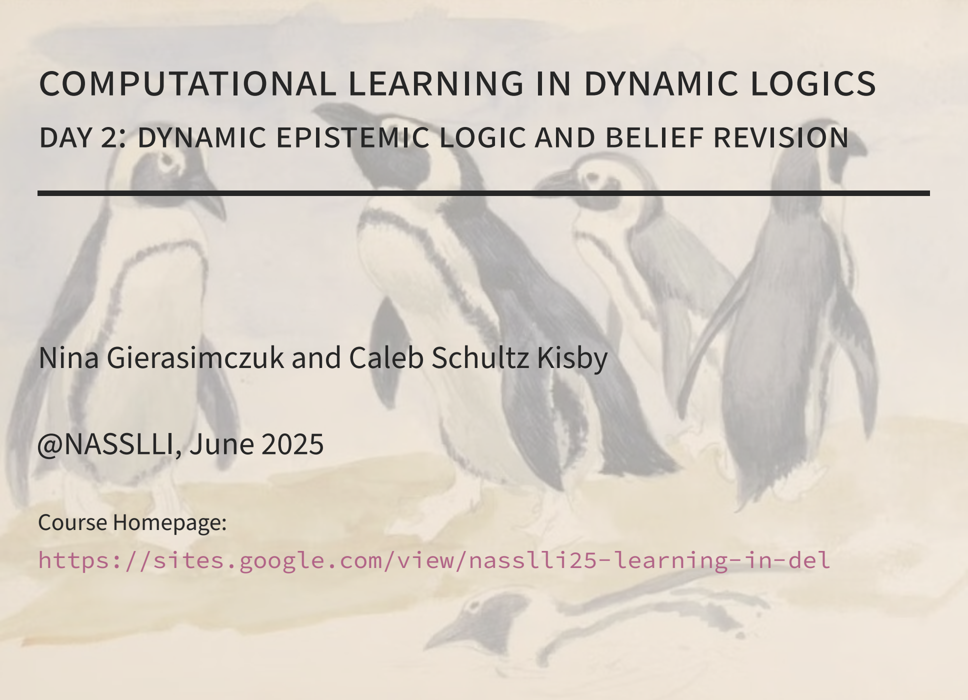

# COMPUTATIONAL LEARNING IN DYNAMIC LOGICS

## DAY 2: DYNAMIC EPISTEMIC LOGIC AND BELIEF REVISION

Nina Gierasimczuk and Caleb Schultz Kisby

@NASSLLI, June 2025

Course Homepage:
https://sites.google.com/view/nasslli25-learning-in-del

# PLAN FOR TODAY

1. Muddy Children Puzzle
2. Dynamic Epistemic Logic
3. Doxastic Logic and Belief Revision

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## MUDDY CHILDREN REVISITED

Imagine $n$ children are playing outside together. Now it happens that during their play some of them, say $k$ get mud on their foreheads. Each can see mud on others but not on his own forehead.

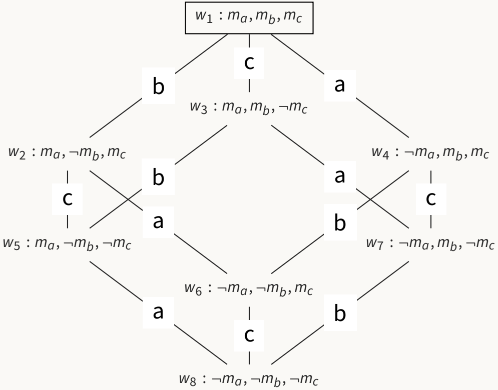
Along comes the father, who says, “At least one of you have mud on your forehead”. The father then asks the following question, over and over: “Does any of you know whether you have mud on your own forehead?” Assuming that all the children are perceptive, intelligent, truthful, and they answer simultaneously, what will happen?

Surprisingly, after the father asks the question for the $k^{th}$ time all muddy children will say “yes”. How come?

## MUDDY CHILDREN: THE UNDERLYING ASSUMPTIONS

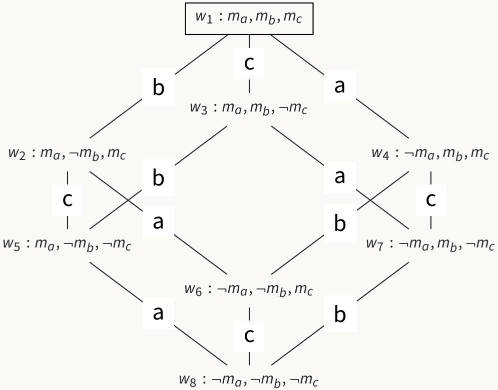
- **Common knowledge** that the father is truthful,
- that all the children hear the father,
- that all the children see each other,
- that none of them can see their own forehead,
- and that all the children are truthful and intelligent.

# MUDDY CHILDREN SCENARIO MODELLED IN EPISTEMIC LOGIC




Before the announcement.

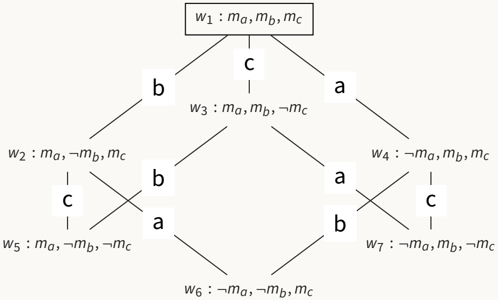
# MUDDY CHILDREN SCENARIO MODELLED IN EPISTEMIC LOGIC




Father says:

“$m_a \vee m_b \vee m_c$”

# MUDDY CHILDREN SCENARIO MODELLED IN EPISTEMIC LOGIC




– Does any of you know?

– No!

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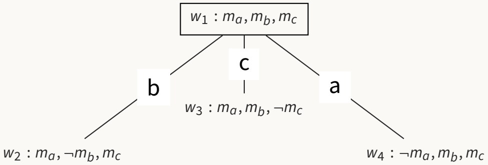


– Does any of you know?

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# MUDDY CHILDREN SCENARIO MODELLED IN EPISTEMIC LOGIC

$\boxed{w_1 : m_a, m_b, m_c}$

– Does any of you know?

– Yes!

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# LOGICS OF PUBLIC ANNOUNCEMENTS

- PAL (Public Announcement Logic) was proposed by Jan Plaza in 1989
- PAC (Public Announcement logic with common knowledge) is PAL + C
- PAL and PAC are examples of **dynamic epistemic logic**.

Dynamic Epistemic Logics formalize informational changes:
the dynamics of knowledge/belief.

## DYNAMIC MODALITIES

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The intended meaning of $[\alpha]\varphi$ is:
if action $\alpha$ is performed, then $\varphi$ will become true.

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$!\varphi$ can only be performed if $\varphi$ is **true**,

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In particular, if a false sentence is “truthfully announced",
then everything is true after that (including contradictions):

$$\neg\varphi \to [!\varphi]\bot,$$

where $\bot$ is any sentence that is always false (contradictory).

# LANGUAGE OF PUBLIC ANNOUNCEMENT LOGIC

**Definition (Syntax)**

$\Phi$ is a set of propositions, with $p \in \Phi$, and $\mathcal{A} = \{1, \ldots, n\}$ is a set of agents.

$$\varphi := \top \;\Big|\; p \;\Big|\; \ldots \;\Big|\; K_i\varphi \;\Big|\; [!\varphi]\varphi$$

where $\top$ abbreviates a tautology and $i \in \mathcal{A}$ is the name of some agent.

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where $\top$ abbreviates a tautology and $i \in \mathcal{A}$ is the name of some agent.

As before, these formulas are interpreted in possible world models.

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LEARNING = ELIMINATING POSSIBILITIES

From now on, we denote by $!\varphi$ the operation of deleting the non-$\varphi$ worlds, and call it **public announcement with** $\varphi$, or **joint update with** $\varphi$.

# SEMANTICS OF PUBLIC ANNOUNCEMENT LOGIC

**Definition**

Let $M = (S, \pi, \mathcal{K}_1, \ldots, \mathcal{K}_n)$.

| | | |
|---|---|---|
| $(M, s) \vDash p$ | iff | $\pi(s)(p) = 1$ |
| | $\ldots$ | |
| $(M, s) \vDash K_i\varphi$ | iff | for all $v$ with $(s, v) \in \mathcal{K}_i$, $(M, v) \vDash \varphi$ |
| $(M, s) \vDash [!\varphi]\psi$ | **iff** | **if** $(M, s) \vDash \varphi$ **then** $(M|\varphi, s) \vDash \psi$ |

where $M|\varphi = (S', \pi', \mathcal{K}'_1, \ldots, \mathcal{K}'_n)$ is defined as follows:

- $S' := \{s \in S \mid (M, s) \vDash \varphi\}$
- $\pi' := \pi$ restricted to $S'$
- for each $i \in \{1, \ldots, n\}$, $\mathcal{K}'_i := \mathcal{K}_i \cap (S' \times S')$

# ARE SENTENCES KNOWN AFTER TRUTHFULLY ANNOUNCED?

Intuitively, it may seem that:

every sentence becomes known after it is truthfully publicly announced.

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If the above were true then, assuming truthfulness, we would also get:

$$(?)\;\; [!\varphi]\varphi, \text{ for any sentence } \varphi.$$

# MOORE SENTENCES

Two stockbrokers Alice and Bob are lunching in a Wall Street café. A messenger comes in and delivers a letter to Alice. On the envelope it is written 'urgently requested data on United Agents'. Alice opens and reads the letter, which informs her of the fact that United Agents is doing well, such that she intends to buy a portfolio of stocks of that company, immediately.

Alice says to Bob:

**'UA is doing well.'**

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Alice says to Bob:

**‘Guess you don’t know it, but UA is doing well.’**

# UNSUCCESSFUL UPDATE

$$
\begin{tikzcd}
\boxed{s : u} \arrow[r, leftrightarrow, "b"] \arrow[d, dotted, "!(u \wedge \neg K_b u)"] & t : \neg u \\
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\end{tikzcd}
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but also epistemic formulas: $[!(\neg K_b u)]$

and make announcements about other announcements: $[!([!u]K_b u)]$.

# CLOSURE OF PUBLIC ANNOUNCEMENT UNDER COMPOSITION

$$\vDash [!\varphi][!\psi]\theta \leftrightarrow [!(\varphi \wedge [!\varphi]\psi)]\theta.$$

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this expresses **closure of public announcements under sequential composition**:

performing successively two public announcements: $!\varphi; !\psi$

is equivalent to performing one: $!(\varphi \wedge [!\varphi]\psi)$.

## REDUCTION LAWS FOR PAL

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Using those reduction axioms, one can translate formulas with announcement modalities into ones without.

This shows PAL can be reduced to Epistemic Logic.

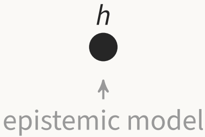
# DYNAMIC EPISTEMIC LOGIC IN GENERAL

- DEL comprises a family of logics.
- Each has syntax and semantics.
- DEL concerns explicit informational actions.
- Corresponding knowledge and belief changes in agents.
- Often uses special **action models**.

## DEL BY EXAMPLE: A HIDDEN COIN TOSS

We use the action models of DEL with postconditions (ontic actions).

$h$:="the coin faces heads up"

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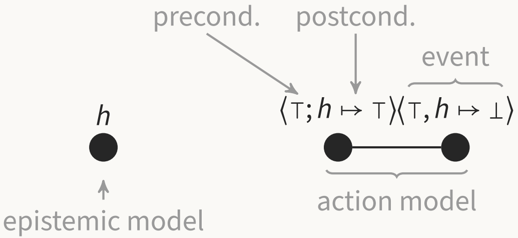


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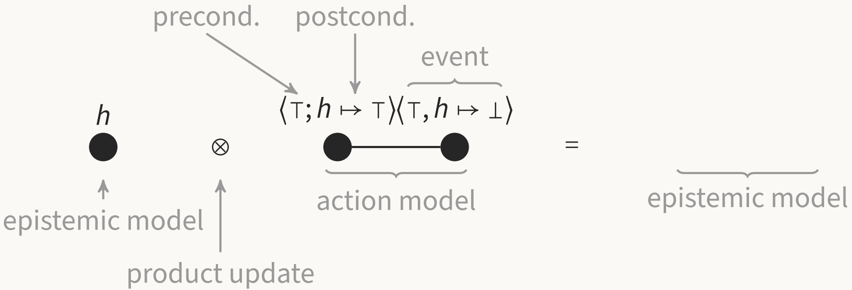



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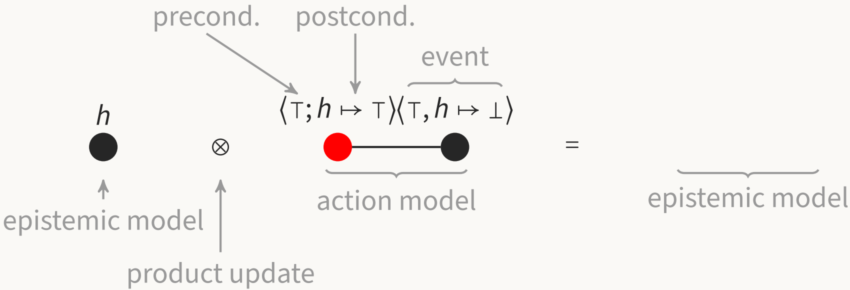


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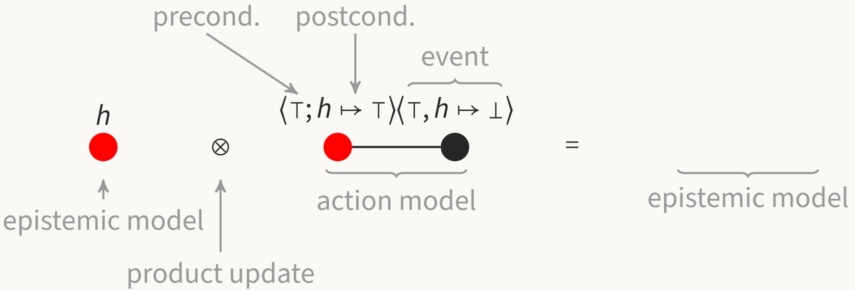


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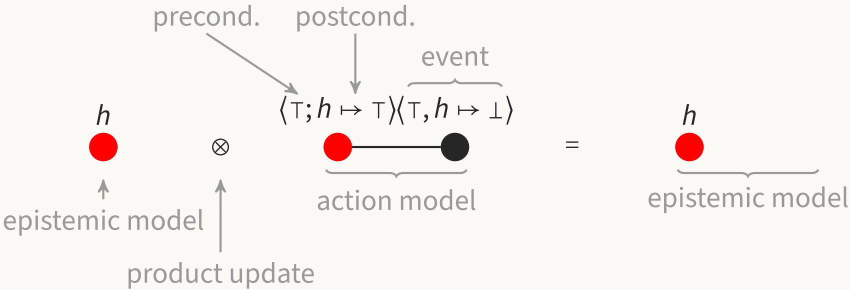



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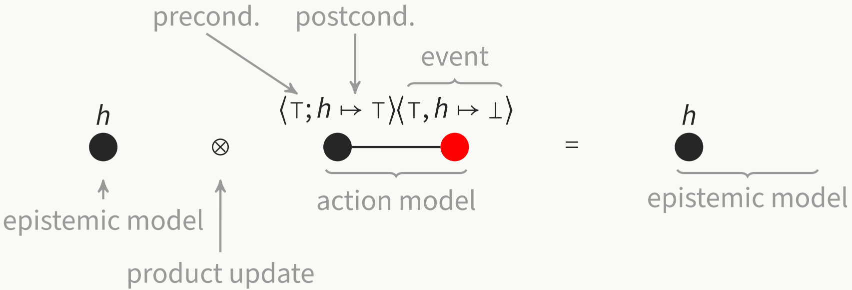


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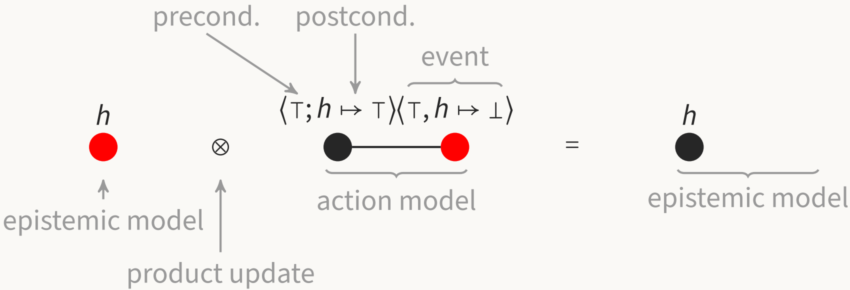


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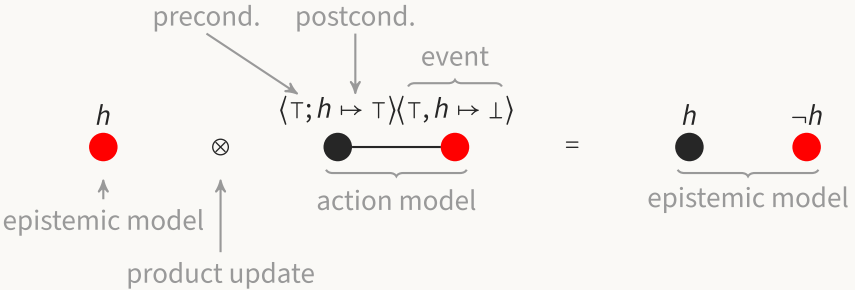


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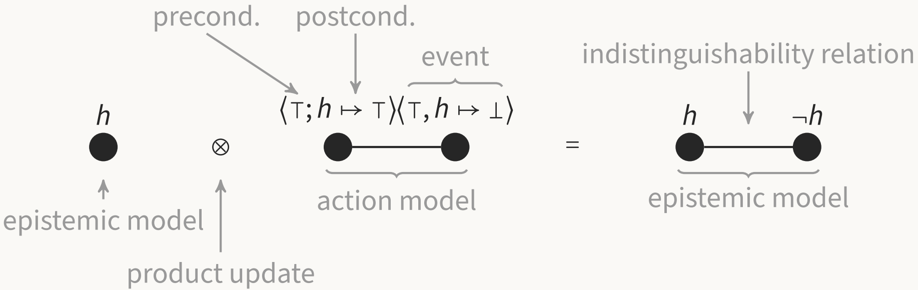


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- So far, we have only talked about learning in terms of *knowledge update*
- **Knowledge is monotonic:** Once $\varphi$ is *known*, it cannot be taken back or revised. (*hard information*)
- **Belief is not monotonic:** Our beliefs are tentative convictions, which allow for exceptions and future revision (*soft information*)
    - An agent can believe false things
    - Beliefs can allow for exceptions, e.g., the belief that birds fly
    - An agent can revise or retract her beliefs

# THE LANGUAGE OF CONDITIONAL BELIEF

**Definition**

Take a countable set of propositions $\mathsf{Prop}$.

$$\varphi, \psi := \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid A\varphi \mid \mathbf{B}^{\psi}\varphi$$

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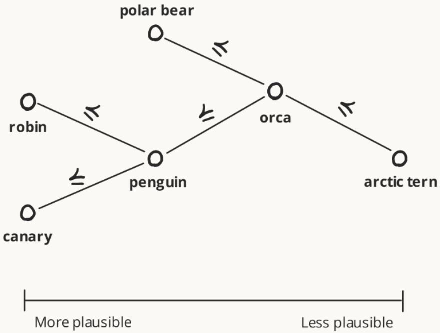
- Example: $\mathbf{B}^{\mathsf{bird}}(\mathsf{fly})$ means "the agent believes that normal birds fly"
  - taking some liberties: "normally, birds fly"
  - allows for exceptional birds that do not fly, e.g. penguins, dodos

# AGENT PLAUSIBILITY ORDERS

- In order to model the agent's concept of "most normal", we use a plausibility order on worlds $\preceq : W \times W$:

  $u \preceq w$ if the agent considers $u$ at least as plausible as $w$




- Let $\text{best}_{\preceq}(S) = \{w \in S \mid \text{for all } u \in S, \text{not } u \preceq w\}$, the set of most plausible worlds over $S$. In cognitive science, this is known as the *prototype* of $S$.

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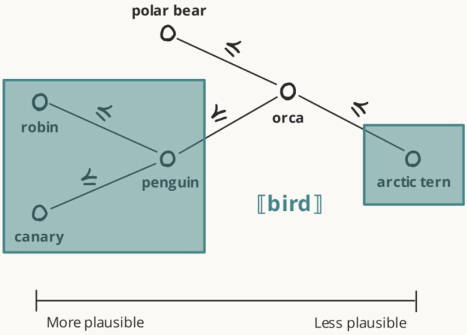


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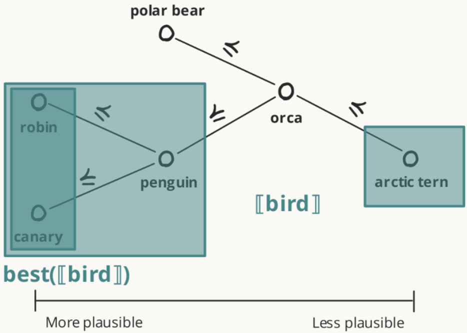


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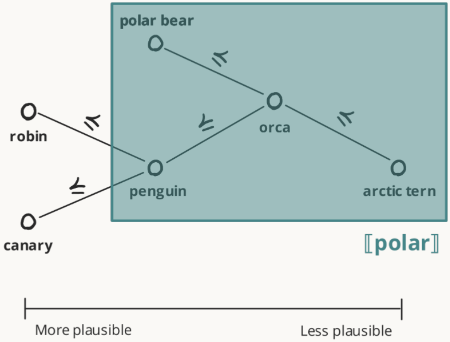


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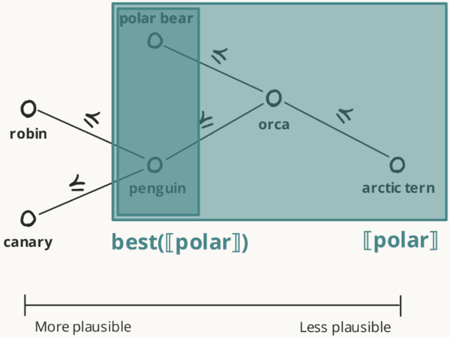


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# SEMANTICS FOR CONDITIONAL BELIEF

**Definition**

Given a plausibility model $M = (W, \preceq, V)$ and a state $s \in W$:

| | | |
|---|---|---|
| $M, s \models p$ | iff | $s \in V(p)$ for each $p \in \mathsf{Prop}$ |
| $M, s \models \neg\varphi$ | iff | not $M, s \models \varphi$ |
| $M, s \models \varphi \wedge \psi$ | iff | $M, s \models \varphi$ and $M, s \models \psi$ |
| $M, s \models A\varphi$ | iff | $M, u \models \varphi$ for all $u \in W$ whatsoever |
| $M, s \models \mathbf{B}^{\psi}\varphi$ | iff | $\mathsf{best}_{\preceq}([\![\psi]\!]) \subseteq [\![\varphi]\!]$ |

"the most normal $\psi$-worlds are $\varphi$-worlds"

Again, $[\![\varphi]\!] = \{u \mid M, u \models \varphi\}$ is the set of $\varphi$-states.

# ADDITIONAL COMMENTS ON CONDITIONAL BELIEF

- The semantics for $\mathbf{B}^{\psi}\varphi$ doesn't depend on the state $s$ at all!
  - We can fix this by having a different plausibility order $\preceq_s$ *per state*.

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- We've dropped the subscripts $i$ — we're more interested in an individual's learning policy here.
  - We could have a different plausibility order $\preceq_{i,s}$ *per agent, per state*
  - This gets messy!
- We can also define ordinary (nonconditional) belief as

$$M, s \vDash \mathbf{B}\varphi \qquad \text{iff} \qquad \text{For all } t \in \mathsf{best}_{\preceq}([\![\varphi]\!]), M, t \vDash \varphi$$

  - But conditional belief is more expressive!

$$\vDash \mathbf{B}\varphi \leftrightarrow \mathbf{B}^{\varphi}\varphi, \text{ since } M, s \vDash \mathbf{B}\varphi \text{ iff } \mathsf{best}_{\preceq}([\![\varphi]\!]) \subseteq [\![\varphi]\!]$$

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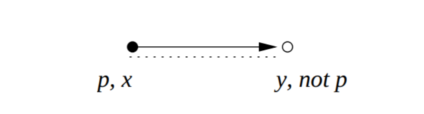

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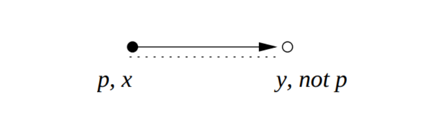


- We need a less destructive idea than world elimination…

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- There are many, many different policies we could use to re-order!
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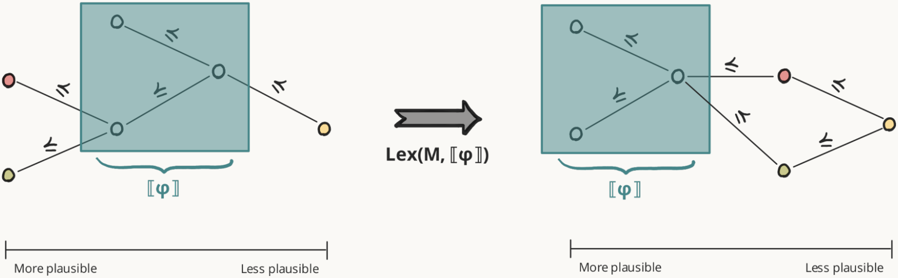
- Each revision policy represents the agent's “style” of response to incoming information (*hard vs soft*, *radical vs minimal*)

# LEXICOGRAPHIC UPGRADE

*Make all $\varphi$-worlds more plausible than $\neg\varphi$ worlds*



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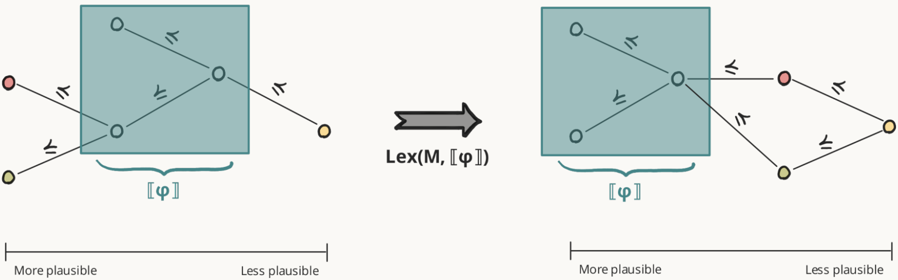
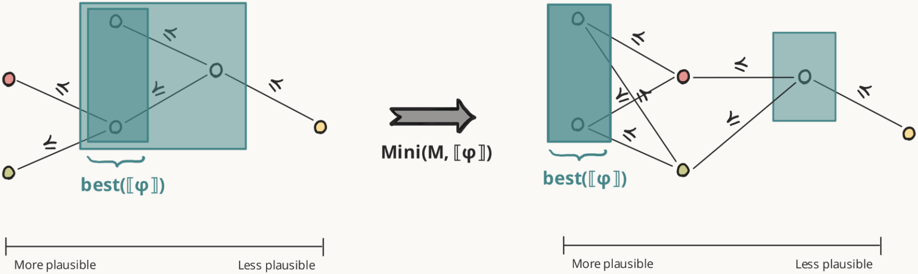



- Formally, $\mathsf{Lex}((W, \preceq, V), [\![\varphi]\!]) = (W, \preceq', V)$, where the plausibility order is replaced with the following:

  All $[\![\varphi]\!]$-worlds are $\preceq'$-better than all $[\![\neg\varphi]\!]$-worlds, but within those two groups the old ordering $\preceq$ remains

# MINIMAL UPGRADE

*Make only the **best** $\varphi$-worlds more plausible than the rest*



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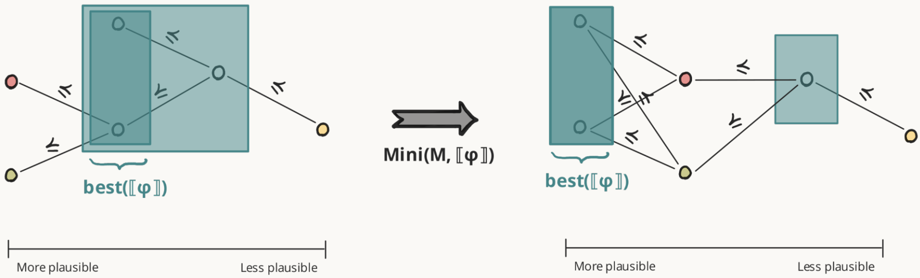


- Formally, $\mathsf{Mini}((W, \preceq, V), [\![\varphi]\!]) = (W, \preceq', V)$, where the plausibility order is replaced with the following:

  The $\mathsf{best}_{\preceq}([\![\varphi]\!])$ worlds come on top, but otherwise the old order remains

**Definition**

We extend our language from before with two new upgrade operators:

$$\varphi, \psi := \top \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid A\varphi \mid \mathbf{B}^{\psi}\varphi \mid [\Uparrow\varphi]\psi \mid [\uparrow\varphi]\psi$$

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- We read them as follows:

“After re-ordering the agent's plausibility order via Lex (or Mini) on input $[\![\varphi]\!]$, $\psi$ holds.”

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- **Note:** $E\varphi$ here just says “there is a world (at all) where $\varphi$ holds”

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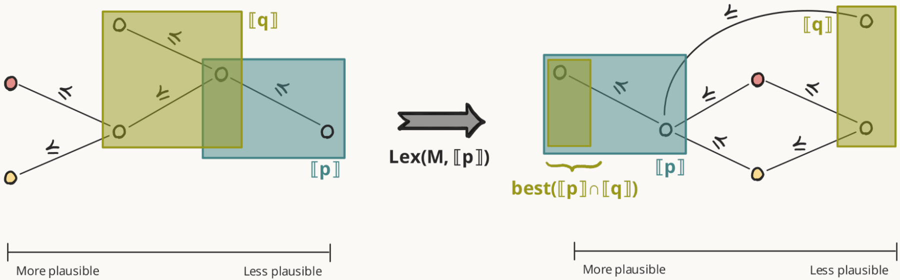

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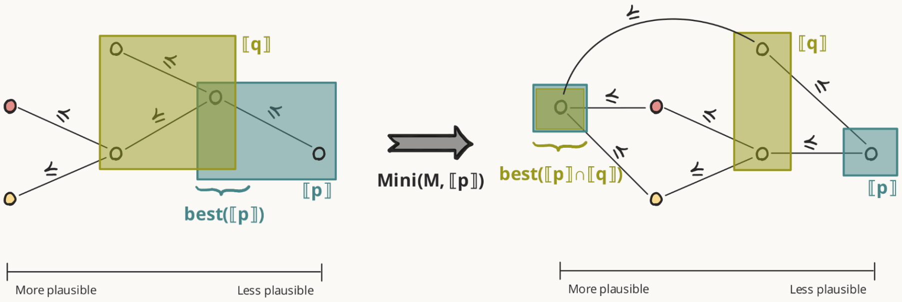
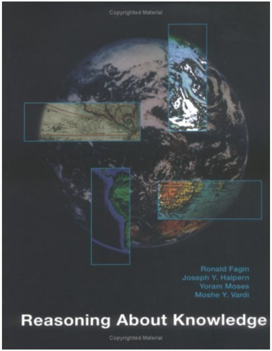
$$[\uparrow p]\mathbf{B}^q r \leftrightarrow (\mathbf{B}^p\neg q \wedge \mathbf{B}^q r) \vee (\neg\mathbf{B}^p\neg q \wedge \mathbf{B}^{p\wedge q} r)$$

- This encodes in our logic a complete description of the effect Mini has on the plausibility ordering. This is the description:

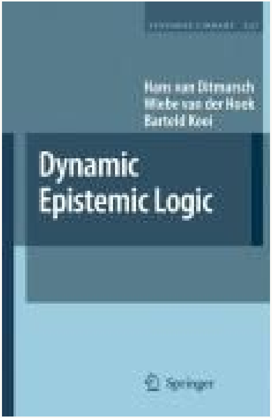
$$\mathsf{best}_{\mathsf{Mini}(\preceq,[\![p]\!])}([\![q]\!]) = \begin{cases} \mathsf{best}_{\preceq}([\![q]\!]) & \text{if } \mathsf{best}_{\preceq}([\![p]\!]) \subseteq [\![q]\!]^{\complement} \\ \mathsf{best}_{\preceq}([\![p]\!] \cap [\![q]\!]) & \text{if } \mathsf{best}_{\preceq}([\![p]\!]) \not\subseteq [\![q]\!]^{\complement} \end{cases}$$



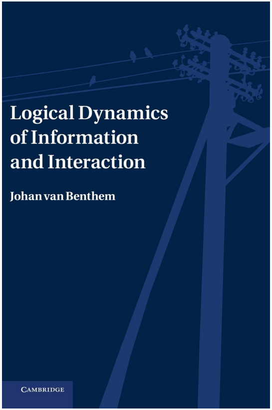
## TO LEARN MORE CHECK OUT...




(a) Reasoning about Knowledge by Halpern, Vardi, Fagin, & Moses

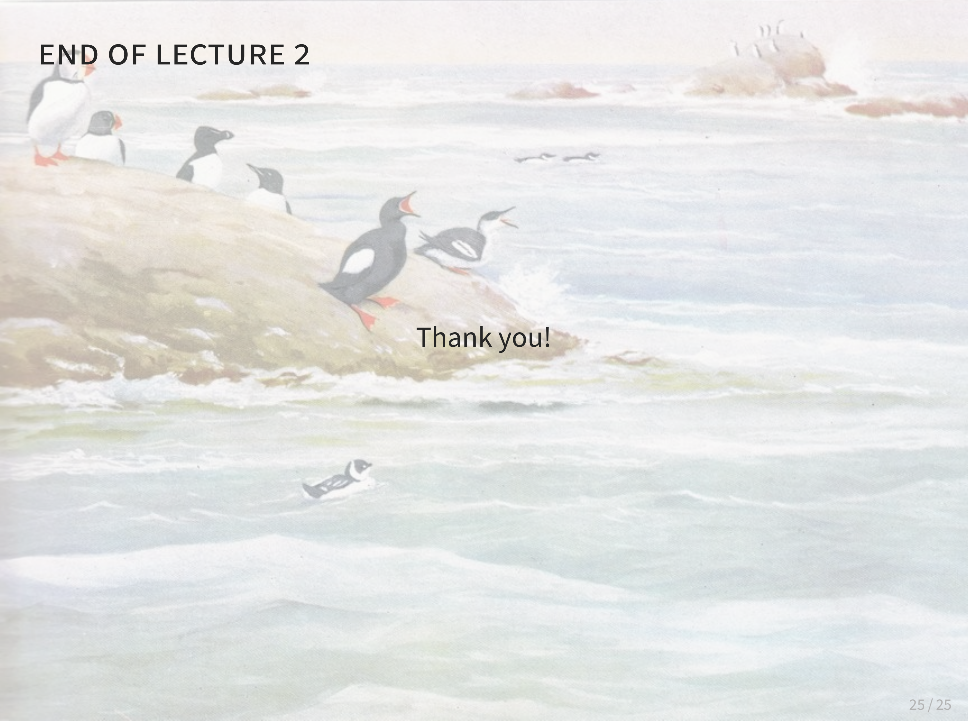



(b) Dynamic Epistemic Logic by van Ditmarsch, van der Hoek, & Kooi




(c) Logical Dynamics of Information and Interaction by van Benthem

# END OF LECTURE 2



Thank you!