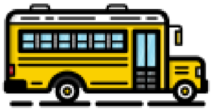
# Optimizing School Bus Operations

Denver Public School



**Group:23**
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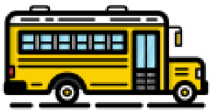
# Appendix

# Background



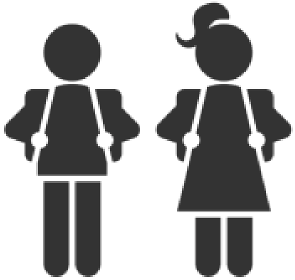
# Overview

Around 90,000 kids are transported by Denver Public Schools (DPS)

A number of states in the US have laws requiring schools to offer transportation to all eligible students

It will cost more than $23 billion to transport K–12 students in the United States each year, who total more than 25 million

Despite efforts to serve all eligible students, school districts face numerous obstacles, including operational costs, bus stop and route designations, and driving time



# Business Problems

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Developing and review bus-route assignments using distance data from Google Maps Application Programming Interface manually has been challenging and time consuming for DPS

Minimizing cost and deviation from the existing system $

Minimize the no. of miles bus travels empty (reposition miles)

Provide transportation to maximum possible eligible students

Optimize the no of buses and bus-drivers required to reduced the need to take contracted third-party services

# Model Description

# Data Consideration

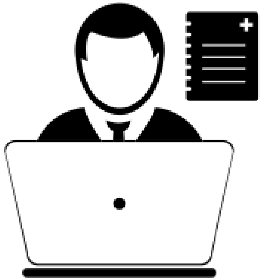
- B: Set of buses (i = 1, . , n)
- S: Set of routes (j = 1, .  , m)
- $S_0$: Set of routes including 0 to represent the bus's terminal (j = 0, 1, . . . , m)

- $m_{i0j}$: Reposition miles from bus i's terminal to the start of route j, $i \in B$, $j \in S$
- $m_{jk}$: Reposition miles from the end of route j to the start of route k, $j \in S$, $k \in S$
- $m_{j0i}$: Reposition miles from end of route j to the terminal of bus i, $i \in B$, $j \in S$
- $a_{jk}$: Slack time if route k is served immediately after route j by the same bus

# Data Consideration

- c: Penalty for using a bus
- e: Penalty for not serving a route
- r: Penalty for reposition miles
- v: Penalty for deviating from the current bus-route assignments
- b: Value under which slack time is penalized in the objective function
- s: Penalty for small slack time between two consecutive routes

- $c_i$: Capacity of bus $i \in B$
- $c_{wi}$ : Wheelchair capacity of bus $i \in B$
- $\hat{y}_{i0j}$ : bus i that covers route j first in the current bus-route assignment
- $\hat{y}_{ijk}$ : bus i that covers route j and then k, where $j \neq k$ in the current bus-route assignment
- $\hat{y}_{ij0}$ : bus i that serves route j last in the current bus-route assignment



# Decision Variables & Objective

**Decision Variables**

- Yi0j : bus i serving route j first?
- Yijk : bus i covering route j and then route k?
- Yij0 : bus i returning from j to its terminal?
- xj : route j is not being covered by any bus?

**Objective Function**

Penalty for deviation from current assignment + Penalizing low slack time below a threshold + Penalty for total repositioning in miles for entire trip of the bus + Penalty for not serving a route + Penalty for utilizing a bus

# Constraints

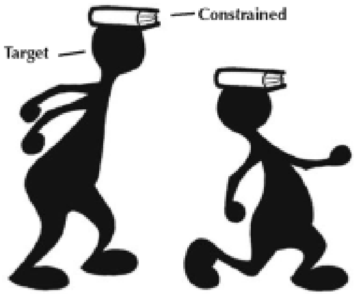
Each bus can only serve one route first : $\Sigma\, Yi0j <= 1 , \forall i \in B$

Flow constraint : $\sum Yi0j = \sum Yijk, \forall i \in B, \forall j \in S$

The route can be either served or cannot be served (sum of DV=1) : $xj + \sum Yi0j = 1, \forall j \in S$

Non Negativity Constraints : $Yi0j >= 0, Yijk >= 0, Yij0 >= 0, xj >= 0$

Binary Constraints : $Yi0j, Yijk, Yij0, xj$ are binary



# Decision Support Tool

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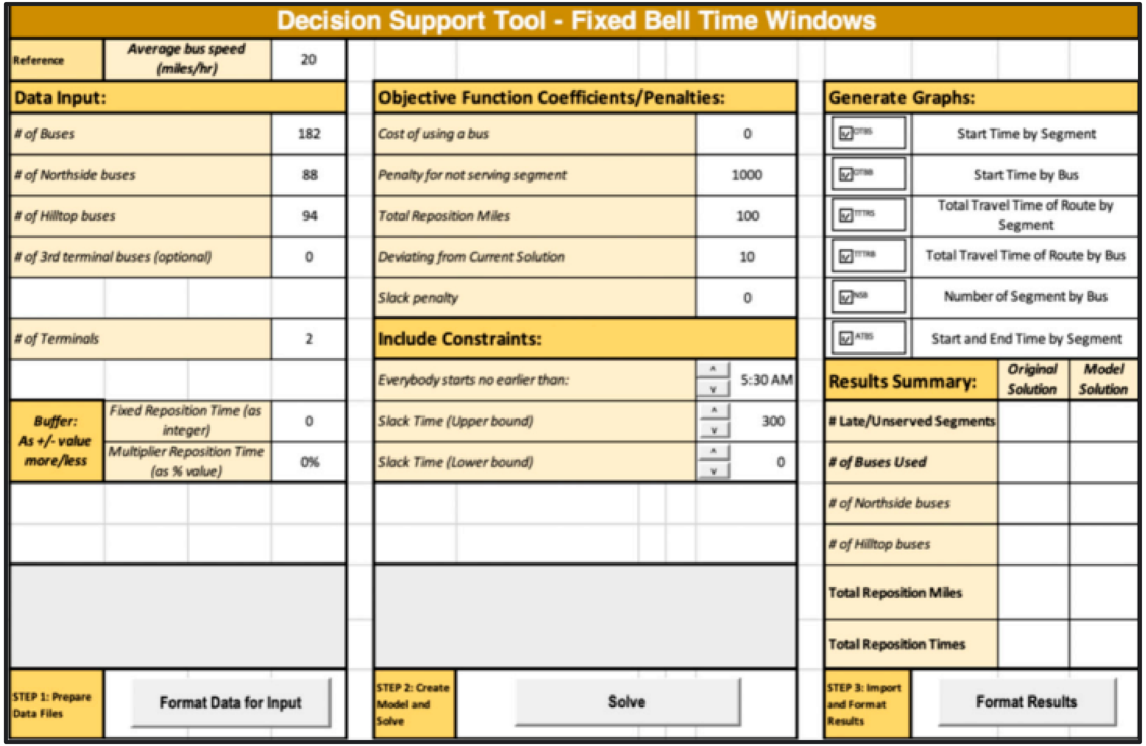



The DST uses models that were written in the mathematical programming language AMPL and is populated with data from the DPS. This allows the DST to call any integer programming (IP) solution. Currently, the DST makes use of the GNU Linear Programming Kit.

# Results & Benefits

# Results & Benefits

Given the strategic objective of minimizing reposition time and miles between routes to meet shifting traffic conditions around Denver, DST assisted DPS in helping it immediately detect possible problems with bus-route assignments.

The DST method would result in an 8% reduction in the number of buses used and a 20% reduction in reposition miles compared to the prior route assignment, according to the pilot testing on a subset of bus routes during the 2017–2018 school year.

Over 700 routes were allocated to 200 buses during the 2017–2018 school year. DPS analysts analyzed and altered these routes while taking into account the viability of adopting DST for time and capacity.

The DST lessened the need for third-party services to fill bus-driver shortages and let DPS begin the new school year with a realistic and solid school bus-route assignment plan (considering the nationwide bus-driver shortage)

The DPS transportation team has been utilizing the DST in the route planning process since 2018 due to its simplicity, effectiveness, and flexibility in producing high-quality bus-route assignments.

# Teaching Example

# Business Problems - Taken In Scope For Model Design

Minimize the no. of miles bus travels (reposition miles)

Provide transportation to maximum possible eligible students

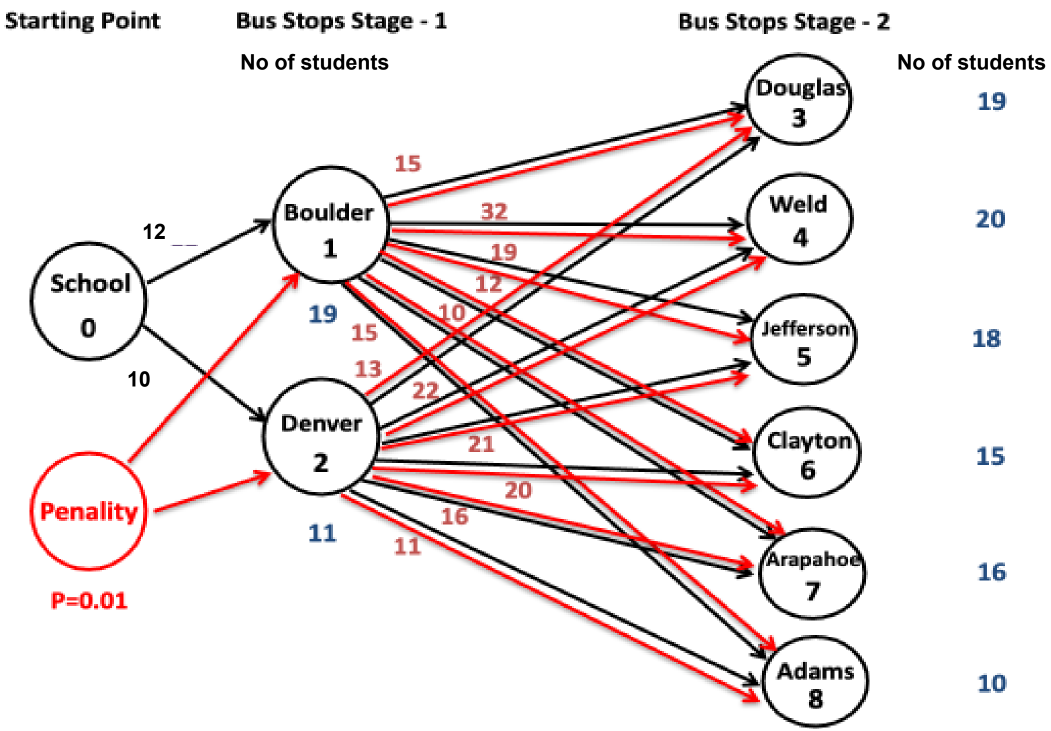
## Data Considerations

**$M_{ij}$** : The reposition miles from location i to j
**$N_{jl}$** : The reposition miles from location j to l
**$C_m$** : No of students at each pickup location in Stage 1
**$D_n$** : No of students at each pickup location in stage 2
**P** : Penalty for not picking up a student

## Model Design

- We have created a micro model considering assignment of route to 1 school bus
- The model's aim is to determine which location is to be covered at Stage-1 and Stage-2
- We have considered school (Node-0) to be the starting point of bus
- Stage - 1: Consists of 2 locations through which the bus will pick up students and traverse to the next stop
- Stage - 2: Consists of 6 end-locations where the bus will pick up the students and return back to school
- A penalty is assigned for not picking up the eligible students

# Bus Route Flow




Overview | Business Problem | Data Consideration | Data Model | Results | Teaching Example

# Decision Variables & Objective

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**Decision Variables**

The decisions to be made: **If the route should be taken or not?**

**Stage 1:**

**$X_{ij}$:** Where i is the starting node and j is the ending node **$i \in \{0\}$, $j \in \{1,2\}$**

**Stage 2:**

**$Y_{jl}$:** Where j is the starting node and l is the ending node **$j \in \{1, 2\}$, $l \in \{3, 4, 5, 6, 7, 8\}$**

**Objective Function (z)**

Minimize the reposition miles

$\Sigma X_{ij} \times M_{ij}$ where **$i = 0$ , $j \in \{1, 2\}$** +

$\Sigma Y_{j\square} \times N_{jl}$ where **$j \in \{1, 2\}$ , $l \in \{3, 4, 5, 6, 7, 8\}$**

+

Minimize the penalty of not picking up students

$\Sigma(1-X_{ij}) * C_m * P$ where **$i=0$ & $j,m \in \{1, 2\}$** +

$\Sigma(1-Y_{jl}) * D_n * P$ where **$j \in \{1, 2\}$ & $l,n \in \{3, 4, 5, 6, 7, 8\}$**

# Constraints

**Binary & Non-Negativity Constraints :**

$X_{ij}$ = Binary,$X_{ij}$ >= 0 where i=0 , j $\in$ {1, 2}

$Yjl$ = Binary, $Yjl$ >= 0 where j $\in$ {1, 2} , l $\in$ {3, 4, 5, 6, 7, 8}

**Bus Capacity Constraint :**

$(\Sigma(X_{ij} \times Cm) + \Sigma(Yjl \times Dn)) <= 40$ where i=0 , j,m $\in$ {1, 2} & l,n $\in$ {3, 4, 5, 6, 7, 8}

**Flow Balancing Constraints :**

Node 0 : $-X_{01} - X_{02} = -1$

Node 1 : $X_{10} - Y_{13} - Y_{14} - Y_{15} - Y_{16} - Y_{17} - Y_{18} = 0$

Node 2 : $X_{20} - Y_{23} - Y_{24} - Y_{25} - Y_{26} - Y_{27} - Y_{28} = 0$

Node 3 : $Y_{31} + Y_{32} <= 1$

Node 4 : $Y_{41} + Y_{42} <= 1$

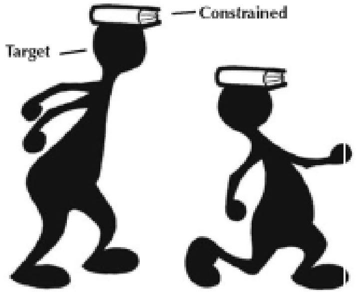
Node 5 : $Y_{51} + Y_{52} <= 1$

Node 6 : $Y_{61} + Y_{62} <= 1$

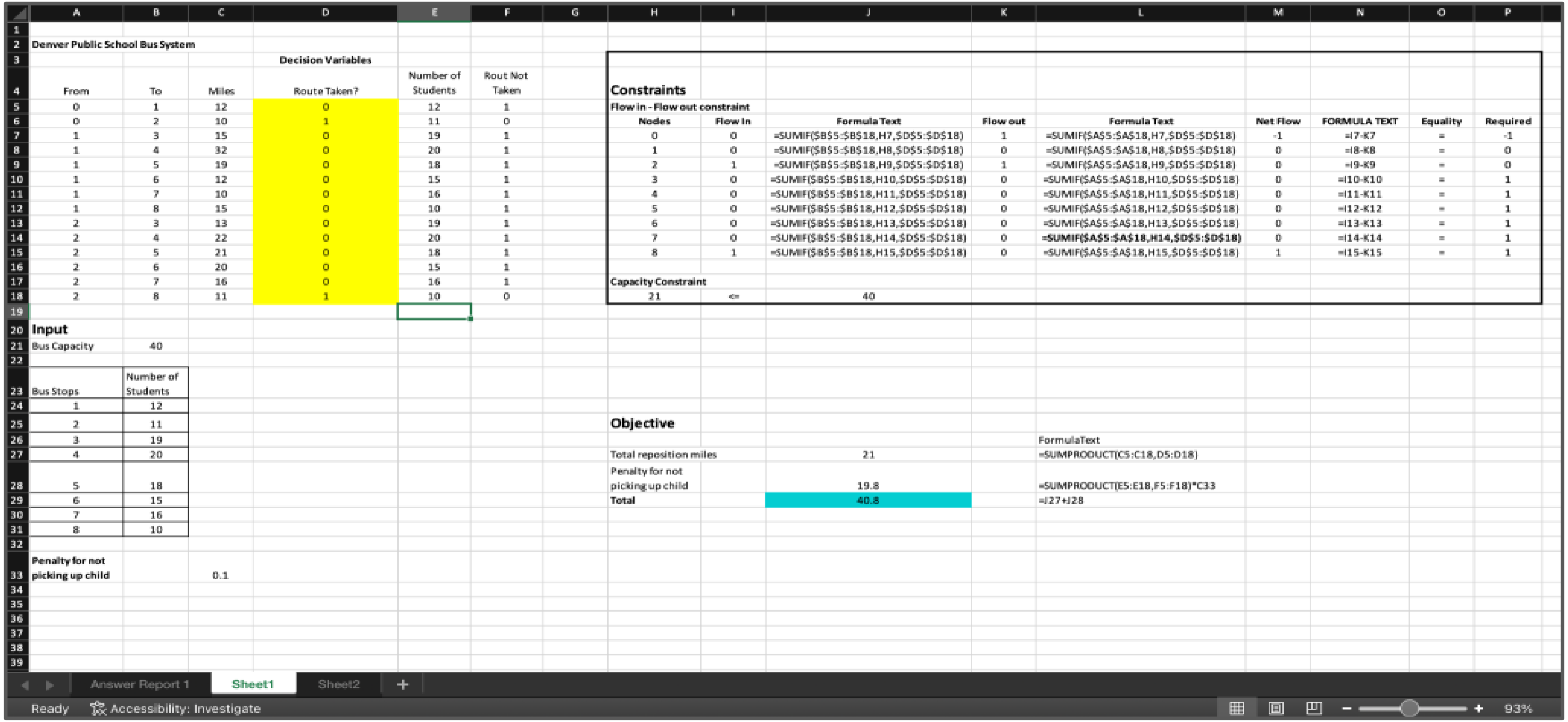
Node 7 : $Y_{71} + Y_{72} <= 1$

Node 8 : $Y_{81} + Y_{82} <= 1$

$Y_{31} + Y_{32} + Y_{41} + Y_{42} + Y_{51} + Y_{52} + Y_{61} + Y_{62} + Y_{71} + Y_{72} + Y_{81} + Y_{82} = 1$



# Model Solution



Denver Public School Bus System

| From | To | Miles | Decision Variables<br>Route Taken? | Number of Students | Rout Not Taken |
|---|---|---|---|---|---|
| 0 | 1 | 12 | 0 | 12 | 1 |
| 0 | 2 | 10 | 1 | 11 | 0 |
| 1 | 3 | 15 | 0 | 19 | 1 |
| 1 | 4 | 32 | 0 | 20 | 1 |
| 1 | 5 | 19 | 0 | 18 | 1 |
| 1 | 6 | 12 | 0 | 15 | 1 |
| 1 | 7 | 10 | 0 | 16 | 1 |
| 1 | 8 | 15 | 0 | 10 | 1 |
| 2 | 3 | 13 | 0 | 19 | 1 |
| 2 | 4 | 22 | 0 | 20 | 1 |
| 2 | 5 | 21 | 0 | 18 | 1 |
| 2 | 6 | 20 | 0 | 15 | 1 |
| 2 | 7 | 16 | 0 | 16 | 1 |
| 2 | 8 | 11 | 1 | 10 | 0 |

**Input**

Bus Capacity 40

| Bus Stops | Number of Students |
|---|---|
| 1 | 12 |
| 2 | 11 |
| 3 | 19 |
| 4 | 20 |
| 5 | 18 |
| 6 | 15 |
| 7 | 16 |
| 8 | 10 |

**Penalty for not picking up child** 0.1

**Constraints**

Flow in - Flow out constraint

| Nodes | Flow In | Formula Text | Flow out | Formula Text | Net Flow | FORMULA TEXT | Equality | Required |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | =SUMIF($B$5:$B$18,H7,$D$5:$D$18) | 1 | =SUMIF($A$5:$A$18,H7,$D$5:$D$18) | -1 | =I7-K7 | = | -1 |
| 1 | 0 | =SUMIF($B$5:$B$18,H8,$D$5:$D$18) | 0 | =SUMIF($A$5:$A$18,H8,$D$5:$D$18) | 0 | =I8-K8 | = | 0 |
| 2 | 1 | =SUMIF($B$5:$B$18,H9,$D$5:$D$18) | 1 | =SUMIF($A$5:$A$18,H9,$D$5:$D$18) | 0 | =I9-K9 | = | 0 |
| 3 | 0 | =SUMIF($B$5:$B$18,H10,$D$5:$D$18) | 0 | =SUMIF($A$5:$A$18,H10,$D$5:$D$18) | 0 | =I10-K10 | = | 1 |
| 4 | 0 | =SUMIF($B$5:$B$18,H11,$D$5:$D$18) | 0 | =SUMIF($A$5:$A$18,H11,$D$5:$D$18) | 0 | =I11-K11 | = | 1 |
| 5 | 0 | =SUMIF($B$5:$B$18,H12,$D$5:$D$18) | 0 | =SUMIF($A$5:$A$18,H12,$D$5:$D$18) | 0 | =I12-K12 | = | 1 |
| 6 | 0 | =SUMIF($B$5:$B$18,H13,$D$5:$D$18) | 0 | =SUMIF($A$5:$A$18,H13,$D$5:$D$18) | 0 | =I13-K13 | = | 1 |
| 7 | 0 | =SUMIF($B$5:$B$18,H14,$D$5:$D$18) | 0 | **=SUMIF($A$5:$A$18,H14,$D$5:$D$18)** | 0 | =I14-K14 | = | 1 |
| 8 | 1 | =SUMIF($B$5:$B$18,H15,$D$5:$D$18) | 0 | =SUMIF($A$5:$A$18,H15,$D$5:$D$18) | 1 | =I15-K15 | = | 1 |

Capacity Constraint

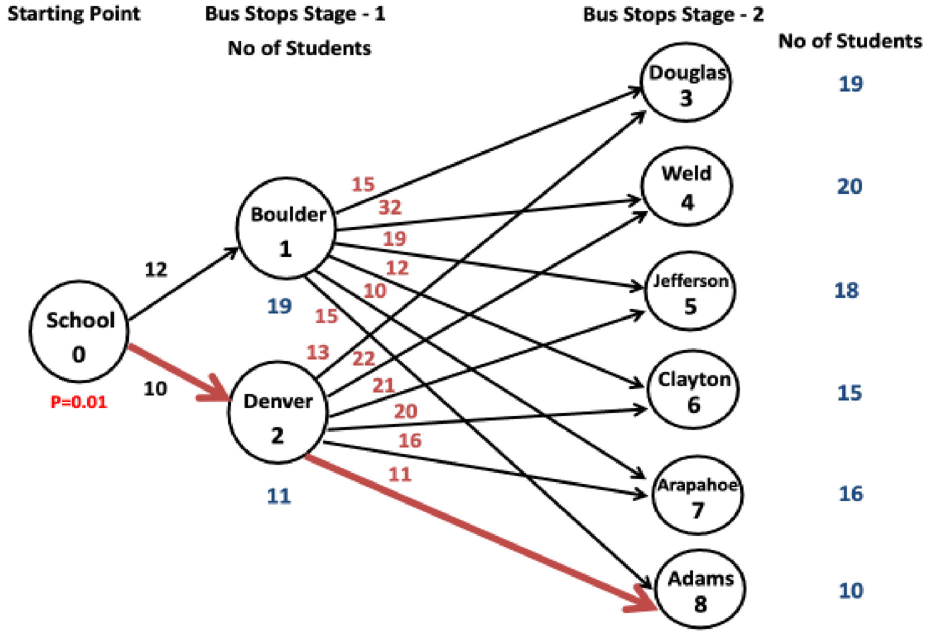
21 <= 40

**Objective**

| | | FormulaText |
|---|---|---|
| Total reposition miles | 21 | =SUMPRODUCT(C5:C18,D5:D18) |
| Penalty for not picking up child | 19.8 | =SUMPRODUCT(E5:E18,F5:F18)*C33 |
| Total | 40.8 | =I27+I28 |

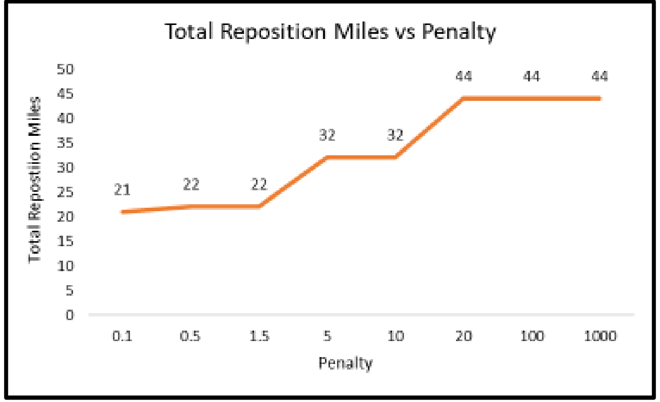
# Optimal Solution






**Route 0 to 2 and 2 to 8 was the optimal solution given by solver!**

Total Reposition Miles vs Penalty




Observed trend of increase in reposition miles as the penalty increases! 😮

# References

# References

Amanda Chu, Pinar Keskinocak, Monica C. Villarreal (2020) Empowering Denver Public Schools to Optimize School Bus Operations. INFORMS Journal on Applied Analytics 50(5):298-312. https://doi.org/10.1287/inte.2020.1042