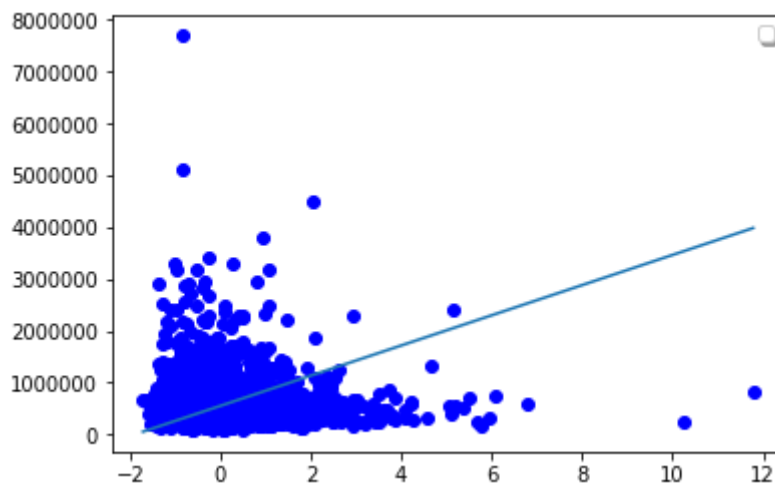
```
In [252]: rmse_train_2 = rmse_calculator(X_2_train,weights_best_model_2,y_2_train)
rmse_test_2 = rmse_calculator(X_2_test,weights_best_model_2,y_2_test)
print("rmse_train",rmse_train_2)
print("rmse_test",rmse_test_2)
```

```
rmse_train 275355.0256953383
rmse_test 264311.94247884274
```

## Graph Train Deg-1

```
In [253]: graph_plotter(X_2_train,weights_best_model_2,y_2_train,graph_label=None)
```

No handles with labels found to put in legend.



## 2-2. Fit the data using 2nd order polynomial regression model (matrix form, gradient descent method).

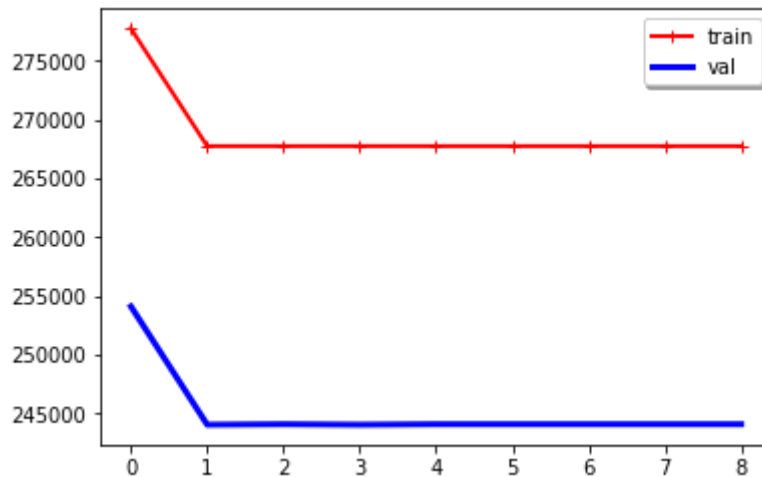
```
In [254]: X_2_poly_d2_scaled_coloumn_added = polynomial_adder(X_2,2)
```

```
In [255]: X_22_train_val, X_22_test, y_22_train_val, y_22_test = train_test_split(
X_2_poly_d2_scaled_coloumn_added, Y_2, test_size=0.2,random_state=1)
X_22_train, X_22_val, y_22_train, y_22_val = train_test_split(X_22_train_val, y_22_train_val, test_size=0.25,random_state=1)
```

```
In [256]: best_model_22 = train_val_best_model(X_22_train, X_22_val, y_22_train, y_22_val, 2)
weights_best_model_22 = weight_optimizer(X_22_train, 2, y_22_train, learn_rate = best_model_22 [1], epochs = best_model_22 [0], regularisation_alpha=0)
```

```
[(1, 254082.7389780978), (2, 244029.3535573043), (3, 244071.59635894437), (4, 244029.2686895008), (5, 244071.5963589144), (6, 244071.59635907054), (7, 244071.59635891087), (8, 244071.59635907042), (9, 244071.59635907054)]
```

Epoch and Learning Rate Combination 4 (10000, 0.001)



## RMSE - Train, test Deg -2

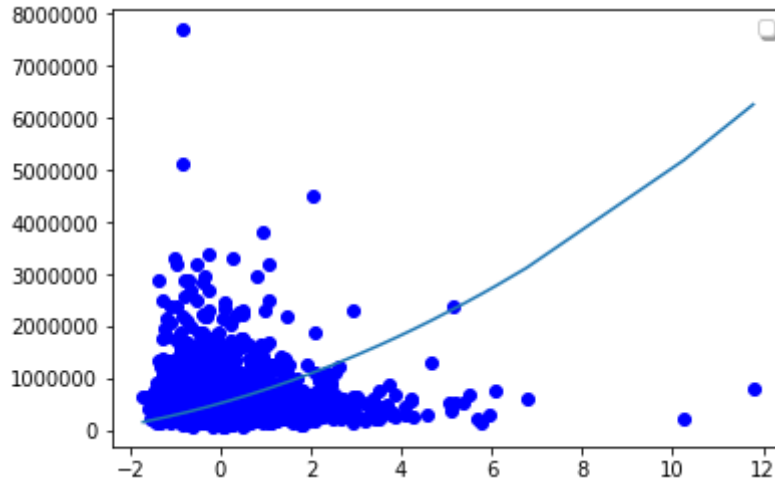
```
In [257]: rmse_train_22 = rmse_calculator(X_22_train, weights_best_model_22, y_22_train)
rmse_test_22 = rmse_calculator(X_22_test, weights_best_model_22, y_22_test)
print("rmse_train", rmse_train_22)
print("rmse_test", rmse_test_22)
```

```
rmse_train 267719.7117411374
rmse_test 254533.92847752557
```

## Graph Train Deg-2

```
In [258]: graph_plotter(X_22_train,weights_best_model_22,y_22_train,graph_label=None)
```

No handles with labels found to put in legend.



## 2-3. Fit the data using 4th order polynomial regression model (matrix form, gradient descent method).

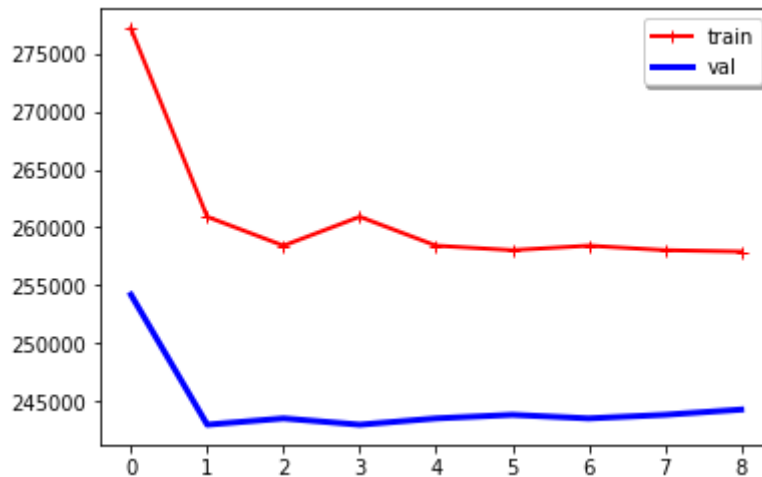
```
In [259]: X_2_poly_d4_scaled_coloumn_added = polynomial_adder(X_2,4)
```

```
In [260]: X_24_train_val, X_24_test, y_24_train_val, y_24_test = train_test_split(
X_2_poly_d4_scaled_coloumn_added, Y_2, test_size=0.2,random_state=1)
X_24_train, X_24_val, y_24_train, y_24_val = train_test_split(X_24_train_val, y_24_train_val, test_size=0.25,random_state=1)
```

```
In [261]: best_model_24 = train_val_best_model(X_24_train, X_24_val, y_24_train, y_24_val,4)
weights_best_model_24 = weight_optimizer(X_24_train,4,y_24_train,learn_rate = best_model_24 [1],epochs = best_model_24 [0],regularisation_alpha=0)
```

```
[(1, 254205.33656847285), (2, 243001.66458493247), (3, 243544.8531816557), (4, 243001.8240493743), (5, 243544.6655575537), (6, 243854.6229242246), (7, 243544.64619277278), (8, 243854.612717776), (9, 244304.69676459354)]
```

Epoch and Learning Rate Combination 2 (1000, 0.01)



## RMSE - Train,test Deg -4

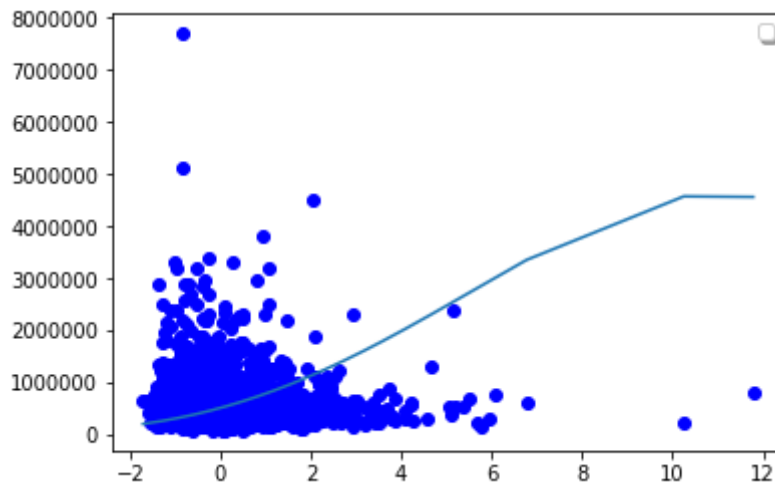
```
In [262]: rmse_train_24 = rmse_calculator(X_24_train,weights_best_model_24,y_24_train)
rmse_test_24 = rmse_calculator(X_24_test,weights_best_model_24,y_24_test)
print("rmse_train",rmse_train_24)
print("rmse_test",rmse_test_24)
```

```
rmse_train 260919.45768952757
rmse_test 251751.42950903866
```



```
In [263]: graph_plotter(X_24_train,weights_best_model_24,y_24_train,graph_label=None)
```

No handles with labels found to put in legend.



## 2-4. Fit the data using 16th order polynomial regression model (matrix form, gradient descent method).

```
In [26]: def weight_optimizer_stoc_batch(data,degree,Label,learn_rate = .01,epochs = 100000,regularisation_alpha=0):
    theta_1 = np.random.randn(degree+1,1)
    data_size = data.shape[0]
    #print (data_size)
    batch_size = data_size // 16
    #print (batch_size)
    for epoch in range(epochs):
        for batch in range(batch_size):
            X_batch, y_batch = resample(data, Label, n_samples=16, random_state=0,replace=False)
            gradients = (2/X_batch.shape[0])* (X_batch.T.dot(X_batch.dot(theta_1)- y_batch)+ \
                                                    regularisation_alpha*theta_1)
            theta_1 = theta_1 - learn_rate * gradients
    return theta_1
```

```

In [21]: def train_val_best_model_stoc_batch(X_train, X_val, y_train, y_val, degree):
    epochs = [10000]
    learning_rate = [.001, .01, .1]
    train_errors, val_errors = [], []
    counter = 1
    combination = {}
    for epoch in epochs:
        for rate in learning_rate:
            weights = weight_optimizer_stoc_batch(X_train, degree, y_train,
            , learn_rate = rate, epochs = epoch, regularisation_alpha=0)
            rmse_train = rmse_calculator(X_train, weights, y_train)
            rmse_val = rmse_calculator(X_val, weights, y_val)
            train_errors.append(rmse_train)
            val_errors.append(rmse_val)
            combination[counter] = (epoch, rate)
            counter += 1
    plt.plot(train_errors, "r-+", linewidth=2, label="train")
    plt.plot(val_errors, "b-", linewidth=3, label="val")
    legend = plt.legend(loc='upper right', shadow=True)
    ## Find the lowest validation error
    counter_val_error = list(enumerate(val_errors, 1))
    counter_val_error_sorted = sorted(counter_val_error, key=lambda x:x[1])
    best_model = combination[(counter_val_error_sorted[0][0])]
    print(counter_val_error)
    print("Epoch and Learning Rate Combination", counter_val_error_sorted[0][0], best_model)
    return best_model

```

```

In [22]: X_2_poly_d16_scaled_coloumn_added = polynomial_adder(X_2, 16)

```

```

In [23]: X_216_train_val, X_216_test, y_216_train_val, y_216_test = train_test_split(X_2_poly_d16_scaled_coloumn_added, Y_2, test_size=0.2, random_state=1)
    X_216_train, X_216_val, y_216_train, y_216_val = train_test_split(X_216_train_val, y_216_train_val, test_size=0.25, random_state=1)

```

**Implemented Mini-batch Gradient Descent as I was getting error with full batch**

```
In [373]: best_model_216 = train_val_best_model_stoc_batch(X_216_train, X_216_val,
y_216_train, y_216_val,16)
weights_best_model_216 = weight_optimizer_stoc_batch(X_216_train,16,y_216_train,learn_rate = best_model_216 [1],epochs = best_model_216 [0],regularisation_alpha=0)
```

3242

202

3242

202

3242

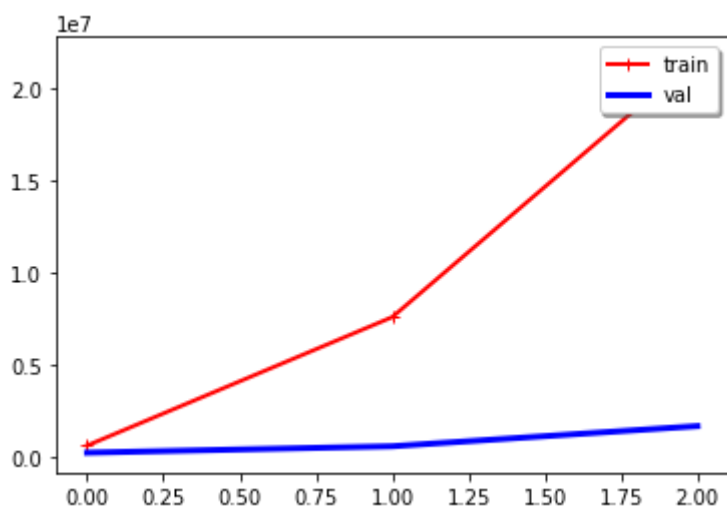
202

[(1, 271431.84548393165), (2, 621086.9719878328), (3, 1715299.1157242693)]

Epoch and Learning Rate Combination 1 (10000, 0.001)

3242

202



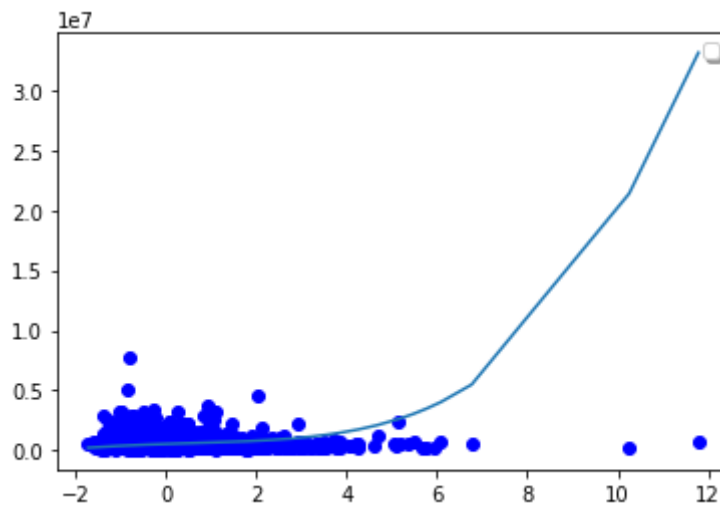
```
In [374]: rmse_train_216 = rmse_calculator(X_216_train,weights_best_model_216,y_216_train)
rmse_test_216 = rmse_calculator(X_216_test,weights_best_model_216,y_216_test)
print("rmse_train",rmse_train_216)
print("rmse_test",rmse_test_216)
```

rmse\_train 656156.7479793766

rmse\_test 274794.112255375

```
In [375]: graph_plotter(X_216_train,weights_best_model_216,y_216_train,graph_label  
=None)
```

WARNING:matplotlib.legend:No handles with labels found to put in legend.



**2-5. Fit the data using 16th order polynomial regression model with ridge (L2 penalty) regularization.**

```

In [27]: def train_val_best_model_ridge_stoc_batch(X_train, X_val, y_train, y_val
,degree):
    epochs = [10000]
    learning_rate = [.01]
    regularisation_alpa = [.1,1,10]
    train_errors, val_errors = [], []
    weights_list = []
    counter = 1
    combination = {}
    for epoch in epochs:
        for rate in learning_rate:
            for alpha in regularisation_alpa:
                weights = weight_optimizer_stoc_batch(X_train,degree,y_t
rain,learn_rate = rate,epochs = epoch, \
                                                    regularisation_alpha=alpha)
                weights_list.append(weights)
                rmse_train = rmse_calculator(X_train,weights,y_train)
                rmse_val = rmse_calculator(X_val,weights,y_val)
                train_errors.append(rmse_train)
                val_errors.append(rmse_val)
                combination[counter] = (epoch,rate,alpha)
                counter += 1
    weights_array = np.asarray(weights_list)
    print (weights_array)
    plt.plot(train_errors, "r-+", linewidth=2, label="train")
    plt.plot(val_errors, "b-", linewidth=3, label="val")
    legend = plt.legend(loc='upper right', shadow=True)
    ## Find the lowest validation error
    counter_val_error = list(enumerate(val_errors, 1))
    counter_val_error_sorted = sorted(counter_val_error, key=lambda x:x[
1])
    best_model = combination[(counter_val_error_sorted[0][0])]
    print(best_model)
    return best_model,weights_array

```

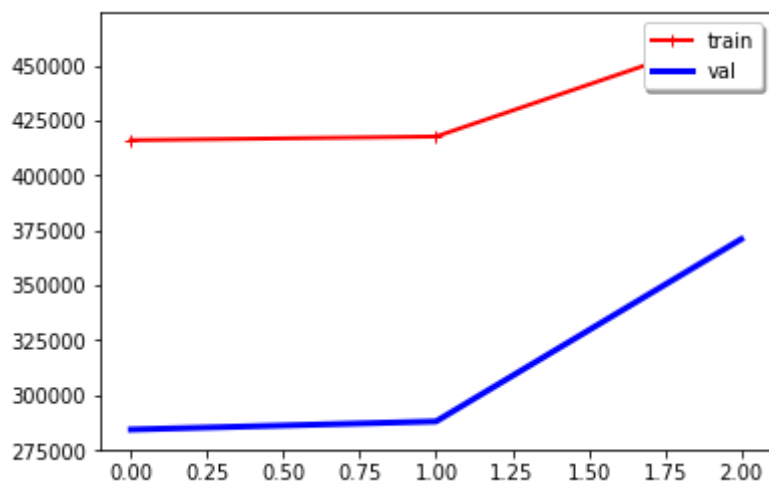
**Implemented Mini-batch Gradient Descent as I was getting error with full batch**

```
In [28]: best_model_216_ridge, weights_array = train_val_best_model_ridge_stoc_batch(X_216_train, X_216_val, y_216_train, y_216_val, 16)
weights_best_model_216_ridge = weight_optimizer_stoc_batch(X_216_train, 16, y_216_train, learn_rate = best_model_216_ridge [1], epochs = best_model_216_ridge [0], regularisation_alpha=best_model_216_ridge [2])
```

```
[ [ 529311.56308421 ]  
  [ 140367.80182524 ]  
  [ -4546.74748686 ]  
  [ -28473.30732643 ]  
  [ -19788.63272868 ]  
  [ -15000.66415285 ]  
  [ -12895.44455603 ]  
  [ -11538.24832723 ]  
  [ -10566.67038328 ]  
  [ -9891.60090774 ]  
  [ -9426.90906073 ]  
  [ -9097.26154915 ]  
  [ -8850.68250377 ]  
  [ -8655.58649547 ]  
  [ -8493.84397512 ]  
  [ -8355.19791351 ]  
  [ -8233.75543243 ] ]
```

```
[ [ 504369.59043868 ]  
  [ 112854.12132627 ]  
  [ 20818.52540674 ]  
  [ -23880.56066294 ]  
  [ -26120.99445878 ]  
  [ -19357.51087343 ]  
  [ -14465.9510496 ]  
  [ -11760.93642951 ]  
  [ -10310.04194258 ]  
  [ -9496.30495061 ]  
  [ -9002.50480531 ]  
  [ -8673.93636613 ]  
  [ -8435.03414097 ]  
  [ -8248.08593394 ]  
  [ -8093.69358333 ]  
  [ -7961.51143409 ]  
  [ -7845.77403797 ] ]
```

```
[ [ 333564.0674735 ]  
  [ 66591.27018116 ]  
  [ 11619.48500806 ]  
  [ -16917.22262927 ]  
  [ -18072.40431746 ]  
  [ -13199.53401866 ]  
  [ -9722.42710617 ]  
  [ -7830.67797699 ]  
  [ -6834.78228822 ]  
  [ -6285.08729544 ]  
  [ -5955.10514574 ]  
  [ -5736.84789641 ]  
  [ -5578.58986315 ]  
  [ -5454.8830961 ]  
  [ -5352.75811019 ]  
  [ -5265.33509663 ]  
  [ -5188.79119887 ] ] ]  
(10000, 0.01, 0.1)
```



```
In [34]: weights_array = weights_array.reshape(3,17)
```

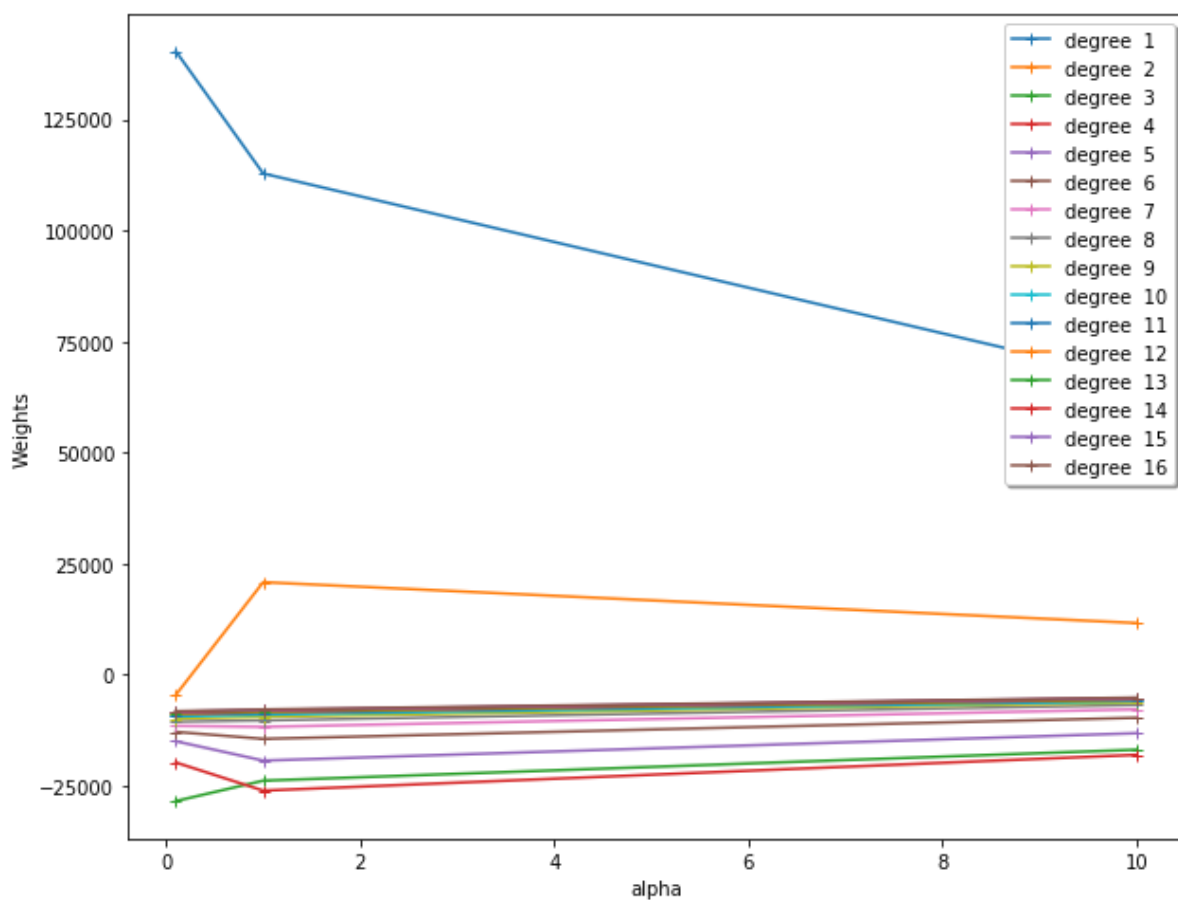
```
In [51]: weights_array[:,1]
```

```
Out[51]: array([140367.80182524, 112854.12132627, 66591.27018116])
```

```
In [89]: def weights_plotter(weights_array):
          fig = plt.figure(figsize=(10,8))
          plt.ylabel('Weights')
          plt.xlabel('alpha')
          for i in range(1,weights_array.shape[1]):
              plt.plot([0.1,1,10],weights_array[:,i],'-+',label="degree % i" %
i)
              legend = plt.legend(loc='upper right', shadow=True)
```



```
In [90]: weights_plotter(weights_array)
```



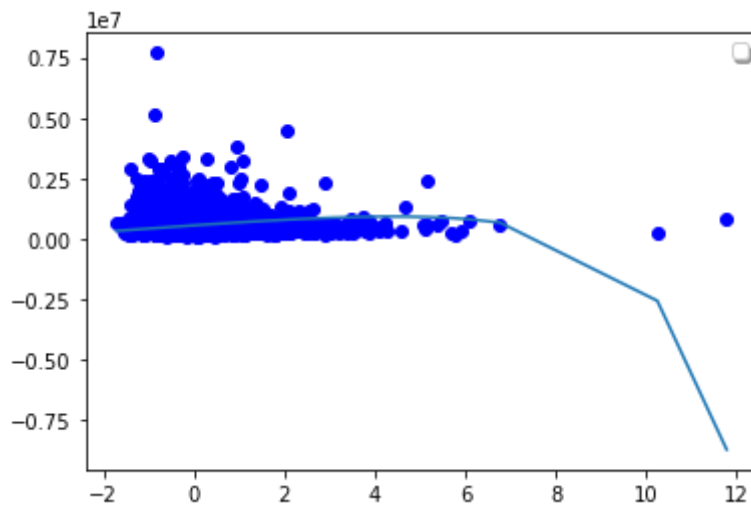
```
In [383]: rmse_train_216_ridge = rmse_calculator(X_216_train,weights_best_model_216_ridge,y_216_train)
rmse_test_216_ridge = rmse_calculator(X_216_test,weights_best_model_216_ridge,y_216_test)
print("rmse_train",rmse_train_216_ridge)
print("rmse_test",rmse_test_216_ridge)
```

```
rmse_train 415963.5773494703
```

```
rmse_test 321415.22175827075
```

```
In [385]: graph_plotter(X_216_train,weights_best_model_216_ridge,y_216_train,graph_label=None)
```

WARNING:matplotlib.legend:No handles with labels found to put in legend.

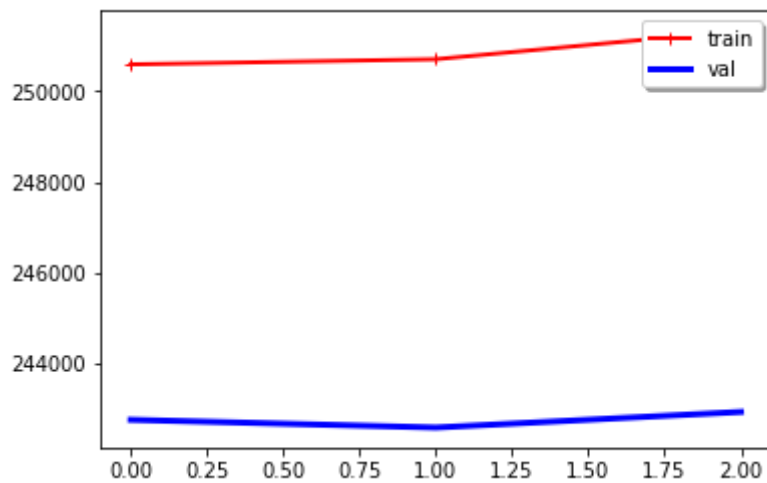


## 2-6. Fit the data using 16th order polynomial regression model with scikit-learn Ridge model.

```
In [64]: def train_val_best_model_scikit_ridge(X_train, X_val, y_train, y_val):
    regularisation_alpa = [0.1,1,10]
    train_errors, val_errors = [], []
    counter = 1
    combination = {}
    weights_list = []
    for alpha in regularisation_alpa:
        ridge_model = ridge_optimized_weight(X_train,y_train,alpha)
        weights_list.append(ridge_model.coef_)
        rmse_train = ridge_lasso_rmse(X_train,y_train,ridge_model)
        rmse_val = ridge_lasso_rmse(X_val,y_val,ridge_model)
        train_errors.append(rmse_train)
        val_errors.append(rmse_val)
        combination[counter] = (alpha,ridge_model)
        counter += 1
    weights_array = np.asarray(weights_list)
    #print (weights_array)
    plt.plot(train_errors, "r-+", linewidth=2, label="train")
    plt.plot(val_errors, "b-", linewidth=3, label="val")
    legend = plt.legend(loc='upper right', shadow=True)
    ## Find the lowest validation error
    counter_val_error = list(enumerate(val_errors, 1))
    counter_val_error_sorted = sorted(counter_val_error, key=lambda x:x[1])
    best_model = combination[(counter_val_error_sorted[0][0])]
    print("alpha",best_model[0])
    return best_model[1],weights_array
```

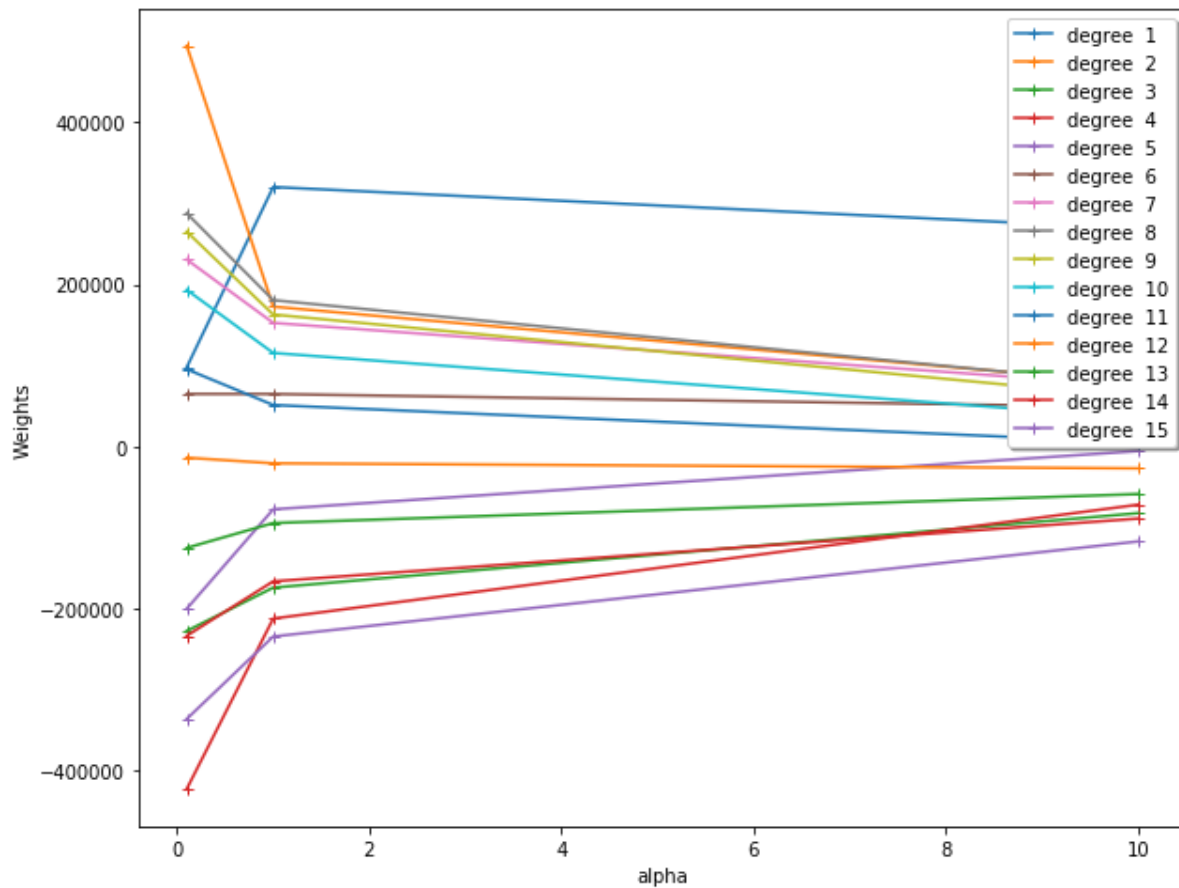
```
In [65]: best_model_216_scikit_ridge, weights_array_scikit_ridge = train_val_best_
model_scikit_ridge(X_216_train, X_216_val, y_216_train, y_216_val)
```

```
Model-Intercept [547704.506878]
Model-Co-ef [[ 47149.02303585]
 [ 97806.15914125]
 [ 494262.89050946]
 [-227237.60901432]
 [-422132.21976636]
 [-199882.87630559]
 [ 64994.59063826]
 [ 230729.38940244]
 [ 287757.59283936]
 [ 265072.70818138]
 [ 193352.45983527]
 [ 95486.86500499]
 [ -13592.57195474]
 [-124860.40617845]
 [-233169.41195629]
 [-335762.59926058]]
Model-Intercept [547718.99272249]
Model-Co-ef [[ -16344.34115262]
 [ 320155.60757134]
 [ 172666.68142153]
 [-173831.65937522]
 [-211992.23283438]
 [ -77256.72585303]
 [ 64972.77614043]
 [ 152621.76687899]
 [ 180680.13409481]
 [ 162984.49861422]
 [ 115562.95467406]
 [ 51361.29686543]
 [ -20522.80095625]
 [ -94191.55415056]
 [-166059.86068628]
 [-234099.56514392]]
Model-Intercept [547547.03238267]
Model-Co-ef [[ 36166.43778387]
 [ 269023.40114305]
 [ 77631.40011951]
 [ -81988.69738317]
 [ -71553.92749472]
 [ -5369.60495591]
 [ 48807.03808062]
 [ 74843.22643076]
 [ 76275.4587262 ]
 [ 60869.18534266]
 [ 35528.32733298]
 [ 5236.22544228]
 [ -26747.06366153]
 [ -58427.80688802]
 [ -88665.61666554]
 [-116867.58253742]]
alpha 1
```



```
In [68]: weights_array_scikit_ridge = weights_array_scikit_ridge.reshape(3,16)
```

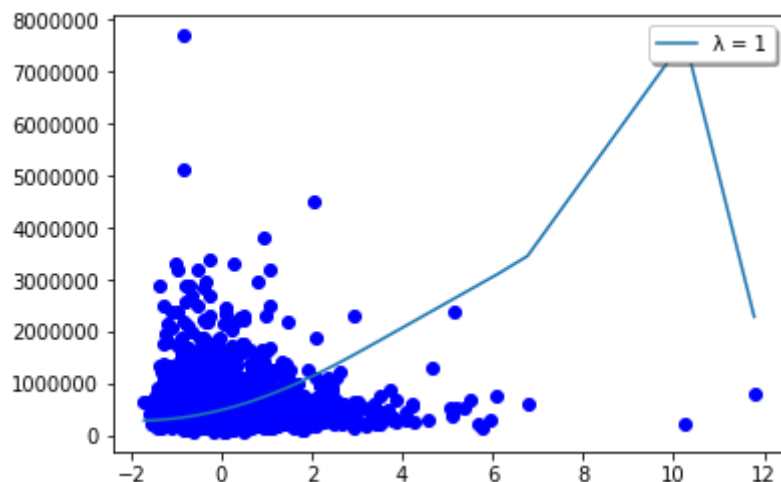
```
In [91]: weights_plotter(weights_array_scikit_ridge)
```



```
In [270]: rmse_train_216_scikit_ridge = ridge_lasso_rmse(X_216_train, y_216_train,
best_model_216_scikit_ridge)
rmse_test_216_scikit_ridge = ridge_lasso_rmse(X_216_test,y_216_test,best
_model_216_scikit_ridge)
print("rmse_train",rmse_train_216_scikit_ridge)
print("rmse_test",rmse_test_216_scikit_ridge)
```

```
rmse_train 250700.05033093243
rmse_test 251065.74316573926
```

```
In [271]: ridge_lasso_graph_plotter(X_216_train,y_216_train,best_model_216_scikit_
ridge,graph_label="λ = 1")
```



**2-7. Fit the data using 16th order polynomial regression model with scikit-learn Lasso model.**

```
In [76]: def train_val_best_model_scikit_lasso(X_train, X_val, y_train, y_val):  
    regularisation_alpa = [0.1,1,10]  
    train_errors, val_errors = [], []  
    counter = 1  
    combination = {}  
    weights_list = []  
    for alpha in regularisation_alpa:  
        lasso_model = lasso_optimized_weight(X_train,y_train,alpha)  
        weights_list.append(lasso_model.coef_)  
        rmse_train = ridge_lasso_rmse(X_train,y_train,lasso_model)  
        rmse_val = ridge_lasso_rmse(X_val,y_val,lasso_model)  
        train_errors.append(rmse_train)  
        val_errors.append(rmse_val)  
        combination[counter] = (alpha,lasso_model)  
        counter += 1  
    weights_array = np.asarray(weights_list)  
    plt.plot(train_errors, "r-+", linewidth=2, label="train")  
    plt.plot(val_errors, "b-", linewidth=3, label="val")  
    legend = plt.legend(loc='upper right', shadow=True)  
    ## Find the lowest validation error  
    counter_val_error = list(enumerate(val_errors, 1))  
    counter_val_error_sorted = sorted(counter_val_error, key=lambda x:x[  
1])  
    best_model = combination[(counter_val_error_sorted[0][0])]  
    print("alpha",best_model[0])  
    return best_model[1],weights_array
```

```
In [77]: best_model_216_scikit_lasso, weights_array_scikit_lasso = train_val_best_
model_scikit_lasso(X_216_train, X_216_val, y_216_train, y_216_val)
```

```
/anaconda2/envs/carnd-term1/lib/python3.5/site-packages/sklearn/linear_
model/coordinate_descent.py:491: ConvergenceWarning: Objective did not
converge. You might want to increase the number of iterations. Fitting
data with very small alpha may cause precision problems.
```

```
ConvergenceWarning)
```

```
Model-Intercept [547780.08000696]
```

```
Model-Co-ef [-125318.97003623  666197.50707237 -261841.01303223 -18730
9.44448817
```

```
77160.31260774  155957.01757717  125786.49836694  69160.91493416
```

```
19708.08956119 -14840.20562329 -36093.17315681 -47445.45636661
```

```
-52259.47895543 -52993.40874272 -51325.09359054 -48343.58909978]
```

```
Model-Intercept [547779.73592156]
```

```
Model-Co-ef [-124992.36217718  665263.70260852 -260753.01353071 -18734
0.17063375
```

```
76345.133887  155967.41656718  125762.63818031  69164.76212111
```

```
19900.84319741 -14276.41829556 -36175.49317507 -47521.03245514
```

```
-52327.97275879 -53055.02152518 -51380.27127648 -48392.87104231]
```

```
Model-Intercept [547776.26865725]
```

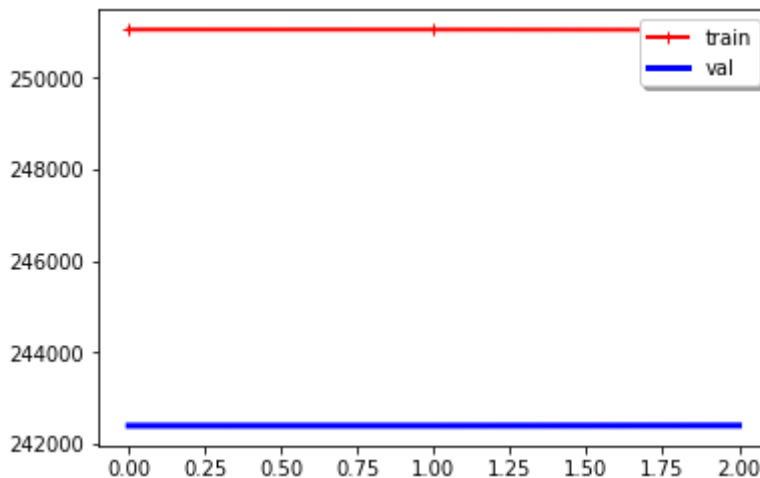
```
Model-Co-ef [-121721.55990706  655932.64030426 -249919.8823818 -18760
2.71597435
```

```
68214.49580173  156107.22185177  125530.49711011  69166.91275706
```

```
21367.94806767 -8139.49732978 -37011.24562008 -48290.1479555
```

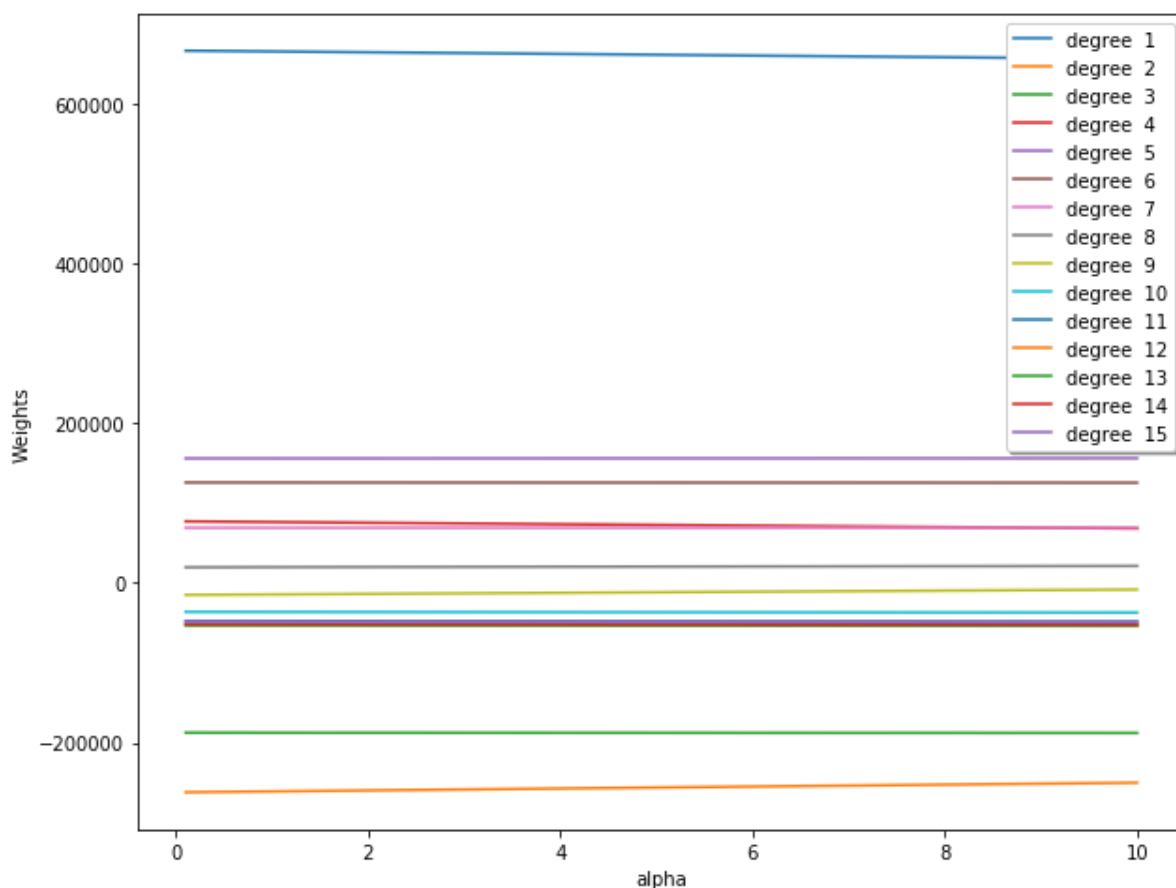
```
-53026.14960449 -53683.77064736 -51943.80255891 -48896.47767207]
```

```
alpha 0.1
```





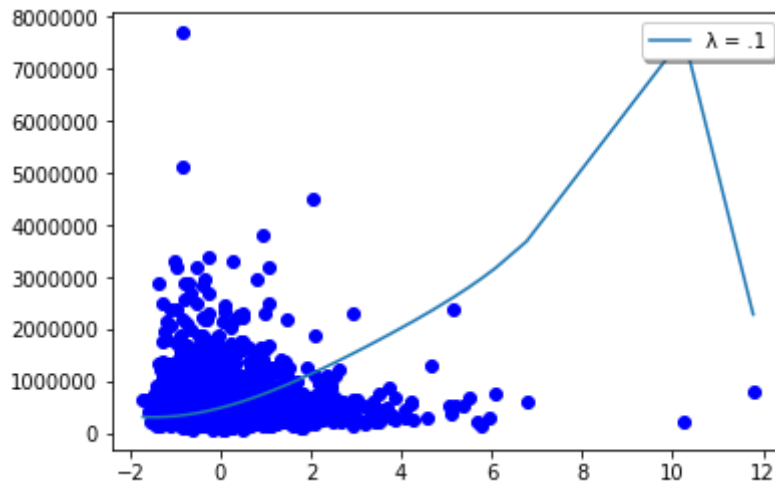
```
In [79]: weights_plotter(weights_array_scikit_lasso)
```



```
In [274]: rmse_train_216_scikit_lasso = ridge_lasso_rmse(X_216_train, y_216_train,
best_model_216_scikit_lasso)
rmse_test_216_scikit_lasso = ridge_lasso_rmse(X_216_test,y_216_test,best
_model_216_scikit_lasso)
print("rmse_train",rmse_train_216_scikit_lasso)
print("rmse_test",rmse_test_216_scikit_lasso)
```

```
rmse_train 251063.94309086326
rmse_test 250161.4085909059
```

```
In [275]: ridge_lasso_graph_plotter(X_216_train,y_216_train,best_model_216_scikit_lasso,graph_label="λ = .1")
```



### 3-1. Fit the training data using regression model with ridge (L2 penalty) regularization with scikit-learn Ridge model.

```
In [80]: data3 = pd.read_csv("ex2data3.csv")
del data3['Unnamed: 0']
```

```
In [81]: X_3 = data3.as_matrix(columns=data3.columns[0:8])
Y_3 = data3.as_matrix(columns=data3.columns[8:])
X_3_scaled = scaler.fit_transform(X_3)
X_3_scaled_coloumn_added = np.c_[np.ones((data3.shape[0], 1)), X_3_scaled]
```

```
In [82]: X_3_train_val, X_3_test, y_3_train_val, y_3_test = train_test_split(X_3_scaled_coloumn_added, Y_3, test_size=0.2, random_state=1)
X_3_train, X_3_val, y_3_train, y_3_val = train_test_split(X_3_train_val, y_3_train_val, test_size=0.25, random_state=1)
```

```
In [92]: best_model_3_scikit_ridge, weights_3rddataset_ridge = train_val_best_model_scikit_ridge(X_3_train, X_3_val, y_3_train, y_3_val)
```

```
Model-Intercept [2.07192364]
```

```
Model-Co-ef [[ 0.82501033]
```

```
[ 0.12438917]
```

```
[-0.23245061]
```

```
[ 0.2778601 ]
```

```
[-0.00178963]
```

```
[-0.03738571]
```

```
[-0.9098153 ]
```

```
[-0.88498937]]
```

```
Model-Intercept [2.07192619]
```

```
Model-Co-ef [[ 0.82499755]
```

```
[ 0.12448827]
```

```
[-0.23231252]
```

```
[ 0.27766071]
```

```
[-0.0017563 ]
```

```
[-0.03738623]
```

```
[-0.90885179]
```

```
[-0.88401781]]
```

```
Model-Intercept [2.07195156]
```

```
Model-Co-ef [[ 0.82484941]
```

```
[ 0.12546401]
```

```
[-0.23091504]
```

```
[ 0.27566139]
```

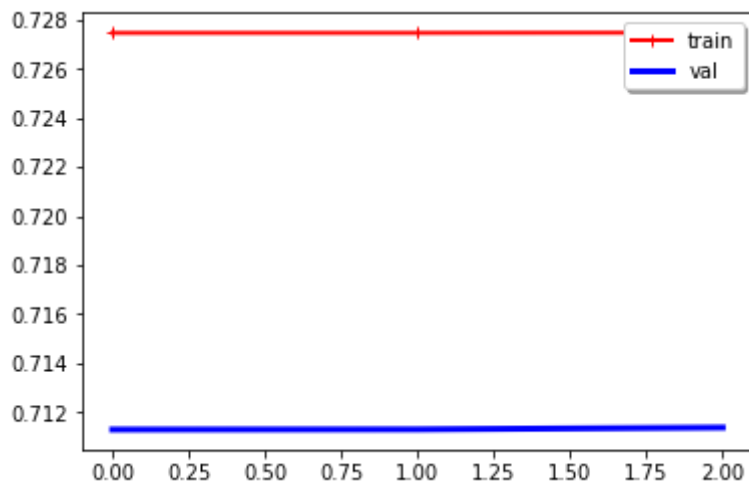
```
[-0.00142815]
```

```
[-0.03739063]
```

```
[-0.89934824]
```

```
[-0.87443278]]
```

```
alpha 0.1
```

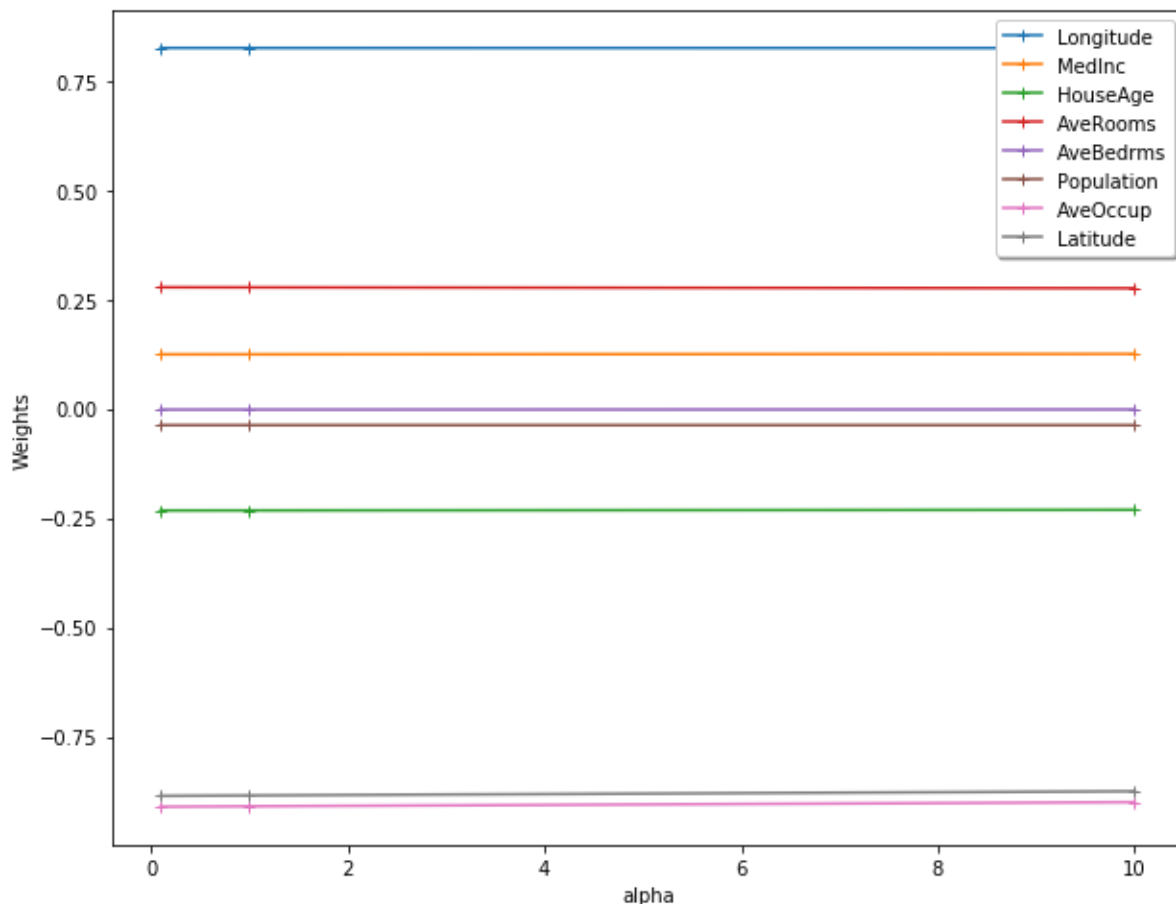


```
In [94]: weights_3rddataset_ridge = weights_3rddataset_ridge.reshape(3,8)
```

```
In [102]: weights_3rddataset_ridge.shape
```

```
Out[102]: (3, 8)
```

```
In [104]: fig = plt.figure(figsize=(10,8))
plt.ylabel('Weights')
plt.xlabel('alpha')
labels = ["MedInc", "HouseAge", "AveRooms", "AveBedrms", "Population", "AveOccup", "Latitude", "Longitude"]
for i in range(len(weights_3rddataset_ridge[1])):
    plt.plot([0.1,1,10],weights_3rddataset_ridge[:,i], '-+', label=labels[i-1])
    legend = plt.legend(loc='upper right', shadow=True)
```



```
In [280]: rmse_train_3_scikit_ridge = ridge_lasso_rmse(X_3_train, y_3_train,best_model_3_scikit_ridge)
rmse_test_3_scikit_ridge = ridge_lasso_rmse(X_3_test,y_3_test,best_model_3_scikit_ridge)
print("rmse_train",rmse_train_3_scikit_ridge)
print("rmse_test",rmse_test_3_scikit_ridge)
```

```
rmse_train 0.7274577804827311
rmse_test 0.7276608037847032
```

### 3-2. Fit the training data using regression model with lasso (L1 penalty) regularization with scikit-learn Lasso model.

```
In [105]: best_model_3_scikit_lasso, weights_3rddataset_lasso = train_val_best_model_scikit_lasso(X_3_train, X_3_val, y_3_train, y_3_val)
```

```
Model-Intercept [2.07432847]
```

```
Model-Co-ef [ 0.71386457  0.11001596 -0.          0.          -0.          -0.          -0.00645422 -0.          ]
```

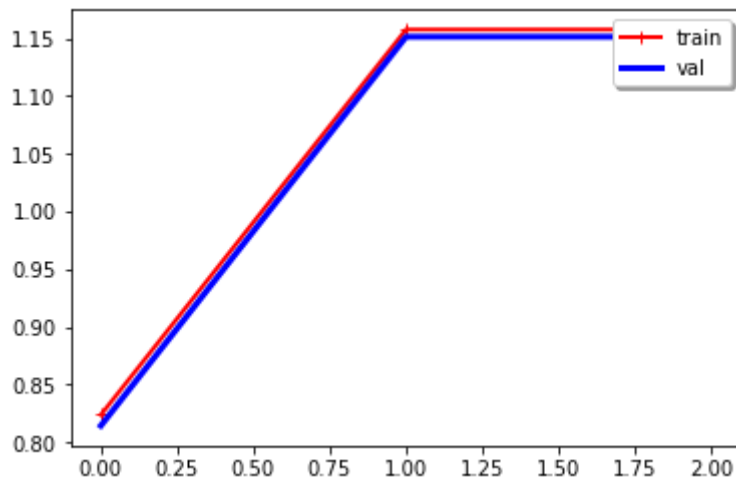
```
Model-Intercept [2.07444727]
```

```
Model-Co-ef [ 0.  0.  0. -0. -0. -0. -0. -0.]
```

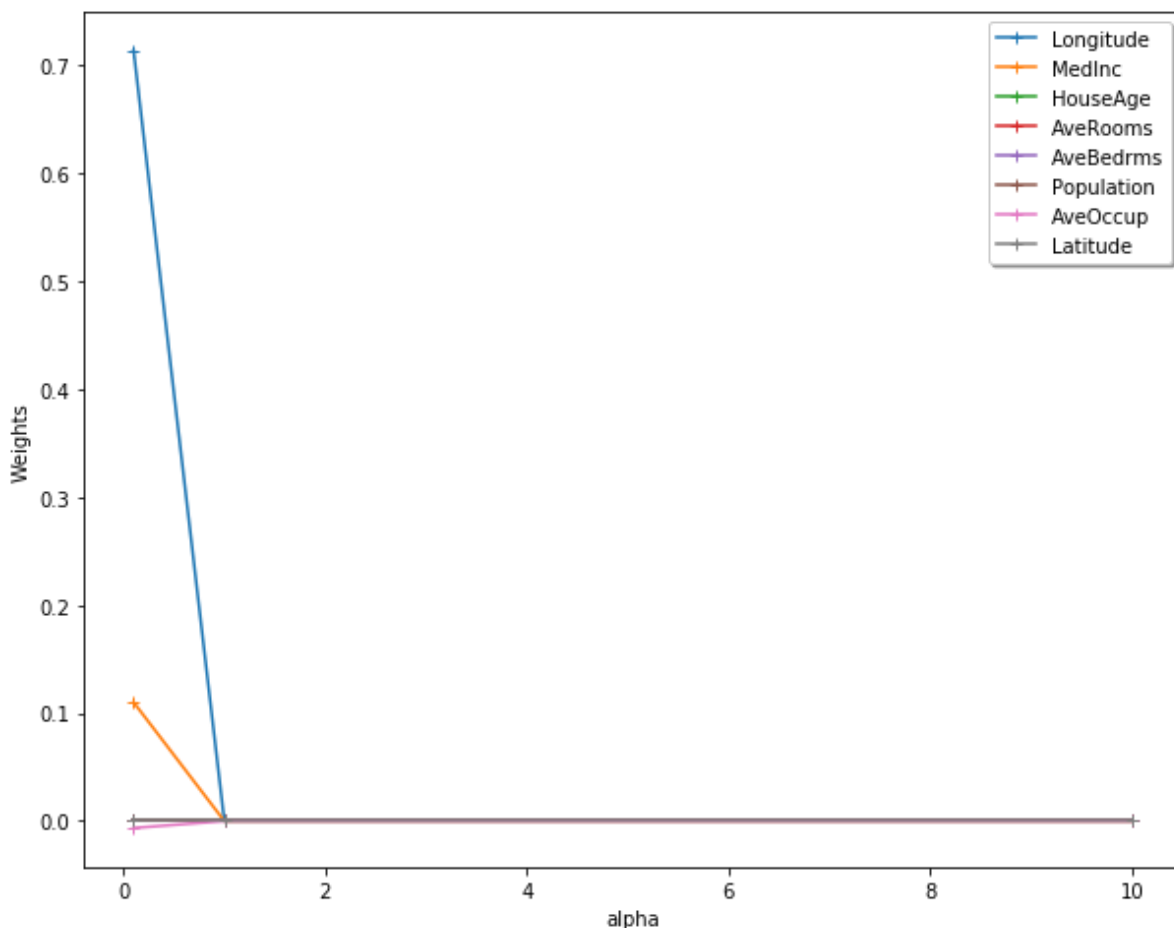
```
Model-Intercept [2.07444727]
```

```
Model-Co-ef [ 0.  0.  0. -0. -0. -0. -0. -0.]
```

```
alpha 0.1
```



```
In [106]: fig = plt.figure(figsize=(10,8))
plt.ylabel('Weights')
plt.xlabel('alpha')
labels = ["MedInc", "HouseAge", "AveRooms", "AveBedrms", "Population", "AveOccup", "Latitude", "Longitude"]
for i in range(len(weights_3rddataset_lasso[1])):
    plt.plot([0.1,1,10],weights_3rddataset_lasso[:,i], '-+', label=labels[i-1])
legend = plt.legend(loc='upper right', shadow=True)
```



```
In [282]: rmse_train_3_scikit_lasso = ridge_lasso_rmse(X_3_train, y_3_train,best_model_3_scikit_lasso)
rmse_test_3_scikit_lasso = ridge_lasso_rmse(X_3_test,y_3_test,best_model_3_scikit_lasso)
print("rmse_train",rmse_train_3_scikit_lasso)
print("rmse_test",rmse_test_3_scikit_lasso)
```

```
rmse_train 0.8239579515519808
rmse_test 0.8147268457966831
```

### 3-3. Fit the training data using regression model with ridge (L2 penalty) regularization using TensorFlow.

```
In [113]: epochs_list = [10000]
          alpha_list = [.1,1,10]
```

```

learning_rate_list = [.01]
weights_list_tensor=[]
#n_epochs = 10000
#learning_rate = 0.01
#alpha = 0.1

tf.reset_default_graph()

X_3 = tf.placeholder(tf.float64)
Y_3 = tf.placeholder(tf.float64)
theta = tf.Variable(tf.random_uniform([data3.shape[1], 1], -1.0, 1.0, dtype=tf.float64, name="theta"))
learning_rate = tf.placeholder(dtype=tf.float64)
alpha = tf.placeholder(dtype=tf.float64)
train_errors = []
val_errors = []
counter = 1
combination = {}

y_pred_3 = tf.matmul(X_3, theta, name="predictions")
error = y_pred_3 - Y_3
mse = tf.sqrt(tf.reduce_mean(tf.square(error), name="mse"))
gradients = 2/tf.shape(X_3)[0] * (tf.matmul(tf.transpose(X_3), error)+alpha*theta)
training_op = tf.assign(theta, theta - learning_rate * gradients)

init = tf.global_variables_initializer()

with tf.Session() as sess:
    sess.run(init)
    for n_epochs in epochs_list:
        for alpha_value in alpha_list:
            for learning_rate_value in learning_rate_list:
                for epoch in range(n_epochs):
                    sess.run(training_op, feed_dict={X_3: X_3_train, Y_3: y_3_train, \
                                                    learning_rate: learning_rate_value, alpha: alpha_value})
                    train_errors.append(sess.run(mse, feed_dict={X_3: X_3_train, Y_3: y_3_train, \
                                                                learning_rate: learning_rate_value, alpha: alpha_value}))
                    val_errors.append(sess.run(mse, feed_dict={X_3: X_3_val, Y_3: y_3_val, \
                                                            learning_rate: learning_rate_value, alpha: alpha_value}))
                    weights_list_tensor.append(theta.eval())
                    combination[counter] = (n_epochs, learning_rate_value, alpha_value)
                    counter += 1
                ## Find the lowest validation error
            plt.plot(train_errors, "r-+", linewidth=2, label="train")
            plt.plot(val_errors, "b-", linewidth=3, label="val")
            legend = plt.legend(loc='upper right', shadow=True)
            counter_val_error = list(enumerate(val_errors, 1))
            counter_val_error_sorted = sorted(counter_val_error, key=lambda x:x[

```



```

1))
    best_model = combination[(counter_val_error_sorted[0][0])]
    print(counter_val_error)
    print("Epoch ,Learning Rate and alpha Combination",best_model)
    ##print train,test error
    print("train_error",sess.run(mse,feed_dict={X_3: X_3_train, Y_3: y_3
_train, \
                                                    learning_
rate: best_model[1],alpha: best_model[2]}))
    print("test_error",sess.run(mse,feed_dict={X_3: X_3_test, Y_3: y_3_t
est, \
                                                    learning_rate
: best_model[1],alpha: best_model[2]}))

```

```

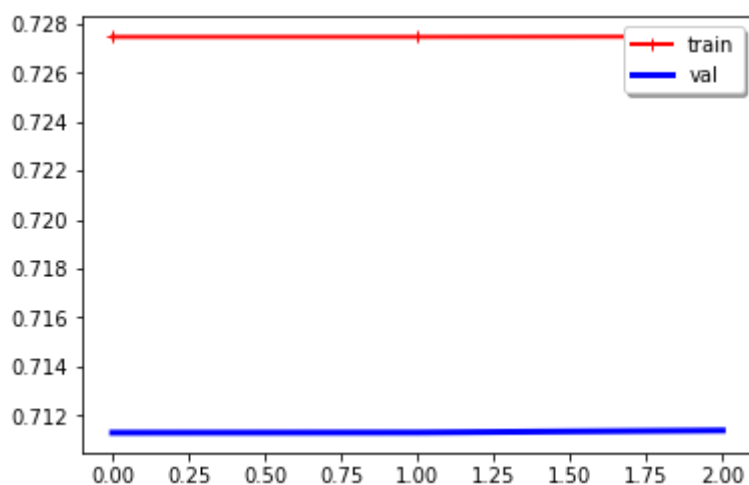
[(1, 0.7112967302993075), (2, 0.7113041668913476), (3, 0.71138890232932
48)]

```

```
Epoch ,Learning Rate and alpha Combination (10000, 0.01, 0.1)
```

```
train_error 0.727470627034165
```

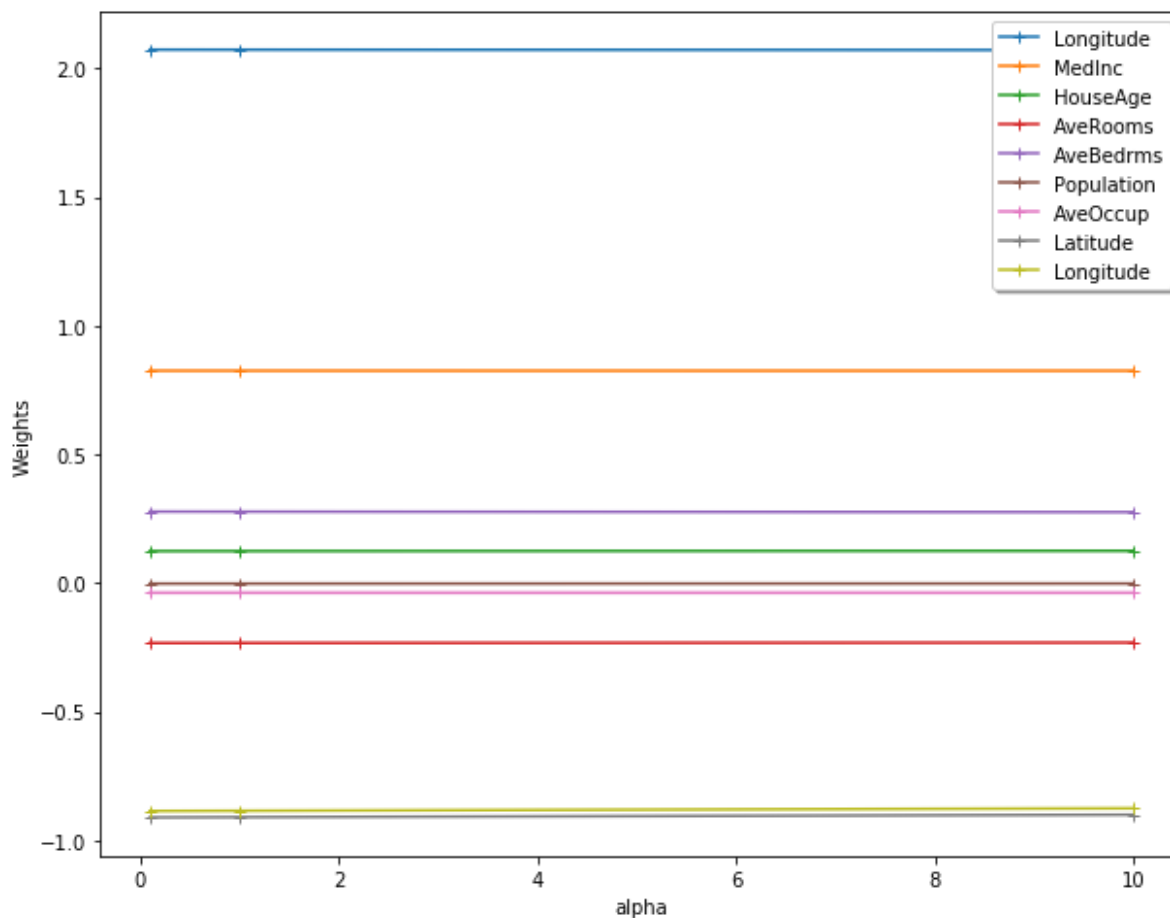
```
test_error 0.7275337209461726
```



```
In [117]: weights_list_tensor_array = np.asarray(weights_list_tensor)
```

```
In [118]: weights_list_tensor_array = weights_list_tensor_array.reshape(3,9)
```

```
In [119]: fig = plt.figure(figsize=(10,8))
plt.ylabel('Weights')
plt.xlabel('alpha')
labels = ["MedInc", "HouseAge", "AveRooms", "AveBedrms", "Population", "AveOccup", "Latitude", "Longitude"]
for i in range(len(weights_list_tensor_array[1])):
    plt.plot([0.1,1,10],weights_list_tensor_array[:,i], '-+', label=labels[i-1])
legend = plt.legend(loc='upper right', shadow=True)
```



**3-4. Fit the training data using regression model with lasso (L1 penalty) regularization using TensorFlow.**