Syllogistic Logic + Sizes of Sets + Union

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Rules for S + ‘more(a, b)’ + ‘atleast(a, b)’

Moss, Larry. “Syllogistic Logic with Cardinality Comparisons”

$$\frac{}{\forall(p,p)}\ (\textsc{axiom}) \qquad \frac{\forall(n,p) \quad \forall(p,q)}{\forall(n,q)}\ (\textsc{barbara})$$

$$\frac{\exists(p,q)}{\exists(p,p)}\ (\textsc{some}) \qquad \frac{\exists(q,p)}{\exists(p,q)}\ (\textsc{conversion})$$

$$\frac{\exists(p,n) \quad \forall(n,q)}{\exists(p,q)}\ (\textsc{darii}) \qquad \frac{\forall(p,q) \quad \exists^{\geq}(p,q)}{\forall(q,p)}\ (\textsc{card-mix})$$

$$\frac{\forall(p,q)}{\exists^{\geq}(q,p)}\ (\textsc{subset-size}) \qquad \frac{\exists^{\geq}(n,p) \quad \exists^{\geq}(p,q)}{\exists^{\geq}(n,q)}\ (\textsc{card-trans})$$

$$\frac{\exists(p,p) \quad \exists^{\geq}(q,p)}{\exists(q,q)}\ (\textsc{card-}\exists) \qquad \frac{\exists^{>}(p,q)}{\exists^{\geq}(p,q)}\ (\textsc{more-at least})$$

$$\frac{\exists^{>}(n,p) \quad \exists^{\geq}(p,q)}{\exists^{>}(n,q)}\ (\textsc{more-left}) \qquad \frac{\exists^{\geq}(n,p) \quad \exists^{>}(p,q)}{\exists^{>}(n,q)}\ (\textsc{more-right})$$

$$\frac{\exists^{\geq}(p,q) \quad \exists^{>}(q,p)}{\varphi}\ (\textsc{x})$$

Rules for S + ‘most(a, b)’

Endrullis, J and Moss, L. “Syllogistic Logic with ‘Most’”

$$\frac{}{\textsf{All } X \textsf{ are } X}\ \textit{axiom} \qquad \frac{\textsf{All } X \textsf{ are } Y \quad \textsf{All } Y \textsf{ are } Z}{\textsf{All } X \textsf{ are } Z}\ \textit{barbara}$$

$$\frac{\textsf{Some } X \textsf{ are } Y}{\textsf{Some } Y \textsf{ are } X}\ \textit{conv} \qquad \frac{\textsf{Some } X \textsf{ are } Y}{\textsf{Some } X \textsf{ are } X}\ \textit{some} \qquad \frac{\textsf{Some } X \textsf{ are } Y \quad \textsf{All } Y \textsf{ are } Z}{\textsf{Some } X \textsf{ are } Z}\ \textit{darii}$$

$$\frac{\textsf{Most } X \textsf{ are } Y}{\textsf{Some } X \textsf{ are } Y}\ m_1 \qquad \frac{\textsf{Some } X \textsf{ are } X}{\textsf{Most } X \textsf{ are } X}\ m_2 \qquad \frac{\textsf{Most } X \textsf{ are } Y \quad \textsf{All } Y \textsf{ are } Z}{\textsf{Most } X \textsf{ are } Z}\ m_3$$

$$\frac{\textsf{Most } X \textsf{ are } Z \quad \textsf{All } X \textsf{ are } Y \quad \textsf{All } Y \textsf{ are } X}{\textsf{Most } Y \textsf{ are } Z}\ m_4$$

$$\frac{\textsf{All } Y \textsf{ are } X \quad \textsf{All } X \textsf{ are } Z \quad \textsf{Most } Z \textsf{ are } Y}{\textsf{Most } X \textsf{ are } Y}\ m_5$$

$$\frac{X_1 \triangleright_{A,B} Y_1 \quad Y_1 \triangleright_{B,A} X_2 \quad \cdots \quad X_n \triangleright_{A,B} Y_n \quad Y_n \triangleright_{B,A} X_1}{\textsf{Some } A \textsf{ are } B}\ \triangleright$$

Figure 1: Rules of the logical system for All, Some, and Most. The last line is an infinite rule scheme, and the syntax is explained in Section 3.

Definition of a ‘Suitable’ Relation

Taken from our Overleaf document. :)

Definition 6. A *suitable pair of relations on Pairs*(n) is a pair of relations[2]

$$(\preceq, \preceq_{subset})$$

such that

1. $\preceq$ and $\preceq_{subset}$ are preorders on $\text{Pairs}(n)$.
2. For all $(i,j),(k,l) \in \text{Pairs}(n)$, either $(i,j) \preceq (k,l)$ or $(k,l) \prec (i,j)$.
3. If $i < j$, then $(i,i) \preceq_{subset} (i,j)$. If $j < i$, then $(i,i) \preceq_{subset} (j,i)$.
4. If $(i,i) \preceq_{subset} (k,\ell)$ and $(j,j) \preceq_{subset} (k,\ell)$ and $i < j$, then $(i,j) \preceq_{subset} (k,\ell)$.
5. If $p \preceq_{subset} q$, then $p \preceq q$.
6. If $p \preceq_{subset} q$ and $q \preceq p$, then $q \preceq_{subset} p$.