

III Cardinality of sets

Card = num of elements in the set

Infinite Cardinality:

$|A|, |B|$

$f: A \rightarrow B$ and $g: B \rightarrow A \rightarrow$ Schroder-Bernstein theorem

$f: A \rightarrow B$ tells us that $|A| \leq |B|$ and

$g: B \rightarrow A$ is $|B| \leq |A|$ so it should be $|A| = |B|$

1) Show that all ints are countable:

$$f(n) = \frac{n}{2} \quad f(n) = \frac{(n-1)}{2}$$

$$\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 0 & 1 & -1 & 2 & -2 & 3 & -3 \end{matrix}$$

2) Prove that rational num is countable

\aleph_0 Aleph Null

$$\{1, 2, 3, 4, 5, 6, 7, \dots\}$$

$\leftrightarrow \mathbb{N} \times \mathbb{N}$ The subset of even number and odd

$\leftrightarrow \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ or subset of present numbers

$\leftrightarrow \mathbb{N} \times \mathbb{N} \times \mathbb{N}$

$\leftrightarrow \mathbb{N} \times \mathbb{N} \times \mathbb{N}$

3) Since room is countable and name is uncountable, we cannot fit people in Hilbert's hotel