Syllogistic Logics with ‘More Than’ and ‘Most’

The Natural Logic Program

- Aristotle, Van Benthem, Moss
- Natural language inference
  - No translating into FOL
- Special-purpose *logic engineering*
  - Complete *and decidable* logics
  - 5 different systems in this talk alone!

Logics about Size Comparison

- Restricted domain: *Reasoning about sizes*
- Why not embed into FOL with Arithmetic?
    - Decidable $\rightarrow$ More cognitively plausible
    - Light $\rightarrow$ More cognitively plausible
    - Counting is not needed:
        - "There is more sand than water in the pond"

$\mathcal{S}(card)$: Syllogistic + ‘More Than’ + ‘At Least as Many’

- $a, b, c, \ldots$ are nouns
- $all(a, b)$
- $some(a, b)$
- $more(a, b)$
- $atLeast(a, b)$

- *No connectives* $\wedge, \vee \neg$
- *No quantifiers* $\forall, \exists$

$\mathcal{S}(card)$: Semantics

- Models: $\mathcal{M}$ consisting of set M.
- For all nouns a, $[\![a]\!] \subseteq M$

- $\mathcal{M} \models all(a, b)$ iff $[\![a]\!] \subseteq [\![b]\!]$
- $\mathcal{M} \models some(a, b)$ iff $[\![a]\!] \cap [\![b]\!] \neq \emptyset$
- $\mathcal{M} \models more(a, b)$ iff $|[\![a]\!]| > |[\![b]\!]|$
- $\mathcal{M} \models atLeast(a, b)$ iff $|[\![a]\!]| \geq |[\![b]\!]|$

- $\Gamma \models \varphi$ iff every **FINITE** model $\mathcal{M}$ which satisfies Γ satisfies φ.

$S(card)$: Natural Deduction Rules

See board

$\mathcal{S}(card)$: Completeness Theorem

Theorem: $\mathcal{S}(card)$ is complete, i.e. $\Gamma \models \varphi$ iff $\Gamma \vdash \varphi$.

- From Larry Moss' “Syllogistic Logic with Cardinality Comparisons”

$\mathcal{S}^\dagger(card)$: $\mathcal{S}(card)$ + Noun Complement

- For a noun a, we allow a to be complemented: $\bar{a}$
- $\bar{\bar{a}} = a$ *at syntax level*
- **Semantics:** $[\![\bar{a}]\!] = M \setminus [\![a]\!]$
- **Rules of Inference:**
  - **See board**

$\mathcal{S}^\dagger(card)$: Completeness Theorem

Theorem: $\mathcal{S}^\dagger(card)$ is complete, i.e. $\Gamma \models \varphi$ iff $\Gamma \vdash \varphi$.

- More complicated than the proof for $\mathcal{S}(card)$!

Bonus Theorem: $\mathcal{S}^\dagger(card)$ is decidable in polynomial time!

$\mathcal{S}(most)$: Syllogistic + ‘Most’

- $all(a, b)$
- $some(a, b)$
- $most(a, b)$

- $\mathcal{M} \models most(a, b)$ iff $|[\![a]\!] \cap [\![b]\!]| > \frac{1}{2}[\![a]\!]$

$\mathcal{S}(most)$: Natural Deduction Rules

See board

Theorem: $\mathcal{S}(most)$ is complete. -- and decidable in polynomial time!

The Hierarchy of Syllogistic Logics

$$
\begin{array}{lll}
\mathcal{A}ll & & \\
\downarrow & & \\
\mathcal{S} & \rightarrow & \mathcal{S}(most) \\
\downarrow & & \downarrow \\
\mathcal{S}(card) & & \downarrow \\
\downarrow & & \downarrow \\
\mathcal{S}^{\dagger}(card) & \rightarrow & \textbf{Holy Grail}
\end{array}
$$

- We want a complete logic involving ‘more than’ *and* ‘most’
- Can’t just combine rules, because of interplay between ‘more than’ and ‘most’

A Solution: Complement *and Intersection*

- $most(a,b) \equiv more(a \cap b, a \cap \bar{b})$

- So we need **noun complement** *and* **noun intersection**
- It is easier to make rules for Union $\cup$
  - $a \cap b \equiv \overline{\bar{a} \cap \bar{b}}$

$\mathcal{S}^{\cup}(card)$: $\mathcal{S}(card)$ + Noun Union

- For *base* nouns a, b, we allow a and b to be unioned: $a \cup b$
- We assume laws of boolean algebra at *syntax level* (?)
- **Semantics:** $[\![a \cup b]\!] = [\![a]\!] \cup [\![b]\!]$
- **Rules of Inference:**
  - **See board**

Conjecture: The set of rules given above for $\mathcal{S}^{\cup}(card)$ is complete.

$\mathcal{S}^{\dagger\cup}(card)$: $\mathcal{S}(card)$ + Noun Union + Noun Complement

- The Holy Grail!
- What is a complete set of rules?
    - Can't just haphazardly combine $\mathcal{S}^{\dagger}(card)$ and $\mathcal{S}^{\cup}(card)$

$\mathcal{S}^{\dagger\cup}(card)$: Proposed Rules

See board

- We can derive the first 6 ‘most’ rules already!
  - Remember that infinite schema!