

```
In [197]: import matplotlib
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
from sklearn import linear_model
from sklearn.preprocessing import StandardScaler
from sklearn.metrics import mean_squared_error
from math import sqrt
import tensorflow as tf
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import Ridge
from sklearn.linear_model import Lasso
from sklearn.model_selection import train_test_split
from sklearn.utils import resample
```

1-1. Fit the data using linear (1st order) regression model (matrix form, gradient descent method).

Plot the data with the fitted line. Print optimized weights. Print Root Mean Squared Error (RMSE).

```
In [198]: data1 = pd.read_csv("ex2data1.csv")
X_1 = data1.as_matrix(columns=data1.columns[0:1])
Y_1 = data1.as_matrix(columns=data1.columns[1:])
X_1_coloumn_added = np.c_[np.ones((data1.shape[0], 1)), X_1]
```

```
In [199]: def weight_optimizer(data, degree, Label, learn_rate = .01, epochs = 100000,
regularisation_alpha=0):
    theta_1 = np.random.randn(degree+1, 1)
    for epoch in range(epochs):
        gradients = (2/data.shape[0]) * (data.T.dot(data.dot(theta_1) - La
bel) + regularisation_alpha * theta_1)
        theta_1 = theta_1 - learn_rate * gradients
    return theta_1
```

```
In [200]: def rmse_calculator(data, weights, actual_label):
    predict_y = data.dot(weights)
    rms = sqrt(mean_squared_error(actual_label, predict_y))
    return rms
```

```
In [201]: # Please input sorted data
def graph_plotter(X_1_poly_dn_scaled_coloumn_added,weights,actual_label,
graph_label=None):
    X_1_poly_dn_scaled_coloumn_added_sorted = X_1_poly_dn_scaled_coloumn
_added [X_1_poly_dn_scaled_coloumn_added[:,1].argsort()]
    plt.plot(X_1_poly_dn_scaled_coloumn_added_sorted[:,1:2], actual_labe
l, 'bo')
    y_1_dn_plot = X_1_poly_dn_scaled_coloumn_added_sorted.dot(weights)
    plt.plot(X_1_poly_dn_scaled_coloumn_added_sorted[:,1:2], y_1_dn_plot
,label=graph_label)
    legend = plt.legend(loc='upper right', shadow=True)
```

```
In [202]: scaler = StandardScaler()
```

```
In [203]: def polynomial_adder(data,degree):
    poly_features = PolynomialFeatures(degree=degree, include_bias=False
)
    X_1_poly_dn = poly_features.fit_transform(data)
    X_1_poly_dn_scaled = scaler.fit_transform(X_1_poly_dn)
    X_1_poly_dn_scaled_coloumn_added = np.c_[np.ones((data.shape[0], 1
)), X_1_poly_dn_scaled]
    ## Sorting for graph plotting
    #X_1_poly_dn_scaled_coloumn_added = X_1_poly_dn_scaled_coloumn_added
[X_1_poly_dn_scaled_coloumn_added[:,1].argsort()]
    return X_1_poly_dn_scaled_coloumn_added
```

Optimized Weight - Deg1

```
In [204]: theta_1 = weight_optimizer(X_1_coloumn_added,1,Y_1)
print (theta_1)

[[ 1.07581132]
 [-1.1887203 ]]
```

RMSE - Deg1

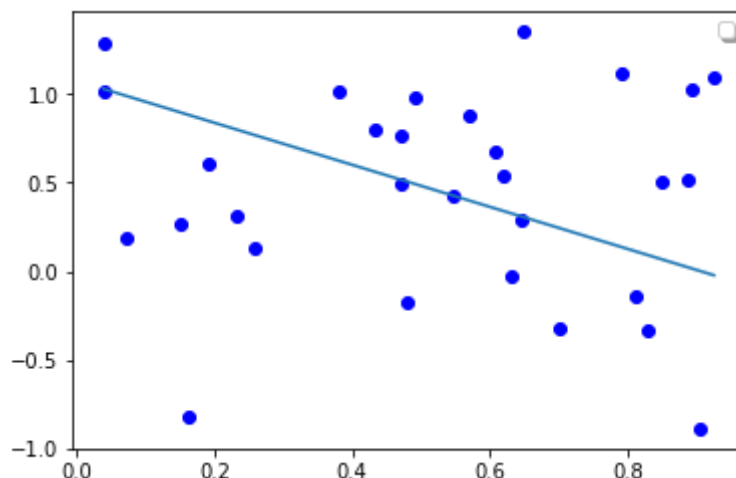
```
In [205]: rms_1 = rmse_calculator(X_1_coloumn_added,theta_1,Y_1)
print (rms_1)

0.47624021947640577
```

Plot - Deg1

```
In [206]: graph_plotter(X_1_coloumn_added,theta_1,Y_1)
```

No handles with labels found to put in legend.



1-2. Fit the data using 2nd order polynomial regression model (matrix form, gradient descent method).

```
In [207]: X_1_poly_d2_scaled_coloumn_added = polynomial_adder(X_1,2)
```

Optimized Weight - deg2

```
In [208]: theta_1_d2 = weight_optimizer(X_1_poly_d2_scaled_coloumn_added,2,Y_1,epo
chs=1000000)
print (theta_1_d2)
```

```
[[ 0.45269151]
 [ 1.13735736]
 [-1.51236155]]
```

RMSE

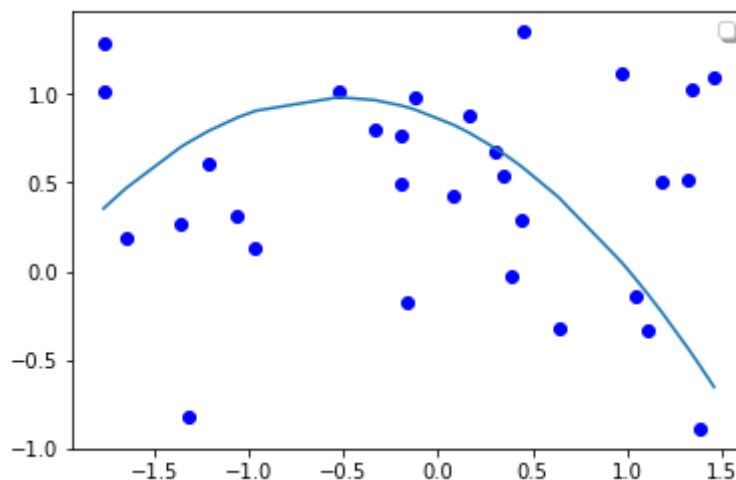
```
In [209]: rms_1_d2 = rmse_calculator(X_1_poly_d2_scaled_coloumn_added,theta_1_d2,Y_1)
print (rms_1_d2)
```

```
0.2856992946970416
```

Plot - deg2

```
In [210]: graph_plotter(X_1_poly_d2_scaled_coloumn_added,theta_1_d2,Y_1)
```

No handles with labels found to put in legend.



1-3. Fit the data using 4th order polynomial regression model (matrix form, gradient descent method).

```
In [211]: X_1_poly_d4_scaled_coloumn_added = polynomial_adder(X_1,4)
```

Optimized Weight - deg4

```
In [212]: theta_1_d4 = weight_optimizer(X_1_poly_d4_scaled_coloumn_added,4,Y_1,epo
chs=1000000)
print (theta_1_d4)
```

```
[[ 0.45269151]
 [ 1.29328167]
 [-1.97124121]
 [ 0.40758024]
 [-0.09772327]]
```

RMSE Deg-4

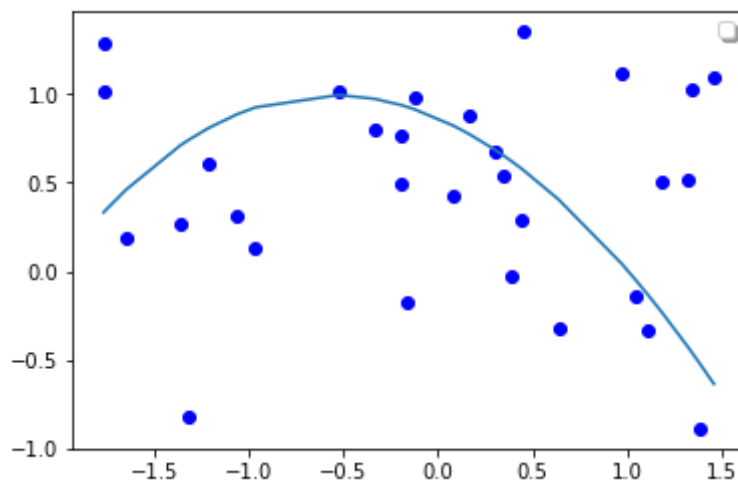
```
In [213]: rms_1_d4 = rmse_calculator(X_1_poly_d4_scaled_coloumn_added,theta_1_d4,Y_1)
print (rms_1_d4)
```

```
0.2853539310798953
```

Graph_Deg 4

```
In [214]: graph_plotter(X_1_poly_d4_scaled_coloumn_added,theta_1_d4,Y_1)
```

No handles with labels found to put in legend.



1-4. Fit the data using 16th order polynomial regression model (matrix form, gradient descent method).

```
In [215]: X_1_poly_d16_scaled_coloumn_added = polynomial_adder(X_1,16)
```

Optimized Weight - deg16

```
In [216]: theta_1_d16 = weight_optimizer(X_1_poly_d16_scaled_coloumn_added,16,Y_1,
epochs=1000000)
print (theta_1_d16)
```

```
[[ 0.45269151]
 [ 0.90569215]
 [-0.63905604]
 [ 0.58174172]
 [-2.18981068]
 [-0.36381962]
 [-0.48679136]
 [ 0.03242304]
 [ 1.22399681]
 [ 1.3783914 ]
 [ 0.51790883]
 [ 0.54350124]
 [-1.12891568]
 [ 0.26915691]
 [ 0.5333662 ]
 [-0.19490954]
 [-1.39357019]]
```

RMSE - Deg4

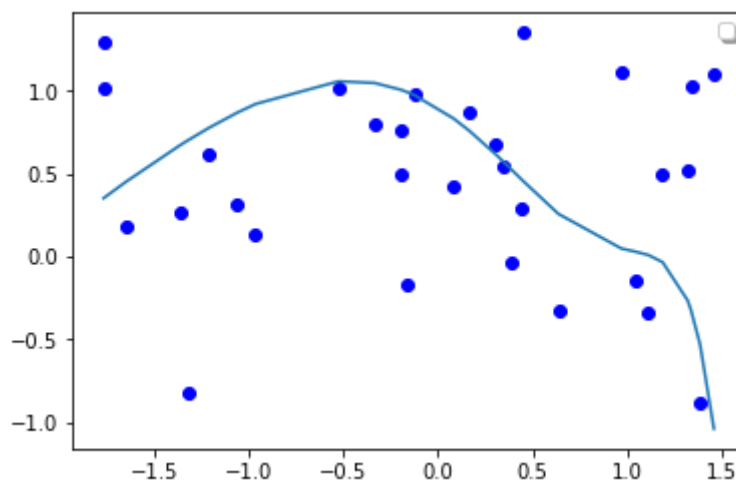
```
In [217]: rms_1_d16 = rmse_calculator(X_1_poly_d16_scaled_coloumn_added,theta_1_d16,Y_1)
          print (rms_1_d16)

0.2625062559634639
```

Graph_Deg 16

```
In [218]: graph_plotter(X_1_poly_d16_scaled_coloumn_added,theta_1_d16,Y_1)

No handles with labels found to put in legend.
```



1-5. Fit the data using 16th order polynomial regression model with ridge (L2 penalty) regularization (matrix form, gradient descent method).

You need to try at least 3 different L2 penalty (for example, $\lambda = 0.1$, 1, 10).

$\lambda = 0.1$

Optimized weights $\lambda = 0.1$

```
In [219]: theta_1_d16_regularised_1 = weight_optimizer(X_1_poly_d16_scaled_coloumn_
          _added,16,Y_1,regularisation_alpha=0.1)
          print (theta_1_d16_regularised_1)

[[ 0.45118755]
 [ 0.78496938]
 [-0.41986364]
 [-0.64770631]
 [-0.48516438]
 [-0.22691241]
 [ 0.00677979]
 [ 0.17744631]
 [ 0.27520327]
 [ 0.30862102]
 [ 0.28991242]
 [ 0.22673522]
 [ 0.12928467]
 [ 0.01020639]
 [-0.12850398]
 [-0.27605485]
 [-0.43514748]]
```

RMSE - $\lambda = 0.1$

```
In [220]: rms_1_d16_regularised_1 = rmse_calculator(X_1_poly_d16_scaled_coloumn_ad
          _ded,theta_1_d16_regularised_1,Y_1)
          print (rms_1_d16_regularised_1)

0.2693872247572115
```

$\lambda = 1$

Optimized weights $\lambda = 1$

```
In [221]: theta_1_d16_regularised_2 = weight_optimizer(X_1_poly_d16_scaled_coloumn_
          _added,16,Y_1,regularisation_alpha=1)
          print (theta_1_d16_regularised_2)

[[ 0.43808856]
 [ 0.30532714]
 [-0.11714469]
 [-0.21555533]
 [-0.18959658]
 [-0.12931817]
 [-0.06945018]
 [-0.02187918]
 [ 0.01069068]
 [ 0.02911397]
 [ 0.03545038]
 [ 0.03198174]
 [ 0.02082246]
 [ 0.00379041]
 [-0.01760999]
 [-0.04215991]
 [-0.06888419]]
```

RMSE - $\lambda = 1$

```
In [222]: rms_1_d16_regularised_2 = rmse_calculator(X_1_poly_d16_scaled_coloumn_ad
          ded,theta_1_d16_regularised_2,Y_1)
          print (rms_1_d16_regularised_2)

0.30190806311237905
```

$\lambda = 10$

Optimized weights $\lambda = 10$


```
In [223]: theta_1_d16_regularised_3 = weight_optimizer(X_1_poly_d16_scaled_coloumn_
          _added,16,Y_1,regularisation_alpha=10)
          print (theta_1_d16_regularised_3)

[[ 0.33951863]
 [ 0.05560105]
 [-0.02707079]
 [-0.0564417 ]
 [-0.06173571]
 [-0.05764355]
 [-0.05057775]
 [-0.04323712]
 [-0.03668487]
 [-0.03126116]
 [-0.02699188]
 [-0.02377544]
 [-0.02146823]
 [-0.01992273]
 [-0.01900323]
 [-0.01859114]
 [-0.01858551]]
```

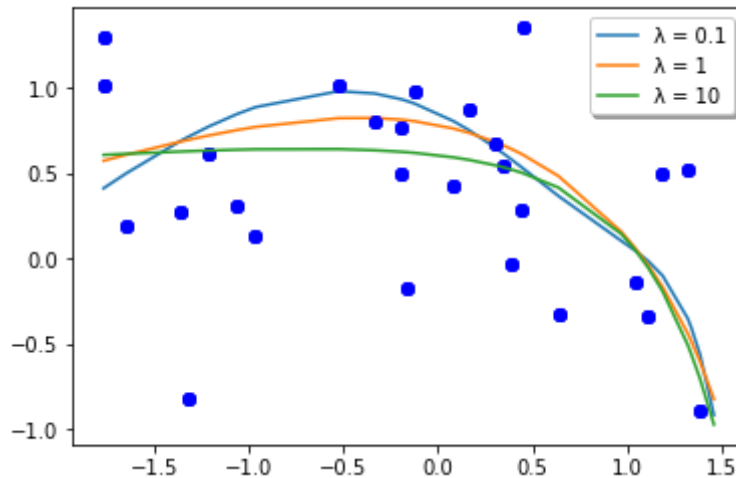
RMSE - $\lambda = 10$

```
In [224]: rms_1_d16_regularised_3 = rmse_calculator(X_1_poly_d16_scaled_coloumn_ad
          _ded,theta_1_d16_regularised_3,Y_1)
          print (rms_1_d16_regularised_3)

0.34575123097106086
```

Graph Plot $\lambda = 0.1,1,10$

```
In [225]: graph_plotter(X_1_poly_d16_scaled_coloumn_added,theta_1_d16_regularised_
1,Y_1,graph_label="λ = 0.1")
graph_plotter(X_1_poly_d16_scaled_coloumn_added,theta_1_d16_regularised_
2,Y_1,graph_label= "λ = 1")
graph_plotter(X_1_poly_d16_scaled_coloumn_added,theta_1_d16_regularised_
3,Y_1,graph_label= "λ = 10")
```



1-6. Fit the data using 16th order polynomial regression model with scikit-learn Ridge model.

You need to try at least 3 different L2 penalty (for example, $\lambda = 0.1$, 1, 10).

```
In [226]: alpha = [0.1, 1, 10]
```

```
In [227]: def ridge_optimized_weight(data,label,alpha=0.1):
    ridge_reg = Ridge(alpha=alpha, solver="cholesky")
    ridge_model = ridge_reg.fit(data[:,1:], label)
    print ("Model-Intercept",ridge_model.intercept_)
    print ("Model-Co-ef",ridge_model.coef_.T)
    return ridge_model
```

```
In [228]: def ridge_lasso_graph_plotter(data,actual_label,ridge_model,graph_label=
None):
    X_1_poly_dn_scaled_coloumn_added_sorted = data[data[:,1].argsort()]
    Y_ridge_model_1_predict = ridge_model.predict(X_1_poly_dn_scaled_col
oumn_added_sorted[:,1:])
    plt.plot(X_1_poly_dn_scaled_coloumn_added_sorted[:,1:2], actual_labe
l, 'bo')
    plt.plot(X_1_poly_dn_scaled_coloumn_added_sorted[:,1:2], Y_ridge_mod
el_1_predict,label=graph_label)
    legend = plt.legend(loc='upper right', shadow=True)
```

```
In [229]: def ridge_lasso_rmse(data,actual_label,ridge_model):
          Y_ridge_model_1_predict = ridge_model.predict(data[:,1:])
          rms = sqrt(mean_squared_error(actual_label, Y_ridge_model_1_predict
          ))
          return rms
```

Optimized_Weights ridge $\lambda = 0.1$

```
In [230]: ridge_model_1 = ridge_optimized_weight(X_1_poly_d16_scaled_coloumn_added
          ,Y_1,0.1)
```

```
Model-Intercept [0.45269151]
Model-Co-ef [[ 0.78485691]
 [-0.41980483]
 [-0.6474652 ]
 [-0.48475686]
 [-0.22708056]
 [ 0.00646183]
 [ 0.17607267]
 [ 0.27503508]
 [ 0.30951311]
 [ 0.28986803]
 [ 0.22699526]
 [ 0.13087521]
 [ 0.01010497]
 [-0.12814542]
 [-0.27800671]
 [-0.43473061]]
```

RMSE ridge $\lambda = 0.1$

```
In [231]: rms_ridge_1 = ridge_lasso_rmse(X_1_poly_d16_scaled_coloumn_added,Y_1,ridge_model_1)
          print(rms_ridge_1)

0.2693814634185468
```

Optimized Weights Ridge $\lambda = 1$

```
In [232]: ridge_model_2 = ridge_optimized_weight(X_1_poly_d16_scaled_coloumn_added
, Y_1, 1)

Model-Intercept [0.45269151]
Model-Co-ef [[ 0.30532714]
 [-0.11714469]
 [-0.21555533]
 [-0.18959658]
 [-0.12931817]
 [-0.06945018]
 [-0.02187918]
 [ 0.01069068]
 [ 0.02911397]
 [ 0.03545038]
 [ 0.03198174]
 [ 0.02082246]
 [ 0.00379041]
 [-0.01760999]
 [-0.04215991]
 [-0.06888419]]
```

RMSE ridge $\lambda = 1$

```
In [233]: rms_ridge_2 = ridge_lasso_rmse(X_1_poly_d16_scaled_coloumn_added, Y_1, ridge_model_2)
print(rms_ridge_2)

0.3015546921663988
```

Optimized Weights Ridge $\lambda = 10$

```
In [234]: ridge_model_3 = ridge_optimized_weight(X_1_poly_d16_scaled_coloumn_added
, Y_1, 10)

Model-Intercept [0.45269151]
Model-Co-ef [[ 0.05560105]
 [-0.02707079]
 [-0.0564417 ]
 [-0.06173571]
 [-0.05764355]
 [-0.05057775]
 [-0.04323712]
 [-0.03668487]
 [-0.03126116]
 [-0.02699188]
 [-0.02377544]
 [-0.02146823]
 [-0.01992273]
 [-0.01900323]
 [-0.01859114]
 [-0.01858551]]
```

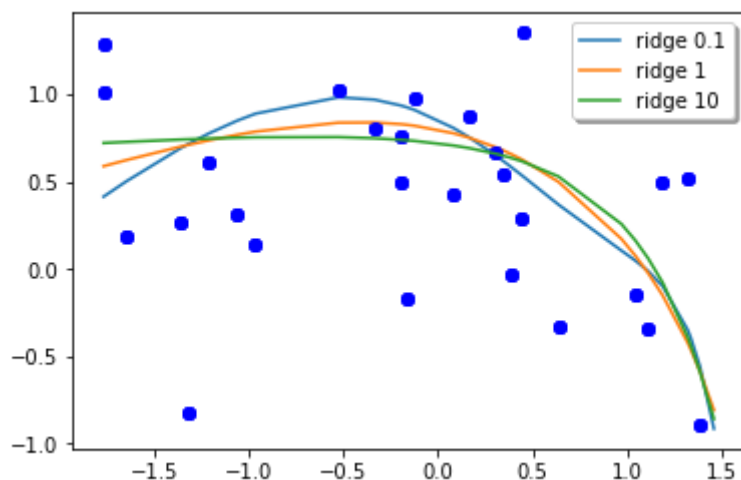
RMSE ridge $\lambda = 10$

```
In [235]: rms_ridge_3 = ridge_lasso_rmse(X_1_poly_d16_scaled_coloumn_added,Y_1,ridge_model_3)
          print(rms_ridge_3)
```

0.3267044743318329

Graph Plot Ridge $\lambda = 0.1, 1, 10$

```
In [236]: ridge_lasso_graph_plotter(X_1_poly_d16_scaled_coloumn_added,Y_1,ridge_model_1,graph_label="ridge 0.1")
          ridge_lasso_graph_plotter(X_1_poly_d16_scaled_coloumn_added,Y_1,ridge_model_2,graph_label="ridge 1")
          ridge_lasso_graph_plotter(X_1_poly_d16_scaled_coloumn_added,Y_1,ridge_model_3,graph_label="ridge 10")
```

**1-7. Fit the data using 16th order polynomial regression model with scikit-learn Lasso model.**

```
In [237]: def lasso_optimized_weight(data,label,alpha=0.1):
          lasso_reg = Lasso(alpha)
          lasso_model = lasso_reg.fit(data[:,1:], label)
          print ("Model-Intercept",lasso_model.intercept_)
          print ("Model-Co-ef",lasso_model.coef_.T)
          return lasso_model
```

Optimized_Weights lasso $\lambda = 0.1$

```
In [238]: lasso_model_1 = lasso_optimized_weight(X_1_poly_d16_scaled_coloumn_added
, Y_1, 0.1)

Model-Intercept [0.45269151]
Model-Co-ef [-0.          -0.          -0.          -0.          -0.
-0.37373874
-0.          -0.          -0.          -0.          -0.          -0.
-0.          -0.          -0.          -0.          ]
```

RMSE lasso $\lambda = 0.1$

```
In [239]: rms_lasso_1 = ridge_lasso_rmse(X_1_poly_d16_scaled_coloumn_added, Y_1, las
so_model_1)
print(rms_lasso_1)

0.34466712593964577
```

Optimized_Weights lasso $\lambda = 1$

```
In [240]: lasso_model_2 = lasso_optimized_weight(X_1_poly_d16_scaled_coloumn_added
, Y_1, 1)

Model-Intercept [0.45269151]
Model-Co-ef [-0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -
0. -0.]
```

RMSE lasso $\lambda = 1$

```
In [241]: rms_lasso_2 = ridge_lasso_rmse(X_1_poly_d16_scaled_coloumn_added, Y_1, las
so_model_2)
print(rms_lasso_2)

0.5772554182087681
```

Optimized_Weights lasso $\lambda = 10$

```
In [242]: lasso_model_3 = lasso_optimized_weight(X_1_poly_d16_scaled_coloumn_added
, Y_1, 10)

Model-Intercept [0.45269151]
Model-Co-ef [-0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -0. -
0. -0.]
```

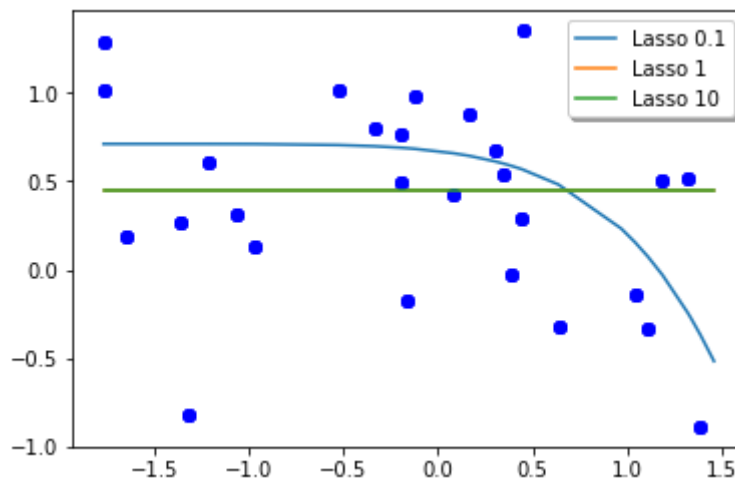
RMSE lasso $\lambda = 10$

```
In [243]: rms_lasso_3 = ridge_lasso_rmse(X_1_poly_d16_scaled_coloumn_added,Y_1,lasso_model_3)
print(rms_lasso_3)
```

0.5772554182087681

Graph Plot Lasso $\lambda = 0.1, 1, 10$

```
In [244]: ridge_lasso_graph_plotter(X_1_poly_d16_scaled_coloumn_added,Y_1,lasso_model_1,graph_label="Lasso 0.1")
ridge_lasso_graph_plotter(X_1_poly_d16_scaled_coloumn_added,Y_1,lasso_model_2,graph_label="Lasso 1")
ridge_lasso_graph_plotter(X_1_poly_d16_scaled_coloumn_added,Y_1,lasso_model_3,graph_label="Lasso 10")
```



2. Polynomial regression with train/validation/test

You need to split the data into training/validation/testing data set as 60% / 20% / 20%.

Please use `np.random.seed(1)` to have consistent data for evaluation

2-1. Fit the data using linear (1st order) regression model (matrix form, gradient descent method).

```

In [245]: # def plot_learning_curves(model, X, y):
#         X_train, X_val, y_train, y_val = train_test_split(X, y, test_size=
0.2)
#         train_errors, val_errors = [], []
#         for m in range(1, len(X_train)):
#             model.fit(X_train[:m], y_train[:m])
#             y_train_predict = model.predict(X_train[:m])
#             y_val_predict = model.predict(X_val)
#             train_errors.append(mean_squared_error(y_train_predict, y_train[:m]))
#             val_errors.append(mean_squared_error(y_val_predict, y_val))
#         plt.plot(np.sqrt(train_errors), "r-+", linewidth=2, label="train")
#         plt.plot(np.sqrt(val_errors), "b-", linewidth=3, label="val")

# lin_reg = LinearRegression()
# plot_learning_curves(lin_reg, X, y)

# from sklearn.pipeline import Pipeline

# polynomial_regression = Pipeline([
#     ("poly_features", PolynomialFeatures(degree=10, include_bias=False)),
#     ("lin_reg", LinearRegression()),
# ])

# plot_learning_curves(polynomial_regression, X, y)

```

```

In [246]: data2 = pd.read_csv("ex2data2.csv")
X_2 = data2.as_matrix(columns=data2.columns[0:1])
Y_2 = data2.as_matrix(columns=data2.columns[1:])
X_2_poly_d1_scaled = scaler.fit_transform(X_2)
X_2_column_added = np.c_[np.ones((data2.shape[0], 1)), X_2]
X_2_poly_d1_scaled_column_added = np.c_[np.ones((data2.shape[0], 1)), X_2_poly_d1_scaled]

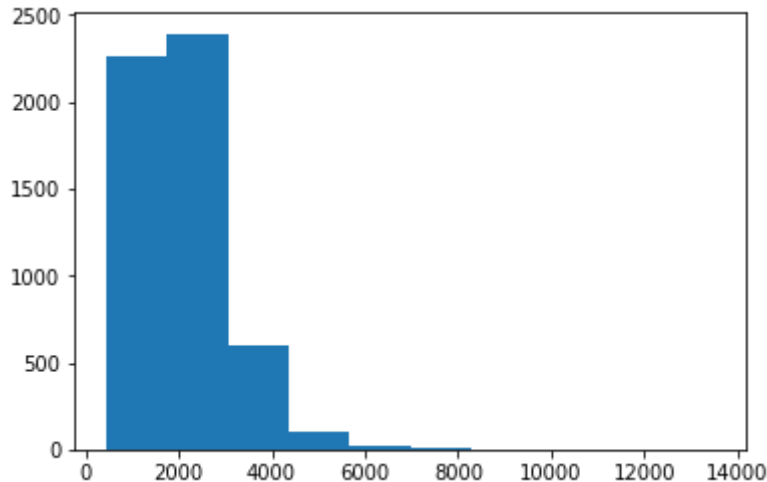
/anaconda2/envs/carnd-term1/lib/python3.5/site-packages/sklearn/utils/validation.py:475: DataConversionWarning: Data with input dtype int64 was converted to float64 by StandardScaler.
warnings.warn(msg, DataConversionWarning)

```



```
In [247]: plt.hist(X_2)
```

```
Out[247]: (array([2.266e+03, 2.393e+03, 6.040e+02, 1.050e+02, 2.100e+01, 1.200e+0
1,
          1.000e+00, 0.000e+00, 1.000e+00, 1.000e+00]),
          array([ 430., 1741., 3052., 4363., 5674., 6985., 8296., 9607.,
10918., 12229., 13540.]),
          <a list of 10 Patch objects>)
```



```
In [248]: X_2_train_val, X_2_test, y_2_train_val, y_2_test = train_test_split(X_2_
poly_d1_scaled_coloumn_added , Y_2, test_size=0.2,random_state=1)
X_2_train, X_2_val, y_2_train, y_2_val = train_test_split(X_2_train_val,
y_2_train_val, test_size=0.25,random_state=1)
print (X_2_coloumn_added.shape)
print (X_2_train_val.shape)
print (X_2_train.shape)
```

```
(5404, 2)
```

```
(4323, 2)
```

```
(3242, 2)
```

```
In [249]: def Mean_Square_Error_calculator(data,weights,actual_label):
          predict_y = data.dot(weights)
          rms = mean_squared_error(actual_label, predict_y)
          return rms
```

```

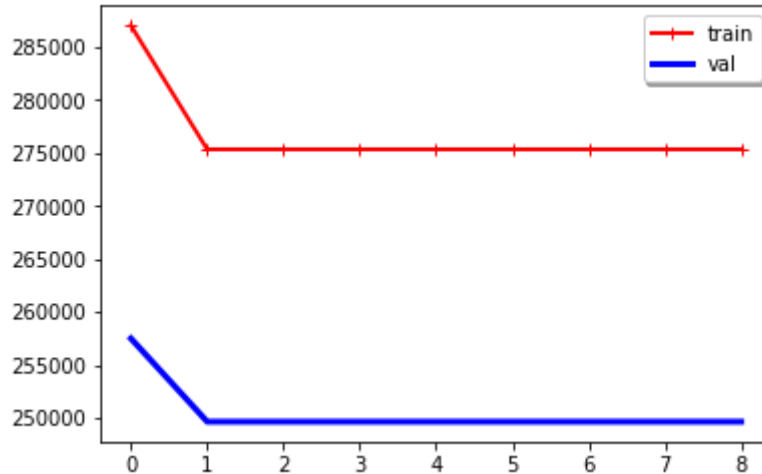
In [250]: def train_val_best_model(X_train, X_val, y_train, y_val, degree):
    epochs = [1000, 10000, 100000]
    learning_rate = [.001, .01, .1]
    train_errors, val_errors = [], []
    counter = 1
    combination = {}
    for epoch in epochs:
        for rate in learning_rate:
            weights = weight_optimizer(X_train, degree, y_train, learn_rate
= rate, epochs = epoch, regularisation_alpha=0)
            rmse_train = rmse_calculator(X_train, weights, y_train)
            rmse_val = rmse_calculator(X_val, weights, y_val)
            train_errors.append(rmse_train)
            val_errors.append(rmse_val)
            combination[counter] = (epoch, rate)
            counter += 1
    plt.plot(train_errors, "r+", linewidth=2, label="train")
    plt.plot(val_errors, "b-", linewidth=3, label="val")
    legend = plt.legend(loc='upper right', shadow=True)
    ## Find the lowest validation error
    counter_val_error = list(enumerate(val_errors, 1))
    counter_val_error_sorted = sorted(counter_val_error, key=lambda x:x[
1])
    best_model = combination[(counter_val_error_sorted[0][0])]
    print(counter_val_error)
    print("Epoch and Learning Rate Combination", counter_val_error_sorted
[0][0], best_model)
    return best_model

```

```
In [251]: best_model_2 = train_val_best_model(X_2_train, X_2_val, y_2_train, y_2_val, 1)
weights_best_model_2 = weight_optimizer(X_2_train, 1, y_2_train, learn_rate = best_model_2 [1], epochs = best_model_2 [0], regularisation_alpha=0)
```

```
[(1, 257488.04094283126), (2, 249622.42769314421), (3, 249622.4277054363), (4, 249622.4276900577), (5, 249622.42770543642), (6, 249622.4277054363), (7, 249622.42770543747), (8, 249622.42770543642), (9, 249622.4277054363)]
```

Epoch and Learning Rate Combination 4 (10000, 0.001)



RMSE - Train, test Deg -1

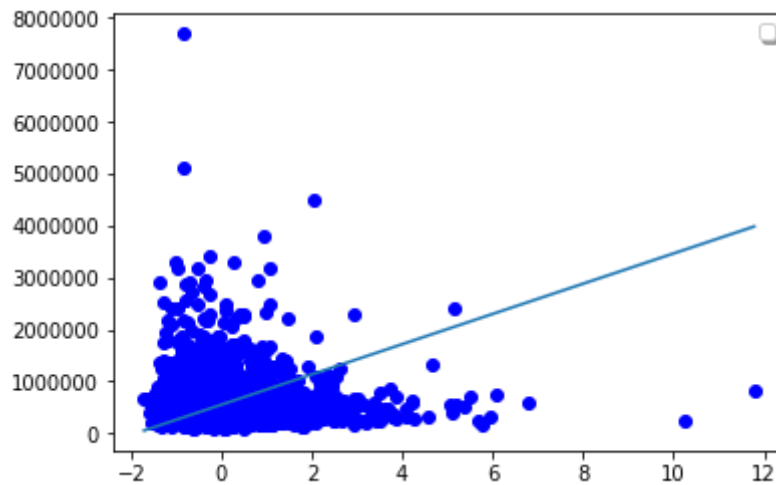
```
In [252]: rmse_train_2 = rmse_calculator(X_2_train, weights_best_model_2, y_2_train)
rmse_test_2 = rmse_calculator(X_2_test, weights_best_model_2, y_2_test)
print("rmse_train", rmse_train_2)
print("rmse_test", rmse_test_2)
```

```
rmse_train 275355.0256953383
rmse_test 264311.94247884274
```

Graph Train Deg-1

```
In [253]: graph_plotter(X_2_train,weights_best_model_2,y_2_train,graph_label=None)
```

No handles with labels found to put in legend.



2-2. Fit the data using 2nd order polynomial regression model (matrix form, gradient descent method).

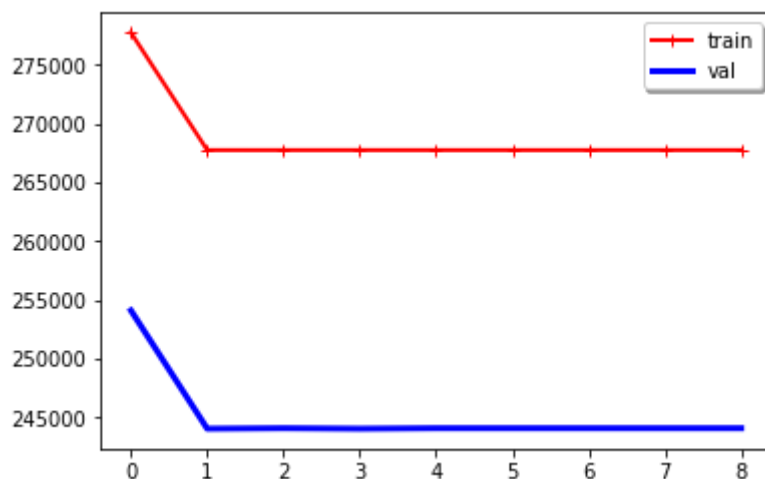
```
In [254]: X_2_poly_d2_scaled_coloumn_added = polynomial_adder(X_2,2)
```

```
In [255]: X_22_train_val, X_22_test, y_22_train_val, y_22_test = train_test_split(
X_2_poly_d2_scaled_coloumn_added, Y_2, test_size=0.2,random_state=1)
X_22_train, X_22_val, y_22_train, y_22_val = train_test_split(X_22_train_val, y_22_train_val, test_size=0.25,random_state=1)
```

```
In [256]: best_model_22 = train_val_best_model(X_22_train, X_22_val, y_22_train, y_22_val, 2)
weights_best_model_22 = weight_optimizer(X_22_train, 2, y_22_train, learn_rate = best_model_22 [1], epochs = best_model_22 [0], regularisation_alpha=0)
```

```
[(1, 254082.7389780978), (2, 244029.3535573043), (3, 244071.59635894437), (4, 244029.2686895008), (5, 244071.5963589144), (6, 244071.59635907054), (7, 244071.59635891087), (8, 244071.59635907042), (9, 244071.59635907054)]
```

Epoch and Learning Rate Combination 4 (10000, 0.001)



RMSE - Train, test Deg -2

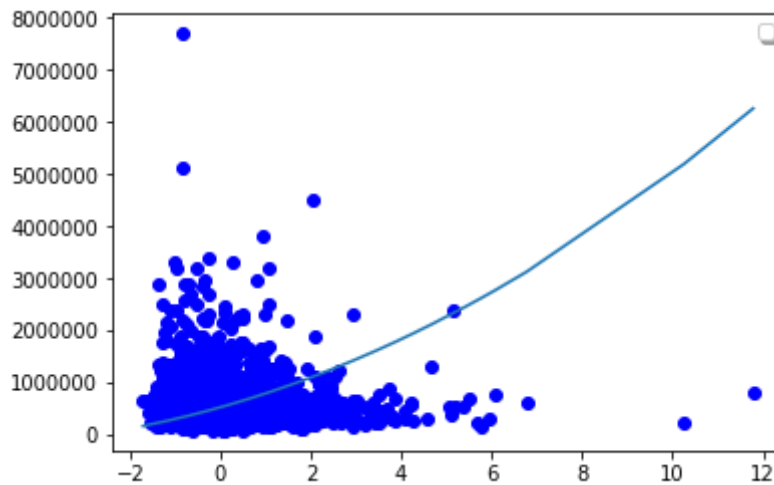
```
In [257]: rmse_train_22 = rmse_calculator(X_22_train, weights_best_model_22, y_22_train)
rmse_test_22 = rmse_calculator(X_22_test, weights_best_model_22, y_22_test)
print("rmse_train", rmse_train_22)
print("rmse_test", rmse_test_22)
```

```
rmse_train 267719.7117411374
rmse_test 254533.92847752557
```

Graph Train Deg-2

```
In [258]: graph_plotter(X_22_train,weights_best_model_22,y_22_train,graph_label=No
ne)
```

No handles with labels found to put in legend.



2-3. Fit the data using 4th order polynomial regression model (matrix form, gradient descent method).

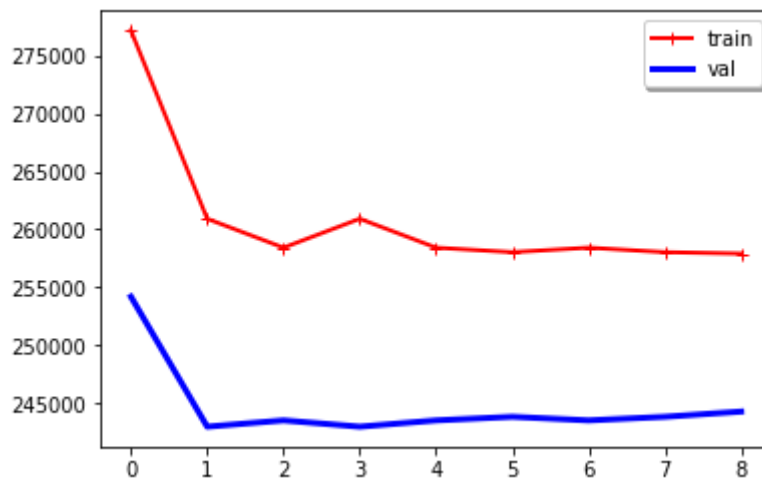
```
In [259]: X_2_poly_d4_scaled_coloumn_added = polynomial_adder(X_2,4)
```

```
In [260]: X_24_train_val, X_24_test, y_24_train_val, y_24_test = train_test_split(
X_2_poly_d4_scaled_coloumn_added, Y_2, test_size=0.2,random_state=1)
X_24_train, X_24_val, y_24_train, y_24_val = train_test_split(X_24_train_val, y_24_train_val, test_size=0.25,random_state=1)
```

```
In [261]: best_model_24 = train_val_best_model(X_24_train, X_24_val, y_24_train, y_24_val,4)
weights_best_model_24 = weight_optimizer(X_24_train,4,y_24_train,learn_rate = best_model_24 [1],epochs = best_model_24 [0],regularisation_alpha=0)
```

```
[(1, 254205.33656847285), (2, 243001.66458493247), (3, 243544.8531816557), (4, 243001.8240493743), (5, 243544.6655575537), (6, 243854.6229242246), (7, 243544.64619277278), (8, 243854.612717776), (9, 244304.69676459354)]
```

Epoch and Learning Rate Combination 2 (1000, 0.01)



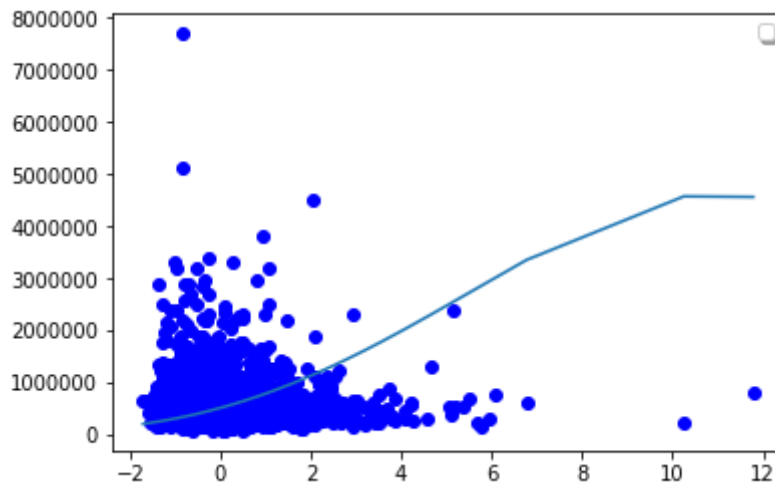
RMSE - Train,test Deg -4

```
In [262]: rmse_train_24 = rmse_calculator(X_24_train,weights_best_model_24,y_24_train)
rmse_test_24 = rmse_calculator(X_24_test,weights_best_model_24,y_24_test)
print("rmse_train",rmse_train_24)
print("rmse_test",rmse_test_24)
```

```
rmse_train 260919.45768952757
rmse_test 251751.42950903866
```

```
In [263]: graph_plotter(X_24_train,weights_best_model_24,y_24_train,graph_label=None)
```

No handles with labels found to put in legend.



2-4. Fit the data using 16th order polynomial regression model (matrix form, gradient descent method).

```
In [357]: def weight_optimizer_stoc_batch(data,degree,Label,learn_rate = .01,epochs = 100000,regularisation_alpha=0):
    theta_1 = np.random.randn(degree+1,1)
    data_size = data.shape[0]
    print (data_size)
    batch_size = data_size // 16
    print (batch_size)
    for epoch in range(epochs):
        for batch in range(batch_size):
            X_batch, y_batch = resample(data, Label, n_samples=16, random_state=0,replace=False)
            gradients = (2/X_batch.shape[0])* (X_batch.T.dot(X_batch.dot(theta_1)- y_batch)+ \
                                                regularisation_alpha*theta_1)
            theta_1 = theta_1 - learn_rate * gradients
    return theta_1
```



```

In [370]: def train_val_best_model_stoc_batch(X_train, X_val, y_train, y_val, degree):
    epochs = [10000]
    learning_rate = [.001, .01, .1]
    train_errors, val_errors = [], []
    counter = 1
    combination = {}
    for epoch in epochs:
        for rate in learning_rate:
            weights = weight_optimizer_stoc_batch(X_train, degree, y_train,
            , learn_rate = rate, epochs = epoch, regularisation_alpha=0)
            rmse_train = rmse_calculator(X_train, weights, y_train)
            rmse_val = rmse_calculator(X_val, weights, y_val)
            train_errors.append(rmse_train)
            val_errors.append(rmse_val)
            combination[counter] = (epoch, rate)
            counter += 1
    plt.plot(train_errors, "r+", linewidth=2, label="train")
    plt.plot(val_errors, "b-", linewidth=3, label="val")
    legend = plt.legend(loc='upper right', shadow=True)
    ## Find the lowest validation error
    counter_val_error = list(enumerate(val_errors, 1))
    counter_val_error_sorted = sorted(counter_val_error, key=lambda x:x[1])
    best_model = combination[(counter_val_error_sorted[0][0])]
    print(counter_val_error)
    print("Epoch and Learning Rate Combination", counter_val_error_sorted[0][0], best_model)
    return best_model

```

```

In [371]: X_2_poly_d16_scaled_coloumn_added = polynomial_adder(X_2, 16)

```

```

In [372]: X_216_train_val, X_216_test, y_216_train_val, y_216_test = train_test_split(X_2_poly_d16_scaled_coloumn_added, Y_2, test_size=0.2, random_state=1)
    X_216_train, X_216_val, y_216_train, y_216_val = train_test_split(X_216_train_val, y_216_train_val, test_size=0.25, random_state=1)

```

Implemented Mini-batch Gradient Descent as I was getting error with full batch

```
In [373]: best_model_216 = train_val_best_model_stoc_batch(X_216_train, X_216_val,
y_216_train, y_216_val,16)
weights_best_model_216 = weight_optimizer_stoc_batch(X_216_train,16,y_216_train,learn_rate = best_model_216 [1],epochs = best_model_216 [0],regularisation_alpha=0)
```

3242

202

3242

202

3242

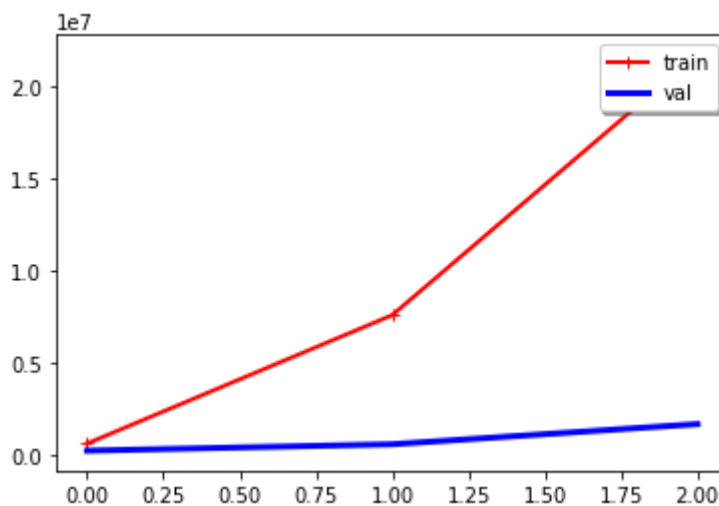
202

```
[(1, 271431.84548393165), (2, 621086.9719878328), (3, 1715299.1157242693)]
```

Epoch and Learning Rate Combination 1 (10000, 0.001)

3242

202



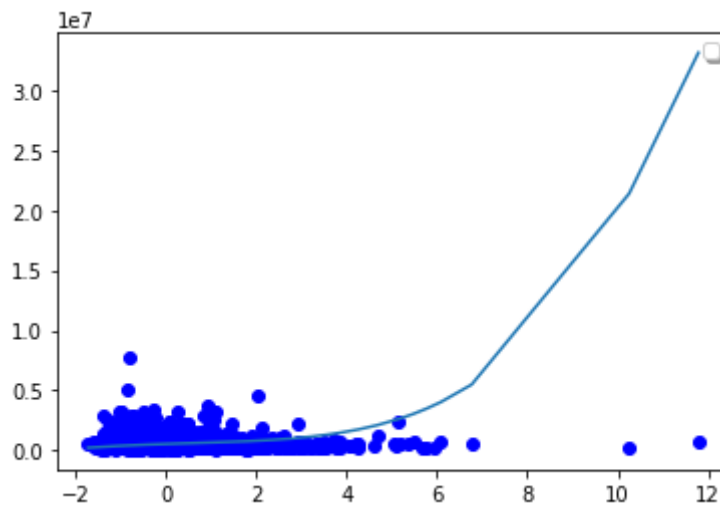
```
In [374]: rmse_train_216 = rmse_calculator(X_216_train,weights_best_model_216,y_216_train)
rmse_test_216 = rmse_calculator(X_216_test,weights_best_model_216,y_216_test)
print("rmse_train",rmse_train_216)
print("rmse_test",rmse_test_216)
```

rmse_train 656156.7479793766

rmse_test 274794.112255375

```
In [375]: graph_plotter(X_216_train,weights_best_model_216,y_216_train,graph_label  
=None)
```

WARNING:matplotlib.legend:No handles with labels found to put in legend.



2-5. Fit the data using 16th order polynomial regression model with ridge (L2 penalty) regularization.

```

In [379]: def train_val_best_model_ridge_stoc_batch(X_train, X_val, y_train, y_val
,degree):
    epochs = [10000]
    learning_rate = [.01]
    regularisation_alpa = [.1,1,10]
    train_errors, val_errors = [], []
    counter = 1
    combination = {}
    for epoch in epochs:
        for rate in learning_rate:
            for alpha in regularisation_alpa:
                weights = weight_optimizer_stoc_batch(X_train,degree,y_t
rain,learn_rate = rate,epochs = epoch, \
                                                    regularisation_alpha=alpha)
                rmse_train = rmse_calculator(X_train,weights,y_train)
                rmse_val = rmse_calculator(X_val,weights,y_val)
                train_errors.append(rmse_train)
                val_errors.append(rmse_val)
                combination[counter] = (epoch,rate,alpha)
                counter += 1
    plt.plot(train_errors, "r-+", linewidth=2, label="train")
    plt.plot(val_errors, "b-", linewidth=3, label="val")
    legend = plt.legend(loc='upper right', shadow=True)
    ## Find the lowest validation error
    counter_val_error = list(enumerate(val_errors, 1))
    counter_val_error_sorted = sorted(counter_val_error, key=lambda x:x[
1])
    best_model = combination[(counter_val_error_sorted[0][0])]
    print(best_model)
    return best_model

```

Implemented Mini-batch Gradient Descent as I was getting error with full batch

```
In [380]: best_model_216_ridge = train_val_best_model_ridge_stoc_batch(X_216_train
, X_216_val, y_216_train, y_216_val,16)
weights_best_model_216_ridge = weight_optimizer_stoc_batch(X_216_train,1
6,y_216_train,learn_rate = best_model_216_ridge [1],epochs = best_model_
216_ridge [0],regularisation_alpha=best_model_216_ridge [2])
```

3242

202

3242

202

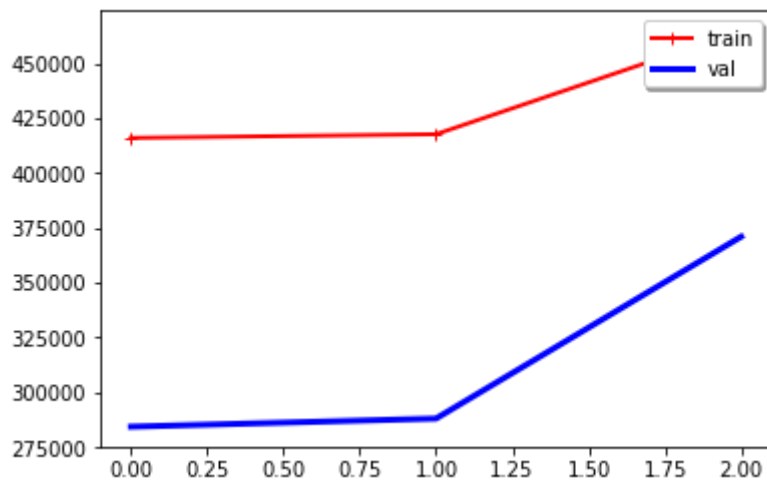
3242

202

(10000, 0.01, 0.1)

3242

202



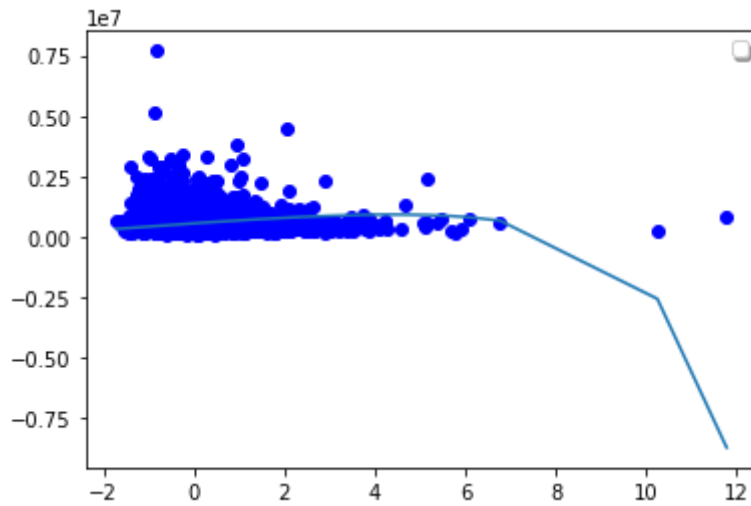
```
In [383]: rmse_train_216_ridge = rmse_calculator(X_216_train,weights_best_model_21
6_ridge,y_216_train)
rmse_test_216_ridge = rmse_calculator(X_216_test,weights_best_model_216_
ridge,y_216_test)
print("rmse_train",rmse_train_216_ridge)
print("rmse_test",rmse_test_216_ridge)
```

rmse_train 415963.5773494703

rmse_test 321415.22175827075

```
In [385]: graph_plotter(X_216_train,weights_best_model_216_ridge,y_216_train,graph_label=None)
```

WARNING:matplotlib.legend:No handles with labels found to put in legend.

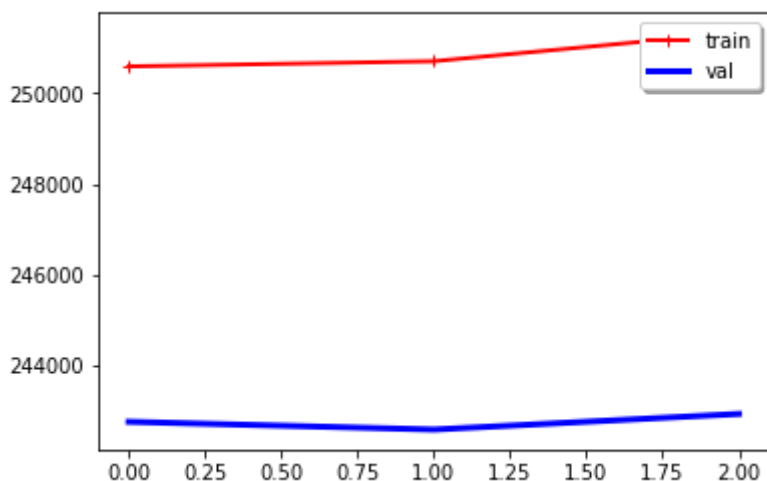


2-6. Fit the data using 16th order polynomial regression model with scikit-learn Ridge model.

```
In [268]: def train_val_best_model_scikit_ridge(X_train, X_val, y_train, y_val):
    regularisation_alpha = [0.1,1,10]
    train_errors, val_errors = [], []
    counter = 1
    combination = {}
    for alpha in regularisation_alpha:
        ridge_model = ridge_optimized_weight(X_train,y_train,alpha)
        rmse_train = ridge_lasso_rmse(X_train,y_train,ridge_model)
        rmse_val = ridge_lasso_rmse(X_val,y_val,ridge_model)
        train_errors.append(rmse_train)
        val_errors.append(rmse_val)
        combination[counter] = (alpha,ridge_model)
        counter += 1
    plt.plot(train_errors, "r-+", linewidth=2, label="train")
    plt.plot(val_errors, "b-", linewidth=3, label="val")
    legend = plt.legend(loc='upper right', shadow=True)
    ## Find the lowest validation error
    counter_val_error = list(enumerate(val_errors, 1))
    counter_val_error_sorted = sorted(counter_val_error, key=lambda x:x[1])
    best_model = combination[(counter_val_error_sorted[0][0])]
    print("alpha",best_model[0])
    return best_model[1]
```

```
In [269]: best_model_216_scikit_ridge = train_val_best_model_scikit_ridge(X_216_train, X_216_val, y_216_train, y_216_val)
```

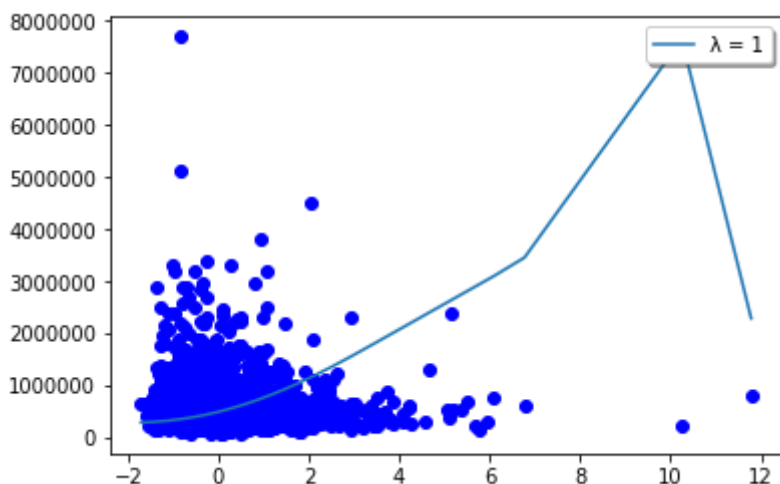
```
Model-Intercept [547704.506878]
Model-Co-ef [[ 47149.02303585]
 [ 97806.15914125]
 [ 494262.89050946]
 [-227237.60901432]
 [-422132.21976636]
 [-199882.87630559]
 [ 64994.59063826]
 [ 230729.38940244]
 [ 287757.59283936]
 [ 265072.70818138]
 [ 193352.45983527]
 [ 95486.86500499]
 [ -13592.57195474]
 [-124860.40617845]
 [-233169.41195629]
 [-335762.59926058]]
Model-Intercept [547718.99272249]
Model-Co-ef [[ -16344.34115262]
 [ 320155.60757134]
 [ 172666.68142153]
 [-173831.65937522]
 [-211992.23283438]
 [ -77256.72585303]
 [ 64972.77614043]
 [ 152621.76687899]
 [ 180680.13409481]
 [ 162984.49861422]
 [ 115562.95467406]
 [ 51361.29686543]
 [ -20522.80095625]
 [ -94191.55415056]
 [-166059.86068628]
 [-234099.56514392]]
Model-Intercept [547547.03238267]
Model-Co-ef [[ 36166.43778387]
 [ 269023.40114305]
 [ 77631.40011951]
 [ -81988.69738317]
 [ -71553.92749472]
 [ -5369.60495591]
 [ 48807.03808062]
 [ 74843.22643076]
 [ 76275.4587262 ]
 [ 60869.18534266]
 [ 35528.32733298]
 [ 5236.22544228]
 [ -26747.06366153]
 [ -58427.80688802]
 [ -88665.61666554]
 [-116867.58253742]]
alpha 1
```

```
In [270]: rmse_train_216_scikit_ridge = ridge_lasso_rmse(X_216_train, y_216_train,
best_model_216_scikit_ridge)
rmse_test_216_scikit_ridge = ridge_lasso_rmse(X_216_test,y_216_test,best
_model_216_scikit_ridge)
print("rmse_train",rmse_train_216_scikit_ridge)
print("rmse_test",rmse_test_216_scikit_ridge)
```

```
rmse_train 250700.05033093243
rmse_test 251065.74316573926
```

```
In [271]: ridge_lasso_graph_plotter(X_216_train,y_216_train,best_model_216_scikit_
ridge,graph_label="λ = 1")
```



2-7. Fit the data using 16th order polynomial regression model with scikit-learn Lasso model.

```
In [272]: def train_val_best_model_scikit_lasso(X_train, X_val, y_train, y_val):  
    regularisation_alpa = [0.1,1,10]  
    train_errors, val_errors = [], []  
    counter = 1  
    combination = {}  
    for alpha in regularisation_alpa:  
        lasso_model = lasso_optimized_weight(X_train,y_train,alpha)  
        rmse_train = ridge_lasso_rmse(X_train,y_train,lasso_model)  
        rmse_val = ridge_lasso_rmse(X_val,y_val,lasso_model)  
        train_errors.append(rmse_train)  
        val_errors.append(rmse_val)  
        combination[counter] = (alpha,lasso_model)  
        counter += 1  
    plt.plot(train_errors, "r+", linewidth=2, label="train")  
    plt.plot(val_errors, "b-", linewidth=3, label="val")  
    legend = plt.legend(loc='upper right', shadow=True)  
    ## Find the lowest validation error  
    counter_val_error = list(enumerate(val_errors, 1))  
    counter_val_error_sorted = sorted(counter_val_error, key=lambda x:x[  
1])  
    best_model = combination[(counter_val_error_sorted[0][0])]  
    print("alpha",best_model[0])  
    return best_model[1]
```

```
In [273]: best_model_216_scikit_lasso = train_val_best_model_scikit_lasso(X_216_train, X_216_val, y_216_train, y_216_val)
```

```
/anaconda2/envs/carnd-term1/lib/python3.5/site-packages/sklearn/linear_model/coordinate_descent.py:491: ConvergenceWarning: Objective did not converge. You might want to increase the number of iterations. Fitting data with very small alpha may cause precision problems.
```

```
ConvergenceWarning)
```

```
Model-Intercept [547780.08000696]
```

```
Model-Co-ef [-125318.97003623  666197.50707237 -261841.01303223 -187309.44448817
```

```
  77160.31260774  155957.01757717  125786.49836694   69160.91493416
```

```
  19708.08956119 -14840.20562329 -36093.17315681 -47445.45636661
```

```
 -52259.47895543 -52993.40874272 -51325.09359054 -48343.58909978]
```

```
Model-Intercept [547779.73592156]
```

```
Model-Co-ef [-124992.36217718  665263.70260852 -260753.01353071 -187340.17063375
```

```
  76345.133887  155967.41656718  125762.63818031   69164.76212111
```

```
  19900.84319741 -14276.41829556 -36175.49317507 -47521.03245514
```

```
 -52327.97275879 -53055.02152518 -51380.27127648 -48392.87104231]
```

```
Model-Intercept [547776.26865725]
```

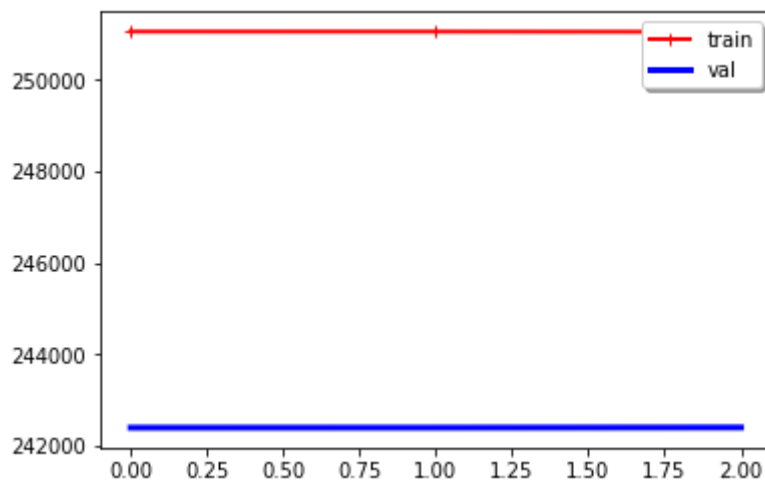
```
Model-Co-ef [-121721.55990706  655932.64030426 -249919.8823818 -187602.71597435
```

```
  68214.49580173  156107.22185177  125530.49711011   69166.91275706
```

```
  21367.94806767 -8139.49732978 -37011.24562008 -48290.1479555
```

```
 -53026.14960449 -53683.77064736 -51943.80255891 -48896.47767207]
```

```
alpha 0.1
```

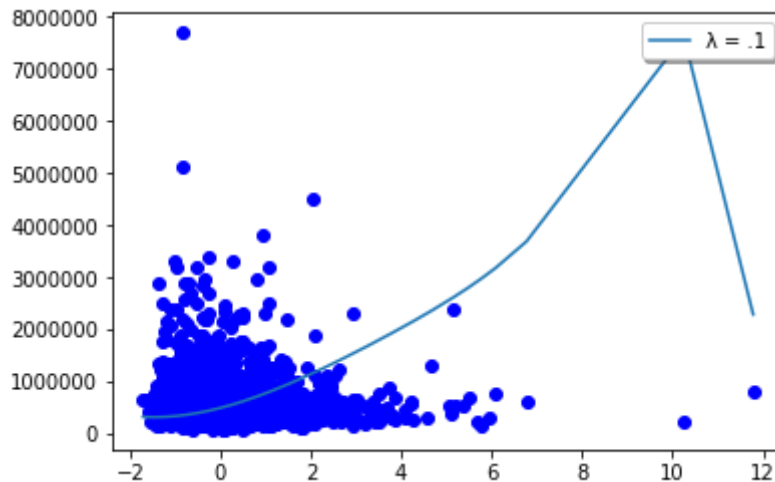


```
In [274]: rmse_train_216_scikit_lasso = ridge_lasso_rmse(X_216_train, y_216_train,
best_model_216_scikit_lasso)
rmse_test_216_scikit_lasso = ridge_lasso_rmse(X_216_test, y_216_test, best_model_216_scikit_lasso)
print("rmse_train", rmse_train_216_scikit_lasso)
print("rmse_test", rmse_test_216_scikit_lasso)
```

```
rmse_train 251063.94309086326
```

```
rmse_test 250161.4085909059
```

```
In [275]: ridge_lasso_graph_plotter(X_216_train,y_216_train,best_model_216_scikit_lasso,graph_label="λ = .1")
```



3-1. Fit the training data using regression model with ridge (L2 penalty) regularization with scikit-learn Ridge model.

```
In [276]: data3 = pd.read_csv("ex2data3.csv")
del data3['Unnamed: 0']
```

```
In [277]: X_3 = data3.as_matrix(columns=data3.columns[0:8])
Y_3 = data3.as_matrix(columns=data3.columns[8:])
X_3_scaled = scaler.fit_transform(X_3)
X_3_scaled_coloumn_added = np.c_[np.ones((data3.shape[0], 1)), X_3_scaled]
```

```
In [278]: X_3_train_val, X_3_test, y_3_train_val, y_3_test = train_test_split(X_3_scaled_coloumn_added, Y_3, test_size=0.2, random_state=1)
X_3_train, X_3_val, y_3_train, y_3_val = train_test_split(X_3_train_val, y_3_train_val, test_size=0.25, random_state=1)
```

```
In [279]: best_model_3_scikit_ridge = train_val_best_model_scikit_ridge(X_3_train,  
X_3_val, y_3_train, y_3_val)
```

```
Model-Intercept [2.07192364]
```

```
Model-Co-ef [[ 0.82501033]
```

```
[ 0.12438917]
```

```
[-0.23245061]
```

```
[ 0.2778601 ]
```

```
[-0.00178963]
```

```
[-0.03738571]
```

```
[-0.9098153 ]
```

```
[-0.88498937]]
```

```
Model-Intercept [2.07192619]
```

```
Model-Co-ef [[ 0.82499755]
```

```
[ 0.12448827]
```

```
[-0.23231252]
```

```
[ 0.27766071]
```

```
[-0.0017563 ]
```

```
[-0.03738623]
```

```
[-0.90885179]
```

```
[-0.88401781]]
```

```
Model-Intercept [2.07195156]
```

```
Model-Co-ef [[ 0.82484941]
```

```
[ 0.12546401]
```

```
[-0.23091504]
```

```
[ 0.27566139]
```

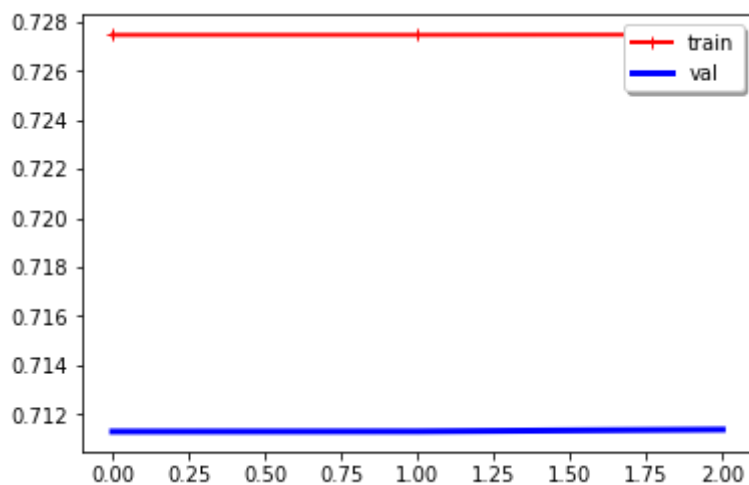
```
[-0.00142815]
```

```
[-0.03739063]
```

```
[-0.89934824]
```

```
[-0.87443278]]
```

```
alpha 0.1
```



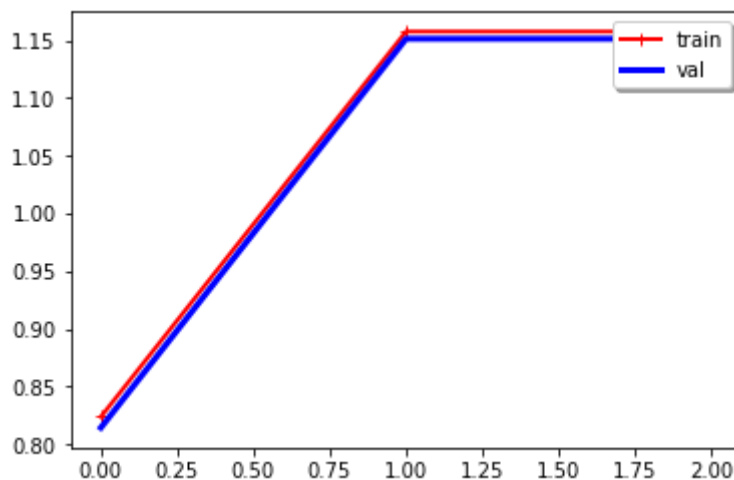
```
In [280]: rmse_train_3_scikit_ridge = ridge_lasso_rmse(X_3_train, y_3_train,best_m
odel_3_scikit_ridge)
rmse_test_3_scikit_ridge = ridge_lasso_rmse(X_3_test,y_3_test,best_model
_3_scikit_ridge)
print("rmse_train",rmse_train_3_scikit_ridge)
print("rmse_test",rmse_test_3_scikit_ridge)

rmse_train 0.7274577804827311
rmse_test 0.7276608037847032
```

3-2. Fit the training data using regression model with lasso (L1 penalty) regularization with scikit-learn Lasso model.

```
In [281]: best_model_3_scikit_lasso = train_val_best_model_scikit_lasso(X_3_train,
X_3_val, y_3_train, y_3_val)
```

```
Model-Intercept [2.07432847]
Model-Co-ef [ 0.71386457  0.11001596 -0.          0.          -0.
-0.00645422 -0.          ]
Model-Intercept [2.07444727]
Model-Co-ef [ 0.  0.  0. -0. -0. -0. -0. -0.]
Model-Intercept [2.07444727]
Model-Co-ef [ 0.  0.  0. -0. -0. -0. -0. -0.]
alpha 0.1
```



```
In [282]: rmse_train_3_scikit_lasso = ridge_lasso_rmse(X_3_train, y_3_train,best_m
odel_3_scikit_lasso)
rmse_test_3_scikit_lasso = ridge_lasso_rmse(X_3_test,y_3_test,best_model
_3_scikit_lasso)
print("rmse_train",rmse_train_3_scikit_lasso)
print("rmse_test",rmse_test_3_scikit_lasso)

rmse_train 0.8239579515519808
rmse_test 0.8147268457966831
```

3-3. Fit the training data using regression model with ridge (L2 penalty) regularization using TensorFlow.

```

In [345]: epochs_list = [10000]
alpha_list = [.1,1,10]
learning_rate_list = [.001,.01,.1]

#n_epochs = 10000
#learning_rate = 0.01
#alpha = 0.1

tf.reset_default_graph()

X_3 = tf.placeholder(tf.float64)
Y_3 = tf.placeholder(tf.float64)
theta = tf.Variable(tf.random_uniform([data3.shape[1], 1], -1.0, 1.0, dtype=tf.float64, name="theta"))
learning_rate = tf.placeholder(dtype=tf.float64)
alpha = tf.placeholder(dtype=tf.float64)
train_errors = []
val_errors = []
counter = 1
combination = {}

y_pred_3 = tf.matmul(X_3, theta, name="predictions")
error = y_pred_3 - Y_3
mse = tf.sqrt(tf.reduce_mean(tf.square(error), name="mse"))
gradients = 2/tf.shape(X_3)[0] * (tf.matmul(tf.transpose(X_3), error)+alpha*theta)
training_op = tf.assign(theta, theta - learning_rate * gradients)

init = tf.global_variables_initializer()

with tf.Session() as sess:
    sess.run(init)
    for n_epochs in epochs_list:
        for alpha_value in alpha_list:
            for learning_rate_value in learning_rate_list:
                for epoch in range(n_epochs):
                    sess.run(training_op, feed_dict={X_3: X_3_train, Y_3: y_3_train, \
                                                    learning_rate: learning_rate_value, alpha: alpha_value})
                    train_errors.append(sess.run(mse, feed_dict={X_3: X_3_train, Y_3: y_3_train, \
                                                                learning_rate: learning_rate_value, alpha: alpha_value}))
                    val_errors.append(sess.run(mse, feed_dict={X_3: X_3_val, Y_3: y_3_val, \
                                                            learning_rate: learning_rate_value, alpha: alpha_value}))
                    combination[counter] = (n_epochs, learning_rate_value

```



```

,alpha_value)
        counter += 1
        ## Find the lowest validation error
        plt.plot(train_errors, "r-+", linewidth=2, label="train")
        plt.plot(val_errors, "b-", linewidth=3, label="val")
        legend = plt.legend(loc='upper right', shadow=True)
        counter_val_error = list(enumerate(val_errors, 1))
        counter_val_error_sorted = sorted(counter_val_error, key=lambda x:x[
1])
        best_model = combination[(counter_val_error_sorted[0][0])]
        print(counter_val_error)
        print("Epoch ,Learning Rate and alpha Combination",best_model)
        ##print train,test error
        print("train_error",sess.run(mse,feed_dict={X_3: X_3_train, Y_3: y_3
_train, \
                                                    learning_
rate: best_model[1],alpha: best_model[2]}))
        print("test_error",sess.run(mse,feed_dict={X_3: X_3_test, Y_3: y_3_t
est, \
                                                    learning_rate
: best_model[1],alpha: best_model[2]}))

```

```

[(1, 0.712454827014997), (2, 0.7112967432688554), (3, 0.711296818273914
8), (4, 0.7113039267849257), (5, 0.711304166648072), (6, 0.711304166488
7844), (7, 0.7113825753435046), (8, 0.7113889004585454), (9, 0.71138889
92949285)]

```

```
Epoch ,Learning Rate and alpha Combination (10000, 0.01, 0.1)
```

```
train_error 0.7274706274995454
```

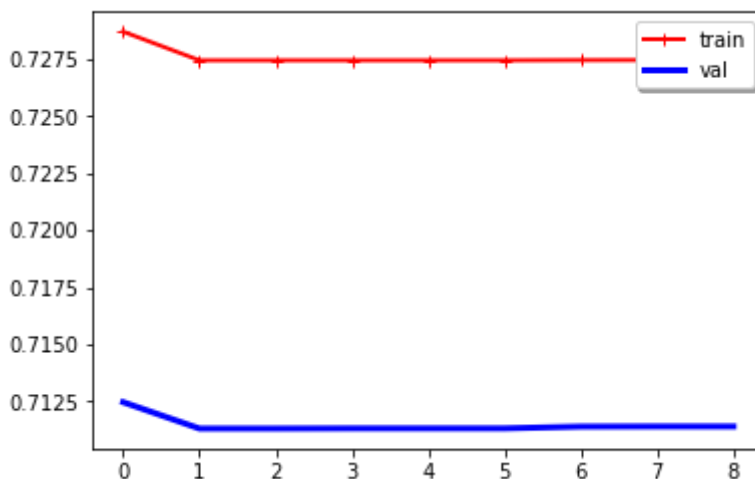
```
test_error 0.7275337175750082
```

```
Exception ignored in: <generator object get_controller at 0x11bd75f10>
Traceback (most recent call last):
```

```
File "/anaconda2/envs/carnd-term1/lib/python3.5/site-packages/tensorflo
w/python/framework/ops.py", line 3523, in get_controller
```

```
% type(default))
```

```
AssertionError: Nesting violated for default stack of <class 'tensorflow.
python.framework.ops.Graph'> objects
```



3-4. Fit the training data using regression model with lasso (L1 penalty) regularization using TensorFlow.