Syllogistic Logics with 'More Than' and 'Most'

The Natural Logic Program

- Aristotle, Van Benthem, Moss
- Natural language inference
 - No translating into FOL
- Special-purpose *logic engineering*
 - Complete and decidable logics
 - o 5 different systems in this talk alone!

Logics about Size Comparison

- Restricted domain: Reasoning about sizes
- Why not embed into FOL with Arithmetic?
 - \circ Decidable \to More cognitively plausible
 - \circ Light \rightarrow More cognitively plausible
 - Counting is not needed:
 - "There is more sand than water in the pond"

$\mathcal{S}(card)$: Syllogistic + 'More Than' + 'At Least as Many'

- a, b, c, \ldots are nouns
- all(a,b)
- some(a, b)
- more(a, b)
- atLeast(a, b)
- No connectives $\land, \lor \lnot$
- No quantifiers \forall , \exists

$\mathcal{S}(card)$: Semantics

- Models: \mathcal{M} consisting of set M.
- For all nouns a, $\llbracket a \rrbracket \subseteq M$

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 \begin{array}{ll} \bullet & \mathcal{M} \models all(a,b) & \text{iff } \llbracket a \rrbracket \subseteq \llbracket b \rrbracket \\ \bullet & \mathcal{M} \models some(a,b) & \text{iff } \llbracket a \rrbracket \cap \llbracket b \rrbracket \neq \emptyset \\ \bullet & \mathcal{M} \models more(a,b) & \text{iff } |\llbracket a \rrbracket | > |\llbracket b \rrbracket | \\ \bullet & \mathcal{M} \models atLeast(a,b) & \text{iff } |\llbracket a \rrbracket | \geq |\llbracket b \rrbracket | \\ \end{array}
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• $\Gamma \models \varphi$ iff every **FINITE** model $\mathcal M$ which satisfies Γ satisfies φ .

$\mathcal{S}(card)$: Natural Deduction Rules

See board

$\mathcal{S}(card)$: Completeness Theorem

Theorem: $\mathcal{S}(card)$ is complete, i.e. $\Gamma \models \varphi$ iff $\Gamma \vdash \varphi$.

• From Larry Moss' "Syllogistic Logic with Cardinality Comparisons"

$\mathcal{S}^{\dagger}(card)$: $\mathcal{S}(card)$ + Noun Complement

- $\bullet \;\;$ For a noun a, we allow a to be complemented: \bar{a}
- $ullet \ ar{ar{a}} = a$ at syntax level
- ullet Semantics: $[\![ar{a}]\!]=M\setminus [\![a]\!]$
- Rules of Inference:
 - See board

$\mathcal{S}^{\dagger}(card)$: Completeness Theorem

Theorem: $\mathcal{S}^{\dagger}(card)$ is complete, i.e. $\Gamma \models \varphi$ iff $\Gamma \vdash \varphi$.

• More complicated than the proof for $\mathcal{S}(card)$!

Bonus Theorem: $\mathcal{S}^{\dagger}(card)$ is decidable in polynomial time!

$\mathcal{S}(most)$: Syllogistic + 'Most'

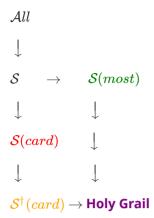
- all(a,b)
- some(a, b)
- most(a, b)
- $\bullet \quad \mathcal{M} \models most(a,b) \text{ iff } |\llbracket a \rrbracket \cap \llbracket b \rrbracket| > \frac{1}{2} \llbracket a \rrbracket$

$\mathcal{S}(most)$: Natural Deduction Rules

See board

Theorem: $\mathcal{S}(most)$ is complete. -- and decidable in polynomial time!

The Hierarchy of Syllogistic Logics



- We want a complete logic involving 'more than' and 'most'
- Can't just combine rules, because of interplay between 'more than' and 'most'

A Solution: Complement and Intersection

- $\bullet \ \ most(a,b) \equiv more(a \cap b, a \cap \overline{b})$
- So we need **noun complement** *and* **noun intersection**
- $\bullet~$ It is easier to make rules for Union $\cup~$
 - $ullet a\cap b\equiv \overline{ar a\cap ar b}$

$\mathcal{S}^{\cup}(card)$: $\mathcal{S}(card)$ + Noun Union

- ullet For *base* nouns a,b, we allow a and b to be unioned: $a\cup b$
- We assume laws of boolean algebra at syntax level (?)
- Semantics: $\llbracket a \cup b \rrbracket = \llbracket a \rrbracket \cup \llbracket b \rrbracket$
- Rules of Inference:
 - See board

Conjecture: The set of rules given above for $\mathcal{S}^{\cup}(card)$ is complete.

$\mathcal{S}^{\dagger \cup}(card)$: $\mathcal{S}(card)$ + Noun Union + Noun Complement

- The Holy Grail!
- What is a complete set of rules?
 - \circ Can't just haphazardly combine $\mathcal{S}^\dagger(card)$ and $\mathcal{S}^\cup(card)$

$\mathcal{S}^{\dagger \cup}(card)$: Proposed Rules

See board

- We can derive the first 6 'most' rules already!
 - Remember that infinite schema!