# Response 1:

## Playing Notakto

## Question 1 – Playing Notakto

Consider the following two-player game on an $n \times n$ board, with $n \leq 6$. The two players take turns filling the board with the same symbol "X." The first player that completes a row, a column, or the main diagonal and anti-diagonal loses the game.

Expectations: For this question, the following is expected.

a) Implement a neural network architecture to play the game. To aid you in your task, you are allowed to research network architectures that have been used for other games and you might use common open-source libraries like tensor flow.
b) You should try different architectures and parameters, and eventually decide on specific choices.
c) You need to write a short (around 3 to 5 pages) report that addresses the choice of architecture, parameters, and how you asses the performance of your network.
d) The report should also include observations you find interesting during the training process. In particular, if the training process provides any insight that can be translated into strategies to play the game.

# The Game of Notakto

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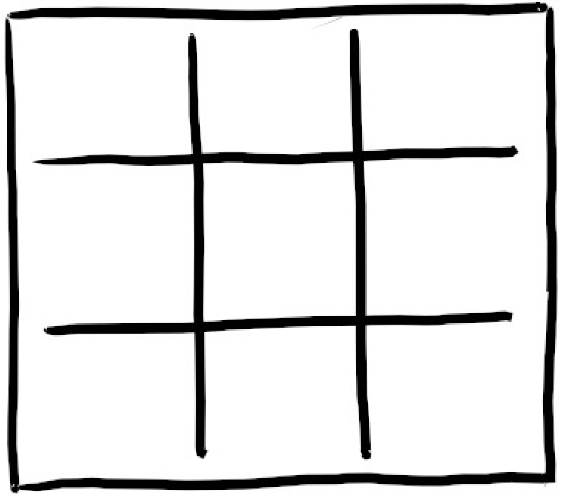
(Plambeck and Whitehead 2013)



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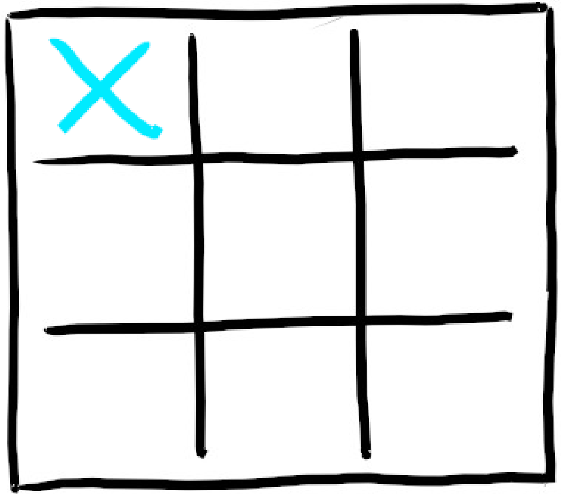

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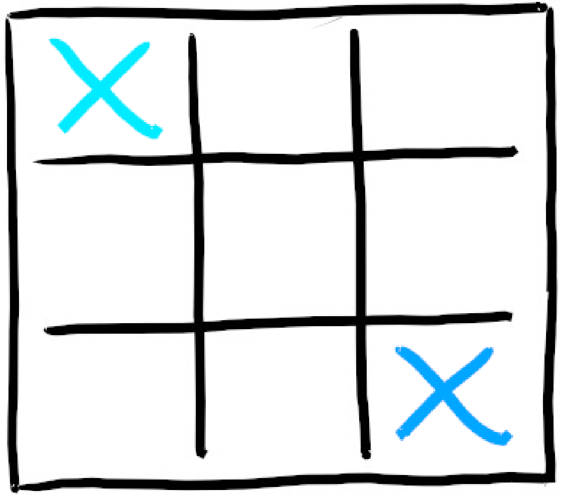

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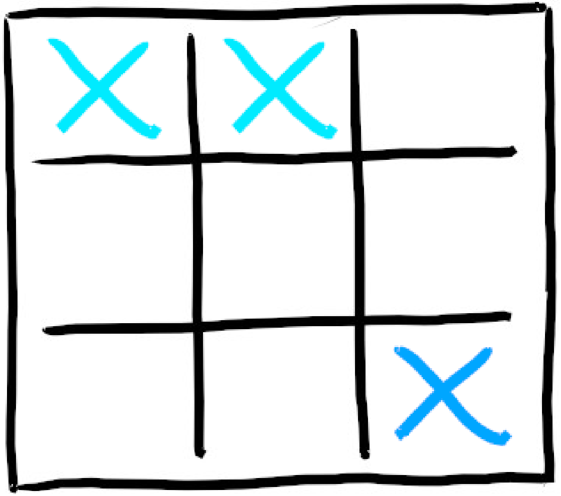

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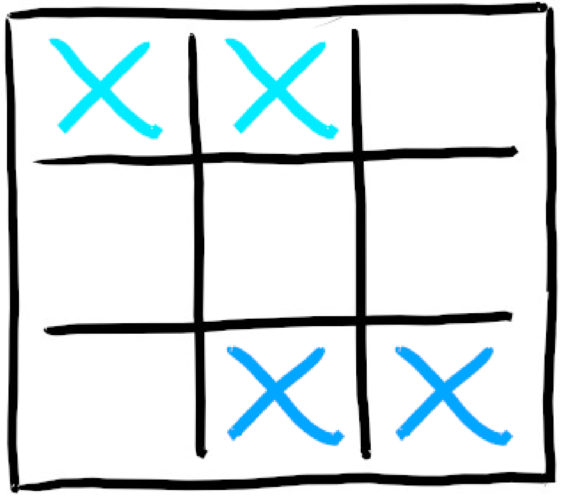

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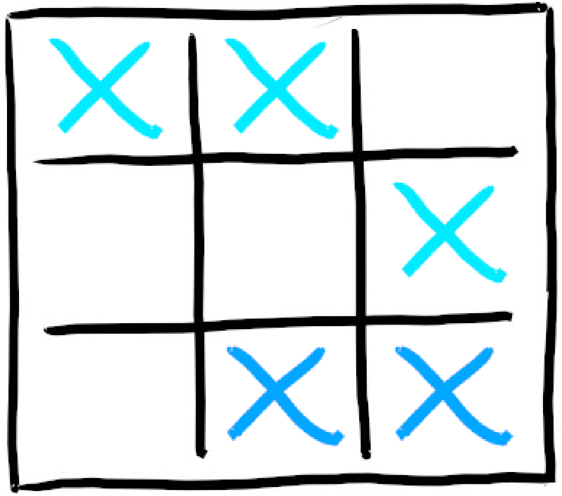

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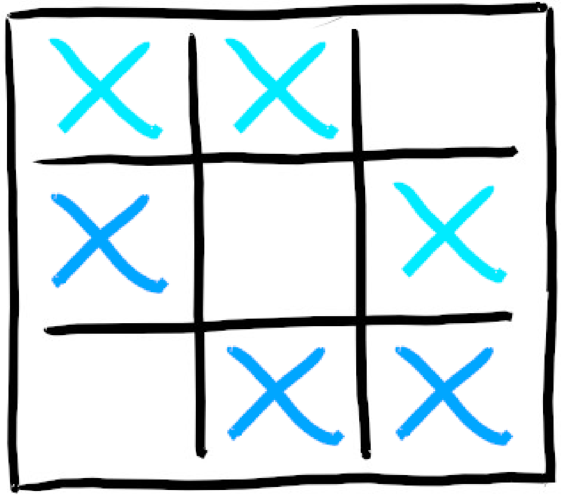

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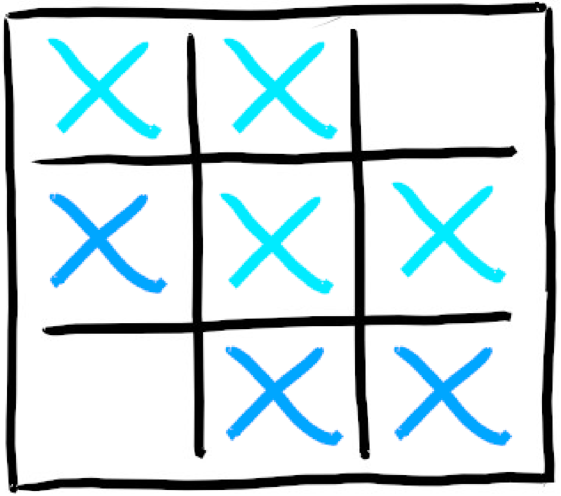

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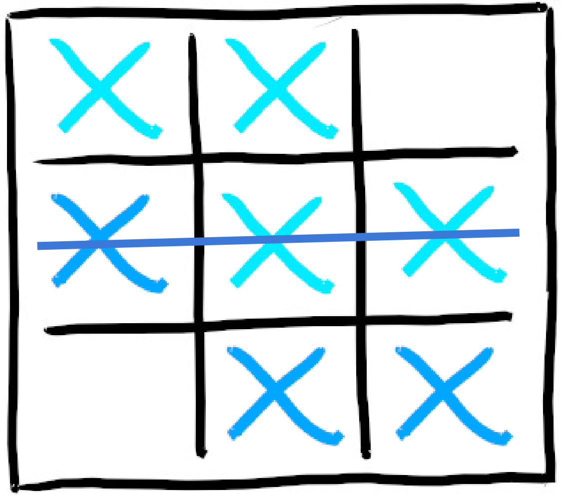

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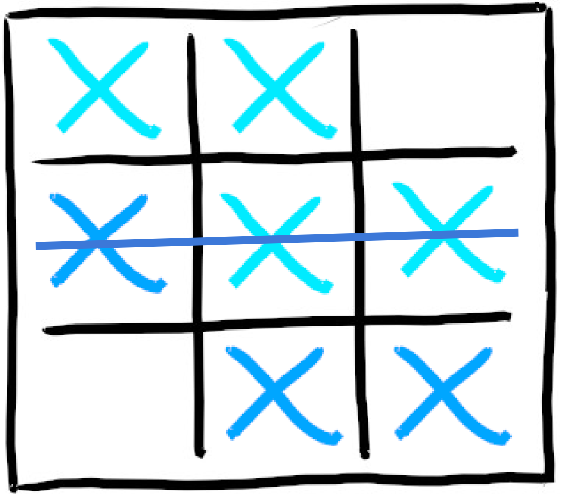

**Player 2 Wins!**

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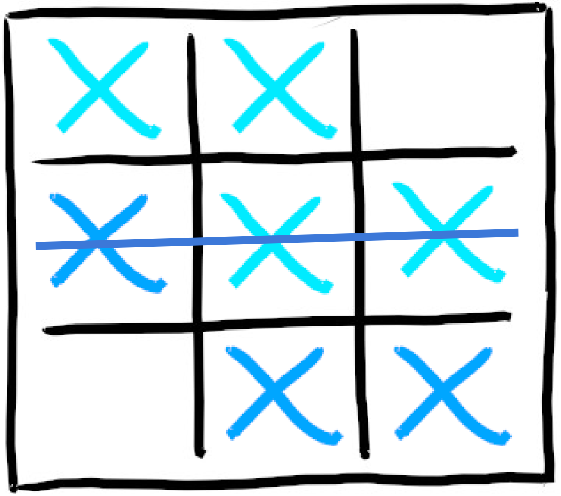

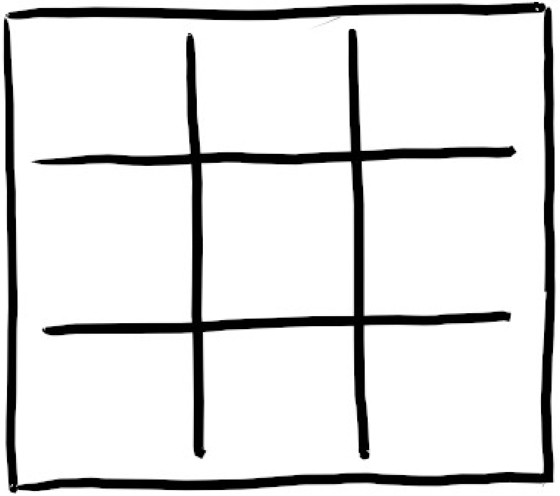

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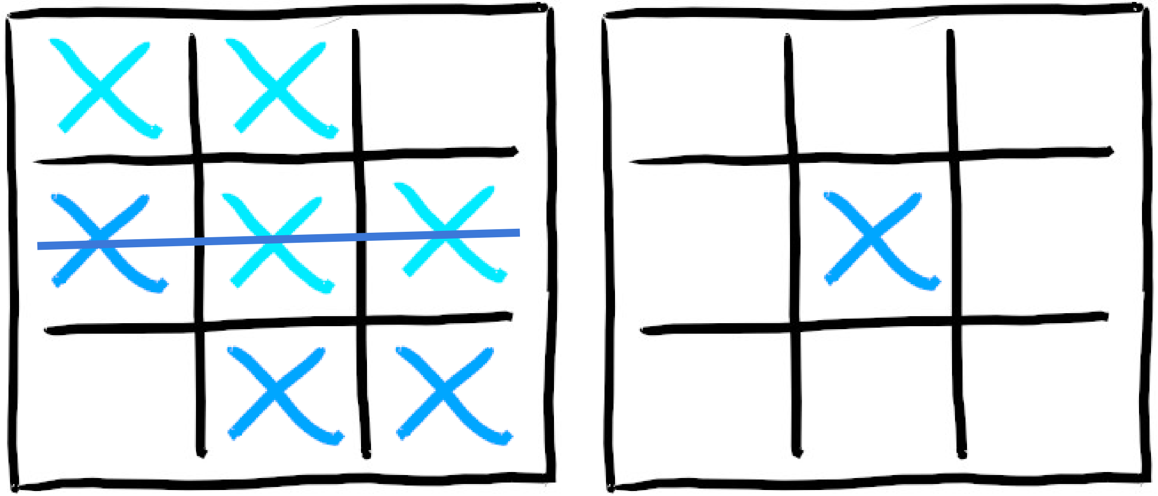

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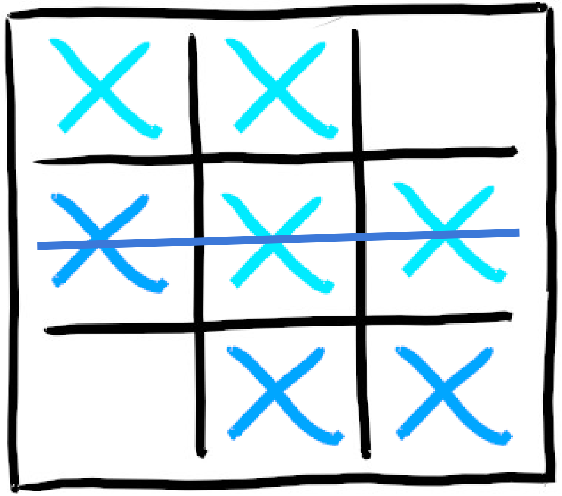

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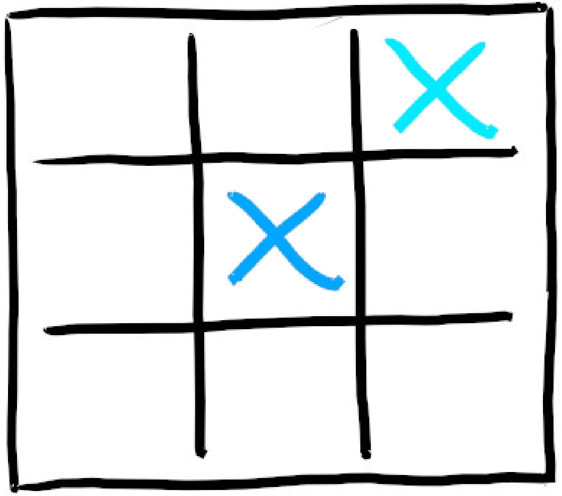

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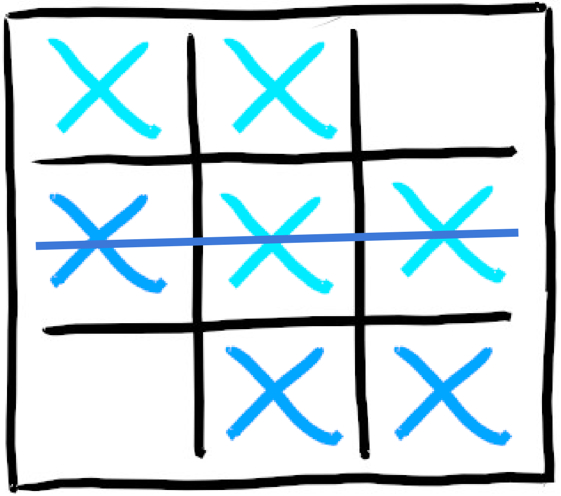

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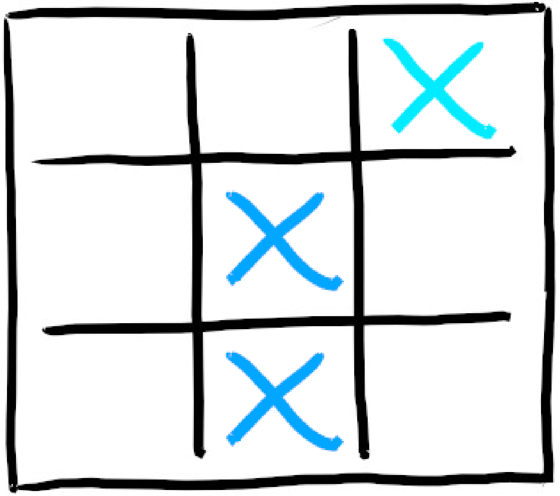

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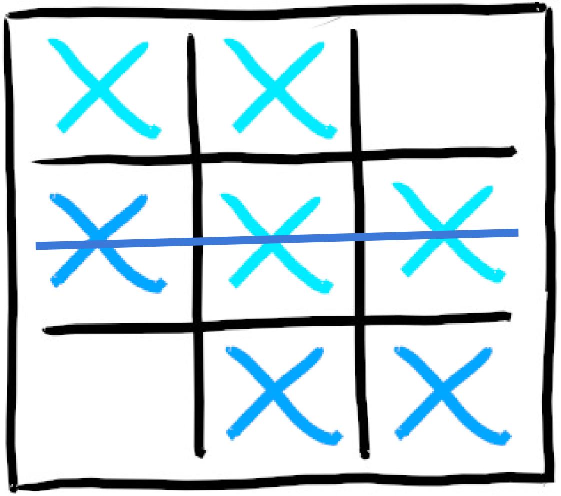

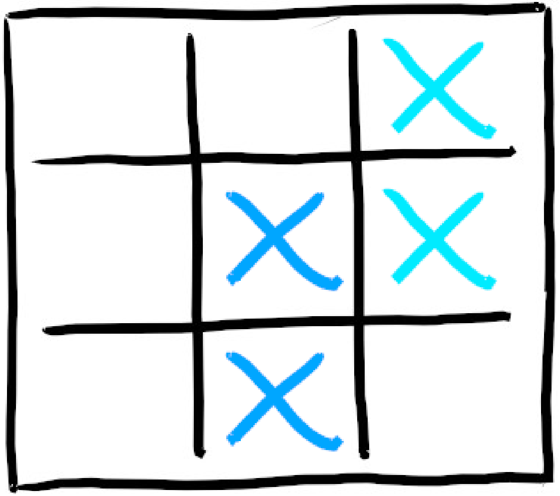

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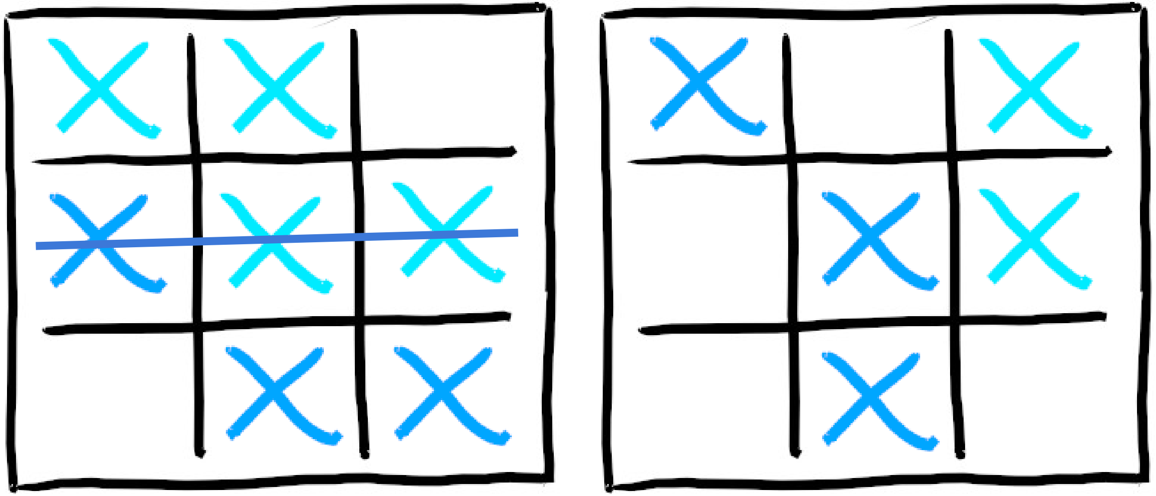

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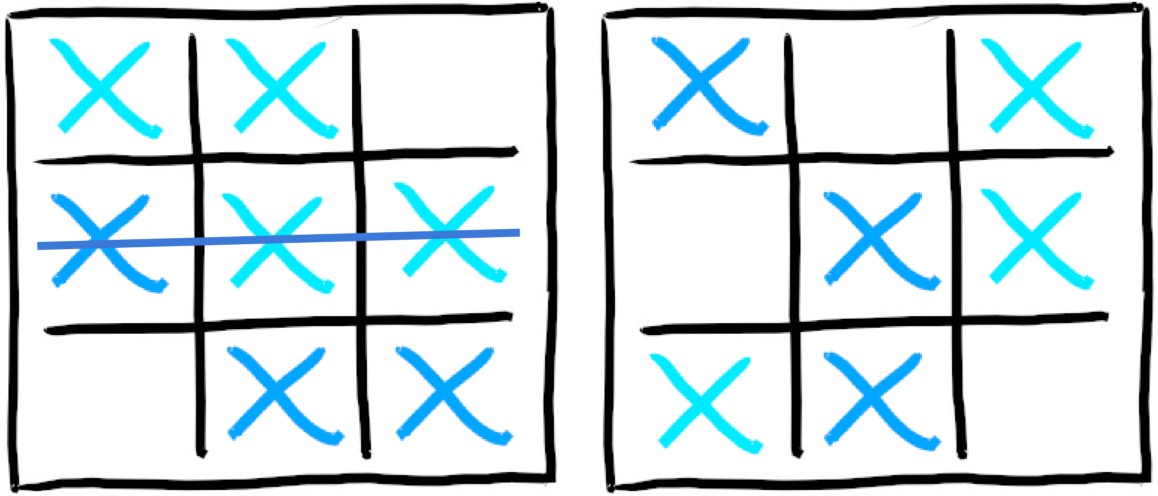

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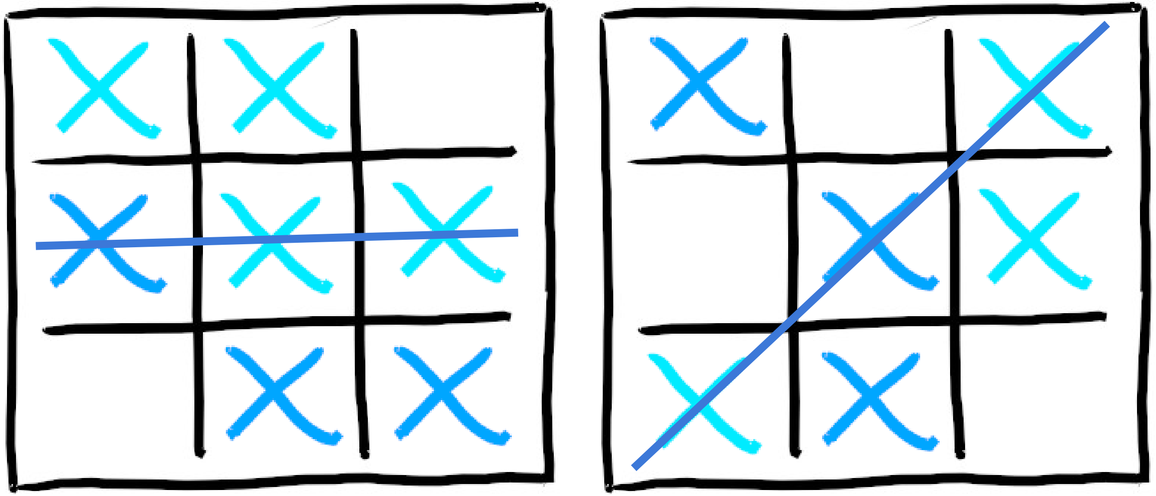

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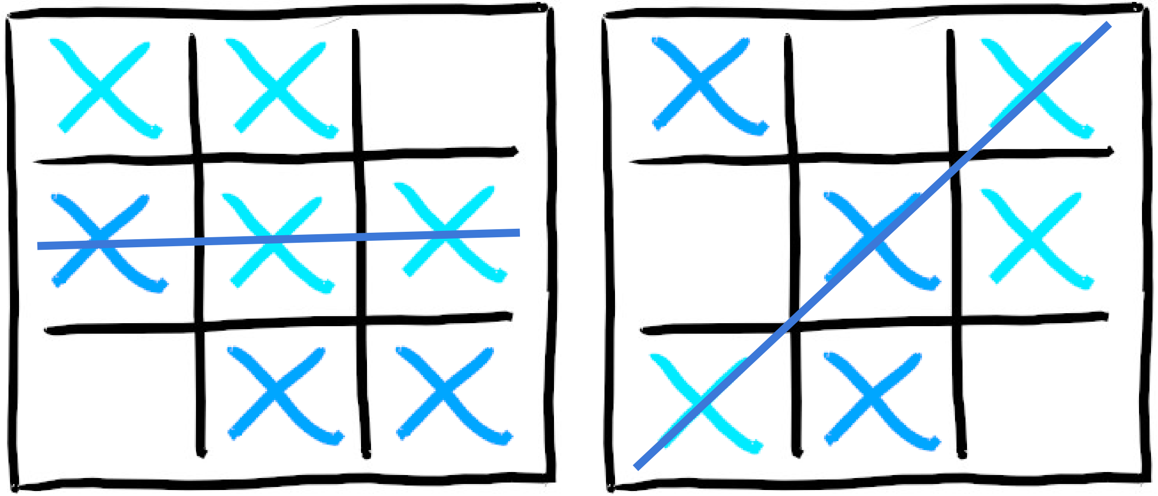

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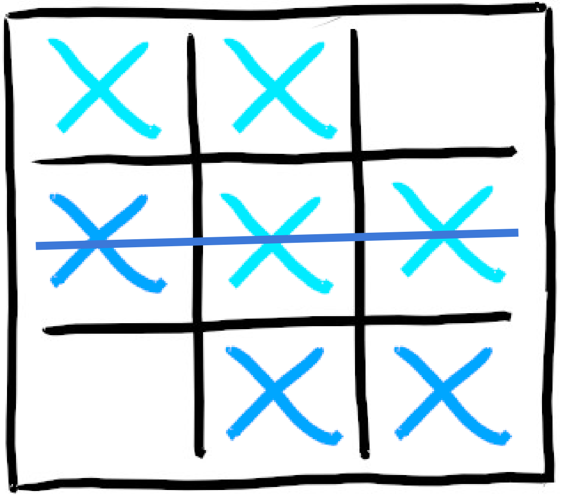

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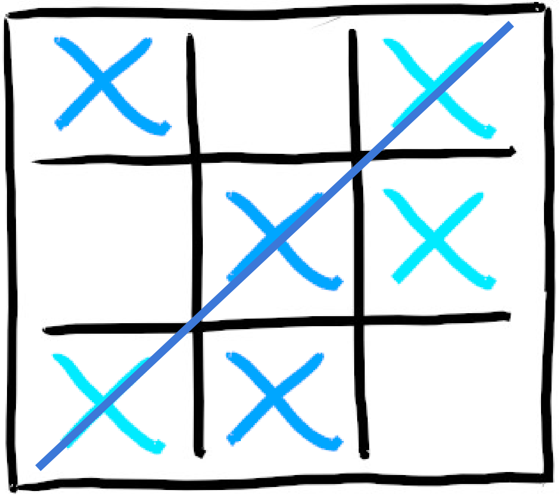

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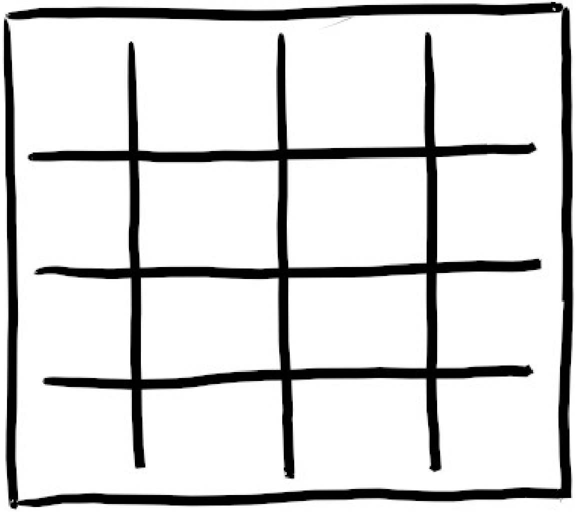

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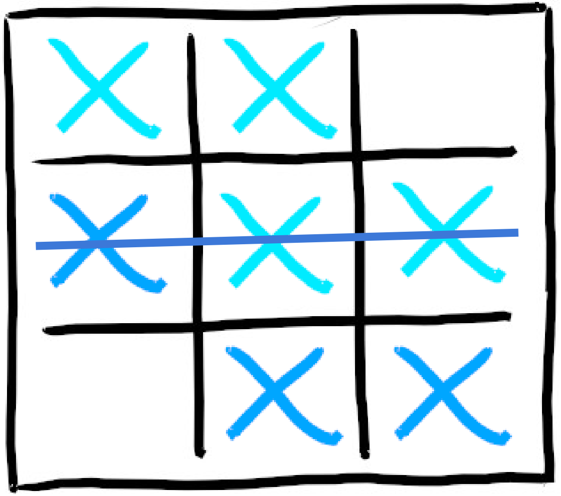

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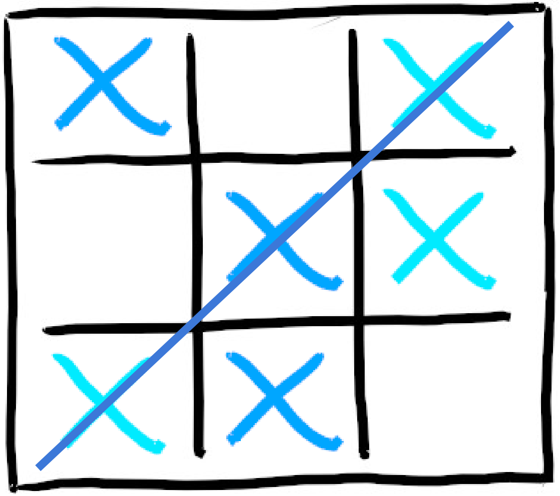

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| | | | |
|---|---|---|---|
| a | b | c | d |
| e | f | g | h |
| b | a | d | c |
| f | e | h | g |

# How to Win at Notakto

| n | Winner under perfect play | Reference |
|---|---|---|
| 1 | 2nd Player | |
| 2 | 1st Player | |
| 3 | 1st Player | |
| 4 | 2nd Player | (Chow 2010) |
| 5 | 1st Player | (Chow 2010) |
| 6 | 1st Player | (Chen et al. 2021) |
| $n = 4k$ | 2nd Player | (Chen et al. 2021) |

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(Chen et al. 2021)

- AlphaGo Zero fails to learn competent play for small board sizes n > 5
- Rare nature of competent Notakto boards
- MCTS and sampling method also encode prior knowledge (Marcus 2018)

“Twin Systems”
(Keane and Kenny 2019)

Symbolic & ANN
Heavy Interaction

Logic ⟷ ANN
(Leitgeb 2005)

Symbolic w ANN
Component

Logic ---> ANN
(Garcez, Lamb, and Gabbay 2008)

**Weakly-Coupled**

**Strongly-Coupled**

# The Spectrum of Neuro-Symbolic Proposals

# Training Boards

(Lai 2015)

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**Generating the Boards**

- Distribution
  - Boards should reflect competent play

- Variety
  - e.g. Boards that are unfairly stacked against player

- Volume
  - Enough boards for ANN to generalize

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(Lai 2015)

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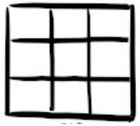

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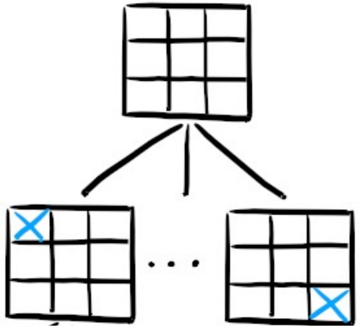

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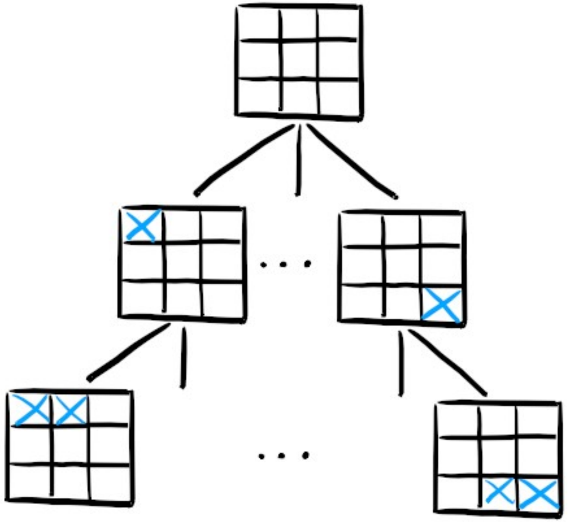

# Feature Representation

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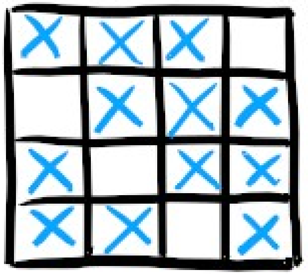
**Turn Flag**



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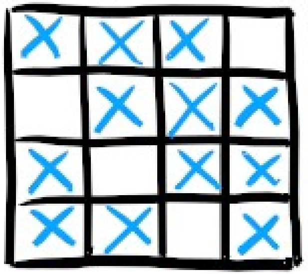
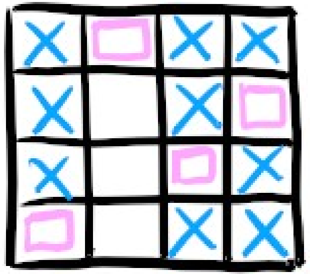


**Dead Squares**



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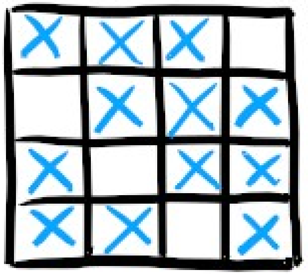

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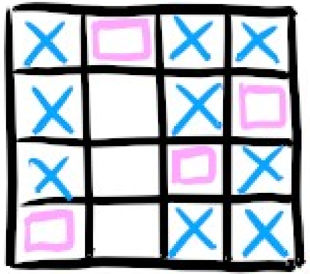
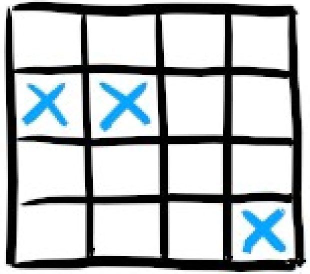


**Row Parity**



# Neural Network Architecture

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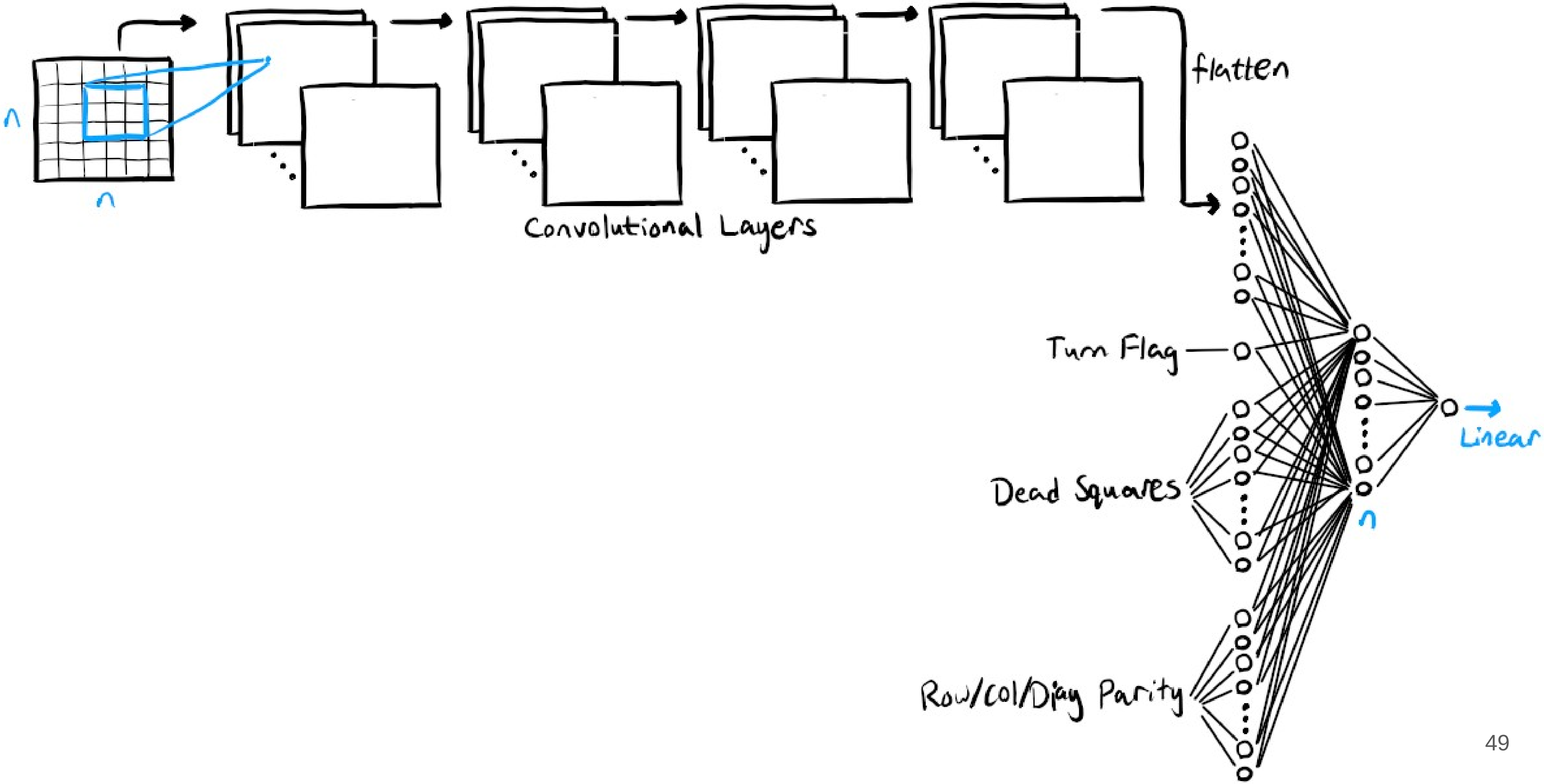

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|---|---|---|---|
| $n = 3$ | Itself | 100% / 0% | 92% / 8% |
| | Featured Net | | 100% / 1% |
| | AlphaZero Net | 99% / 0% | |
| | Random | 94% / 82% | 90% / 84% |
| | Greedy | 79% / 26% | 88% / 20% |
| $n = 4$ | Itself | 8% / 92% | 92% / 8% |
| | Featured Net | | 18% / 82% |
| | AlphaZero Net | 18% / 82% | |
| | Random | 93% / 94% | 91% / 96% |
| | Greedy | 37% / 55% | 49% / 54% |
| $n = 5$ | Itself | 59% / 41% | 41% / 59% |
| | Featured Net | | 47% / 49% |
| | AlphaZero Net | 51% / 53% | |
| | Random | 99% / 99% | 99% / 96% |
| | Greedy | 54% / 48% | 60% / 39% |
| $n = 6$ | Itself | 23% / 77% | 69% / 31% |
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| $n = 6$ | Itself | 23% / 77% | 69% / 31% |
| | Featured Net | | 33% / 53% |
| | AlphaZero Net | 47% / 67% | |
| | Random | 100% / 99% | 99% / 98% |
| | Greedy | 52% / 47% | 55% / 59% |

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- We try to extract the ANN’s strategy from a heat map
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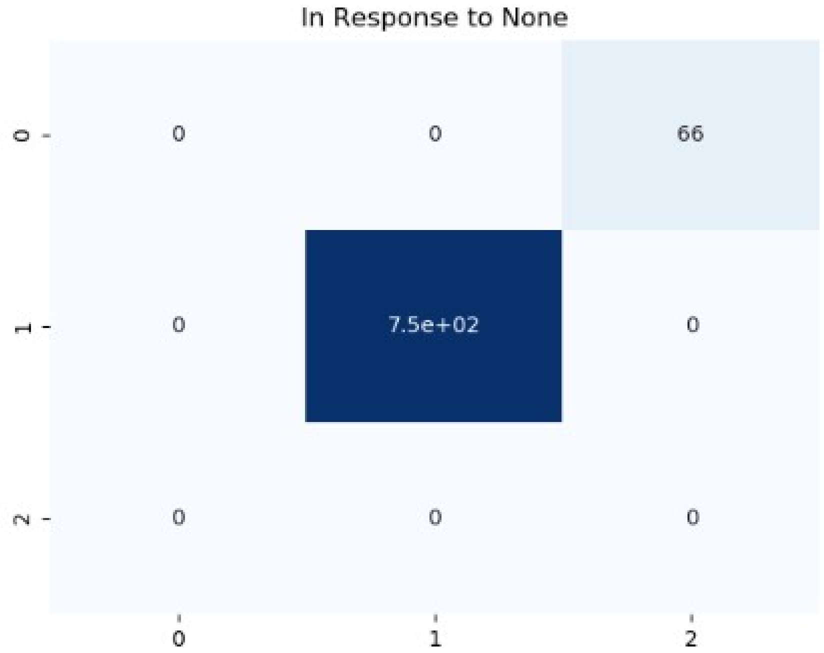

| | | | |
|---|---|---|---|
| a | b | c | d |
| e | f | g | h |
| b | a | d | c |
| f | e | h | g |

## Using an LSTM Instead
(Near-Future Work)

**Note:**

- Winning strategies depend on
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- Contrast with Chess, Go, Tic-Tac-Toe, ...

**Hypothesis:**

- Can’t evaluate a single static board
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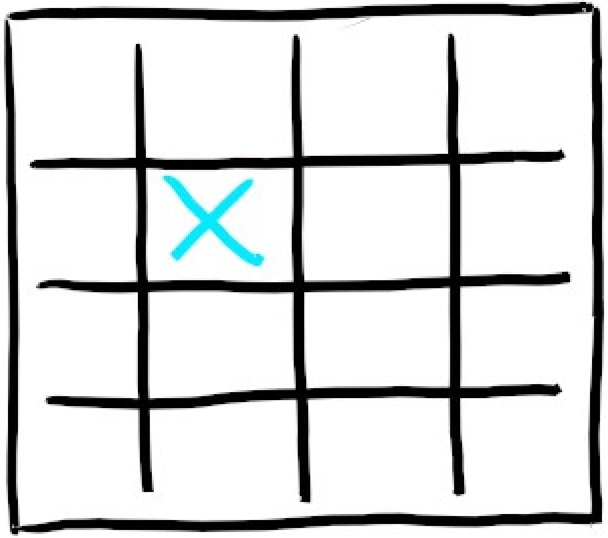

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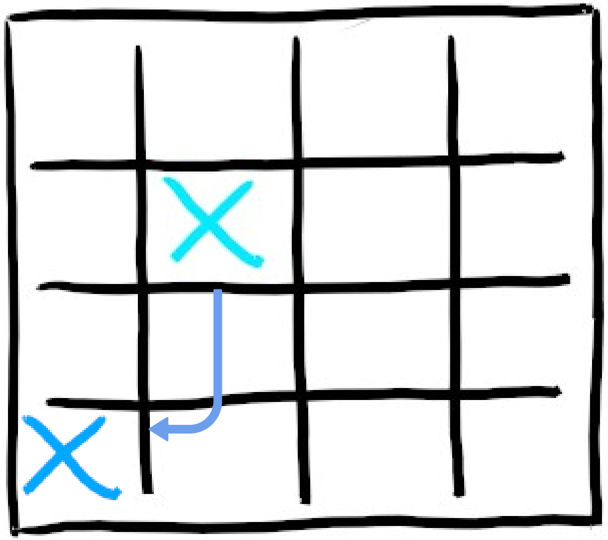

## Reconsidering Features
(Near-Future Work)

(Lai 2015)

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“For neural networks to work well, the feature representation needs to be relatively smooth in how the input space is mapped to the output space. Positions that are close together in the feature space should, as much as possible, have similar evaluations.”

# Q&A

**References:**


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## Github:

https://github.com/ais-climber/notakto-player/tree/main/v1_feedforward_net