Aishameriane Venes Schmidt

Assessing the impact of conventional monetary policy on the capital-labor ratio in Brazil

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Aishameriane Venes Schmidt[1]

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Esta Dissertação foi julgada adequada para obtenção do Título de "Mestra" e aprovada em sua forma final pelo Programa de Pós Graduação em Economia.

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Para o Lilo, em memória.

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Naturally, I am the solo responsible for any of the remaining errors and omissions.

"Ever tried. Ever failed. No matter. Try again. Fail again. Fail better."

(Samuel Beckett)

RESUMO

A crise financeira internacional de 2008 intensificou o debate sobre a existência de efeitos redistributivos da política monetária, e, em particular, sobre quais são os efeitos sobre a renda, riqueza e consumo. Com relação à primeira, uma vez que a composição da renda entre diferentes fontes irá variar entre os indivíduos, eles perceberão de maneira distinta os efeitos das mudanças nas taxas de juros. Além disso, as taxas de juros também afetarão de maneira heterogênea as famílias de acordo com a maturidade dos seus ativos e passivos, além de ter um efeito indireto através do seu impacto na inflação. No entanto, o efeito líquido da política monetária através dos seus canais redistributivos é incerto, considerando tanto a teoria como os estudos empíricos existentes na literatura atual. Com o objetivo de investigar os efeitos redistributivos da política monetária convencional no Brasil, nós utilizamos as séries de dados mensais da razão capital-trabalho, PIB per capita, taxa de inflação, taxa de câmbio e taxa de juros Selic em um modelo de vetor autoregressivo bayesiano com parâmetros variando no tempo. Este modelo é uma proposta deste trabalho e é uma extensão do VAR bayesiano com coeficientes estáticos e volatilidade estocástica Wishart desenvolvido no trabalho de Uhlig (1997). O período de análise compreende os meses entre Março de 2000 até Outubro de 2018 e corresponde à quase todo o período de regime de metas, que foi implementado na economia Brasileira no ano de 1999. Os resultados mostram uma resposta positiva e significativa da razão capital-trabalho a um choque de política monetária contracionista e que dura pelo menos um semestre, isto é, um aumento na taxa de juros desloca a renda do trabalho para o capital, sugerindo que o aumento na taxa de juros tem um efeito redistributivo não-negligível. Este resultado não é estável ao longo do tempo e foram observadas mudanças no comportamento das funções impulso resposta ao longo da amostra, resultantes da mudança temporal de alguns dos parâmetros do modelo. Isto implica que a relação entre a política monetária e a distribuição da renda entre os fatores de produção se alterou e, mais especificamente, ficou mais fraca nos últimos anos, considerando o período de tempo analisado.

Palavras-chave: Política monetária. Distribuição de renda. Econometria Bayesiana. Vetores autoregressivos. Volatilidade estocástica Wishart.

ABSTRACT

The aftermath of the Great Recession intensified the debate on the heterogeneous effects of monetary policy (MP), specially on income, wealth and consumption. Regarding the first, changes in the interest rates can have various effects on individuals depending on the household income composition; the maturity of their liabilities and assets and, indirectly, through inflation. However, the net effect of the monetary policy through these channels is uncertain, considering both theoretical and empirical studies. In order to investigate the redistributive effects of conventional monetary policy in Brazil, we used the series of the capital-labor ratio, as well as monthly data for GDP, inflation rate, exchange rate and interest rate in a Bayesian autoregressive vector model using our proposed extension of Uhlig's (1997) BVAR to a time-varying parameter framework. The data used for posterior computations comprises the monthly observations between March 2000 to October 2018, which corresponds to the inflation targeting regime in the Brazilian economy. The results show a positive and significant response of the capital-labor ratio to contractionary monetary shocks, which lasts at least a semester, i.e., a contractionary MP shock shifts income from labor to capital, which suggests that interest rate shocks have a non-negligible redistributive effect. This result is not stable over time and changes in the impulse response functions across the sample were observed due to a time-varying behavior of some of the model parameters. This implies that the relationship between the monetary policy and distribution of income between production factors has changed over time. Moreover, it became less intense in the last years of the sample period analyzed.

Keywords: Monetary Policy. Income distribution. Bayesian Econometrics. Vector autoregressions. Wishart stochastic volatility.

JEL classification: E52; E25; E64; C11.

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LIST OF ABBREVIATIONS AND ACRONYMS

ACF	Autocorrelation function
BACEN	Central Bank of Brazil
BVAR	Bayesian vector autoregression
CB	Central Bank
CODACE	Economic Cycle Dating Committee
Copom	Monetary Policy Committee of the Central Bank of Brazil
HANK	Heterogeneous Agent New Keynesian (model)
IBGE	Brazilian Institute of Geography and Statistics
IT	Inflation targeting
MP	Monetary Policy
MSV	Multivariate stochastic volatility
CMN	National Monetary Council
PNAD	National Household Sample Survey
rv	Random variable
RANK	Representative Agent New Keynesian (model)
Selic	Short-term interest rate, the interest rate for overnight interbank loans collateralized by government bonds registered with and traded on the Sistema Especial de Liquidação e Custódia
SV	Stochastic volatility
Swap	Swap reference rate - preset DI rate (BM&F) - 90-day term.
TVP-VAR	Time-varying vector autoregressive
VAR	Vector autoregressive

LIST OF SYMBOLS

$^{K}/_{L}$	Capital-labor ratio
y_t	Vector of observed variables
α_t	Vector of latent time-varying states
Ω_t	Stochastic time-varying covariance matrix
$\mathcal{N}_n(\mu, \Sigma)$	The multivariate normal distribution (of dimension n) with mean vector μ and covariance matrix Σ
$\mathcal{W}_p(S, \nu)$	The Wishart distribution (of dimension $p \times p$) with ν degrees of freedom and scale matrix S
$\mathbb{I}_n$	The $(n \times n)$ identity matrix

CONTENTS

1 INTRODUCTION

"There may be wide difference of opinion as to the significance of a very unequal distribution of wealth, but there can be no doubt as to the importance of knowing whether the present distribution is becoming more or less unequal."

MAX O. LORENZ

The 2008's financial crisis jolted the apparent agreement on the roles of monetary and fiscal policy authorities, causing a profound impact on the economic research agenda (CARVALHO et al., 2015). Concurrently, the emerging flow of data showing disparities in the concentration of wealth and income brought the holophotes to economic inequality while disputing the idea behind the Kuznets' inverted U curve, in which economic growth is preceded by an increase in inequality that starts to decline after the Economy attains a certain level of per capita income (PIKETTY; SAEZ, 2003). In the intersection, lies the problem of lack of theoretical models linking monetary policy (MP) to redistribution, which likely reflects the general understanding that the issue on redistribution should be seen as a separated topic from the aggregation stabilization (AUCLERT, 2017). On top of these, the number of empirical studies relating monetary policy and income distribution is still small and does not allow us to draw generalised conclusions (DAVTYAN, 2017; FURCERI; LOUNGANI; ZDZIENICKA, 2018). At first all these layers may seem almost randomly entangled but, as we shall argue in this thesis, there is a string that binds them together and at the same time challenges the conventional view on monetary policy.

After the massive shocks and hyperinflation episodes that ravaged some economies in the last century followed by the unsuccessful efforts from fiscal policy to solely promote economic growth and stability in the seventies, there is a general agreement regarding the relevance of monetary policy in the economy as well as the importance that the Central Banks (CB) have on price stability. In many existing Central Banks, to maintain a stable inflation level is perceived as the major/main goal of monetary policy and it may be found in companion of seeking control of unemployment rates and/or output gap as well. This view is intrinsically bounded to the understanding that prices exert a major influence in the economy through its effects on consumption and savings decisions as well as investment spending decisions. If the uncertainty induced by the out of control price level is too high, the risk associate to economic decisions is higher as well and people could be discouraged to choose future consumption over present comsumption or could change their investments decisions in favor to a more secure behavior (MISHKIN, 2010).

In some sense, it appears to exist an implicit idea of wellbeing behind the monetary policy decisions: if the inflation is under control, the economic agents will be better off. However, it is also an agreement that in order to best achieve its targets, the Central Banks should pursue few goals and, in general, this does not include distributional concerns. But this leads us to a puzzling situation: it has been more common to find Central Bankers talking about income, wealth and consumption inequality and distribution, as in Deutsche Bundesbank (2016), Bernanke (2015a), Cœuré (2012), Yellen (2014), Panetta (2015) and Bernanke (2015b), just to name a few. If inequality and distribution are not CB targets, what is going on with all these policy makers talking about the subject?

A simplified answer to this question lies in the structure of economic models. It is undeniable that representative agent models were one of the major contributions to macroeconomic modelling and grew beyond academic boundaries to be an important engine in modern macroeconomic policy. During the 1990's, there was also an intensive research agenda on explaining the mechanisms behind monetary policy as clear as possible and there was no dispute in the effects of the interest rates over inflation - see, for example, Mishkin (1995) or Bernanke & Gertler (1995). The combination of those two elements resulted in the incorporation of monetary policy rules in macroeconomic models using representative agents, most of which could successfully replicate stylized facts of the economy, but none of which including redistributional effects, an obvious consequence of such models. However, this approach was challenged in the aftermath of the Great Recession, with the monetary policy and the role of Central Banks being directly questioned not only by academics but also by the non-academic public, since the unconventional monetary policy seemed to favor some at the cost of others. Concurrent to this, the developments of heterogeneous agent models started to show that the efficacy of monetary policy could depend on the degree of heterogeneity in the population (KAPLAN; MOLL; VIOLANTE, 2018). Results from these models show that there is a link between monetary policy and redistribution that goes beyond the effects of inflation over consumption.

By allowing models to have more than one type of household, it became possible to understand the role of the marginal propensities to consume in the transmission mechanism of monetary policy. People will lose or gain from changes in the interest rates depending on their assets composition and maturity, which would allow them to spend more or less in consumption (AUCLERT, 2017). Depending on the structure of a given economy, it is most likely that the positive and negative effects on households balance sheets will not cancel out, causing the aggregate effect of the monetary policy to be uncertain. Empirical works addressing the interaction between monetary policy and distribution/inequality also emerged in the literature, but there is still no consensus on the subject (FURCERI; LOUNGANI; ZDZIENICKA, 2018).

Although the literature addressing the redistribution channels of monetary policy is useful to understand how interest rates can affect inequality and distribution in a *ceteris* *paribus* environment, it is imperative to have empirical studies looking at the combined effect altogether. As pointed out by Casiraghi et al. (2017), the same framework that holds everything else constant except by one or another channel which allows to evaluate individual effects can be misleading because the interactions between different channels when functioning simultaneously can change the overall result in comparison to only one effect at a time. Bernanke (2015a) also advocates in favor of an aggregate approach, emphasizing that the best course of action to investigate the distributional effects of MP is by comparing changes in the flow of income from capital investments to labor (and vice-versa).

Regarding the growth in empirical studies relating monetary policy and income distribution, there are no such studies for the Brazilian economy. Thus, with this thesis we sought to investigate the joint dynamics of monetary policy shocks on the capital-labor ratio ($^K/_L$), which accounts for the ratio between the capital income and the labor income using data from Brazil. The idea behind it is that given an uneven composition of income, shifts in the share of national income allocated to wages or to profits imply a redistributional effect. We estimated a Bayesian time-varying parameter vector autoregression (TVP-VAR) with Wishart innovations, which is our generalized version of the model first proposed by Uhlig (1997). This model is flexible enough for us to verify if exogenous shocks on the interest rate can affect $^K/_L$ through the impulse response analysis. In addition, by allowing time variation in the model coefficients, we can capture changes in the relationship between the variables, while the stochastic volatility can better fit the changes in the behavior of the shocks that affect the economy.

Results show that the effect of monetary shocks over $^K/_L$ is positively significant and lasts at least fifteen months, suggesting that there is a non-negligible redistributive effect of monetary policy over income distribution. Expansionary monetary shocks lead to the increase of the capital-labor ratio, which can signify either the increase in the share of capital income or the reduction of the share of labor income.

Our contribution to the literature goes in two directions. First, we are unaware of similar studies relating monetary policy and functional income distribution using data from Brazil. Moreover, the core part of international literature that is emerging on this area is focused in addressing the monetary impact on income, wealth or consumption inequality. Given that, our empirical approach provides a different way to see the aggregate redistributive affects of monetary shocks, focusing on the remuneration of the production factors (capital and labor). Second, we are proposing an extension of the bayesian vector autoregressive (BVAR) Wishart model first proposed by Uhlig (1997) that has technical advantages regarding specification and estimation in comparison to the concurrent TVP-VAR models that are currently used in the literature.

It is equally important to stress what this work does not ought to answer. Income distribution has a link with economic inequality which can lead to other forms of inequality and vice-versa. For example, some forms of discrimination based on, but not limited to, gender, race, religion that lead to unequal access of opportunities, employment, education, etc, can promote income inequality and the other way around. Although this is a relevant topic, it is not our aim to make the analysis of the monetary policy social or economic implications on income inequality. Similarly, we are not making any policy prescriptions based on the results found. The present work is bounded to the investigation of the existence (or not) of a phenomena - impact of monetary shocks in the functional distribution of income - and, if it is the case, quantify it, using the appropriate econometric tools. Regarding our econometric model, since it is in some way a novel proposal, it would have been interesting to make performance tests to verify how our solution works with synthetic data and compare to the results obtained by concurrent models. However, this thesis is an empirical work and such analysis, although very intellectually compelling, would deviate largely from our main goal and would clash with the time constraint that a master program imposes.

This thesis, besides the introduction and concluding remarks, is divided in three main parts. The first part is dedicated to the economical background concerning monetary policy and its relationship with income distribution. It has a discussion regarding the traditional view on the monetary transmission channels and it presents the redistribution channels that have been proposed in recent years in the economic academic literature. This part also contains a summary of some empirical studies that link monetary policy to income, wealth and/or consumption inequality. Part II presents a review on the main specifications of TVP-VARs that are currently being used in the literature as well as the BVAR model from Uhlig (1997). Then, we introduce our extension to this model and how to use the propositions from Windle & Carvalho (2014) to estimate the model parameters. In part III we empirically test our hypothesis of monetary redistribution effects. We start by presenting the data then proceed to the results and discussion. This is complemented by some data descriptive analysis and convergence diagnostics that are explained in the Appendix. Also in the Appendix, we present a brief introduction to Bayesian Inference, enough for the non-bayesian reader understand the main ideas of our estimation procedure.

Part I

Economics Background

2 MONETARY POLICY, HETEROGENEITY AND INCOME DISTRIBUTION

> *"Monetary policy is a powerful tool, but one that sometimes has unexpected or unwanted consequences."*
>
> Frederic S. Mishkin

The distribution of income has been an important subject in the economic debate and there is an ever-growing literature trying to address what are the impacts that macroeconomic factors, including interest rates, have on inequality and distribution of income, wealth and consumption (see, for example, Lucas, 2000; Anand & Segal, 2008; Areosa & Areosa, 2016; Benhabib, Bisin & Luo, 2017).

Specifically regarding monetary policy, the predominant idea before the Great Recession in 2008 was that an expansionary policy could reduce inequality in the short run, but for long lasting results in the well-being of the poor, it would be better for the monetary authority to aim for inflation control and stability of the aggregate demand (ROMER; ROMER, 1998). This view started to be challenged in the aftermath of the financial crisis, leading to theoretical and empirical studies investigating the redistributional aspects of conventional and unconventional monetary policy. However, there is no consensus regarding what are the redistributional channels, about the direction of the redistributional effects of the monetary policy and neither the magnitude of this relationship, i.e., it remains to be decided whether an increase (or decrease) in interest rates can rise or shrink inequality - or even significantly affect the distribution of income and wealth among different groups of individuals (FURCERI; LOUNGANI; ZDZIENICKA, 2018).

In this chapter, we start reviewing the conventional monetary policy and its transmission channels with a focus on inflation targeting regimes, which is followed by a discussion about redistribution channels that appeared more recently in the literature. Then, findings from empirical studies relating monetary policy to inequality (either in wealth, income or consumption) are discussed.

2.1 MONETARY POLICY

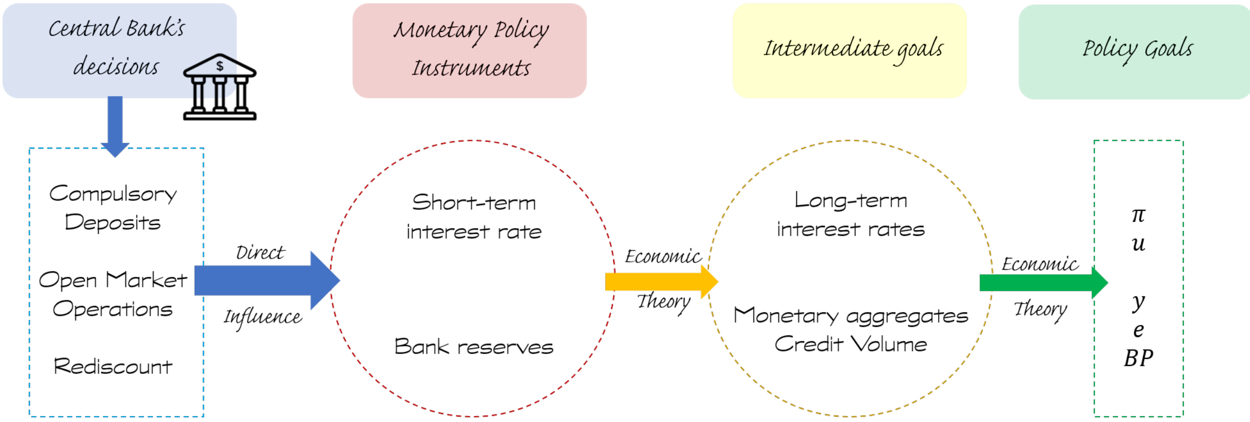
Monetary policy (MP) and central banks (CB) are intrinsically bounded to an extension that it is almost impossible to talk about one without at least mentioning the other. For the sake of conciseness, in the present work we will not enter in a discussion regarding money or the central banks' role in currency emission, supervision of the financial markets or holding international reserves. Our focus will remain to the aspects concerning the conduction of the monetary policy, including the instruments and transmission channels. For a comprehensive text on the aspects of monetary policy that were not included in here, we recommend reading Mishkin (2010) or Carvalho et al. (2015).

The origin of the first central banks lies in the first commercial banks that started to fund governments and economic development, obtaining, in exchange, the rights to issue money or other concessions (CARVALHO et al., 2015). With the increasing complexity of market economies, the role of Central Banks expanded and they became pivotal in the price stability and, ultimately, in helping to promote economic growth. A major tipping point in the course of monetary policy occurred with the reformulation of its theory led by Friedman and the rational expectations revolution. Until mid-sixties, the Keynesian recipe for boosting the effective demand via public spending was widely accepted while the monetary policy was seen as innocuous in influencing the economic development (CARVALHO et al., 2015). This view was challenged after the rise of inflation and unemployment combined with a falloff in economic growth that succeeded the supply shocks at the beginning of the seventies (SNOWDON; VANE, 2005). Nowadays, there is no dispute with regard to the power exerted by the central banks in the economy (MISHKIN, 1995).

Changes in the conduction of monetary policy are not at random, i.e., the central banks have goals that give purpose to their decisions. The CB then uses its instruments in order to achieve such goals, given the constraints imposed by the economy (GALÍ, 2015). However, the instruments cannot directly affect the policy goals: for example, it is not possible for the Central Bank to directly establish prices by setting the interest rate, assuming an economy without central planning. Broadly speaking, it is not really the inflation that is directly affected by the monetary shocks, but other aggregates that play a role in defining the price level, such as consumption. One of the most common cases is the intertemporal substitution: when interest rates are high, households have an incentive to postpone their present consumption in exchange of higher consumption in the future. Then, the demand for goods and services fall which causes the prices to go down as well, changing the inflation path. But this is not the only way which interest rate affects inflation. Formally, there are 4 well-established transmission channels that are consolidated in the literature and we will discuss them in the following subsections.

Figure 1 has a diagram representing the general trajectory of the monetary policy, from the Central Bank making the decisions until reaching it(s) target(s), which will vary depending on the policy goals. In practice, after taking the decision regarding its policy goal, the Central Bank makes open market operations (buying and selling of bonds), changes the percentage required of compulsory deposits for commercial banks and/or changes the aliquot for loans made to commercial banks (rediscount). Those directly drives the MP instruments[1], which can be the short-term interest rate - Selic, in Brazil's case - or the monetary base.

Figure 1 – Monetary policy transmission path



Notes: The Central Bank can use compulsory deposits, open market operations or rediscount to affect either the short-term interest rate (Selic rate, in the case of Brazil) or the Monetary basis, being the former the most common instrument of monetary policy - it is not possible to operate on both instruments simultaneously. Economic theory explains how the short-term rate can influence the long-term ones, the credit volume in the economy or the monetary aggregates, which will ultimately impact the policy targets. These can be the inflation (π) and/or unemployment (u) for the majority of cases, but monetary policy can also aim at other aggregates such as output/output gap (y), exchange rate (e) and balance of payments (BP).

Source – Own elaboration based on Carvalho et al. (2015).
Icon from the Noun Project <https://thenounproject.com/>.

Although the CB aims at final targets, denoted as *policy goals* in Figure 1, the results of MP do not take place immediately, i.e., the economy takes some time until responding to the Central Bank's actions. And there is no guarantees that the results will be the ones expected: it is possible, depending on the economic conditions, the credibility of the monetary authority and/or many other factors, that the resultant effect of the MP will be higher, lower or innocuous. Instead of waiting to see the final results of its policy (when it might be too late to take action), the CB then sets intermediate goals that help to see if the course of the monetary policy is converging towards the right final destination. Those intermediate goals are usually the long-term interest rates, the monetary aggregates, and the credit volume.

Economic theory gives reasoning on how the instruments affect the intermediate goals and how this is transmitted to policy goals. In the case of the long-term interest rates, they represent the equilibrium of the market's decisions in terms of demand and supply for bonds, which are driven by the agents' preferences and expectations regarding

[1] Carvalho et al. (2015) refer to the percentage of compulsory deposits, open market operations or rediscount rate as being the *instruments* of monetary policy whilst denoting the short-term interest and monetary base as *operational variables*. Since our focus is in the Brazilian economy who adopts the inflation targeting regime and we are not making a distinction on how the CB changes the interest rate in our model, whenever we talk about monetary policy instruments, we will be referring to the short term interest rate - unless explicitly stated otherwise.

the future. For example, if the economic agents believe that the short-term interests are too low and the inflation will rise, the long term interest rate will be higher, indicating that the market expects a future policy action to control the prices. The long term rates are those that will affect most of the households and firms decisions, which will have an impact on consumption and investment, changing or maintaining the course of inflation, output, unemployment and other related aggregates. As for the bank reserves, it will affect the amount of capital available for commercial banks to make loans which affect the money supply and credit volume in the economy. Those, by its turn, will also have an impact on the policy goals. As put by Carvalho et al. (2015), the intermediate goals will serve as indicators of MP results as they have a direct effect on the credit and capital's opportunity cost and availability or the aggregate level of spending.

Price stability, in the last decades, has received more attention from the central bankers, due to the awareness that inflation takes a heavy toll on society. The uncontrolled rise of the prices leads to more uncertainty regarding the economy, hindering economic growth (MISHKIN, 2010). Accordingly to Central Bank of Brazil (Bacen), *"keeping the inflation low, stable and predictable is the best contribution that the monetary policy can do in order to promote economic growth and welfare"* (Banco Central do Brasil, 2019d). The argument to support this idea is that the stabilization of prices allows households and firms to plan better, improving the conditions for business and general spending. There is also a strong argumentation in favor of the poor since it is believed that they suffer the most when inflation rises due to their lack of protection against inflationary tax. These claims are not exclusively used in Brazil and many other countries use that to justify the implementation of the inflation targeting (IT) regime, which is described in the next topic.

2.1.1 Inflation targeting regime

Before going into the details regarding the transmission mechanisms of monetary policy, we will briefly discuss the functioning of inflation targeting, since this is the current regime adopted by the Central Bank of Brazil. The adoption of this regime implies that the CB uses the short-term interest rate as its primary instrument and will have as its main policy goal the price level control. Until 2012, there were 27 countries who officially adopted inflation targeting and various others who were committed to some variation, including the United States, that adopts several recommendations that are inherent to this framework (HAMMOND, 2012).

The first country to adopt inflation targeting was New Zeland in 1989, and in the following years this regime was ratified by several other countries[2]. The inflation targeting,

[2] In 2012, the countries who followed the inflation targeting regime were: Armenia, Australia, Brazil, Canada, Chile, Colombia, Czech Republic, Ghana, Guatemala, Hungary, Iceland, Indonesia, Israel, Mexico, New Zealand, Norway, Peru, Philippines, Poland, Romania, Serbia, South Africa, South Korea, Sweden, Thailand, Turkey and United Kingdom (HAMMOND, 2012).

as the name suggests, consists in the government or other institution announcing publicly an inflation target, that needs to be recognized as the primary goal of monetary policy, and committing to use its instruments to achieve such goal. Other characteristics are the transparency in the communication of the CB with the general public and accountability mechanisms (HAMMOND, 2012). Lastly, a necessary condition for any economy to adopt inflation targeting is the independence of its Central Bank.

There are other alternatives to inflation targeting, such as the fixed exchange rate, or *currency peg*, which consists in tying the domestic currency to another currency, preferably from a low inflation country. Although functional, this setup leaves the domestic economy prone to speculative attacks and comes at the cost of the international reserves. Other options include using the money supply to control the prices, but this also has drawbacks: the money supply is harder to adjust than the short term interest rates and depends on the stability of the demand for money. In comparison to these two, the inflation targeting policy using the interest rate seems to provide more control to the CB while letting the economy less vulnerable to shocks. Another advantage of the IT, accordingly to Hammond (2012) is the possibility of an “hybrid” policy, combining both rules and discretion elements in the CB decisions. This allows, for example, to use some type of Taylor-rule to achieve the medium-term goals whilst reacting to unanticipated shocks in the economy.

In Brazil, the inflation targeted value (average and tolerance interval) for a three-year spam is defined by the National Monetary Council (CMN) and those values are the policy goal that the CB should pursue. Then, the Monetary Policy Committee of the Central Bank of Brazil (Copom) defines in its meetings the target for the short term interest rate (Selic rate), considering the macroeconomic scenarios (internal and external) and the forecasts produced by the bank’s internal models. These meetings are equally spaced in 45 days intervals between each other and, in the following day after a meeting, it is already possible to observe the changes in the Selic rate. Those occur as a response of the market to the Central Bank open market operations. What happens next is the transmission of the monetary policy to the economy through the transmission channels, which will be explained in the next section (Banco Central do Brasil, 2006).

Hammond (2012) highlights that there are some competing conclusions in empirical results regarding the performance of IT regimes around the world. Nevertheless, it appears to be the case that in Latin America economies it improved the inflation control achieving lower price levels with lower volatility. A collateral (beneficial) effect in those countries was the improvement of their fiscal policies.

Although employed by many central banks, the IT regime is not immune to critiques. After the 2008 crisis, questions regarding the limits and efficacy of monetary policy under inflation targeting became more frequent. Smaghi (2016) emphasizes how much from the pre-crisis policy should be retained to be used in the following periods, citing both the apparent limitation of the regime to guarantee price stability in a bubble scenario and its limitations to keep up with the challenges of modern economies, some of them not present a few decades ago. Nevertheless, the author defends the inflation targeting policies arguing that the founding principles of the regime (one instrument for each target and efficient allocation of the instruments) are still relevant.

The same line of reasoning is followed by Woodford (2016), who claims that not all knowledge regarding monetary policy should be discarded since there is no evidence of wrongdoing in the Central Banks pursuing explicit inflation targets. It is undeniable that many modern economies faced periods of large moderation and there is no dispute that it would be worse if along the rise in unemployment experimented in the American economy in the last years there was also an acceleration in the price.

From this point on, our analysis will focus on an inflation targeting monetary policy with the same characteristics of the Brazilian regime: inflation as the only policy goal, short term interest rates as the monetary instrument and an independent Central Bank. Having the policy goal well-established, we can now discuss how the CB will make its decisions reach the real economy in order to achieve its target.

2.1.2 Transmission channels

Successful conduction of monetary policy is completely dependent on the transmission channels through which the decisions of the monetary authority effectively goes by until making effects in the real economy. The CB, after having decided its goals, needs to use its instruments in order to achieve the targets. Inflation control is the policy goal of the Central Bank's who adhere to inflation targeting regimes, as explained in the previous section, but the other quantities could be a target in some other cases, for example, the unemployment rate is for the FED in the United States (CARVALHO et al., 2015). As mentioned at the beginning of this chapter, there are four most usual and traditional transmission channels that are consolidated in the literature[3]: the interest rate channel; the exchange rate channel; the credit channel and the asset valuation channel.

The first one, the **interest rate channel**, has been standard in economic texts for the last 70 years. When a contractionary monetary policy is at play, there is an increase in the nominal interest rate. Then, this raises the capital cost, lowering the investment spending. What happens next is the decrease in the aggregate demand and consequential diminish of output. Although this channel has been taught even at the undergraduate level and has been known for many years, there is no consensus in the literature regarding

[3] It is possible that depending on the reference, ones find these channels grouped, ungrouped or under different names. We are basing our descriptions in the paper of Mishkin (1995). Additions to it will be duly referenced.

the magnitude of this channel's effect on the economy.

Some particularities surround the **exchange rate channel** because it will play a larger or minor role depending on the size and degree of openness of each economy. In addition, when the exchange rate is fixed, it makes no sense to discuss it as a transmission mechanism of MP. So, given an open economy with flexible exchange rates, a contractionary monetary policy that raises the nominal interest rate will raise the real interest rate as well. In the case of Brazil, it will make investments in Brazilian Reais (R$) more attractive, i.e., the domestic currency becomes appreciated (in comparison to US Dollars, for example) and the exchange rate falls[4]. This will have a direct impact in lowering net exports (either by increasing imports given the R$ appreciation or lowering exports) which will lower output.

The **credit channel** can be seen as the set of two related channels: the commercial banks (bank lending channel) and the balance sheet channel. In the first, the CB affects the resources available for loans in commercial banks by either changing the banking reserves (through buying or selling public bonds) or changing the interest rate of rediscount loans (CARVALHO et al., 2015). The effect remains the same: a contractionary monetary policy will decrease the resources available for loans in commercial banks, which will affect households and small firms who do not have access to financial markets in order to finance their consumption/investments and the aggregate result will be a lower output. The balance sheet channel has an analogous effect and it is based on the existence of asymmetries in credit markets. In general lines, when there is a contractionary monetary policy, what follows is a fall in equity prices and/or in the cash flow (because of the rise in the interest rate), and both risk of adverse selection and moral hazard rise. The banks will become more strict to mitigate their risks and the loan volume will diminish, causing a lowering in investment spending which affects output.

Lastly, the **asset valuation channel** operates based on the assumption of a stable relationship between the interest rates Carvalho et al. (2015). We can use Tobin's q theory to explain what happens when there is a rise in the short term interest rate. First, remember that q is given by

$$q = \frac{\text{market value}}{\text{replacement cost of capital}} \quad , \tag{2.1}$$

which can be seen as the relationship between the cost of buying an existing firm and the cost of starting a new business. Moreover, it works as an indicator of investment spending. Without loss of generality, suppose that the economy is in crisis and the market value is low. Assuming that the replacement cost of capital is given, then q will be less than 1, indicating that it is better to buy an existing firm than starting a new one. On the other

[4] In this work we are defining the exchange rate as the amount of Brazilian Reais necessary to buy one American Dollar.

hand, if the market value of a company is too high, from the investor's perspective is better to start a new company, subsiding the investment in new plant and capital equipment through stock issuing. In this case, there would be an increase in investment spending (MISHKIN, 2001).

Having established the link between Tobin's q and investment, we need to understand the role of the monetary policy in this. When the short interest rate goes up, the asset prices go down, due to the relationship between short and long term rates. This will lower the firm's market value, decreasing q, which indicates less investment and, ultimately, less output. The same effect will be observed through the wealth effect: if the stock prices go down, the owners of equities will have an impact in their wealth, which decreases the capital available for consumer spending, which also diminishes the output.

Carvalho et al. (2015) raise questions on the functioning of the MP transmission channels in Brazil, citing characteristics that make it a special case. First, the average short maturity of bonds make our term structure of interest rate limited. This is related to the period of hyperinflation and economic instability that prevailed for many years and ceased only a couple of decades ago. As a measure of protection, contracts, even with longer duration, are indexed on the Selic rate, which in practice turns those into short term contracts. The same mechanisms clog the credit channel: the financial sector follows an inertial pattern of high interests in loan contracts, which makes the credit volume low in comparison with the size of the Brazilian economy. The natural question that arises is *"how it is possible that monetary policy succeeded in such harsh environment?"* They offer two explanations. One plausible answer would be the importance of the exchange rate channel. Opening the capital accounts in the middle of the nineties and the subsequent adoption of the inflation targeting regime while abandoning the fixed exchange rate system by the end of the same decade seems to have contributed in creating a good environment, together with high-interest rates, to attract foreign investments. The overall effect was that the exchange rate channel had a significant contribution in transmitting the monetary policy in the place of the asset valuation and credit channels. The other explanation offered is related to the term structure. Households and firms got used to macroeconomic instability and became more sensitive to surprises in the short run, almost like exclusively look at the Selic or one-year interest rates to make long-term decisions. In fact, Bacen states that the time between observing the first significative effects of a Selic change over inflation is, on average, six to nine months (Banco Central do Brasil, 2019b). Nevertheless, the monetary policy, in particular after the inflation targeting adoption, has been fairly successful in controlling Brazil's inflation, in comparison to past decades (AFONSO; ARAÚJO; FAJARDO, 2016).

Back to the monetary transmission channels, the theory takes little or no information regarding the different types of agents in the economy. Having this in mind, the following question arises: "*is it really reasonable to assume that the effects of monetary policy will* *be homogeneous among households and, if not, in what extension the aggregate effects of the policy are dependent on the degree of heterogeneity present in the economy?*". In order to address this, we will need to extend our scope to beyond the traditional transmission channels, which will be done in the next subsection.

2.1.3 Redistribution channels

As suggested by Mishkin (1995) in the quote that opens this chapter or by Tobin (2005), who said that some whys and hows in monetary policy remain a mystery even to economists and central bankers[5], it seems that the transmission mechanisms of monetary policy described in the previous section may not be a perfect explanation for what follows after a monetary shock. In the last subsection, we discussed how it is possible for Brazil, given its unique characteristics, to have benefited from the monetary policy in an inflation targeting regime since the traditional explanations did not seem to work well. We can conclude from this that the theory regarding the transmission channels of monetary policy is solid enough to have explained a fairly large portion of the reality, but when some deviations occur, unexpected things can happen and other explanations are needed.

With roots in the real business cycles models, where the monetary policy had no place at all, the dynamic stochastic general equilibrium models came as a powerful tool to portrait the economy while embodying rational expectations in a general equilibrium setup. The incorporation of nominal rigidities, market power and nominal variables (wages, prices, and nominal interest rate) in this structure was the dawn of the New Keynesian models and they quickly became the workhorse of macroeconomic analysis and it is widely taught in the classrooms, used in academic research and in macroeconomic policy modeling. But these models were under scrutiny after failing to predict the crisis. Other critiques point to the utilization of unrealistic assumptions that can lead to some doubtful conclusions, such as the existence of an infinitely lived representative agent, perfect information and rational expectations (GALÍ, 2015).

An intrinsic characteristic of the most traditional macroeconomic models, including the ones used for monetary policy evaluation, is the assumption of a representative agent (AREOSA; AREOSA, 2016). In the New Keynesian model with representative agents (RANK), this would imply that almost all effect of monetary policy will be through the intertemporal decision, either postponing/anticipating consumption and/or investment (KAPLAN; MOLL; VIOLANTE, 2018). However, this contradicts the empirical evidence that, in general, households are less sensitive to momentary changes in their incomes. The work of Kaplan, Moll & Violante (2018) shows that considerable differences between the results of a RANK and a heterogeneous agent New Keynesian (HANK) model can exist

[5] *"Why does the monetary policy works? How? It is a mystery, one that is not fully understood neither by the central bankers or the economists"* (TOBIN, 2005).

due to the consumption behaviors of households, affecting the aggregate consumption.

Although the discussion on the necessity of incorporating heterogeneity in macroeconomic models gained strength only in recent years, the discussion regarding different behaviors among economic agents that could have an impact in aggregate variables is not new. For example, Tobin wrote the following, back in 1982:

> “Aggregation would not matter if we could be sure that the **marginal propensities to spend** from wealth **were the same** for creditors and debtors. (...) There are indeed reasons for expecting or at least for suspecting, just that. **The population is not distributed between debtors and creditors randomly**. Debtors have borrowed for good reasons, most of which indicate a high marginal propensity to spend from wealth of from current income or from any liquid resources they can command” (TOBIN, 1982).

Tobin’s quote has a key element to understand why it is not reasonable to analyze macro effects without considering the characteristics that arrange the economic agents in different groups. By employing a representative agent model, we are averaging the effects over the whole population, which carries the underlying assumption that the different effects “cancel out” and, on average, everyone will experience more or less the same effect. But what if this “overall effect” depends exactly on the composition of the population, like it is implicit in Tobin’s quote and explicitly in Kaplan, Moll & Violante (2018)’s results? In the case of monetary policy transmission, this would call for changes in the theory as we know it.

According to Yellen (2016), before the 2008 financial crisis it would not be reasonable to expect a discussion on “many instruments” and/or “many objectives” of monetary policy: all the focus was on the sole instrument, the short term interest rate, whose function was to help control the price level[6]. The financial crisis shed light on the problem of conducting monetary policy with conventional instruments in a zero-lower-bound environment, which led to the increase of adoption of unconventional policies in many economies. As works assessing the implications of this type of policy emerged, a branch of research linking monetary policy (conventional and unconventional) to inequality and distribution gained strength in both theoretical and empirical sides.

Auclert (2017) investigates the redistribution effects of MP on consumption using a HANK model and compared the theoretical results from the model to sufficient statistics from Italian and U.S. data. He investigates five redistributional channels, being the first two already present in RANK models: the intertemporal substitution due to changes in the interest rates[7] and the changes in consumption induced by the rise in the aggregate

[6] It is true that in some cases, such as the FED, the monetary policy is also looking at unemployment rate. But since this is not the case of Brazil, at least not explicitly, we will remain to focus on the Central Bank’s objective of price stability.

[7] In our previous section, this would be the interest rate channel.

income [8]. His main contribution lies in the proposal of the remaining three redistribution channels: (i) *the earnings heterogeneity channel*; (ii) *the Fisher channel*; and (iii) *the unhedged interest rate channel*.

In general lines, (i) is related to the sources of income among households - some people depend more on labor income than capital income, which will make the gains and losses regarding changes in the interest rates to be unevenly distributed among the population. The Fisher channel, (ii), is related to the inflation and the net nominal positions. Unexpected changes in inflation will have an impact on households with more or less intensity depending on them having assets whose price will change with changes in the aggregate price level or not. The last channel has a more subtle mechanism: the interest rate can have different effects on two households who possess assets and liabilities with the same net present value. This can happen due to differences in the maturities of assets and liabilities: someone whose wealth is composed primarily by short-term bonds is going to experience different effects of the MP than another person whose investments are long-term contracts such as mortgages since the former can change its position easier than the latter.

Accordingly to Auclert (2017), depending on the characteristics of the economy, the effects of the monetary policy assuming a heterogeneous environment will vary accordingly to the marginal propensities to consume, leading to different aggregate effects. His findings suggest that heterogeneity play a role in amplifying the effects of MP on aggregate consumption. It is important to notice that although these channels relate monetary policy to consumption, there is an underlying income and/or wealth effect in play here as we shall discuss below.

A different approach would be to link MP shocks directly to changes in the distribution of income and/or wealth, which, in an economy where capital is not evenly distributed among households and among types of investments, would imply a change in inequality. Doepke, Schneider & Selezneva (2015) builds a framework based on a life-cycle model with housing to assess the effects of expected and unexpected monetary shocks in wealth. Differently from Auclert (2017), they investigated the effect of a commitment to a long-term inflation target and they are not proposing any different transmission channels. For them, the redistributional effects will be a consequence of the Fisher channel and the interest rate channel: borrowers lose while lenders benefit from increases in the interest rate. Their conclusion is that when a contractionary policy takes over, the middle-aged households with liabilities in the form of mortgages will benefit from the low interest rates, while wealthy retirees will see a shrinkage in their wealth. These gains and losses do not cancel out and the aggregate consumption will have a different outcome than the one

[8] We are not considering this a cannel *per se*, since it can be seen as a consequence of the other channels given that each one of the four traditional channels can lead to raises in output that could induce consumption.

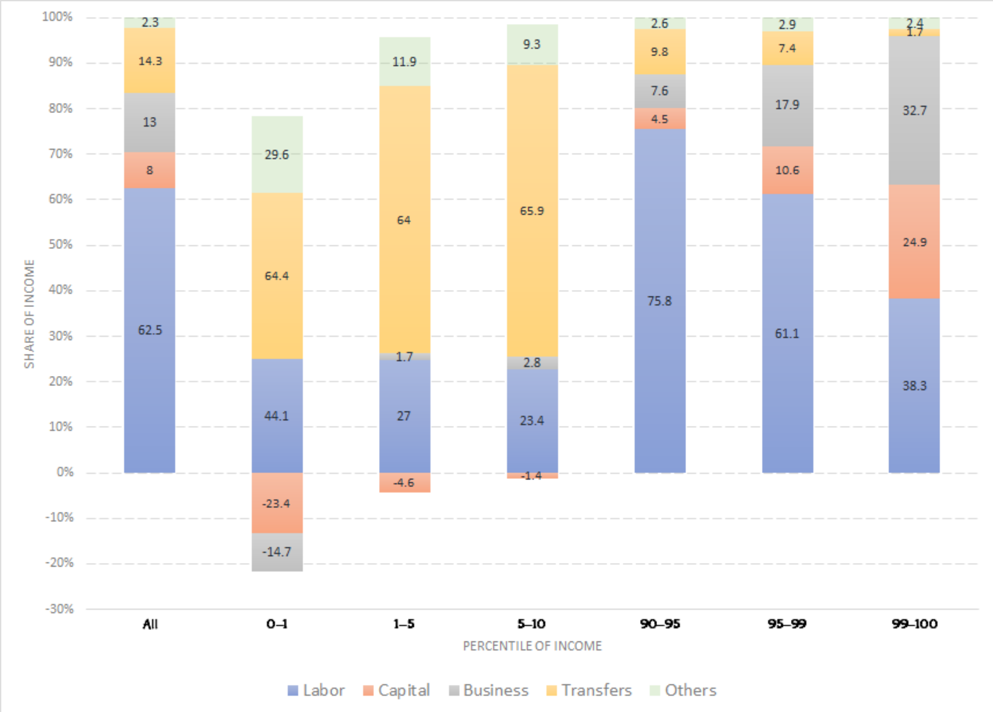
expected from a representative agent model.

Areosa & Areosa (2016) built a calibrated DSGE model with heterogeneous agents in order to investigate the relationship between inequality, inflation and conventional monetary policy. Their model has an inequality curve based on the Gini index for consumption, a slope-modified intertemporal IS curve and what they call "inequality augmented" Phillips curve. The Gini index interacts with inflation because it responds to the interest rate shocks, creating an inequality channel. Summarizing, the main lesson we can take from HANK models is that heterogeneity can affect the aggregate effects of monetary policy, leading to a different path than the one that would be dictated by a RANK framework.

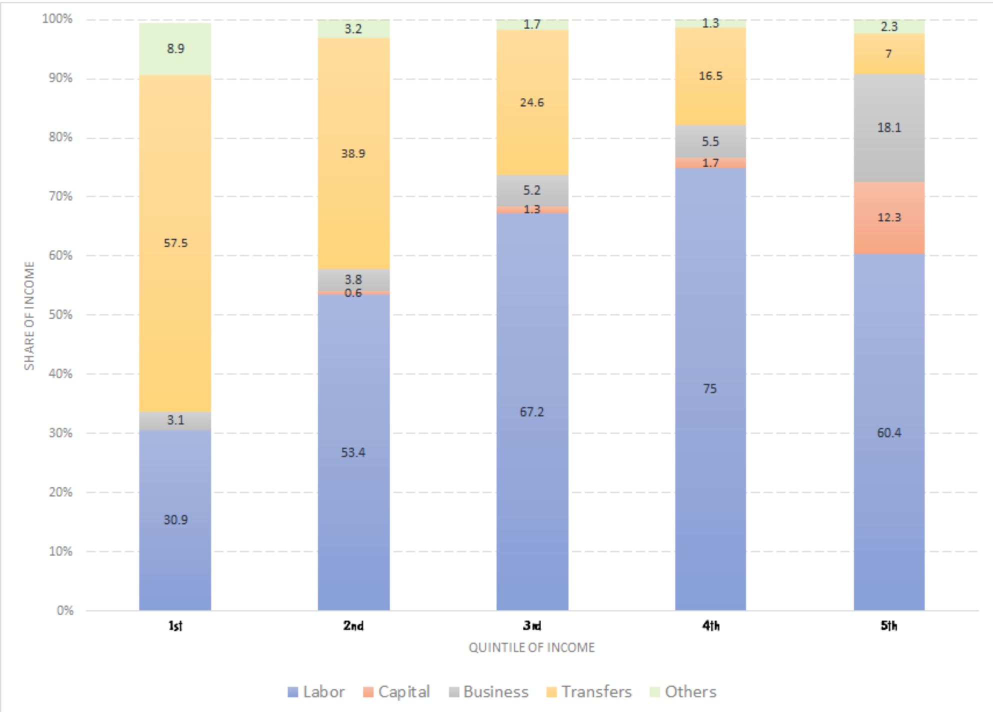
In this thesis we are not using a model where it would be possible to fully discriminate between each channel, instead, we are seeking to find aggregate effects. It is true that conducting analysis of each effect individually in a *ceteris paribus* environment have the advantage of providing a better understanding of the underlying mechanisms, but this comes with a cost: when the channels respond very differently from each other to changes in the interest rates, it is not possible to know what will be the overall effect from the monetary policy (*heterogeneity between channels*). In addition, as we will argue in the remaining of this section, it may not always be possible to anticipate the effects of a single channel because it will depend on the characteristics of the households (*heterogeneity within channel*). Our proposal, then, goes in line with Bernanke (2015a) affirmation that *"The better way to look at the distributional effect of monetary policy is to compare changes in the income flowing from capital investments with the income from labor"*. However, as our results from the empirical model will show, it does not seem to be the case that the "monetary policy tends to affect capital and labor incomes fairly similarly", contrarying Bernanke's claims.

Although our main interest lies on the aggregate impact of monetary shocks in the functional distribution of income, it is also important to understand the theoretical redistributional channels to income and wealth distributions. From the previous literature, we can summarize two main channels that make a direct link between monetary policy and income/wealth heterogeneity: (i) the *income composition channel* (or earnings heterogeneity channel) and (ii) the *interest rate exposure channel*, which combines both the effect of the net nominal positions and the unhedged interest rate exposure. In addition to these, there is a third redistributive channel that is related to the inflation, the (iii) *inflation tax channel*. Together, they provide a theoretical basis to what would be the driving forces behind our empirical model.

Figure 2 – Income source as a share of household income. United States, 2013.



(a) Values for the entire population (left) and for selected percentiles.



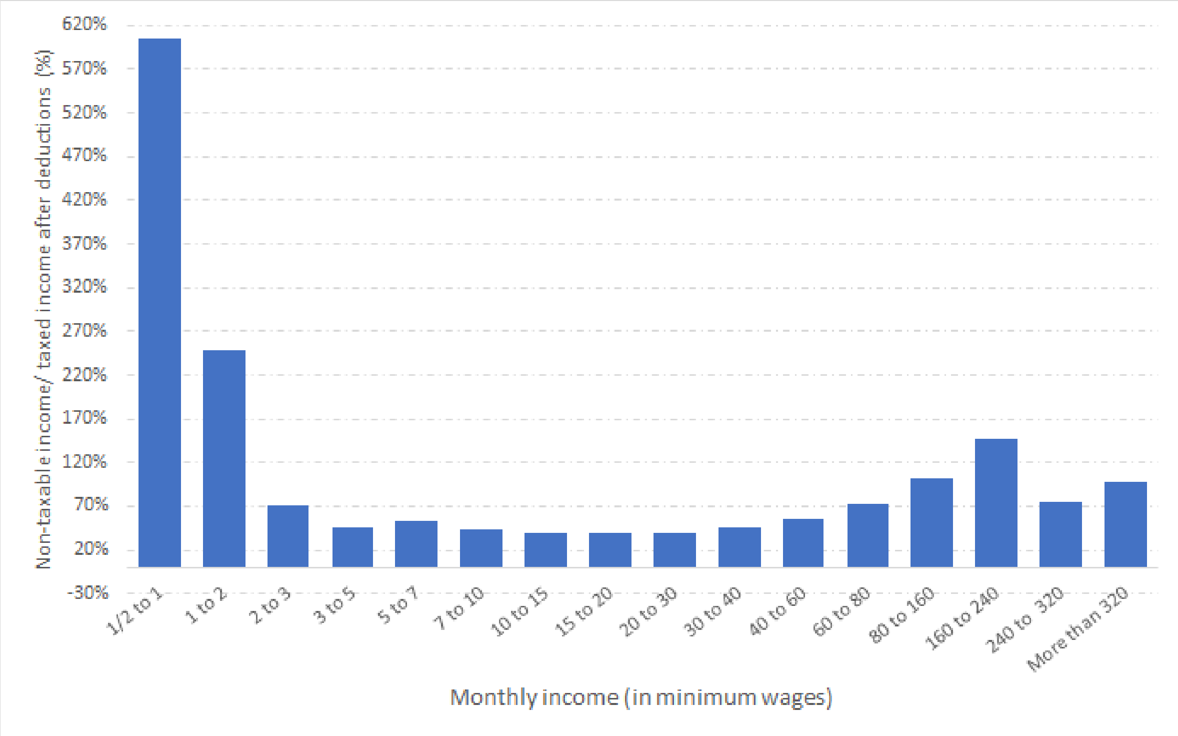
(b) Values grouped by quintiles.

Notes:
(2a) - The first bar represents each income source as a share of the overall household income. Labor represents the majority of income (62.5%), followed by transfers (14.3%) and business (13%), with capital representing 8% of total income. These proportions change when we break the analysis between selected percentiles. Households in the 0th to the 10th percentile of lowest incomes have approximately 65% of their income from transfers, while labor represents a larger fraction of income of the poorest (0th to 1st percentile): 44.1%. Most likely this is related to the fact that those have a large debt in capital and business. As for the ten percent richest, we can see that labor has a predominant role in income from those from the 90th to 99th percentiles, but this falls for the richest 1%, who has a larger share of capital and business income.
(2b) - By grouping the data in the five quintiles of income from left to right, the pattern becomes more evident: transfers start at 57.5% of the income of the 20 percent poorest households whilst representing only 7% of income from the top 20 percent. We see a rise in the importance of wages until reaching the fourth quintile. In the last quintile, there is an increase of the capital and business income.
Source – Adapted from Kuhn & Rios-Rull (2016), using 2013 Survey of Consumer Finances.

The first channel works on the assumption that the income of households will be a result of the combination of different sources (capital, labor and/or transfers). More specifically, it is reasonable to expect that many households will have more than one source of income, for example, wage and dividends from financial applications, making hard to know in advance what would be the role of interest rates in their income. However, from survey and tax data, it seems to be the case that the poorest households will have a large fraction of their income from transfers, while capital income plays a larger role for the richest. Figure 2 has the share of income sources with respect to the total income of American households, grouped by percentiles and quintiles and compared with the averages found when considering the population as a whole (KUHN; RIOS-RULL, 2016). In Figure 2b the evolution of the pattern becomes clear: the poorer the household is, more transfers as a share of income and less income from labor s/he will have. As we move right, towards richer quintiles, we see labor income gaining strength until reaching the top 20% richest, who have still the majority of their incomes coming from labor, but capital and business income play a role as well. Although there is no similar data for the composition of income of the Brazilian households, we can analyze the share of the non-taxable (exempt) income from the taxed income (after deductions), using data from the Brazil's IRS. Figure 3 shows that the poorest households, the ones who receives less than 2 minimum wages per month, have a higher amount of non-taxable income probably due to government cash transfers that are not taxable in Brazil. Then, for households who receive from 3 to 60 minimum wages per month we can observe a constant fraction between non-taxable and taxed income and finally for the richest, the non-taxable income starts to rise again. Most likely this is related to the fact that interest and dividends are also tax-exempt. Although not conclusive like Figure 2, the behavior exhibited in Figure 3 also suggests the existence of three different income groups.

Piketty & Saez (2003) found a similar pattern to the one from Kuhn & Rios-Rull (2016). They analyzed historical series of income of the top 10% richest in the US and also reported a declining behavior of wages' share in income and rise in capital gains when moving from lower deciles to higher percentiles from the income distribution. More specifically, differently from Kuznets original work whose richest groups was the top 10%, Piketty & Saez (2003) analyzed the households within the top decile, including the richest 5%, 1%, 0.1% and 0.01% and concluded that the participation of wages in the overall income of these groups have a negative behavior with respect to the rise of income: more income, less the participation of the wages in it. This implies that a contractionary monetary shock, through the income composition channel, will increase the return of capital, benefiting individuals who have financial assets whose payments rise when the interest rate is higher, while wages and transfers income will stay the same. The effect of this channel can arise also due to the indirect effect that interest rate has on unemployment, affecting mostly people who depend more on wage income. According to Bunn, Pugh &

Figure 3 – Non-taxable income as proportion of the taxed income by groups of total declared income, Brazil, 2016.



Notes: This figure shows on the y-axis the ratio of the non-taxable by the taxable income after deductions (such as medical expenses and education) among groups of total declared income (x-label). The minimum wage in Brazil for 2016 was approximately the equivalent to 1 USD per hour (R$ 4.00). It is possible to observe a downward behavior in the non-taxable (or exempted) income as the amount of taxed income rises, but this inverts for the groups with total income above 60 minimum wages.

Source – Own elaboration using data from BRASIL. Ministério da Fazenda. Receita Federal do Brasil (2018).

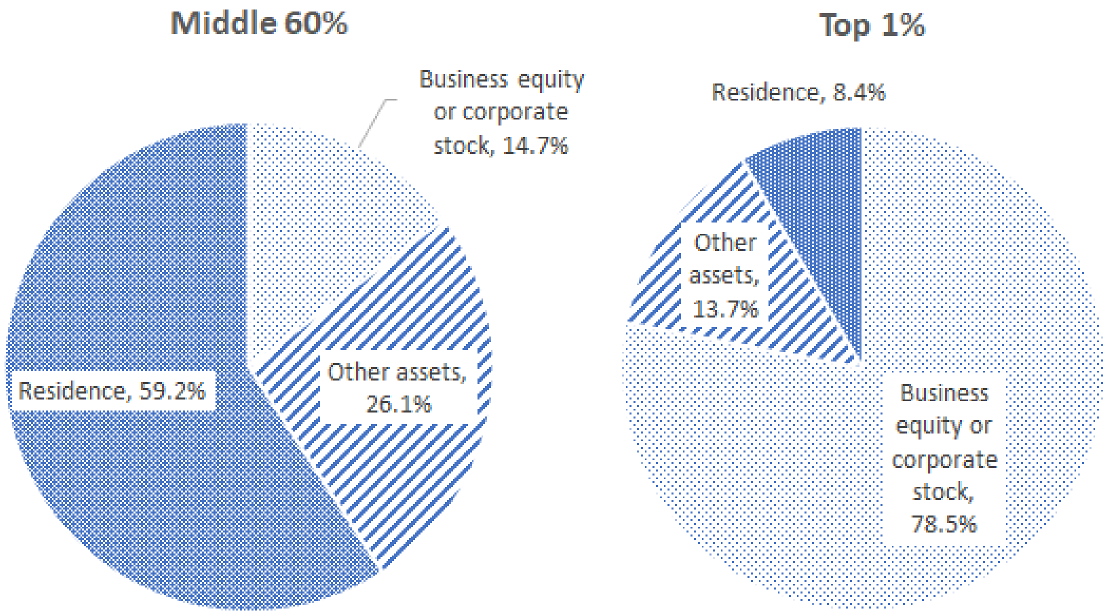
Yeates (2018), this will also differ conditional on age and education of the households.

In the interest rate exposure channel[9], monetary policy will have different effects on the households conditional to their assets' net present value (NPV). This takes into account both the net nominal value of the assets but also its maturities. When the interest rate goes up, the value of an asset goes down (similarly, if we are talking about a pre-fixed loan, the amount to be paid will fall) and due to differences in the asset composition, households will face differently this effect. For example, in the United States in 2001, the households from the middle quintiles of the wealth distribution (from the 20% to the 80% poorests) had almost 60% of their wealth in housing, whilst financial assets such as equity and stocks represented roughly 15% of their assets, while the 1% wealthiests had almost 80% of their wealth in equities and stocks and less than 10% in housing (Figure 4). However, to correct assess if a household will benefit or not from the contractionary monetary policy, it is important to know whether the present values of its assets surpasses the present value of its liabilities. Since poor individuals will have less possessions, they will not feel the impact of interest rates on assets, but on the other hand, they are more

[9] Bunn, Pugh & Yeates (2018) explicit a *Net Interest Income* that has a similar mechanism to the interest rate exposure channel.

susceptible to low term loans in comparison to individuals with more income and/or wealth, that can borrow higher sums with more time to pay back. Therefore, if all loans have fixed interest rates, households who have a longer-term debt structure will suffer the most when the interest rate falls. Put in other words, long term net borrowers tend to benefit from decreases in interest rates while long term net savers loose[10].

Figure 4 – Wealth composition for different household groups in the USA, 2001.



Notes: Wealth composition comparison between the typical household from the middle of the wealth distribution (left, middle 60%) to the top 1% households with the larger wealth (left, top 1%), when classifying the nominal assets in three different categories: (i) business equity or corporate stocks; (ii) residence; (iii) other assets.
Source – Edwards, Roosevelt & Bowles (2005).

The inflation channel or simply savings channel, although also related to assets and liabilities, operates differently. Its effect is directly on the changes in the real price of assets as a consequence of unexpected inflation. Bunn, Pugh & Yeates (2018) divides this channel in two, *net financial wealth* and *net property wealth*, and their argument for doing this differentiation is that financial assets will respond quicker to the market fluctuations. The other redistribution effect of inflation relies on the fact that people from the lowest deciles of income tend to have less access to financial markets and rely mostly on money thus, they see their purchase power diminishing more when inflation rates goes up.

There is no way to fully anticipate what will be the global redistribution outcome of the monetary policy when taking into account all the channels listed above (FURCERI; LOUNGANI; ZDZIENICKA, 2018). The magnitudes and directions of the responses will depend on the population characteristics and, theoretically, it is even possible that the individual effects of each channel cancel one another. This calls for empirical studies trying

[10] The terms *net borrowers* and *net savers* refers to individuals whose liabilities, when taking account maturity, exceeds the assets and vice-versa.

to understand the aggregated effect of these channels altogether, as suggested by Bernanke (2015a).

2.2 EMPIRICAL EVIDENCE

From the empirical side, there is also no consensus on whether monetary policy has an effect on income distribution (or, in the cases where there is an effect, the magnitude and direction is not the same depending on the study). Bivens (2015), considering the American post-crisis case, argues that a monetary expansion does not necessarily increase inequality and the effects of quantitative easing would be distinct depending if housing prices were more/less affected than equities. He adds that if the policy reduces unemployment then inequality would diminish as well. Still for the American economy but using data since 1980 and conventional MP, the work from Coibion et al. (2017) suggests that contractionary shocks in the interest rate historically increased inequality of consumption, total income, labor earnings, and total expenditures, while expansionary shocks are related with inequality reduction. On the other hand, Ludvigson, Steindel & Lettau (2002), using a structural vector autoregressive (SVAR) model, found that the wealth channel had a minor role in transmitting the FED policy to household consumption.

For the Italian economy, Casiraghi et al. (2017) found a negligible effect of unconventional monetary policy on income inequality due to compensation of the redistribution effects (positive and negative effects cancel out), whilst Guerello (2017), using data from the Euro area, investigated the effect of both conventional and unconventional MP on income distribution using a VAR model. Results show that expansionary conventional MP decreases income inequality, but the conclusions may go in the opposite direction when the fiscal policy is redistributive and the household's portfolio is composed by investments with short maturities.

On the study of O'Farrell, Rawdanowicz & Inaba (2016) using data from OECD countries, accommodative MP had little effect on income and net wealth inequality. The differences between countries could be explained through the inherent characteristics of each economy. Increases in housing prices had importance in reducing net wealth inequality, whilst raises in equities and bonds' prices were associated with an increase in inequality. High levels of inequality could decrease the effect of monetary policy over consumption, but this effect was reportedly small. Bunn, Pugh & Yeates (2018) evaluated the effect of the monetary policy easing that followed the 2008 crisis using household panel data from the UK and concluded that the overall effect of the policy on wealth and income inequality was relatively small, but there were important differences between households when analyzing the results in cash terms. With respect to asset price inflation and inequality, Adam & Tzamourani (2016) used survey data from households in the Euro Area. They divided the analysis between three types of assets: housing, bonds and equities. They found out that when bond prices increase (in comparison to housing prices), there is no change in wealth inequality, but when equity prices change, this will benefit households in the top wealth percentiles.

Furceri, Loungani & Zdzienicka (2018) reported asymmetric responses from inequality regarding tightening or easing MP, using panel data from 32 economies in the period from 1990 to 2013. Contractionary policy increases inequality more than the equivalent expansionary measures decreases inequality and the magnitude of the effects depend on the state of the business cycle, the share of labor income and can be diminished if other fiscal redistributional measures are taking place at the same time.

Mumtaz & Theophilopoulou (2015) used a Bayesian mixed-frequency structural vector autoregressive model to explore the role of monetary policy shocks on inequality observed in the UK in the 1968-2008 period. The authors used microdata from national surveys to build annual indexes of inequality for wages and income. Their model showed that contractionary monetary policy shocks lead to a significant increase in inequality in both wages and income. This observed effect was higher in the period before the inflation target policies and was higher in income inequality than wages inequality. Similar results were found in a later paper also by Mumtaz & Theophilopoulou (2017): using quarterly data, they estimated a structural VAR for the period between 1969 and 2012 to see the response of four different Gini indexes (total consumption, consumption of non-durables, disposable income, and gross wage). Furthermore, they investigated if there were some changes in the VAR coefficients when estimating a TVP-VAR for the period of unconventional monetary policies that followed after the 2008 financial crisis. From this last model, they found out evidence that the quantitative easing policy in UK played a role in increasing the inequality on the period of the Great Recession. Davtyan (2017) employed a VEC with data from the 1% richest households in the United States to investigate the effect of contractionary monetary shocks. However, differently from Mumtaz & Theophilopoulou (2017), this study concludes that contractionary shocks decreased inequality.

Notice that although there is an egalitarian appealing in the study of redistributive effects of monetary policy that may alone be a strong enough reason to justify studies in this area, there is an equally strong justificative based on the constraints that a redistributive reaction of the economy can impose over monetary policy. More specifically, depending on the marginal propensities to consume in the population, the aggregate effect of the monetary policy on consumption can change, as shown by Auclert (2017). This result deviates from what would be expected in the traditional models that do not incorporate heterogeneity, making possible that the policymaker takes action expecting a result that will not be verified in practice, i.e., the redistributive effects do not cancel out and the overall result of the policy could deviate from what was initially planned.

Part II

Econometrics Background

3 TIME VARYING VECTOR AUTOREGRESSIONS AND WISHART STOCHASTIC VOLATILITY

> *"Statisticians, like artists, have the bad habit of falling in love for their models."*
>
> George E. P. Box

Having defined in Part I that our interest lies in evaluating the monetary effects on the income distribution using data from Brazil, we will now turn our attention to the methodology. Empirical macroeconomics nowadays is heavily supported by statistical[1] models that, in a roughly categorization, can be divided in two groups: microstructured dynamic stochastic general equilibrium (DSGE) models and vector autoregressive (VAR) models. Both have its roots in the macroeconomics literature that emerged after the breach left by the rational expectations revolution and the Lucas critique (1976) and both can be used to evaluate macroeconomic aggregates joint behavior.

While DSGE models have the advantage of modelling the economy starting from the individual decisions of families and firms while explicitly incorporating economic theory in the form of Taylor rules for the monetary authority or fiscal spending constraints, VAR models are more data-driven and let the data speak in a more freely way. But this does not mean that VAR models are completely absent of economic foundations: in the case of structural VARs, one can impose identification restrictions in order to make impulse response analysis. We refer the reader to Kilian & Lütkepohl (2017) chapter 6 for a detailed comparison between these two types of models. Since our research interest lies in the aggregate effect of monetary policy shocks without necessarily identifying the transmission mechanism, we opted by the latter approach for this initial work.

The vector autoregressive model was first presented by Sims (1980) and consists, broadly speaking, in a group of equations that allows to simultaneously model the dynamics of a given time series set (DEL NEGRO; SCHORFHEIDE, 2013). This framework became

[1] It is debatable whether statistics can really help in finding causal relations in Economics, given that most data is not from experiments and even statistical methods using controlled data can be misleading in its conclusions. Although this is certainly a valid and important discussion not only in Economics but also in other sciences that use this kind of statistical approach, it is undeniable that statistical methods can shed light on a wide range of problems, enlightening our understand of the world. We also acknowledge that there are other methods, such as agent based models, that could also be employed to tackle our research question, but considering the problem at hand and the human capital available, we considered the TVP-VAR approach as the best tool to be used in this work.

widely used in economics and, in particular, in monetary economics, serving as an apparatus to assess the impacts of monetary policy on the real economy (WALSH, 2010). In this chapter we will discuss in relatively depth an extension of the VAR models - one with time-variation in the parameters - and will skip the most basics on the original model. For an exaustive presentation of VARs, we recommend Lütkepohl (2005) or Hamilton (1994).

Evidence that VAR parameters for macroeconomic data should be time-varying (the so-called TVP-VARs) can be found in many works, such as Cogley & Sargent (2001), who used a TVP-VAR to verify if there were changes in the FED's response to inflation in the period between 1948 to 2000. In a posterior work, Cogley & Sargent (2005) added stochastic volatility to their original model, in accordance to the critiques made by Sims (2001) and Stock (2001). The ideia behind a TVP-VAR with multivariate stochastic volatility (MSV), in Cogley & Sargent (2005) context, is that changes in the coefficients would capture switches in the FED's view, whilst alterations in the volatility pattern would imply changes in the exogenous shocks that affect the economy (KOOP; KOROBILIS, 2010).

There are several empirical papers using TVP-VAR models with MSV to model macroeconomic data. For example, in their paper, Mumtaz & Zanetti (2015) use a TVP-VAR to model changes in the employment search patterns and rate of separation in the job market; Baumeister & Peersman (2013) analyzed the economy response to oil supply shocks in the United States, while Galí & Gambetti (2015) studied the effect of monetary policy on financial markets bubbles. The works from Mumtaz & Theophilopoulou (2015, 2017) and Davtyan (2017) use VAR and TVP-VAR models with bayesian estimation to verify the impact of monetary policy shocks on inequality.

Although VAR models with time-varying parameters and multivariate stochastic volatility[2] (TVP-VAR w/ MSV) are useful tools to describe relationships between macroeconomic aggregates and make predictions, there is no consensus on what would be the proper specification neither the best estimation approach for these models. Regarding the latter, one problem that arises is related to the large number of parameters: VAR models are known for having many parameters and when we include temporal variation on top of that, the complexity only increases. Other complexity source is related to the stochastic volatility because its presence makes the whole equation system non-linear, which precludes the direct use of the Kalman Filter.

Primiceri (2005) states that Bayesian methods are most suitable for estimating such high-dimensional and non-linear models, for three main reasons: i) due to its interpretation of parameters as random variables, bayesian methods are more suitable to deal with problems with latent states since they naturally incorporate the uncertainty regarding

[2] From now on, unless explicitly stated otherwise, we will use TVP-VAR to refer to TVP-VARs with MSV.

the parameters in the modelling procedure; ii) in a model with so many parameters, it is possible that some of them will present low variances. In such cases, the frequentist estimate for the volatility would be zero, even though they have some time variation. Argument iii) favours the Bayesian approach due to the non-linearity and high-dimensional characteristics of the likelihood function, because this poses a challenge for an optimization procedure such as the maximum likelihood estimation (MLE). Classical methods can produce results at a local minimum, specially when the surface of the likelihood is unknown. On other hand, Bayesian methods explore the full posterior and Markoc Chain Monte Carlo (MCMC) methods such as the Gibbs Sampler involves taking thousands of draws that together will be used to produce measures like averages, credible intervals, among other measures. For the uninitiated reader, the Appendix A has some basic concepts on Bayesian inference that might be helpful to understand the general ideas of the model estimation procedure described later in this chapter.

The following sections will discuss the TVP-VAR specifications from Cogley & Sargent (2001), Cogley & Sargent (2005) and Primiceri (2005), exploring their characteristics and limitations. Then, we will present the Bayesian VAR (BVAR) from Uhlig (1997) and our extension of this model to a time-varying setup. The estimation procedure of this model is discussed in the sequence, where the method of Windle & Carvalho (2014) is combined with the Carter & Kohn (1994) algorithm into a Gibbs sampler scheme.

3.1 TVP-VAR MODELS

Consider the following state-space representation of a TVP-VAR:

$$y_t = Z_t\alpha_t + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}_k(\mathbf{0}_k, \Omega_t^{-1}) \quad \text{(measure eq.)}, \tag{3.1}$$

$$\alpha_t = \alpha_{t-1} + u_t \qquad u_t \sim \mathcal{N}_p(\mathbf{0}_p, Q) \quad \text{(state transition eq.)}, \tag{3.2}$$

where y_t is a $k \times 1$ vector of endogenous variables observed at time t; Z_t is a matrix with lagged y_t values plus a constant term, i.e., $Z_t = \mathbb{I}_k \otimes [1, y'_{t-1}, \ldots, y'_{t-\ell}]$, with $\otimes$ being the Kronecker product and $\mathbb{I}_k$ the k-dimensional identity matrix; α_t is a vector containing $p = k(k\ell + 1)$ time-varying coefficients that evolve according to (3.2); Ω_t^{-1} and Q are covariance matrices with dimensions $k \times k$ and $p \times p$, respectively and Ω_t^{-1} is stochastic. We use $x \sim \mathcal{N}_n(\mu, \Sigma)$ to indicate that the $n \times n$ random vector x follows a multivariate normal distribution with location or mean vector μ (which is also of dimension n) and covariance matrix Σ, which has dimension $(n \times n)$.

Equations (3.1)-(3.2) define a dynamical system that is formed by a measurement equation, similar to a regression model, and one state transition equation, which describes the system's trajectory through an autoregressive relationship. Fearnhead (2011) says that the idea behind a state space model is the existence of states that evolves with time, but are not directly observable. In (3.1)-(3.2), the α_t coefficients are called *latent states*, since only the measurements y_t are observable.

Cogley & Sargent (2005) define the term ϵ_t of the measurement equation (3.1) as $\epsilon_t = \Omega_t^{-1/2}\xi_t$, where ξ_t follows a standard normal distribution and $\Omega_t^{-1} = B^{-1}H_tB^{-1\prime}$, with:

$$B = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \beta_{21} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{k1} & \beta_{k2} & \cdots & 1 \end{bmatrix} \quad \text{and} \quad H_t = \begin{bmatrix} h_{1t} & 0 & \cdots & 0 \\ 0 & h_{2t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{kt} \end{bmatrix}. \tag{3.3}$$

The elements of the lower triangular matrix B are constants and the elements of the diagonal matrix H_t follows a driftless geometric random walk given by:

$$ln(h_{it}) = ln(h_{it-1}) + \sigma_i\eta_{it}, \quad \eta_{it} \sim \mathcal{N}(0,1). \tag{3.4}$$

The way Cogley & Sargent (2005) define Ω_t, B and H_t restrains the behavior of the covariances among the residuals ϵ_t, causing them to vary as a fixed proportion of the variances. This implies that the shock caused by the i-th variable on the j-th variable is constant through time (KOOP; KOROBILIS, 2010). This was criticized by Primiceri (2005), who argued that simultaneous interactions among variables are essential to capture the effect of time-variation in TVP-VAR models. Moreover, this model is not invariant to the order of the variables in the VAR: permutations of the same variables will potentially result in different estimates (BOGNANNI, 2016). It is important to notice that this is not related to the restrictions that should be imposed to structural VAR models to allow identification of structural shocks. Here (and, as we shall see, in Primiceri (2005) model), the order will matter disregarding the impulse response analysis being performed (for example, in the case of a model used for forecasting purposes only).

In Primiceri's (2005) work, the covariances evolve following a random walk. This means that the matrix B of equation (3.2) now has the specification below:

$$B_t = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \beta_{21,t} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{k1,t} & \beta_{k2,t} & \cdots & 1 \end{bmatrix} \quad \text{with} \quad \beta_t = \beta_{t-1} + \upsilon_t, \quad \upsilon_t \sim \mathcal{N}(0,Q). \tag{3.5}$$

However, as noted by Primiceri, this model is also not invariant with respect to the ordering of the variables that entered the VAR, meaning that the estimation solution is not unique for a given data set. To see this, consider $\tilde{y}_t$ as a permutation of the y_t values (from equation 3.1 and $\tilde{\Omega}_t^{-1}$ its respective covariance matrix (which is obtained by the permutation of lines and columns). So, it is possible to show that there are not a triangular matrix $\tilde{B}_t$ and a diagonal matrix $\tilde{H}_t$ satisfying $\tilde{B}^{-1}\tilde{H}_t\tilde{B}^{-1'} = \tilde{\Omega}_t^{*-1}$ where the elements of $\tilde{B}_t$ follow a Normal distribution, the elements of $\tilde{H}_t$ follow a log-normal distribution while having $\tilde{\Omega}_t^{*-1}$ with the same distribution as $\tilde{\Omega}_t^{-1}$. This is a consequence of both the triangular structure of $\tilde{B}_t$ and the probability distributions chosen to model the stochastic volatility, as illustrated below.

Without loss of generality, consider the case with two equations in the TVP-VAR, y_{1t} and y_{2t}. Then, define the matrices $\tilde{B}_t$ and $\tilde{H}_t$ as follows:

$$\tilde{B}_t = \begin{bmatrix} 1 & 0 \\ \beta_{21,t} & 1 \end{bmatrix} \qquad \textit{and} \qquad \tilde{H}_t = \begin{bmatrix} h_{1,t} & 0 \\ 0 & h_{2,t} \end{bmatrix}, \tag{3.6}$$

and movement laws given by (3.4) and (3.5). Now, we compute $\tilde{\Omega}_t$ using the relation $\tilde{\Omega}_t^{-1} = \tilde{B}_t^{-1}\tilde{H}_t\tilde{B}_t^{-1'}$:

$$\begin{aligned} \tilde{\Omega}_t &= \begin{bmatrix} 1 & 0 \\ -\beta_{21,t} & 1 \end{bmatrix} \cdot \begin{bmatrix} h_{1,t} & 0 \\ 0 & h_{2,t} \end{bmatrix} \cdot \begin{bmatrix} 1 & -\beta_{21,t} \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} h_{1,t} & 0 \\ -\beta_{21,t}\ h_{1,t} & h_{2,t} \end{bmatrix} \cdot \begin{bmatrix} 1 & -\beta_{21,t} \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} h_{1,t} & -\beta_{21,t}\ h_{1,t} \\ -\beta_{21,t}\ h_{1,t} & \beta_{21,t}^2\ h_{1,t} + h_{2,t} \end{bmatrix}. \end{aligned} \tag{3.7}$$

Note that the first element of $\tilde{\Omega}_t$ matrix in (3.7) is $h_{1,t}$, which follows a log-normal distribution as consequence of the law of motion of the $\tilde{H}_t$ matrix (Equation 3.4). After permuting y_{1t} with y_{2t}, the corresponding covariance matrix will be given by:

$$\tilde{\Omega}_t^* = \begin{bmatrix} \beta_{21,t}^2\ h_{1,t} + h_{2,t} & -\beta_{21,t}\ h_{1,t} \\ -\beta_{21,t}\ h_{1,t} & h_{1,t} \end{bmatrix}. \tag{3.8}$$

Now, the term in the $(1,1)$ position of $\tilde{\Omega}_t^*$ is a product of a random variable (rv) that follows a squared normal distribution, $\beta_{21,t}^2$, with another log-normal rv, added to a third one that is also log-normal. Obviously, this will *not* follow a log-normal distribution, which shows that $\tilde{\Omega}_t^*$ and $\tilde{\Omega}$ are not from the same distribution.

Although Primiceri (2005) model has a more flexible covariance structure when compared to Cogley & Sargent (2005), in the paper empirical application a simpler setup was used, by assuming independence of the shocks affecting the covariances of B_t (KIM, 2014). As pointed out by Primiceri, it is possible to generalize this, which requires for the estimation a *multi-move Gibbs sampler* to obtain the posterior estimates. However, in Primiceri (2005) a mistake was made in the algorithm[3] and was only corrected ten years later by Del Negro & Primiceri (2015), evidencing the difficulties that arise when

[3] More specifically, Primiceri (2005)'s algorithm employs a mixture of gaussians to approximate the volatilities, and he failed to include the full history of these mixtures when making the conditional set in the Gibbs blocks, which resulted at the resultant draws not being from the correct jointly posterior (DEL NEGRO; PRIMICERI, 2015).

estimating such a complex model.

Since the invariance and estimation problems from Primiceri (2005) and Cogley & Sargent (2005) are due to the multivariate stochastic volatility specification, we can search for a different evolution for the covariances of the measurement equation. An alternative is to make use of an Wishart model. In these models, the precision matrix from the measurement equation follows an Wishart distribution, which automatically will assure the positiveness condition for the volatility while having nice conjugation properties that are useful for the estimation of dynamical models such a TVP-VAR. Although there are not many works using such specification for TVP-VAR models, Wishart MSV models appears in the theoretical and empirical literature, with an emphasis in finance applications (CHIB; OMORI; ASAI, 2009).

For example, Philipov & Glickman (2006b, 2006a) work with a MSV model whose specification for the precision matrix Ω_t is a Wishart density with dimension k and $\nu > k$ degrees of freedom:

$$\Omega_t|\Omega_{t-1} \sim \mathcal{W}_k(\nu, S_{t-1}). \tag{3.9}$$

The scale matrix S_{t-1} follows a stochastic process and its transition equation is given by:

$$S_t = \frac{1}{\nu} A^{1/2} \Omega_t^d A^{1/2'}, \tag{3.10}$$

where A is a positive definite matrix containing the parameters that govern the temporal sensibility of the S_t elements and d is a persistence parameter. A MSV model defined by a measuremente equation plus (3.9)-(3.10) allows that the covariances between the variables are time-varying without incurring in the ordering problem inherent to Primiceri (2005) specification. Even with a more flexible specification, this model does not contain an extra layer of complexity by adding too many parameters when compared to Primiceri's: besides d (a scalar), there will be n terms in the A matrix, corresponding to the diagonal, that will need to be estimated.

However, like the models from Primiceri (2005) and Cogley & Sargent (2005), a Wishart MSV model whose volatility that follows (3.9)-(3.10) is also a non-linear state space model with a likelihood function that cannot be treated analytically. Philipov & Glickman (2006a) resorted to a Bayesian procedure by using a Gibbs Sampler, but, due to difficulties in jointly estimating the matrix A and the persistence parameter d, they ended up proposing using a fixed value for d. Asai, McAleer & Yu (2006) also worked with a Gibbs sampler scheme for the same model but reported convergence problems of their algorithm.

Another Wishart model is present in the work from Uhlig (1997), where the author proposes a BVAR model (with no time-varying coefficients) with Wishart stochastic volatility. As we shall see in the next section, this model will posses the nice properties of invariance due to permutations of variables and parsimony in comparison to the volatility specification of Cogley & Sargent (2005) and Primiceri (2005). At the same time, a Bayesian procedure from Windle & Carvalho (2014) that exploits the conjugacy of the Wishart distribution can be used in Uhlig (1997)'s model and would avoid the estimation problems from Philipov & Glickman (2006a) and Asai & McAleer (2009). Given that, our proposal is to extend Uhlig (1997)'s model to a TVP-VAR one and use this extension in our empirical application.

3.2 UHLIG'S WISHART BVAR MODEL

The BVAR model with constant coefficients and stochastic volatility proposed by Uhlig (1997) assumes that the precision matrix of the errors in the measurement equation receive shocks that come from a multivariate Beta distribution. More specifically, Uhlig's model is a VAR(k) with m equations where the errors' precision matrices are time varying. This model is defined through a measurement equation,

$$y_t = Z_t \beta + \epsilon_t, \text{ where } \epsilon = U(\Omega^{-1})' \xi_t \text{ and } \xi_t \sim \mathcal{N}(0, \mathbb{I}_m), \tag{3.11}$$

plus a single state transition equation,

$$\Omega_{(t+1)} = \frac{U(\Omega_t)' \Theta_t U(\Omega_t)}{\lambda} \quad \text{with } \Theta \sim \mathcal{B}_m\left(\frac{\nu + c + km}{2}, \frac{1}{2}\right), \tag{3.12}$$

with m variables that are observed in t periods $(t = 1, \ldots, T)$; $\lambda > 0$ and $\nu = m - 1$ are parameters; c is the number of deterministic regressors (such as constant or drift); the shocks Θ are independent; $U(\cdot)$ is the superior Cholesky decomposition and $\mathcal{B}_m(p, q)$ is the multivariate Beta distribution. The parameter ν allows variation in the precision matrix Ω_t: the lesser ν, highest the variation of H_t across time and vice-versa. Asymptotically, when $\nu \to \infty$, the model converges to a VAR without SV, because the multivariate Beta density from (3.14) goes, in this case, to a identity matrix of order m (KIM, 2014). Note that by using the Wishart distribution we automatically have that the precision matrix Ω_t will always be positive definite.

Although the parameter ν can be estimated using maximum likelihood, Kim (2014) argues that in a regression model where the innovations come from a t distribution, there is almost no benefit in estimating the degrees of freedom in comparison of using a pre-fixed value for ν, which ended up being the same strategy used by Uhlig (1997) ($\nu = 60$ for monthly data and 20 for quarterly data). λ is used as tuning parameter and it is related to the evolution of the covariance matrices. Uhlig (1997) recommends a fixed value that depends on ν, i.e., his suggestion is to set $\lambda := \nu/(\nu+1)$, which implies that a fixed ν will result in a fixed λ. On the other hand, Windle & Carvalho (2014) advise against imposing constraints on the value of λ since it will play an important role by controlling the degree of the smoothness imposed on the non-observed covariance matrices when updating the filter.

The prior choice for parameters and innovations made by Uhlig explores the conjugacy between the Beta and Wishart distributions, allowing the posterior update in closed-formula for the latent covariance states[4], differently from Philipov & Glickman (2006b)'s algorithm, improving the efficiency of the posterior results. Another good characteristic of this model is that the Wishart density allows that variances and covariances to move freely, without tying them together as was the case with the model from Cogley & Sargent (2005). Also, in Uhlig's model there is a natural extension from the scalar variances to covariance matrices (instead vectors of log-variances from other models), meaning his model is a multivariate extension to the scalar case. This model also allows for the conditional volatility of a given variable to depend not only on its past volatility but also on past covariances with other variables, meaning that this model formulation incorporates the observed contagion among variables into the covariance structure (KIM, 2014). Finally, the system described in (3.11)-(3.12) is invariant to the order of the variables (UHLIG, 1997, p. 65).

Considering that the BVAR model from Uhlig possess good characteristics in comparison to other similar models as the ones described above, it is natural do ask ourselves if this model can be generalized to allow time variation in the coefficients of the measurement equation (in addition to the variances). Not only this is possible but also there is more than one way to estimate this new model. In the next section we will discuss further details regarding the specification of this extension and the estimation procedure.

3.3 EXTENDING UHLIG'S MODEL

The model described in (3.11)-(3.12) can be extended by adding time-variation in the state coefficients α in the first of the system's equations and adding a third equation for the state's evolution (similar to Equation (3.2)), which results in:

$$y_t = Z_t \cdot \beta_t + \epsilon_t, \text{ with } \epsilon = U(\Omega_t^{-1})'\xi_t \text{ and } \xi_t \sim \mathcal{N}(0, \mathbb{I}_m), \tag{3.13}$$

$$\beta_t = \beta_{t-1} + u_t, \text{ with } u_t \sim \mathcal{N}(0, Q^{-1}), \tag{3.14}$$

$$\Omega_{(t+1)} = \frac{U(\Omega_t)'\Theta_t U(\Omega_t)}{\lambda}, \text{ with } \Theta \sim \mathcal{B}_m\left(\frac{\nu + c + km}{2}, \frac{1}{2}\right). \tag{3.15}$$

A major issue incurs in the estimation of the model defined in (3.13)-(3.15): differently from the model from Cogley & Sargent (2005) or the one from Primiceri (2005), where it is possible to reduce the problem of estimating Ω_t in a series of univariate simpler problems, there is no transformation in the measurement equation (3.13) that will simplify the problem. Uhlig (1997) proposed an extended Kalman filter for the innovations and an

[4] To estimate the non time-varying β coefficients, Uhlig (1997) used importance sampling.

importance sampling scheme to obtain estimates of the coefficients. Although functional, this method cannot be directly applied to a TVP-VAR framework.

To estimate a version of Uhlig (1997) with time-varying coefficients, one could generalize the extended Kalman filter, following a maximum likelihood with closed formula approach, such as proposed by Moura & Noriller (2019). Or, for a Bayesian flavor, we have two alternatives. The first would be the sequential Monte-Carlo algorithm developed by Bognanni (2016), but this method was implemented only for the static coefficients VAR model, which would require the adaptation for the TVP case. Alternatively, one could employ the method from Windle & Carvalho (2014) to the multivariate stochastic Wishart volatility in order to forward the filter, do the backward sampling and to sample from the posterior distribution, which was our choice, since its modular form is ideal to be used as a block into a Gibbs Sampler algorithm. Then, to close the blocks from the Gibbs sampler, we can draw the coefficients β_t, conditional to the innovations using the algorithm from Carter & Kohn (1994) and sample the covariances Q from (3.14) using a conjugate prior.

The bayesian solution proposed here has some advantages over the frequentist approach from Moura & Noriller (2019). First, the model described in (3.13)-(3.15) can be further generalized to allow stochastic volatility in (3.14) in a more flexible way than the proposed form by Moura & Noriller (2019), whose model defines this covariance matrix as a function of the one in the measurement equation (3.13). This simplifies the calculations but imposes a dependence in the model that may not be realistic. Our Gibbs sampler can handle this situation for a more general Q_t, that could be specified with a similar (but independent) law of motion like (3.15). This generalization was not implemented here because there is not a strong economic justification in our empirical application that justifies this extra layer of complexity in the model - not to mention that this could lead to further identification problems due to the inclusion of the extra parameters. Secondly, by using Windle & Carvalho (2014)'s method, we will obtain filtered and smoothed trajectories for all states, so our estimates will contain the information of the whole sample and not only the information from past periods - something absent from Moura & Noriller (2019) ML approach. Lastly, due to the modular nature of the Gibbs Sampler algorithm, we can further extend the estimation procedure to allow shrinkage methods for the coefficients, such as the ones from Bitto & Frühwirth-Schnatter (2018) or Eisenstat, Chan & Strachan (2016).

The next subsection contains the general ideas of a Gibbs sampler and the specification of the blocks used in our algorithm to estimate (3.13)-(3.15). For the reader who is not familiar with the basic Bayesian terminology, we recommend first reading the notes from the Appendix A.

3.3.1 Gibbs sampler structure

Before discussing the specific details regarding the estimation of the model described in (3.13)-(3.15), we will briefly introduce the intuition and motivation behind the use of a Gibbs sampler.

Bayesian inference focuses on finding the posterior density of a vector of parameter θ, that we are going to denote $f(\theta|y)$.However, in many applications, this posterior will not have closed formula and it will not be feasible to draw sample posterior values to compute sample moments. This may happen either due the complexity of the model that imposes a likelihood that cannot be analytically treated, or due to computational constraints or because the posterior did not follow a known probability density. Markov Chain Monte Carlo methods are numerical algorithms that allow us to obtain draws from the posterior distribution, either sampling from the full conditional posterior densities (which is the case of the Gibbs sampler) or from another density that will produce draws similar to the ones that we would obtain from the true posterior. The basic idea is to find an ergodic Markov Chain whose stationary distribution is $f(\theta|y)$. For a sufficiently large number of draws from this chain, we would expect to achieve convergence and the draws of the chain from this point on would be considered realizations of f. Finally, we use Monte Carlo integration to approximate expected values of the posterior density using sample means.

In the case of the Gibbs sampler, we combine the full conditional posterior to the remaining parameters and the data to obtain a candidate density that will have as stationary distribution the joint posterior that we are interested in. These densities can be organized in "blocks" that not necessarily will estimate one single parameter at a time: in some situations we can have a block where two or more parameters are conditional on the others. The important thing is that each block is completely conditional on **all** others, in other words, we need to make sure that we are indeed working with the full conditional posteriors.

The blocks of the Gibbs sampler for the model (3.13)-(3.15) are described below. Denote as B^T the collection of all values of $\{\beta_t\}_{t=1}^T$ (the same for Ω^T).

1. **Initialize Ω^T, B^T, Q, set the hyperparameters and initial values.**

2. **Draw B^T conditional on Ω^T and the other parameters:** In this part we employed the algorithm by Carter & Kohn (1994).

3. **Draw Q from a inverse Wishart distribution**

4. **Draw jointly (ν, Ω^T) conditional on B^T:**

   After estimating the coefficients, we begin the volatility block. Propositions 1, 2 and 3 from Windle & Carvalho (2014) are used in a sequential manner to run the forward filter, backward sampler, predict one step ahead and estimate n, k and m. For the next results, consider the following notation: the collection of data until time t is $\mathcal{D}_t \equiv \{y_t\} \cup \mathcal{D}_{t-1}$ for $t \in \{1, \ldots, T\}$ with $\mathcal{D}_0 \equiv \{\Omega_0\}$, where Ω_0 is an arbitrary covariance matrix. The prior for the data in the first period, $(Z_1|\mathcal{D}_0)$ follows a Wishart distribution and it is given by $\mathcal{W}_m\left(n, (k\Omega_0)^{-1}/\lambda\right)$. All three propositions implicitly conditions the results to the parameters n, k, λ.

a) (WINDLE; CARVALHO, 2014, Proposition 1) **Forward filtering:** Suppose $(Z_t|\mathcal{D}_{t-1}) \sim \mathcal{W}_m\left(n, (k\Omega_{t-1})^{-1}/\lambda\right)$. After observing y_t, the updated distribution is

$$(Z_t|\mathcal{D}_t) \sim \mathcal{W}_m\left(n, (k\Omega_{t-1})^{-1}\right) \tag{3.16}$$

with

$$\Omega_t = \lambda\Omega_{t-1} + y_t. \tag{3.17}$$

To make 1-step ahead predictions, we use

$$(Z_{t+1}|\mathcal{D}_t) \sim \mathcal{W}_m\left(n, \frac{(k\Omega_t)^{-1}}{\lambda}\right) \tag{3.18}$$

b) (WINDLE; CARVALHO, 2014, Proposition 2) **Backward sampling:** The joint density of all data until time T conditional to $\mathcal{D}_T$, $(\{Z_t\}_{t=1}^T|\mathcal{D}_T)$, with respect to the T-fold product of S_m^+ embedded in $\mathbb{R}^{m(m+1)/2}$ with Lebesgue measure, can be decomposed as,

$$p(\{Z_t\}_{t=1}^T|\mathcal{D}_T) = p(Z_T|\mathcal{D}_T)\prod_{t=1}^{T-1} p(Z_t|Z_{t+1}, \mathcal{D}_t). \tag{3.19}$$

The last term in (3.19) is a shifted Wishart distribution given by

$$p(Z_t|Z_{t+1}, \mathcal{D}_t) = \lambda \cdot Z_{t+1} + U_{t+1}, \quad U_{t+1} \sim \mathcal{W}(k, (k\Omega)^{-1}). \tag{3.20}$$

c) (WINDLE; CARVALHO, 2014, Proposition 3) **Marginalization:**The joint density of the observables $\{y_t\}_{t=1}^T$ is given by

$$p(\{y_t\}_{t=1}^T|\mathcal{D}_0) = \prod_{t=1}^{T} p(y_t|\mathcal{D}_{t-1}), \tag{3.21}$$

where $p(y_t|\mathcal{D}_{t-1})$ is defined both for the full-rank case and rank-deficient case. See Windle & Carvalho (2014) for more details.

Part III

Empirical Model

4 ASSESSING THE IMPACT OF CONVENTIONAL MONETARY POLICY ON THE CAPITAL-LABOR RATIO IN BRAZIL

"The power of a theory is exactly proportional to the diversity of situations it can explain."

ELINOR OSTROM

To assess the aggregate impact of conventional monetary policy on income distribution that would be resultant from the redistribution channels described in the previous sessions, our proposal is to evaluate the effect of the Selic shocks in the capital-labor ratio ($^{K}/_{L}$) in a small open economy. We are defining $^{K}/_{L}$ as the ratio between the capital income and the labor income - both are monthly series made available to general public by Brazil's IRS. The remaining series to close the model are the short term interest rate (Selic), real effective exchange rate, price index and per capita GDP, that were used in a TVP-VAR(2) with the Wishart specification discussed in the previous section. The number of lags (2) was chosen accordingly to the AIC criteria.

Overall, we used monthly data from January 1996 to October 2018, totalizing 274 periods over a 22-year spam, of which the first 48 months were used as the model's prior and the remaining 224 were used to compute the posterior results. In this section, we will present the data in detail, followed by the analysis of the results.

4.1 DATA

Table 1 contains all source pertinent information about the data series that were gathered from the Brazilian Central Bank Time Series Management System[1] (SGS) or the Brazilian Institute of Geography and Statistics (IBGE). All series are available for the general public and, except for the Population estimates that were downloaded directly at IBGE's website, all series are available in the SGS.

Where pertinent, the reported graphics have shaded areas indicating recession periods in the Brazilian economy, following the classification by the Brazilian Business Cycle Dating Committee (CODACE). For the Committee, a recession is "*characterized by an expressive decline in economic activity and occurs simultaneously in many sectors during a period of time*" (Business Cycle Dating Committee, 2017). Table 2 has the

[1] Available at: <https://www3.bcb.gov.br/sgspub>. Last visit: January 02, 2019.

Table 1 – Data information

Series	**Series number**[♠]	**Series name (in Portuguese)**	**Beginning**[✧]	**End**[★]	**Unity**	**Source**
Labor income[☆]	7620	Rendimento do trabalho	Jan. 1992	Oct. 2018	u.m.c. (millions)	BCB-DSTAT[1]
Capital income[✪]	7621	Rendimento do capital	Jan. 1992	Oct. 2018	u.m.c. (millions)	BCB-DSTAT[1]
Interest rate[▲]	4390	Taxa de juros SELIC	Jul. 1986	Dec. 2018	%/month	BCB-Demab[2]
Inflation rate[❖]	433	IPCA	Jan. 1980	Nov. 2018	Monthly % var.	IBGE[3]
Exchange rate[✳]	11752	Taxa de câmbio efetiva real	Jan. 1988	Dec. 2018	Index	BCB-Depec[4]
GDP[❄]	4380	PIB mensal	Jan. 1990	Nov. 2018	R$ million	BCB-Depec[4]
Population[✣] (monthly)	-	Estimativa da População	Jan. 2000	Dec. 2030	People	IBGE[3]
Population[†] (yearly)	-	Estimativa da População	1980	2050	People	IBGE[3]

Notes:
[♠] Numeric code that identifies the series in the Brazilian Central Bank Time Series Management System (SGS).
[✧] Month and year of the first available value of the series in the SGS.
[★] Month and year of the last available value in the SGS when consulted in January 2, 2019.
[☆] Tax revenues - Accrual basis - Income tax - Withholdings - Labor earnings.
[✪] Tax revenues - Accrual basis - Income tax - Withholdings - Capital earnings.
[▲] Interest rate - Selic accumulated in the month
[❖] Broad National Consumer Price Index (IPCA)
[✳] Real effective exchange rate index (IPCA) - Jun/1994=101
[❄] GDP monthly - current prices (R$ million)
[✣] Population estimate. Available at: <https://www.ibge.gov.br/estatisticas-novoportal/sociais/populacao/9109-projecao-da-populacao.html?edicao=9116&t=resultados>. Last visit: December 20, 2018.
[†] Population estimate. Available at: <https://www.ibge.gov.br/estatisticas-novoportal/sociais/populacao/9109-projecao-da-populacao.html?edicao=17996&t=resultados>. Used to complete the monthly series. Last visit: December 20, 2018.
[1] BCB-DSTAT Brazilian Central Bank, Statistics Department.
[2] BCB-Demab Brazilian Central Bank, Department of Open Market Operations.
[3] IBGE Brazilian Institute of Geography and Statistics.
[4] BCB-Depec Brazilian Central Bank, Economics Department.

Source – Own construction using information from the SGS and IBGE.

Brazilian Business Cycles quarterly chronology accordingly to CODACE. The relevant time horizon for this work starts in the Expansion period from 1999 Q2 to 2001 Q1. From that point, Brazil faced three more Expansion waves, including the biggest one reported by the CODACE (in terms of Cumulative Growth from Peak to Trough) that occurred from 2003 to 2008. From 2014 to 2016, the Brazilian economy faced a drawback due to a recession that lasted two years and a half and was at the same time of a major political crisis that culminated in the deposition of the then President Dilma Rousseff.

4.1.1 The capital-labor ratio

If one wants to investigate the relationship between the monetary policy, income and wealth distribution for Brazil, there are monthly series of capital and labor income based on the tax revenues from individuals and firms collected by the Brazilian IRS. This series represents an aggregate measure (only the total income considering all population are available, there is no possibility to divide in deciles), thus it can be seen as the functional income distribution between the two production factors, capital and labor.

Table 2 – Duration and Amplitude of Brazilian Business Cycles Quarterly Chronology between 1980 and 2017.

From[1]	**To**	**Duration**[2]	**% Growth**[3]	**% Quarterly Growth**[4]	**Type**[5]
1981Q1	1983Q1	9	-8.5	-3.9	R
1983Q2	1987Q2	17	30.0	6.4	E
1987Q3	1988Q4	6	-4.2	-2.8	R
1989Q1	1989Q2	2	8.5	17.7	E
1989Q3	1992Q1	11	-7.7	-2.9	R
1992Q2	1995Q1	12	19.2	6.0	E
1995Q2	1995Q3	2	-2.8	-5.6	R
1995Q4	1997Q4	9	8.0	3.5	E
1998Q1	1999Q1	5	-1.5	-1.2	R
1999Q2	2001Q1	8	7.5	3.7	E
2001Q2	2001Q4	3	-0.9	-1.2	R
2002Q1	2002Q4	4	5.3	5.3	E
2003Q1	2003Q2	2	-1.6	-3.1	R
2003Q3	2008Q3	21	30.5	5.2	E
2008Q4	2009Q1	2	-5.5	-10.8	R
2009Q2	2014Q1	20	23.0	4.2	E
2014Q2	2016Q4	11	-8.6	-3.2	R

Notes:
[1] Q_i denotes the i-th quarter of a given year.
[2] In quarters.
[3] Cumulative growth from Peak to Trough in Recessions and otherwise in Expansions.
[4] Average Quarterly Growth (annualized).
[5] **E** denotes an expansion and **R** a recession period.
Source – Brazilian Business Cycle Dating Committee (CODACE).

More specifically, if we divide the income from capital by income from labor (obtaining the capital-labor ratio), what we have is the share of the capital with respect to the national income divided by the share of labor w.r.t. the national income. This means that increases in this measure represents a higher payment for capital when compared to labor. Concerning the capital income, broadly speaking, the monetary policy can affect the capital invested in production (which generates dividends) by unbalancing the opportunity cost of capital and its marginal product. On the other hand, both short term rate and the term structure of interest rate influence the present value of financial assets. And, with respect to labor income, the interest rate has an influence on unemployment, which directly impacts wages and salaries. Therefore, we can infer the overall effect of the income composition and the interest rate exposure channels by analyzing the shocks of interest rates on these series.

If capital and labor incomes were evenly distributed among families and firms, a shift in the capital-labor ratio would have zero effect on the income distribution, but this is an unrealistic scenario (BENGTSSON; WALDENSTRÖM, 2017). Although it is true that the richest have income from wages, they also have a considerable amount of income coming from capital gains when compared to the households in the lowest deciles of income that rely heavily on wages and transfers (MEDEIROS; DE SOUZA, 2015). This latter effect could be underestimated because households whose income is less than a certain amount (yearly defined by the IRS) don't need to specify their earnings in the annual tax return. Nevertheless, as pointed out by Souza (2016), there are advantages from the tax data over population surveys, specially concerning income information, such as reduced amount of missing data and sampling issues.

Using taxpayer data from the Brazilian IRS, we constructed the capital-labor ratio ($^K/_L$), which is the quotient between the capital income and the labor income, as declares in tax revenues. **Labor income** refers to all declared income from wages, obtained in the form of wages by working in Brazil or overseas (labor income declared to Brazil's IRS), or obtained through laboring without formal contract, income from private retirement savings and indirect income. On the other hand, the **capital income** comprehends the interests over owner's equity (calculated from companies accounts); profits from fixed income assets; stocks; real state; cultural and artistic investment funds; day-trade operations; income from rents and royalties; Swap operations and operations with investment funds (BRASIL. Ministério da Fazenda. Receita Federal do Brasil, 2015b; 2015a).

Those two series can be used to compute the share of GDP allocated in each production factor (capital or labor). Note that this implies that the capital-labor ratio is the quotient between the share of capital by the share of labor, both with respect to GDP:

$$\frac{K}{L} = \frac{^K/_{GDP}}{^L/_{GDP}} = \frac{\text{Capital share in GDP}}{\text{Labor share in GDP}}. \tag{4.1}$$

The $^K/_L$ ratio can be seen as a measure of distribution of income between two production factors, labor and capital. Since those are not evenly distributed among individuals, shifts in the capital-labor ratio imply a redistribution of income.

4.1.2 Other variables

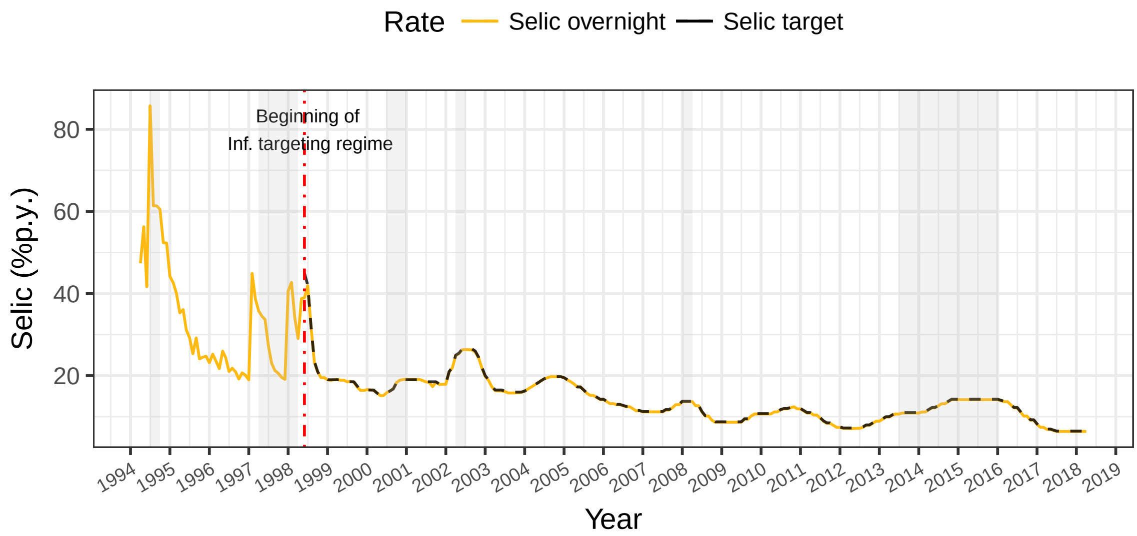
The Central Bank of Brazil (Bacen) uses the monetary policy to affect the capital cost and money supply in the economy. Its main instrument is the Selic rate, which is defined by the Monetary Policy Committee (Copom) of the Central Bank. Brazil formally adhered to the inflation-targeting regime in June of 1999. Under this system, the monetary decisions made by the Copom are aimed to maintain inflation around the target established by the National Monetary Council (CMN).

This process works as follows. First, the CMM sets the target for the inflation in the following three years, which reduces uncertainty and, at the same time, allows households and firms to better plan in advance their decisions regarding consumption, savings and investment. The price index used in the CMN's decisions is the IPCA acummulated in a twelve months period of time. The inflation target always have a tolerance interval, which in the last years was the target plus or minus 1.5%. The Copon then takes the decisions regarding the Selic target rate, accordingly the macroeconomic scenario presented at the time of the Committee meting (Banco Central do Brasil, 2019c). What the Copom does is to set the target for the short term interest rate, Selic, which is the overnight interest rate used by banks to make interbank operations. It is important to note that the target Selic is defined by the Copom and stays the same between any two Copon Meetings, where it can remain the same or change. This is different from the Selic rate. The latter represents an equilibrium between borrowers and lenders in the overnight interbank market, which is basically the transactions with government bonds (Banco Central do Brasil, 2019a). In practice, the values of Selic overnight rate are slightly smaller than the Selic target (about 0.01%).

Figure 5 has the values of the Selic rate from January of 1995 onwards and the values of the Selic target from the dawn of the inflation targeting regime to nowadays. It is possible to see that, prior the inflation targeting regime, the values of the Selic rate were higher and had way more volatility. After the implementation of the new regime, there was a constant downward trend in the inflation rate. A relatively prominent peak in Selic ocurred between the fourth quarter of 2002 and the end of the first quarter of 2003. This period corresponds to the Presidential elections and the first months of the Worker's Party Government in Brazil, where the CB adopted a tight policy in response to the inflation rise expectation projected for the period. This was followed by a rise in the compulsory deposits at the beginning of 2003. Since that period, the interest rate level has been constrained below 20% p.y. and has been lower than 10% since 2017.

The other variables that enter in the model are the usual ones for a small open economy model: inflation index, per capita GDP and exchange rate. The consumer price index rate used is the IPCA accumulated in twelve months with base month is March 2018. The index was used to deflate both exchange rate and the GDP. Regarding the latter, we used the method of Hamilton (2018) as a substitute to the HP filter. Finally, the exchange rate is the real effective one and we used the first difference of the original data logarithm. The final series (after filtering for seasonality when needed) are stationary. All transformations are described in detail in Appendix B.

Figure 5 – Selic rate and Central Banks's Selic target. Brazil, 1995-2018.

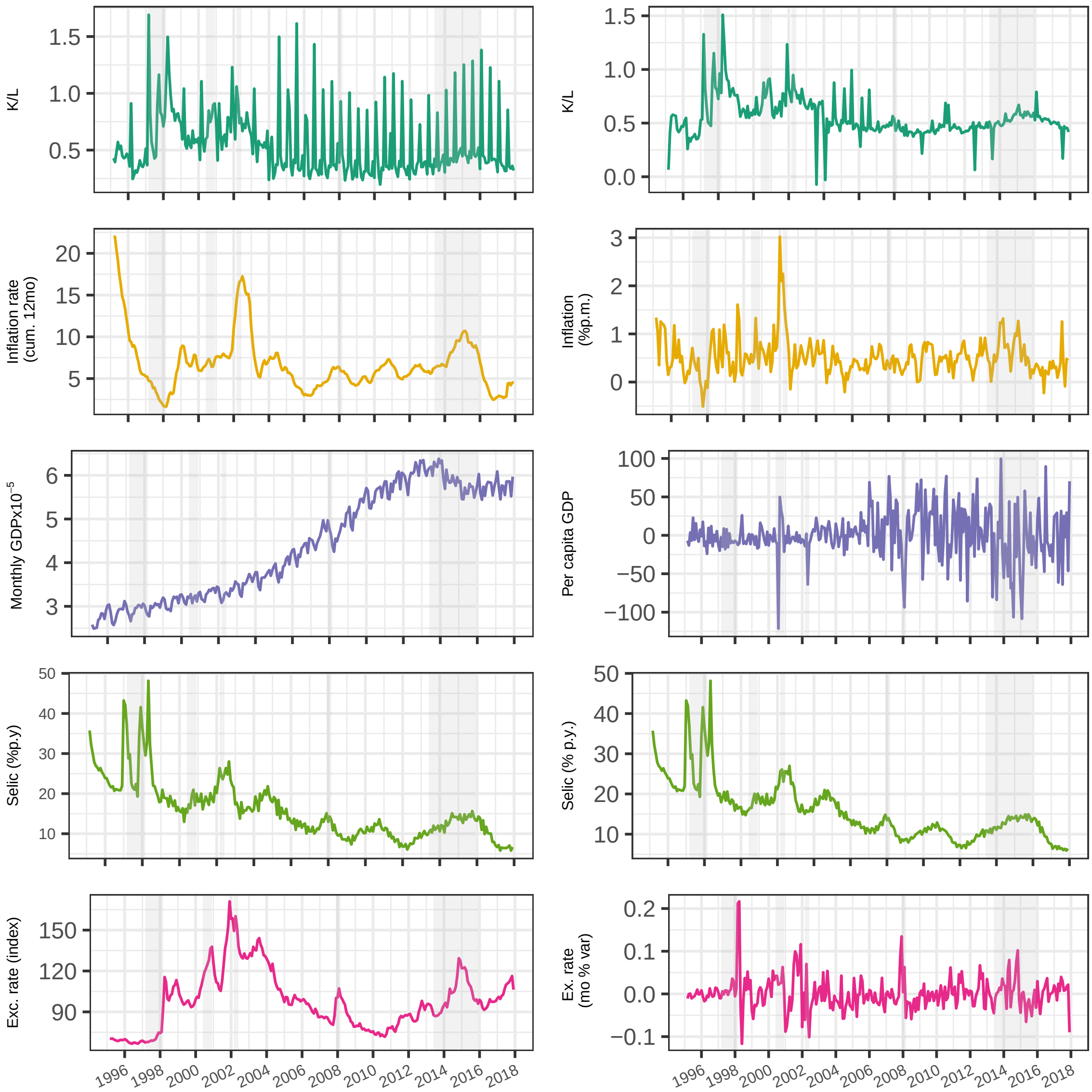


Comparison between the Selic target (black, dashed line) and the Selic overnight (yellow, continuous line). Shaded areas indicate recession periods, accordingly to the Economic Cycle Dating Committee (CODACE) and x-label marks denote the October month for each year. The inflation targeting regime formally initiated in June of 1999, after a period of recession in the Brazilian Economy that followed the Asian and Russian crisis. The Selic overnight is used in the interbank market and its values are always slightly below the target.
Source – Own elaboration using data from BCB-Demab.

A comparison between the raw data and the data after the transformations and adjustments can be seen in Figure 6. Table 3 has the descriptive measures for the five series.From the right column of Figure 6, we observe a higher volatility in the beginning of the series, specially in the years before 2000. It is possible to notice, by looking at the data from Table 3, that the share of capital was smaller than the share of capital in more than 75% of times. We also see the effect of previous turmoil periods in the interest rate: even though in the last 2 years showed a major decline in the SELIC rate, the fact that more than 75% of the values are higher than 10% p.a. is a reminder that the double digit interest rate is not that far away from the present. The GDP per capita is the variable that presents the second largest range (loosing only for the interest rate). Its smaller value occur in 1995, almost a year later after the implementation of the Brazilian real, which could have caused a drop in GDP until the economy adjusted itself to the new scenario.

The final series were all considered stationary for the ADF test with an alpha equal to 10%. Although the model theoretical specification allows for the presence of unitary roots, we considered that stationary series were better suitable for us given that we are interested in estimating a long-term relationship through the impulse response functions and economies, in general, are stable, indicating that most likely the underlying behaviors of the macroeconomic aggregates are indeed stationary.

Figure 6 – Data series used in the TVP-VAR, before (left) and after (right) transformations. Brazil, 1996-2018.



Comparison between the five series used in the model before (left) and after (right) the seasonal adjustment and other transformations. Shaded areas indicate recession periods, accordingly to the Economic Cycle Dating Committee (CODACE) and the vertical divisions marks the November month of each year indicated in the horizontal axis. The series where a seasonal treatment were made are the capital-labor ratio and the interest rate. GDP per capita was treated to remove the trend using the regression method described in Hamilton (2018) and whenever necessary, the ARIMA-X13 filter was used to remove seasonality.

Source – Own construction using data from BCB-Depec, BCB-Demab, IBGE, B3 and BCB-DSTAT.

4.2 MODEL SPECIFICATION

4.2.1 The prior

The prior distributions follow the specification needed to employ the method of Windle & Carvalho (2014) and Carter & Kohn (1994) for the variances and coefficients of the measurement equation, respectively, as well as the conjugate prior for the state transition equation for the coefficients. This means that the initial coefficients β_0 follows a multivariate normal distribution, the prior for the covariance matrix of the shocks over

Table 3 – Descriptive statistics of the series used in the model (n=274 months).

	K/L	Interest rate	Per capita GDP	Exchange rate	Inflation rate
Minimum	−0.0541	−138.5647	−13.7417	−0.1169	−0.5100
1st quartile	0.4530	−11.4516	−0.2265	−0.0183	0.2800
Mean	0.5544	0.0000	15.9358	0.0015	0.5172
Median	0.5154	−1.6502	14.1000	−0.0012	0.4600
3rd quartile	0.6239	12.4888	5.7616	0.0153	0.6900
Maximum	1.5100	104.6974	12.3665	0.2167	3.0200
Stand. Dev.	0.1882	30.1136	4.5896	0.0388	0.3939

Notes:
* All series series start on January, 1996 and finish on October, 2018, totalizing 274 time periods;
* K/L means the capital-labor ratio and is the quotient between the capital income by the labor income.
* Both K/L and GDP had their seasonality component removed using the X-13ARIMA-SEATS algorithm.
* The consumer price index used is the monthly IPCA.
* GDP per capita is the de-trended series, while the exchange rate is the monthly variation of the real effective exchange rate.
* The population estimate used to compute per capita GDP is provided by IBGE. For the first 48 values, we used the yearly estimates as approximation to each month within a year. After that, the monthly estimates with the 2000s correction were used.

Source – Own construction using data from IBGE, BCB-Depec, BCB-Demab, IBGE, B3 and BCB-DSTAT.

the β_t is an inverse wishart and the prior for the covariance Ω_t is a wishart. The initial coefficients and covariance in the measurement equation were inflated by the multiplication of a scalar and the prior hyperparameters were obtained through the OLS quantities from the first 48 observations (that were not included in the period used to compute the posterior).

Choosing the first 48 observations for the prior was not a coincidence. This period corresponds to the subsequent years after the stabilization of the economy through the real plan, that, among other things, implemented the a new currency (Brazilian reais) that at first had a fixed exchange rate. For the first years (1995 to 1998), inflation was still high and the Central Bank spent many resources trying to maintain the exchange rate fixed. The fixed exchange rate started to consume the country's dollar reserves, which was aggravated with the Asian and Russian crisis in 97-98, causing pressure for changes in the conduction of the monetary policy. This happened in 1999, when the government officially committed to the inflation targeting regime. Given that, the period from January 1996 to the beginning of 2000 was marked by shocks from various sources which increased volatility during that time. This implies that the removal of the first 48 observations and subsequent inflation of the prior distributions was a necessary action to account for the high uncertainty of the period.

4.2.2 Identification of the shocks

Given that our interest lies in investigating the impact of interest rate shocks, it is necessary to compute impulse response functions (IRF). For TVP-VAR models, at any instant t we have a set of coefficients, implying that different impulse response functions

Table 4 – Model Priors, initial values and parameters

Parameter	**Description**	**Prior family (or value)**♠	**Coefficient(s)**
β_0	Initial Coefficients	$\mathcal{N}(\hat{\beta}_{OLS},\ k_\alpha \cdot \hat{V}(\hat{\beta}_{OLS}))$	$k_\beta = 4$
Q	Covariance matrix of shocks in β_t	$\mathcal{IW}(1/4\ k_Q^2 \cdot p_Q \cdot \hat{V}(\hat{B}_{OLS}),\ p_Q)$	$k_Q = 0.01$, $p_Q = 48^{☆}$
Ω_1	Initial Covariance	$\mathcal{W}_m(\nu_\Omega, \Sigma_0^{-1}/\lambda_\Omega)^{✣}$	$k_\Omega = 0.01$, $p_\Omega = 6^{❋}$
ν_{H0}	Parameter	$\nu_{H0}^{❋}$	-
λ	Parameter	$\lambda = \frac{\nu}{\nu+1}$	-

Notes:
✣ - Initial value Σ_0 will be estimated based on a Wishart conjugate prior.
♠ - Variables with a hat ând subscript OLS are the ordinary least squares estimates, which were evaluated using the first 48 observations from the sample. $\mathcal{N}(\mu, \theta)$ e $\mathcal{IW}(\Psi, \nu)$ denote the Normal distribution with mean μ e variance θ and the Inverse-Whishart distribution with scale Ψ and ν degrees of freedom, respectively.
$\mathbb{I}_n$ denotes the identity matrix of rank n.
☆ - 48 refers to the number of total observations used to calculate the prior parameters.
❋ - ν_{H0} is an equally-spaced grid with values ranging from 4 to 70.
Source – Own construction.

can be computed at each period. We selected periods considering changes in the Central Bank Governor, Ministry of Finance and also dates related to economic and or political major events in the Brazilian economy that occurred in the sample period. They are described in detail at the results section.

We here are imposing a Cholesky decomposition of the shocks in order to isolate structural effects. Since the results obtained will depend on the order of the variables, we are assuming the following identification scheme: the capital-labor ratio has a structural characteristic (see Herran, 2005), therefore, it is not contemporaneously affected by any other shock; inflation and GDP affect contemporaneously the interest rate, which affects the exchange rate. This last relation is a consequence of the assumptions on our small open economy: the monetary authority will have to choose as nominal anchor either the exchange rate or the interest rate. Considering that Brazil adopts the inflation targeting regime, the latter seems to be more realistic in our application, since it is not possible to have an active monetary policy with a fixed exchange rate for a small open economy.

TVP-VARs such as the ones from Cogley & Sargent (2005), Primiceri (2005) or even the one defined in system (3.13)-(3.15) have the common characteristic that the law of movement for the coefficients is a random walk. This means that we are accepting the possibility of a non-stationary process in the sense that these coefficients can lead to explosive trajectories, which would imply unbounded impulse response functions. Cogley & Sargent included in the model their belief that the Fed follows a meaningful rule through the adoption of a stability condition. In practice, their Gibbs Sampler has a rejection sampling algorithm to discard explosive draws from the posterior. Koop & Potter (2011) proposes an alternative algorithm to avoid the explosive trajectories, arguing that methods such as Cogley & Sargent (2005) could perform poorly and approximate some results, whereas their method is exact. In this work we followed Primiceri (2005)'s approach and did not include restrictions in the model. None of our impulse response functions presented explosive behavior. Given that the sample excludes the Brazilian hyperinflation period prior to 1994 and all series were pre-treated to eliminate any non-stationary behavior, we considered that the restrictions were not essential in our case.

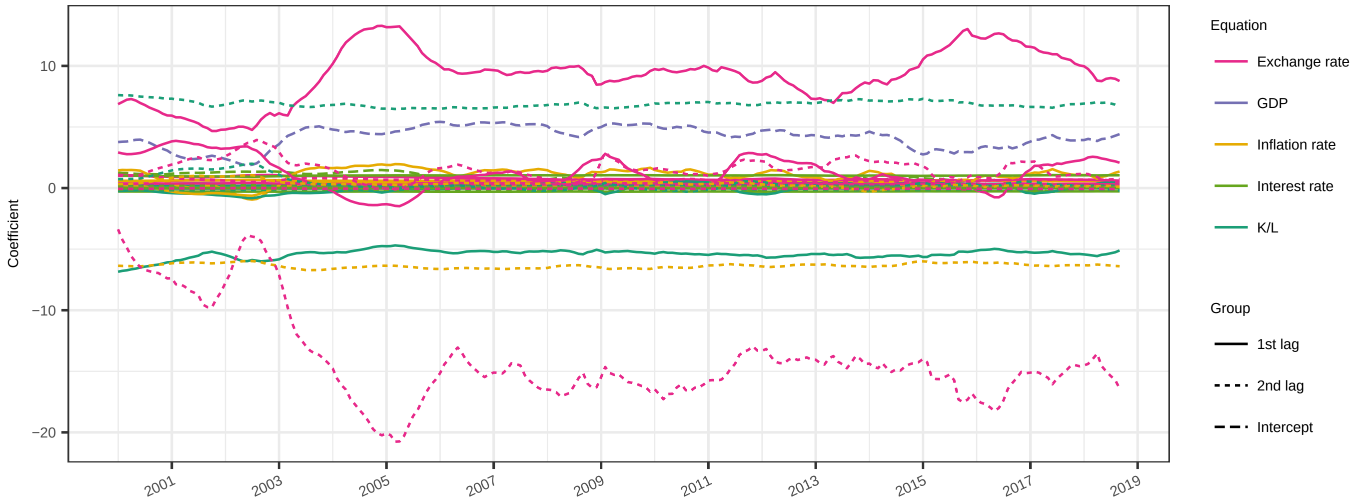
4.3 RESULTS

Our posterior results are based on 226 months of data, scattered from 2006 to 2018, a burn-in of 200.000 interactions and 10.000 draws after burn-in. We divided this section between the analysis of the coefficients and the volatilities and the impulse response analysis.

4.3.1 Coefficients and Volatiliity

At any given time t, the system of equations presents 55 of the α coefficients, which correspond to five intercepts, 25 first lag coefficients and another 25 second lag coefficients. For each one of them, we could draw a whole trajectory from $t = 1$ to $t = 226$, which is represented in Figure 7. The graph contains all trajectories for the coefficients of the intercepts, first and second lags of the VAR model, which are indicated by the legend. Except for the outliers associated with the exchange rate series, the remaining coefficients cannot be distinguished neither by the equation that they are part of (indicated by the colors of the graphs) or the type of coefficient (intercept, first lag or second lag - indicated by the line type). Since the analysis of all coefficients together could represent a challenge, we opted to focus the analysis only at some of the coefficients. The majority of the coefficients have low values or are over the zero line. We observe that there are three types of coefficients: the ones that are time varying, such as the first and second lag coefficients that appear in the exchange rate equation (pink color); the ones who are not time varying but don't appear to be equal zero (such as the first and second lag coefficients of the capital-labor ratio in its own equation in dark green color) and lastly, the ones that are almost equal to zero.

Figure 7 – Posterior means for all coefficients (intercepts, first and second difference) of the model, 2000-2018.

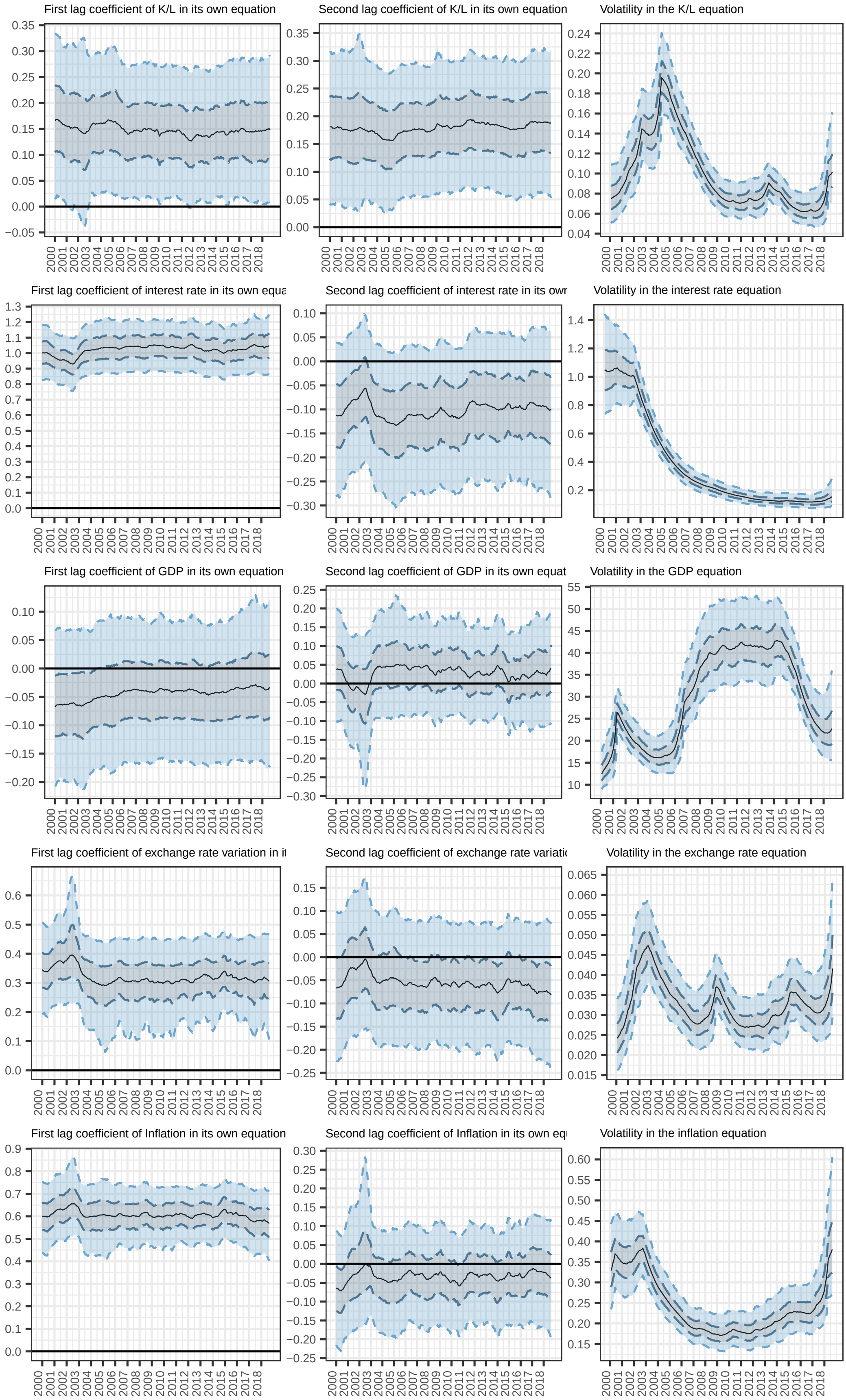


Posterior means obtained through 10.000 draws from the posterior distribution.
Source – Own elaboration with results from the model estimated.

Figure 8 shows the first and second lag coefficients of each variable in their own equations. For example, the first two graphs from the top line are the coefficients that multiplies the capital-labor ratio in $t-1$ and $t-2$ in the capital-labor ratio equation, and so on for the other four equations. The third column on the right contains the estimated standard deviations for each VAR equation. The solid black line is the median of 2500 posterior draws for each one of the parameters, at each given time. The darker gray area comprises the interval between the 25th and 75th percentiles, while the light blue area ranges from 5th to 95th percentile. The coefficient for the second lag of the capital labor ratio has zero included in the 90% credible interval and the same happens for the second-lag coefficient of the exchange rate. In general, the coefficients plotted in Figure 8 have low to nonexistent time variation behavior with some exceptions discussed below.

However, some coefficients show moving patterns in Figure 8. The first lag of interest rate in its own equation has a quick descend behavior until 2003, where its median is below 1 and then after 2003 it rises to stay stable around the unit. This period was shortly after the adoption of inflation targeting by the Central Bank of Brazil and it was followed by the uncertainty that followed the Russian crisis and the confidence crisis during the Presidential elections in October, 2002. Only in 2003 the Brazilian economy started to recover from this events and it may be possible that the monetary policy was weakened until there. Also, the first lag coefficient of the exchange rate in its own equation and the inflation rate present a similar pattern (in different magnitudes). They rise until 2003 and then became stable around a value (0.3 and 0.6, respectively). It is possible that the increase in the autoregressive component in this two equations is related to the decrease in the interest rates, specially in the case of the inflation index.

Figure 8 – First and second lag coefficients of each variable in its own equation and estimated volatilities (median, 50% and 90% centered around the median credible intervals) for 2000-2018.



First and second coefficients of each variable in their own equations (first and second columns) and standard deviation for each VAR equation (third column on the right). Due to lack of space, the interpretation is in the text only.

Source – Own elaboration with results from the model estimated.

Regarding the changes in the exchange rate behavior, it could be an inertial behavior due to the crisis and the end of the exchange rate anchor that resulted in the depreciation of the Brazilian real in 1999.

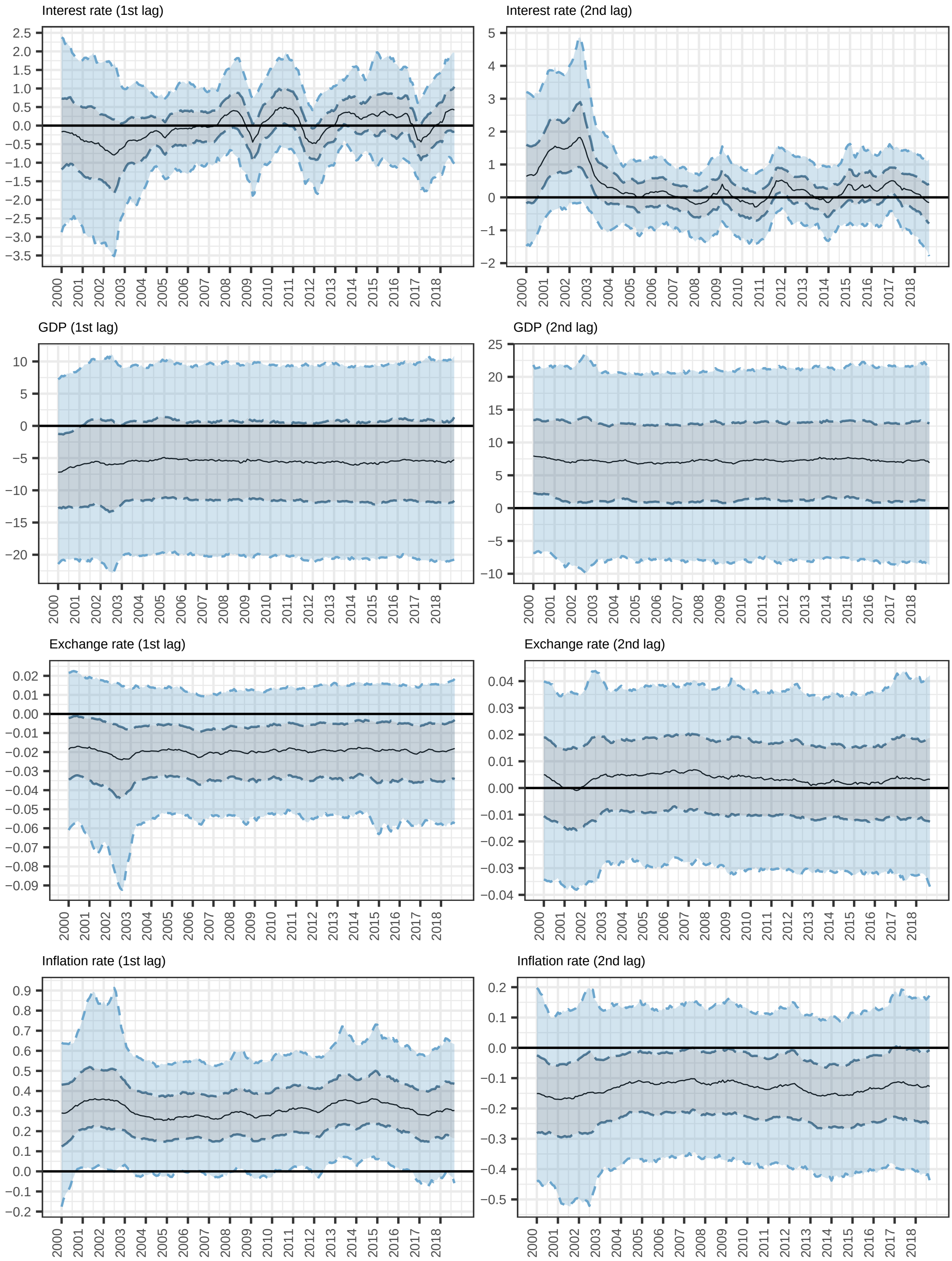
It is clear from Figure 8 that the model volatilities are not constant. The GDP equation presents a spike in its variance by the end of 2002, but the higher volatility period was from 2008 to 2015. This may be related to the Great Recession, since shortly after the economic crisis, Brazil increased economic spending to counter the recession effects - plus, according to the CODACE, the period between 2009 and 2014 was an economic expansion (see Table 2 for all economy cycles). This was followed by the recession period that started in 2014 and has not ended until now. All remaining volatilities in Figure 8 present higher peaks in the first five to six years, with strong evidence of changes in the volatility pattern. In particular, the interest rate volatility, after the turmoil period in the beginning of the 2000s, decreased and remained stable, most likely as a reflex from the inflation targeting adoption by the end of the previous decade. The volatility in the exchange rate equation achieved its peaks in three different periods: around 2003, 2009 and a less prominent rise in 2016. The first was during the first year of the Worker's party presidential mandate, while the second peak occurred while the global economies were still under the effects of the international crisis. The last rise in volatility is coincident with the political crisis in Brasil that culminated with the President Rousseff impeachment.

The coefficients of all other variables in the capital labor ratio equation are in Figure 9. It is possible to observe that many coefficients are not different from zero, when we consider the 90% credible interval. This suggests that, at least in the capital-labor ratio, the coefficients are non time-varying and have values statistically non-different from zero. The exception, besides the coefficients of the first and second lag of the capital-labor ratio (that are in Figure 8), would be the coefficient associated with the first lag of the inflation rate. Considering all the sample period, we can see that only a few times the lower bound of the credible interval crosses the x axis. Nevertheless, its behavior is almost constant, maybe with exception of the spike between 2001 and 2003. The median values of both coefficients of the interest rate in the $^{K}/_{L}$ equation exhibit a rougher trajectory and crossing the zero line several times. Nevertheless, the 90% credible interval always contains the zero.

4.3.2 Impulse response analysis

To verify the impact of monetary shocks over the other model's variables, we calculated the impulse response functions (IRF) for each period, considering the estimated values of β_t, Ω_t and a time horizon of 25 months.

Figure 9 – First (left column) and second (right column) lags' coefficients of the interest rate, per capita GDP, exchange rate and inflation rate in the equation of the capital-labor ratio (median, 50% and 90% centered around the median credible intervals), estimated for the 2000-2018 period.



Source – Own elaboration with results from the model estimated.

Figure 10 presents the median of the IRF using the estimated values at t equal to October 2018 (the last period of the sample) with the intervals from the 5th to 95th percentile (yellow light area) and 25th to 75th percentile (brown darker area). There is no significative effect of the monetary shocks in any of the other variables in the VAR, at least for this period. Although small in magnitude (0.011 at the spike in the fourth month), the effect from the rise of the interest rate over the capital-labor ratio is positive and lasts at least one year, suggesting that in fact there is a redistributive effect of monetary policy over income. It is not possible to tell if the increase is due to a rise in the capital income, a diminution of the labor income or both. Either way, considering the existing heterogeneity in the economy with respect to capital owners and workers, it is most likely that the first group is benefiting more from a contractionary policy, which could ultimately lead to an increase in inequality.

In order to assess whether the relation between monetary policy and K/L is stable or varies across time, we estimated the IRF from a monetary shock in the capital-labor ratio for different periods, which is exhibited on Figure 11. The values are significantly different from zero from the second month after the initial shock and lasts for at least 10 months, for all periods, using the interval between the 5th and the 95th percentiles. The median of the IRF in July, 2008 is slightly higher than the others, suggesting that the effect of monetary shocks on the capital-labor ratio was higher during that period, in line with the results found by Mumtaz & Theophilopoulou (2017) for the Great Recession. We see that the is a change in the pattern of the response to monetary shocks, with the effect being smaller in more recent years (from 2008 onward there is a fall in the values until they reach zero when the IRF is evaluated in 2016 and 2018. The periods elected as the ones to recalculate the IRF are described in Table 5.

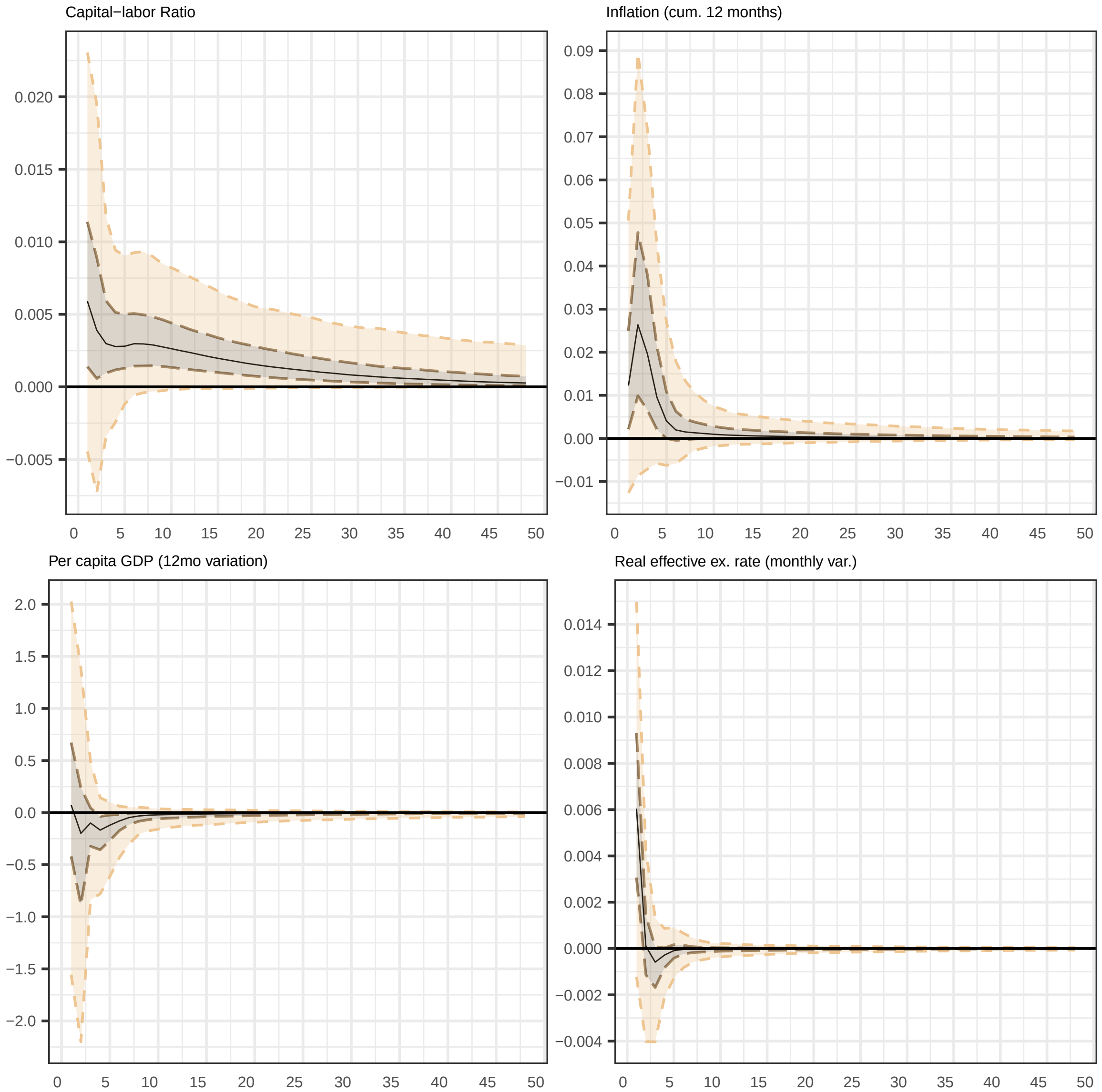
Table 5 – Selected periods for the FIR calculation in Figures 11 and 18

	Date	t	Context		Date	t	Context
01	Mar/00	1	Beginning of the data series[1]	06	Jul/05	65	Political crisis[6]
02	Jun/01	16	Electrical crisis[2]	07	Jun/08	100	International economic crisis
03	Jun/02	28	Economic Expansion period[3]	08	Jun/10	124	Economic Expansion period[3]
04	Oct/02	32	Presidential elections[4]	09	Mar/16	193	Impeachment process [7]
05	Jan/03	35	Presidential Inauguration[5]	10	Oct/18	224	Last observation period

Notes:
[1] - Although we have data since January 1995, due to the use of the first 48 observations for the prior, the model just starts in March 2000.
[2] - The electrical crisis in Brazil occured from July 2001 to February 2002 and affected the power supply all over the country.
[3] - This classification is nade by the Brazilian Business Cycle Dating Committee (CODACE).
[4] - There was great political instability with spillovers to the economy due to Lula's (the candidate from the Worker's Party to presidency of the country) rise in the pools.
[5] - Luis Inácio Lula da Silva, the elected president in the 2002's elections, was nominated the 35*th* Brazil's president in January 1st, 2003.
[6] - The political crisis in 2005 is called *Mensalão Crisis* and was a scandal involving many parties, including the President's. It culminated, later, with the trial and prison of several politicians acused of corruption.
[7] - The impeachment process of Dilma Rousseff started in December 2015.
Source – Own construction.

The next exercise involves checking the reactions of the model variables to monetary shocks considering changes of the Governor of the Central Bank and/or in the head of the Finance Ministry during the sample period (2000-2018). The idea here is that different

Figure 10 – Impulse response functions (median, 50% and 90% centered around the median credible intervals) of the model variables to monetary shocks using the estimated values for October, 2018.

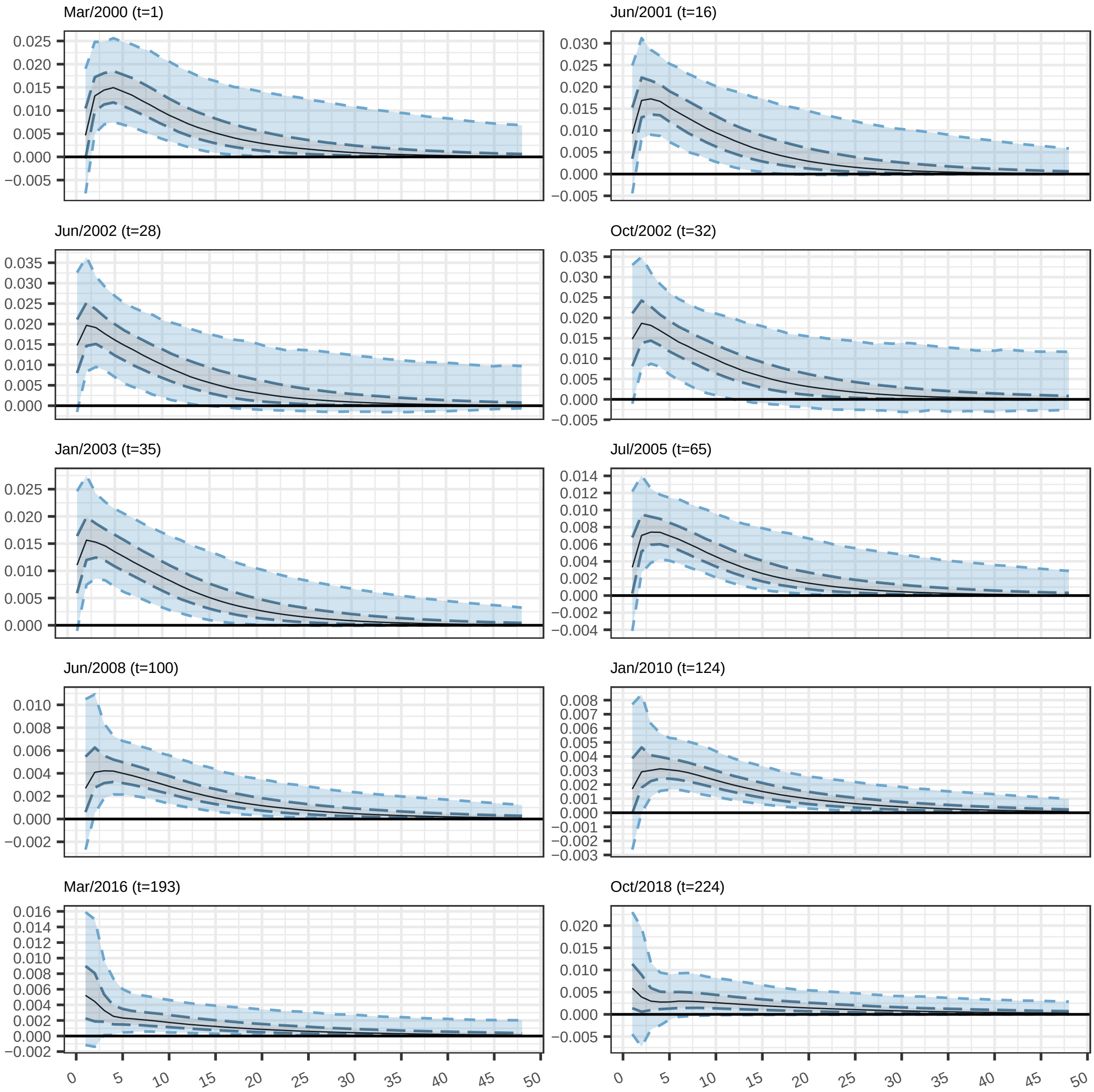


Impulse response functions (IRF) of K/L (%), Inflation (monthly %), per capita GDP and Exchange rate (monthly % variation) to monetary shocks using the estimated coefficients and volatilities for October, 2018. The functions are not significantly different from zero considering the interval between the 5th and 95th percentiles (yellow area). The dark area corresponds to the interval between the 25th and the 75th percentile.
Source – Own construction.

combinations of monetary-fiscal policies could have different outcomes in the income distribution. The dates used to compute the IRFs with the respective names of the Central Bank Governor and Minister of Finance are shown in Table 6. As control, we included the period of June 2008 (Great Recession).

Figure 12 shows median changes in the responses in all variables to monetary shocks when analyzing different periods. It is interesting to note that the peaks occurred at the first posterior period for all variables. It could be the case that the central-bank changed its behavior in a way that indirectly affected these two quantities. Nevertheless,

Figure 11 – Impulse response functions (median, 50% and 90% centered around the median credible intervals) of the capital-labor ratio to monetary shocks for selected periods.



Impulse response functions (IRF) of K/L to a 1% monetary shock using the estimated coefficients and volatilities for each indicated time, selected accordingly to major political and/or economical events that occured between 2000-2018 in Brazil. The blue area corresponds to the interval between the 5th and 95th percentiles, whilst the gray area corresponds to the interval between the 25th and 75th percentiles, based in 2500 posterior draws. For the Capital-Labor ratio, most intervals do not include zero from the second month until the 15th month after the initial shock.
Source – Own construction.

this is a slightly higher median value and considering the credible intervals there is no difference between the curves in different periods.

As for inflation, the highest response to monetary shocks were also in January 2003. This is not surprisingly since it was exactly what the Central Bank aimed when raised the interest rate that time due to the economic instability of that period.The interest rate reached its maximum in September 2005 (19.51% p.y.) and entered in a declining trajectory ever since, whilst inflation was kept under control (Banco Central do Brasil,

Table 6 – Selected periods for the impulse response calculation TVP-VAR from Figure 12

	Month, Year	t	CB[1] Governor & Min.[2] of Finance
01	Jun, 2003	40	Meirelles and Palocci
02	Jul, 2006	77	Meirelles and Mantega
03	June, 2008	100	Financial crisis*
04	July, 2012	149	Tombini and Mantega
05	July, 2015	185	Tombini and J. Levy
06	April, 2016	194	Tombini and N. Barbosa
07	June, 2017	208	Goldfajn and Meirelles

Notes:
1 - *Central Bank.*
2 - *Minister of Finance.*
* - In the thunderstorm of the Great Recession the Central Bank's head was Henrique Meirelles and the Minister of Finance was Guido Mantega. Brazil was considered a success case because the effects of the crisis were not severe here as they were in other economies.

Source – Own elaboration based on information from the Brazil's Central Bank (2018) and Ministry of Economy (2018).

2006). This can be seen in Figure 6. The remaining periods were similar with the exception of January 2015 and 2016. This period was marked by a turmoil in Brazilian politics and economy, which culminated in the Impeachment of the then President Dilma Roussef. If we pay attention to Figure 6, it is clear that a peak in inflation occurred during this time. This increase in prices was resulting from the frozen government controlled prices in energy and oil that were practiced in the previous years and become unsustainable when the international scenario became less favorable. By the end of the first quarter in 2006, the President was removed from office to wait the Impeachment process to come to an end, and an interim government, led by the then vice-president Michel Temer, was established. Mrs. Roussef had replaced the Finance Minister Joaquim Levy by Nelson Barbosa, who stayed until June of 2016 where Mr. Temer replaced both by Mr. Ilan Goldfajn and Mr. Henrique Meirelles, respectively (see Table 6). These findings are supported by Mumtaz & Surico (2015)'s work, where they found that there are asymmetries in the propagation mechanism across good (such as 2006) and bad (2015, 2016) times. When the economy is in expansion (above its long-term average), the estimates of the degree of forward-lookiness and interest rate semi-elasticity are significantly larger than the estimates in recessions (below-average periods), which suggests that monetary policy is more effective during periods of expansions.

Regarding the annual variation of the per capita GDP's response to monetary shocks, we see that it responds negatively to monetary contractions (as one would expect, since raise in interest rates will discourage the firms to make investments in their productions. We can observe in Figure 6 that there was an abrupt decrease in the GDP variation exactly in the end of 2009, beginning of 2010 (the gridlines represent the months of November in each year). This means that the level of GDP in January 2010 was way below it was one year before (it recovered a few months later). Accordingly to the Brazilian Bureau of Geography and Statistics, 2009 had a major decrease in the GDP which was followed by an overheated economy during the last three quarters of 2010. This movements in the economy help to explain the rise in prices that followed in the upcoming years, due to a combination of fiscal stimulus, heated economy and international trade retraction.

Table 7 – Percentage of types of income (of total income) from the annual household tax declarations, 2007-2016.

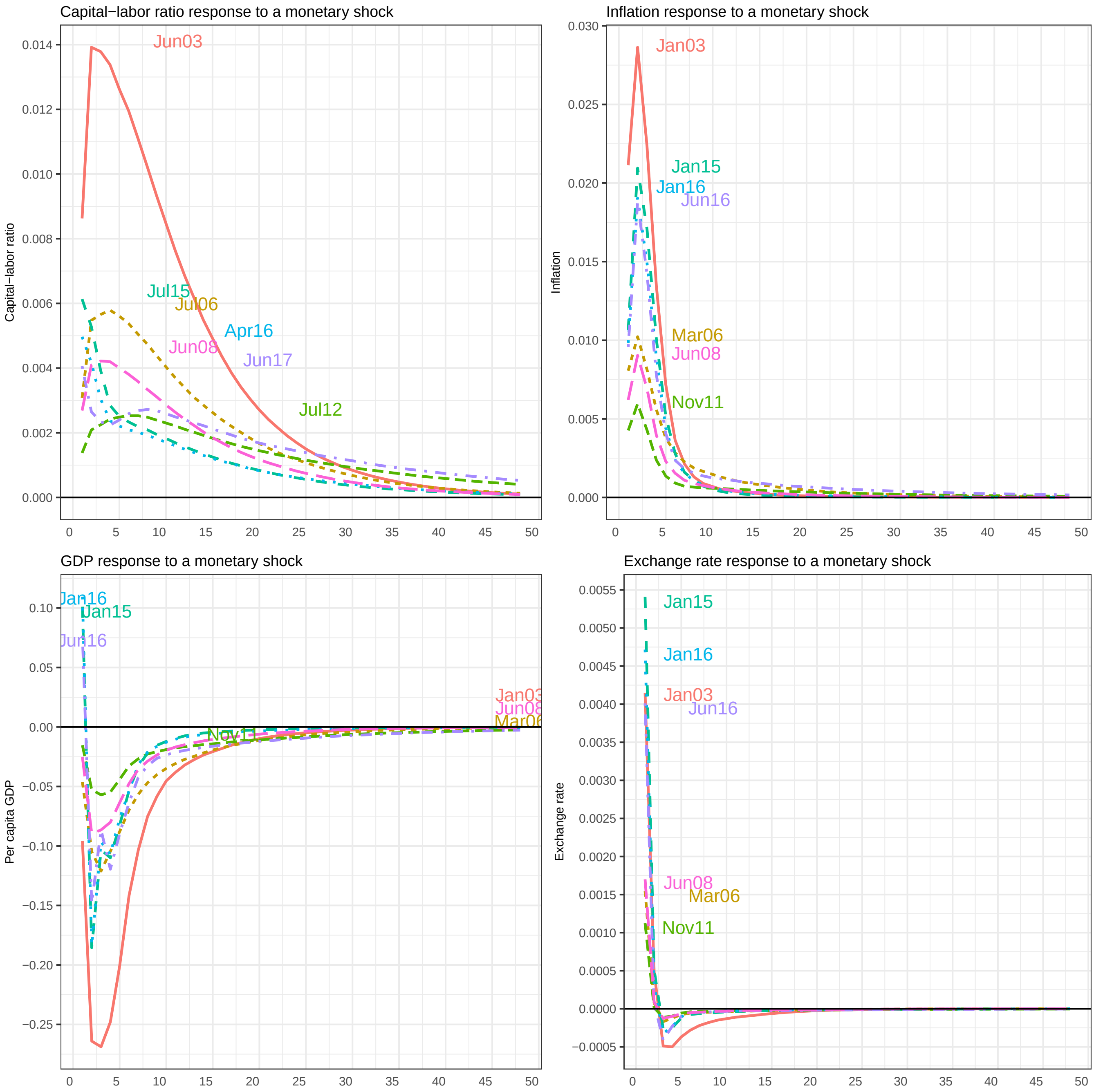
Type of income	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**
Non-taxable income	*29,3*	*37,7*	*36,4*	*38,2*	*39,7*	*38,4*	*39,3*	*40,1*	*41,1*	*41,0*
Tax-exempt	21,4	28,7	28,1	29,2	29,4	29,1	29,6	30,7	31,3	30,7*
Profit and dividend	10,4	10,3	9,9	10,7	10,9	10,7	10,8	10,7	10,0	9,8
Pensions	2,2	3,9	3,9	3,8	3,5	3,5	3,6	3,6	3,8	4,0
Donations and bequests	2,3	3,8	3,5	3,3	3,1	2,8	2,8	3,1	4,0	5,6
Others	6,5	10,8	10,9	11,4	12,0	12,1	12,4	13,3	13,5	10,6
Taxable at the source	7,9	9,0	8,3	9,0	10,3	9,3	9,7	9,4	9,8	10,2
13o wages	2,2	3,0	3,3	3,2	3,1	3,1	3,2	3,2	3,1	3,2
Financial applications	2,4	2,8	2,5	2,5	2,9	2,6	2,1	2,1	2,7	3,0
Capital Gains	2,5	2,4	1,6	2,0	2,9	2,3	2,0	1,5	1,5	1,4
Others	0,8	0,9	0,9	1,3	1,5	1,3	2,4	2,6	2,5	2,7
Taxable income	*70,7*	*62,3*	*63,6*	*61,9*	*60,3*	*61,5*	*60,6*	*59,9*	*58,9*	*59,0*

* - the data for 2016 has disparities between the total tax-exempt income and presented in more than the report. It is possible that this value is slightly overestimated.

Source – Own construction based on BRASIL. Ministério da Fazenda. Receita Federal do Brasil (2018).

By analyzing the pattern of the impulse response functions of the capital-labor ratio in both Figures 11 and 12, there is evidence of a weakening in the relationship between K/L and the Selic rate for the last years in the sample. Regarding the first years (2000-2010), where a contractionary monetary policy shock does have a significant effect on the capital-labor ratio, a possible explanation would be the stable composition between the tax-exempt incomes, incomes taxed “exclusively at the source” and the taxable income. Since there was no big shifts in the shares of each one of these categories with respect to the total amount of declared taxes, it is natural that little or no changes in its response to interest rates would be observed. Table 7 has the share of different taxed exclusively at the source and non-taxable personal income for the period of 2007-2015. Note, however, that this does not correspond entirely to the capital income series used in this paper, since it does not contains the tax declared by firms.

Figure 12 – Medians of the impulse response functions to monetary shocks at different selected periods.

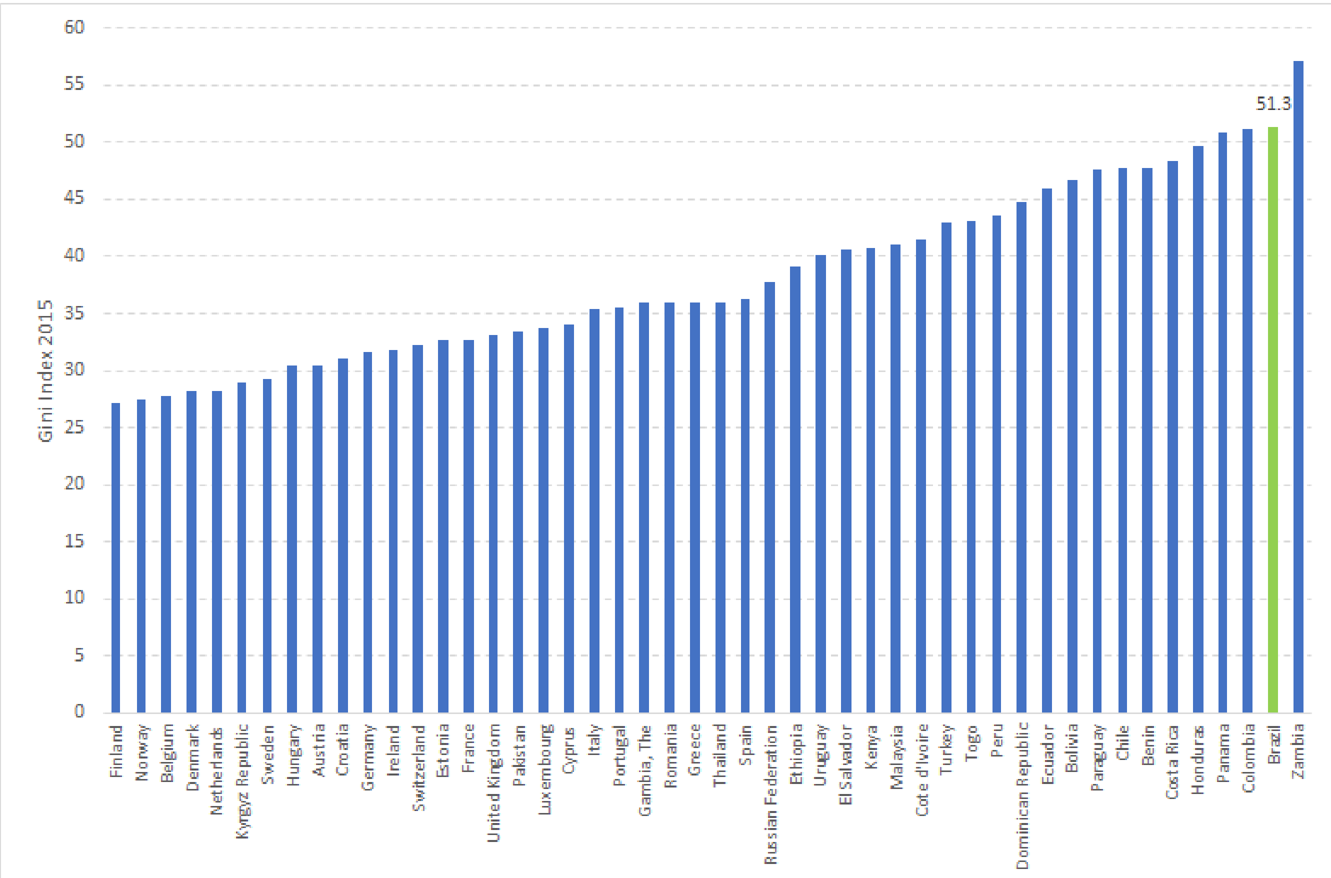


Medians of the impulse response functions (IRF) of the capital-labor ratio (K/L) to monetary shocks at six different times, where the combination between the Governor of the Central Bank and the Ministry of Finance changes from one to another. Plus, there is a 7th line correspondingly to June, 2008 (period of the Great Recession). For all curves at the botton left graph (corresponding to the response of the capital-labor ratio), the 95th percent interval dos not include zero until the 15th month after the initial shock..
Source – Own construction.

From our impulse response analysis, it is clear that the effect of monetary policy on ${}^{K}/{}_{L}$, at least on the period before 2010 has three main characteristics: (i) it is a negative relationship, i.e., a contractionary MP increases the capital-labor ratio; (ii) the magnitude of the effect is small, but lasts almost a year; (iii) it changed through time, being more expressive at the beginning of the sample period until fading away. Moreover, it is important to note that our impulse response analysis only states that there is an effect of monetary policy over K/L, but in the case where capital and labor is evenly distributed among households, this would not implicate a redistributive effect. What we shall argue now is that the income and wealth composition in Brazil is heterogeneous, therefore the shifts from labor income to capital income are indeed representing a redistributive effect.

Although it is not possible to know exactly what is the redistribution channel that is playing a major role in our results, some hypothesis emerge. We discussed in Chapter 2 how heterogeneity in population could be linked to redistributive effects of monetary policy and some redistributive channels were presented. The effects of those channels necessarily depend on the characteristics of the population, in special regarding the distribution of income and wealth as well as the net nominal positions of households. One way to look at this is analyzing the income and wealth inequality using the Gini index. Brazil, in 2015, had a income Gini index equal to 51.3 (Figure 13), almost twice the index calculated for Finland (although twice the Gini does not implicate twice the inequality).

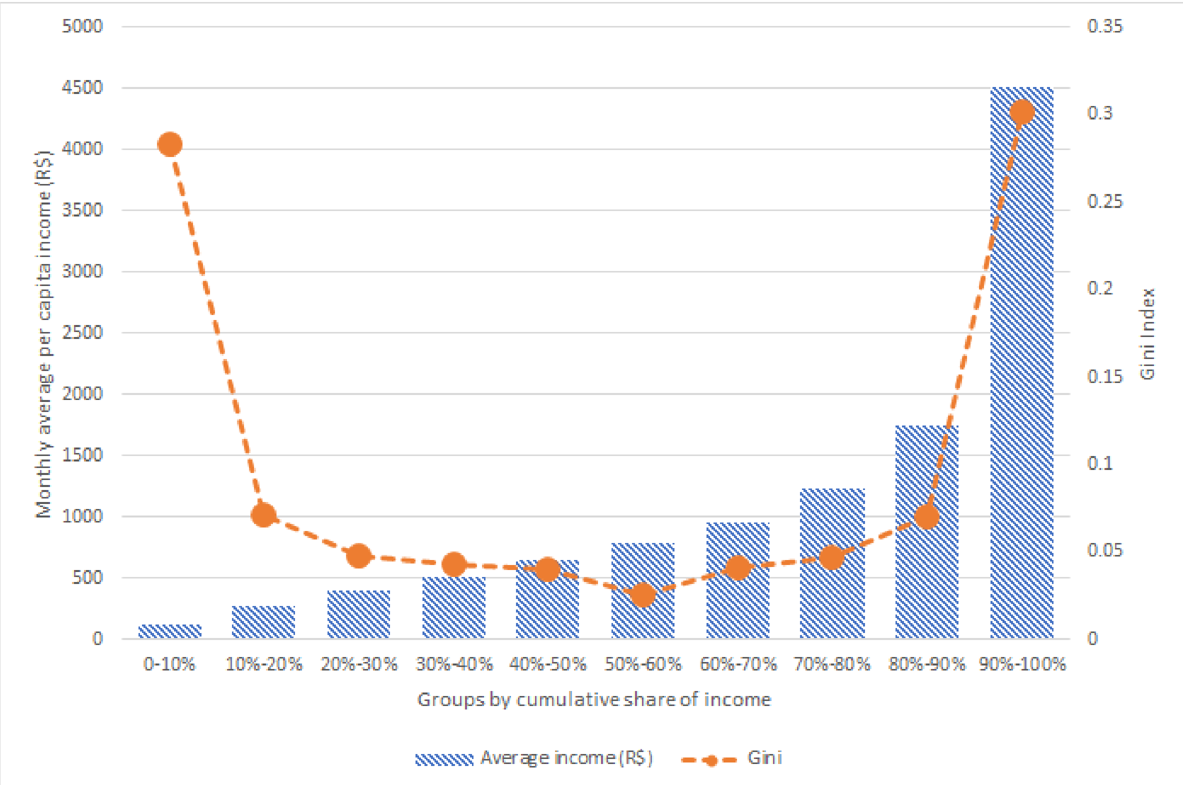
Figure 13 – Gini Index for income. Selected countries, 2015.



The Gini index measures the distribution of a certain variable (in this case income) among a given population. In egalitarian societies, we would expect that the share of income with each population decide was approximately 10%. The Gini index varies from 0 to 100 and higher values are associated with income concentration. However, it is not possible only by looking at the value of the index, to say exactly how is the pattern of the distribution.
Source – Own elaboration using data from The World Bank (2018).

But only knowing the income Gini for Brazil in a given year is not very helpful to understand how the income is really distributed among Brazilian citizens. To that end, we can explore the income Gini within groups ordered by the share of income, as displayed in Figure 14. The data used to build the graph is from a survey that is not necessarily the same population from the tax data used to obtain the capital-labor ratio, but it is a good representation of the economic active households from the country, from which a part is the households who declare taxes. It is possible to see two different patterns in the graph. The first is the U-shaped behavior of the Gini points. Both groups with the lowest and the highest share of income have a Gini of approximately 0.3, whilst the remaining eight groups have Gini below 0.1, indicating that there is great inequality amongh the poorest households and among the richest. Regarding the inequality among the top 10% richest, Piketty (2014) shows that this is a common pattern in many economies. The second pattern is the concentration of income: the top richest group has an average income almost 140% higher than the group with the lowest average income. This concentration is also present in the wealth distribution (data from 2016): the top 1% richest alone has 47.9% of the national wealth and the top 9% has 26.3%, meaning that the remaining 90% of the population gets 25.8% of the national wealth (GEORGES, 2017).

Figure 14 – Gini index and average income organized by deciles of income. Brazil, 2015.

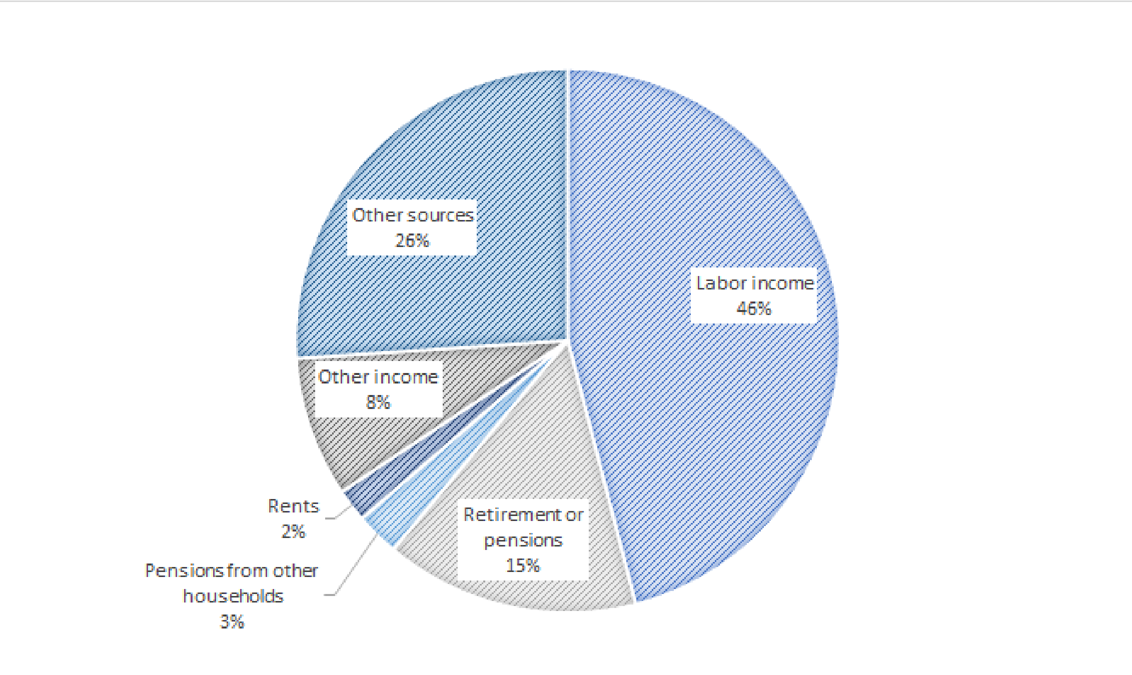


The x-label indicates the groups by share of income, from the bottom 10% poorest to the top 10% richest. The right y-label unity is Brazilian reais and the height of each bar represents the average income within each income group. For example, the bottom 10% poorest had an average monthly income of R$117 (≈ 25 USD) and the Gini index within this group was 0.2829.

Source – Own elaboration using data from IBGE/PNAD (2015) and Georges (2017).

Nevertheless, data like the one in Figure 14 cannot alone explain the heterogeneous effect of MP on income through the income channel, since it is necessary to know the composition of income within these income groups. Figure 15 has information from the average income from different sources, extracted from the National Household Sample Survey (PNAD) for the year of 2017. We can see that on average, the Brazilian household has about half of his or her income in the form of labor income, while other sources or other incomes represent 34%. Although there are differences between different States regarding the GDP or other socio-economic indicators, the income composition for different regions is similar to the national distribution.

Figure 15 – Income composition. Brazil, 2017.



Source – Own elaboration using data from IBGE/PNAD (2017).

Now we are getting closer to understand what are the types of income, but the heterogenous effect of the MP through the income composition channel is derived from the fact that different households will have different compositions of their income (AUCLERT, 2017). In Chapter 2 we used data from the United States to show that it really exists differences in income composition depending on the overall income share (Figure 2). Moreover, we argued, based on the non-taxable income, that Brazilian taxpayers seemed to be divided in three distinct groups as well (Figure 3).

Since the K/L series comes from the tax declarations, we can take a look on the type of income declared separated by types of household's occupation. This is an indirect way to verify the existence of different income composition (in comparison to the data in Figure 2) since here we are not able to separate in groups by the deciles of income. We computed, for each occupation category, the total income, which is the sum of the taxable, non-taxable and taxable at the source incomes. After that, we calculated the share of each one of these income types with respect to the total and also the ratio of assets, loans, donations and bequests by the total income. Results are reported in Figure 16.

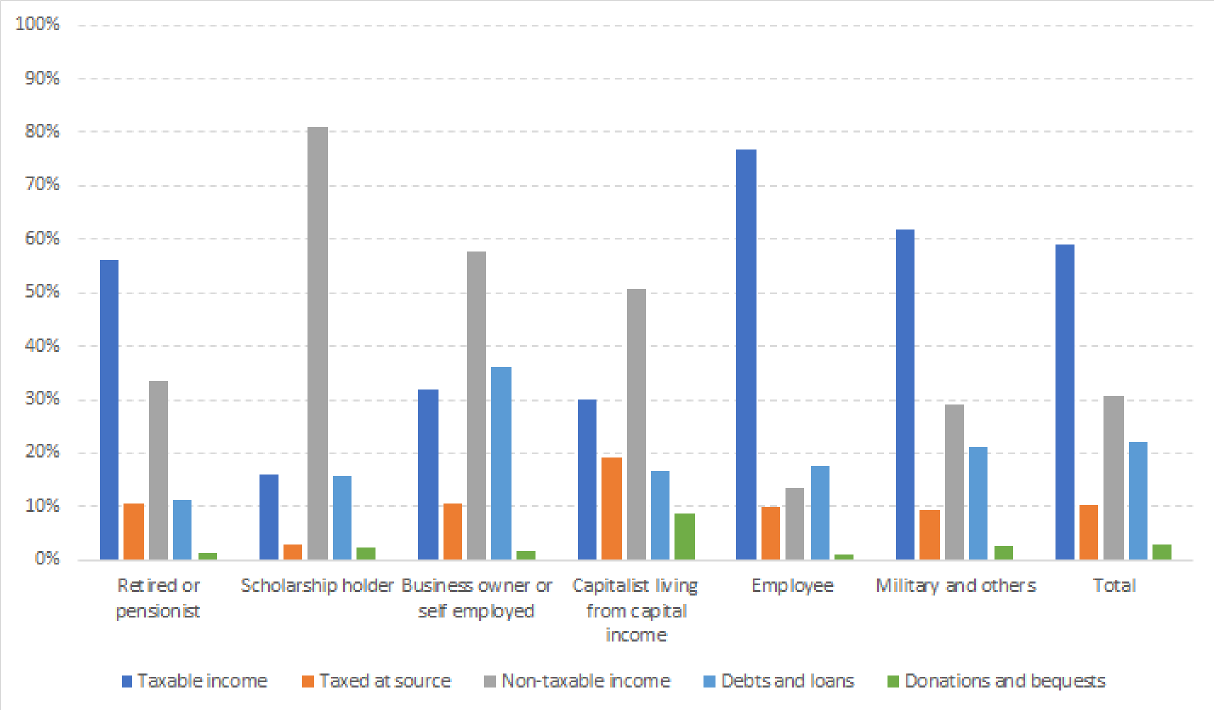
From Figure 16, we observe distinct income compositions across different categories of occupations. Business owners and capitalists who live on capital income have a large share of their total income as income exempt taxes. This is not comparable to scholarship owners that also are exempt from taxes. On average, when we analyze the raw data, scholarship holders receive R\$60,000 in total income per year, against R\$112,000 from the capital owners and over R\$233,000 from the ones living on capital (Receita Federal do Brasil, 2016). Back to Figure 16, it seems that the retired and pensioners have a very similar distribution of income when compared to the military and others category and they are the categories that best mimics the average shares in the entire population (first group from right to left). Business owners and capitalists have also the higher shares of the assets as a fraction of their total income. Although the graphs in Figure 16a cannot be organized by income deciles, they provide good evidence that in fact the composition of income is heterogeneous among households in Brazil, and, consequently, the income composition channel is playing a role in the results from our impulse response analysis[2].

Hallak Neto & Saboia (2014) investigated the evolution of the factors income in Brazil and one of their main findings was that prior 2010, from the beginning of the real plan in 1995 until 2009, there was a decrease of the labor income participation on the national income. Then, the fall in the premium wages for skilled work Development Finance International; OXFAM mostly likely played a role as well. This fall in premium wages was accompanied by an increase in the number of unskilled workers who entered the labor market since 2003, which resulted in a lowering of the Gini index since wages from low-paid jobs or very unskilled workers increased and the wages from the better paid workers not. Afonso, Araújo & Fajardo (2016) used income from tax data and concluded that there was a decrease in the concentration of capital incomes that started in 2008. This could explain why our impulse response functions are changing across time and the effect of the interest rate on the capital-labor was pratically zero when calculating for the last sample periods.

One additional source of changes in the capital-labor ratio could be the inflation channel. Inflation can hurt those with labor income in comparison to those with capital income, since households who exclusively depend on labor and are located in the lowest deciles of the income distribution may not have access to financial markets, which would protect their purchase power. Adam & Zhu (2015) studied the relationship between price level changes and the redistribution of nominal wealth for the Euro Area and one conclusion is that middle class households, due to their negative net nominal positions , are losers

2 Unfortunately, there is no data for asset and liabilities maturity in order to analyze the interest exposure.

Figure 16 – Income taxpayers composition, separated by different occupations. Brazil, 2017.



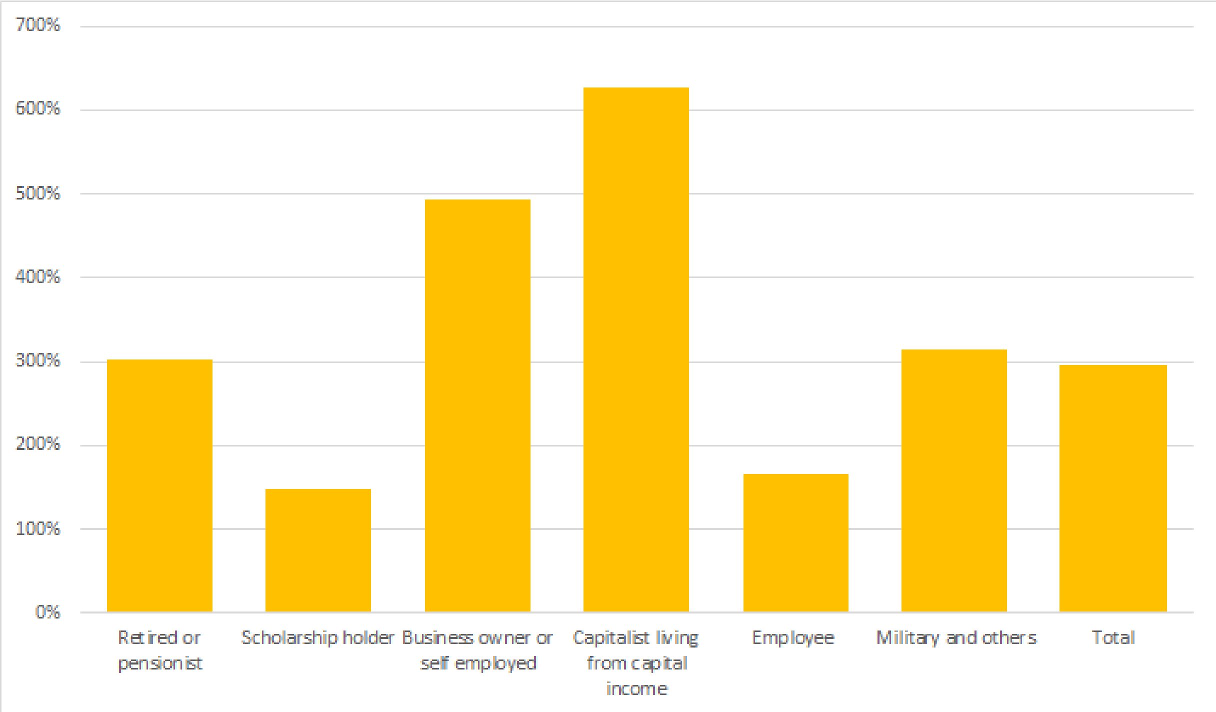
(a) Share of taxable, non-taxable and taxable at the source incomes with respect to the total income, and loans, donations and bequests divided by the total income, separated by groups of occupations.

16a - The percentages were calculated with respect to the total income, which is the sum of the taxable, non-taxable and taxed at the source incomes. Therefore, the sum of these three income shares must be equal to 1. Debts and loans are declared in tax payers but they are not taxed. Regarding bequests, they have an special taxation.
Source – Own elaboration using data from Receita Federal do Brasil (2016).

with respect to increases in the price level, whilst richer households are winners. This result is in line with the findings of Auclert (2017), although the inequality was with respect to consumption and he used a microfounded model. In the present work we are investigating income and not wealth, but those two are intrinsically tied: households who have more income can accumulate more, which generates wealth. If they have financial wealth, this will generate an income stream for the following periods, creating a cycle between wealth and income.

From Figure 18, we conclude that there is no evidence of significant impact of the inflation shocks in the capital-labor ratio. This could be due to the specification of the shocks, and further discussion about the precedence of the shocks would be needed, but also can mean that the capital-labor ratio is not responding to inflation after all. By looking at the stable composition of the types of income described in Table 7, this second explanation is more plausible. If there are no changes in the structure of flows, then the inflation will not play a role. Note, however, that this is not the same for the interest rates: when the interest rate goes up, the capital gains also rise. It is different than the nominal value of money: if there is inflation and the money flow from capital is hurt, also is the

Figure 17 – Income taxpayers composition, separated by different occupations. Brazil, 2017. (cont.)

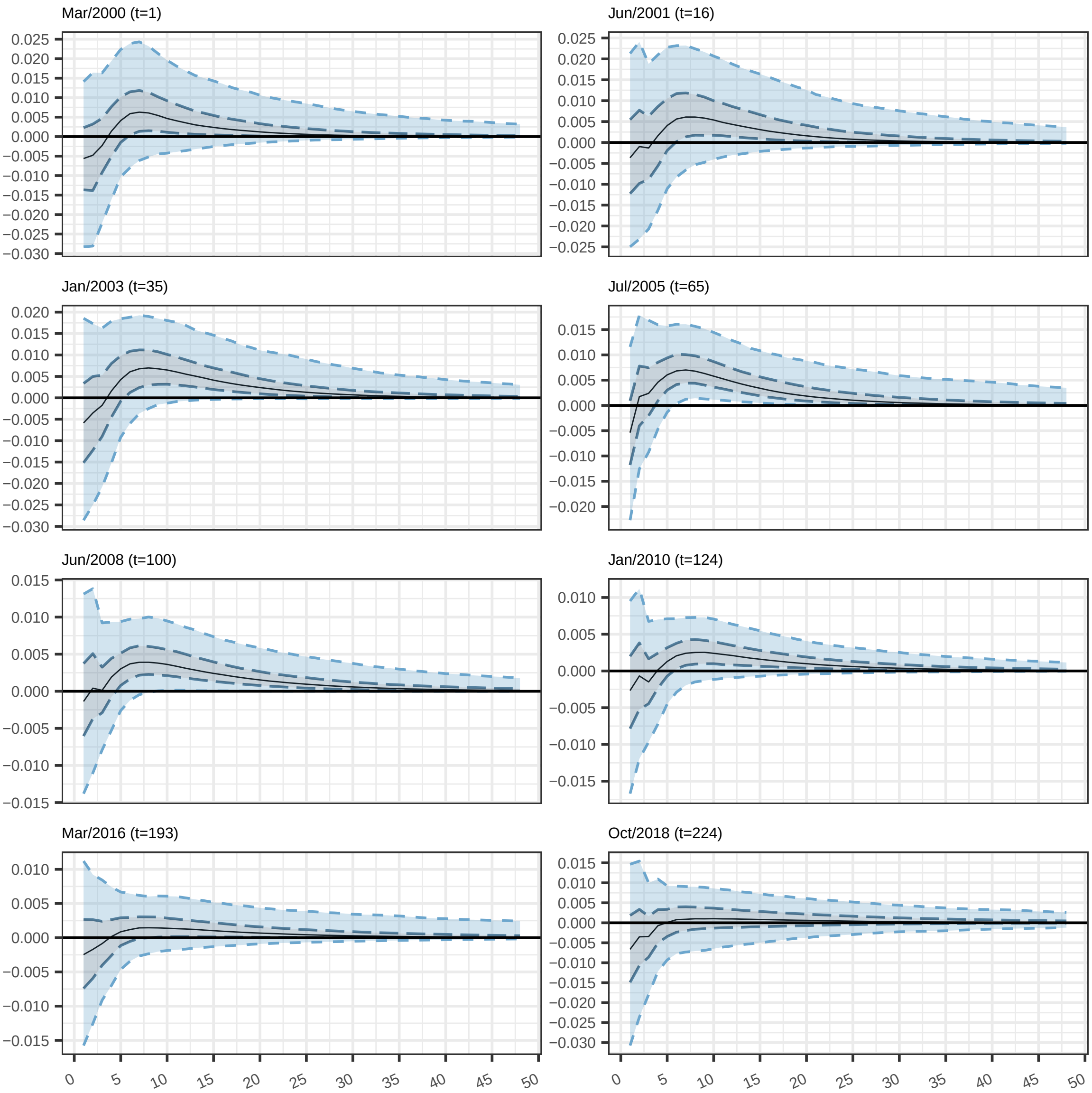


(a) Assets as fraction of the total income.

17a - This graph is similar to the one in figure 16a and is separated because of the scale on the y-axis.
Source – Own elaboration using data from Receita Federal do Brasil (2016).

nominal value of labor income, which does not affect the proportion between them.

Figure 18 – Impulse response functions (median, 50% and 90% centered around the median credible intervals) of the capital-labor ratio to inflation shocks for different periods. Brazil, 2000-2018.



Impulse response functions (IRF) of K/L to a 1% inflation shock using the estimated coefficients and volatilities for each indicated time, selected accordingly to major political and/or economical events that occured between 2000-2018 in Brazil, as described in table 5. The blue area corresponds to the interval between the 5th and 95th percentiles, whilst the gray area corresponds to the interval between the 25th and 75th percentiles, based in 2500 posterior draws. For the Capital-Labor ratio, all intervals include zero in the 48 periods after the initial shock.

Source – Own construction.

5 CONCLUDING REMARKS

"As long as poverty, injustice and gross inequality persist in our world, none of us can truly rest."

NELSON MANDELA

In this work, we investigated whether there is a redistributive effect of the monetary policy in Brazil. Using a TVP-VAR with Wishart innovations we found out a suggestion that contractionary shocks in the interest rates leads to a rise in the capital-labor ratio, meaning that monetary shocks induce a redistribution between capital and labor income, favoring capital owners at cost of people who relies on income from labor. As far as we know, this empirical exercise was not done before for the Brazilian economy. In particular, the relationship between monetary policy and inequality, despite the crescent tide of studies for other countries, appears to have not been not fully investigated using Brazil's data. One possible explanation for this is the lack of microdata for Brazilian households: the surveys that has data on consumption, wealth and income that would allow to estimate inequality measures are from the beginning of the last decade and were collected on an annual basis (MORGAN, 2017). In 2016 a new monthly survey covering a representative sample of the country residents was started, but there is not enough data to be used in an econometric model yet. Therefore, this paper contributes to shed some light on the question by bringing new empirical results to the debate. As a second contribution, we proposed an extension of Uhlig (1997)'s model who allows the coefficients of the VAR to have time-variation, which is a reported phenomena in works dealing with macroeconomic time-series. Our extension has some good characteristics when compared to the existing models, such as fewer parameters to estimate and closed formula for filtering and smoother.

As any study, this one is subject to limitations. One is concerning the data used in the model, or, more specifically, the tax data used to compute the capital-labor ratio. Although there is no dispute regarding the exact correspondence between what is reported in tax files and the data available by the Brazilian IRS, it is undeniable that tax evasions are a problem in Brazil. A 2015 report by the fiscal auditors' union indicated that the volume of fiscal evasions could be as large as 15 billion Reais, which was around 0.25% of that year's GDP. Therefore, one could argue that tax data may reflect poorly the income earnings for the very poor and/or the very rich. From one side, people with precarious or informal jobs will not declare taxes, creating a false effect that on average the labor income presented on data is higher than in reality. In this work, since we used the share of the totals, this average effect would not appear. We did not investigate how the results behave if the per capita capital-labor ratio was used, but this seems to be an interesting investigation in future developments. On the other hand, the sub notification on the tax declarations from the very rich, who we expect to have higher capital income, could imply underestimation of our main results. Still, these information on capital and labor income are, at the moment, the best data source that could be used for our work.

An important (and maybe necessary) observation is that this work is not intended to attack the conventional monetary policy or to promote the idea that the Central Banks should have an active role in conducting redistributional policies. First of all, the effects found by our model, although statistically significant, are small in magnitude, so it is most likely that any attempt to redirect the monetary policy to aim redistribution would do more harm than good. In this sense, we tend to agree with Woodford (2016) when he states that the independence of the Central Banks would be at risk in the case they entered in the grounds of fiscal and redistributive policies. Second, simply because there is an increase in the capital-labor ratio, it is not possible to draw precise conclusions on the redistributive effect being harmful to the economy, especially considering that we did not investigate in detail who would be the winners or losers of such phenomena and what would be the losses and gains. In addition, the inflation targeting regime is aimed to price stabilization and, at least in Brazil, this has been verified since 1999, with few exceptions. In this sense, the Bacen has been successful to achieve its mission. We could discuss what should be the Central Bank targets, in line with Yellen (2016), Smaghi (2016), Woodford (2016), but it is not the scope in here. What we did was to investigate the existence of a phenomena and assess its magnitude. Whether it is good or bad for the economy or if this should exist at all is another research topic.

By all means this work does not give a final mark on the subject of monetary policy and income distribution. However, as put by Blanchard (2018), there is space and need for various types of macroeconomic models and, in particular, empirical results, which justifies our efforts and calls for more investigation on the subject. This is especially true for research on the interactions between MP and inequality or distribution, that is still in its early days (Deutsche Bundesbank, 2016) and much is to be done yet. Having said that, future developments of this particular work may go in two ways: towards the econometric aspects or in the economic direction. The first path leads us to a second bifurcation, one where spin-offs of the proposed model are further investigated and another making modifications in the model to better accommodate the data. Regarding our proposal to extend Uhlig (1997)'s model, this has not been yet discussed in the literature and this model can be a competitive benchmark for some widely known TVP-VAR specifications like the one from Primiceri (2005) or the large TVP-VAR from Koop & Korobilis (2013). In terms of performance, a technical comparison between our Bayesian approach and the Maximum Likelihood from Moura & Noriller (2019) would be of interest as well. Regarding the technicalities our empirical application, the results suggests the presence of three types of parameters in the model: time-varying; non time-varying equal to a constant and zero, which is a perfect setup for the so-called *shrinkage methods*. In broader lines, they incorporate in the estimation procedure an on-the-fly method to decides whether the parameter is time varying or not, improving the posterior estimates in terms of the variance, as reported by Bitto & Frühwirth-Schnatter (2018), Huber, Kastner & Feldkircher (2018), Eisenstat, Chan & Strachan (2016), among others. Another development that is attainable to the current model concerns the goodness-of-fit. Our algorithm provides filtered and smoothed errors, which are not suitable for diagnosis. The economic path presents equally interesting options. The first is to compare our results with a similar model using Gini-like data and other variables representing income distribution. It is obvious that given the lack of a large series, major adaptations would be needed and this idea needs more refinements before being implemented. Finally, an interesting line of research would be the construction of a HANK model for the Brazilian economy. Needless to say, this idea alone would be enough for another thesis.

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Appendix

APPENDIX A – BAYESIAN ECONOMETRICS: A PAPER PLANE TOUR

In this appendix, we will present some introductory concepts on Bayesian Econometrics that might be useful for uninitiated readers. We are assuming that the reader has a fair idea on what is a random variable, a probability density function and joint densities, as well as basic probability calculations. These notes are not intended to give a very formal introduction and did not follow a specific author or book. Instead, our focus here is to introduce the main terminology, in the simplest possible notation, and walk the reader through the first steps into Bayesian reasoning, while contrasting with the traditional frequentist approach - enough to understand in general lines the estimation procedure used in this thesis[1].

Loosely speaking, the fundamental problem of the statistical inference lies in drawing conclusions about unknown **parameters** after observing a **sample**. *Parameter* is an unknown characteristic of a population of interest, for example[2], mean wages from males and females in a country in a given year. If one had the time and financial resources to interview all workers, then a simple tabulation would give the precise value of these means and a comparison between the two exact values (men and women mean wages) would provide a straight answer to which one is higher. However, in real life we often cannot perform a census: either financial, logistics and/or time constraints play a role and we have access to only a fraction of the population, which we are calling the *sample*[3].

In our wages example, assume that a given country has 1000 economic active households who receive wages and, from those, 450 are men and the remaining are women. You have a budget that allows surveying 40 men and 40 women with the intent to determine if there is a gender wage gap in this population[4]. Using a more formal notation, say that μ_F and μ_M are the mean wages for female and males in the population; $X_1, \ldots, X_{40}$ are the 40 potential surveyed men's wages (before collecting the data) and $Y_1, \ldots, Y_{40}$ represent the wages from the women before collecting the data. Note that, before selecting the sample,

1 For a full helicopter ride, we recommend Zellner (1971).

2 We are using the “survey” example due to its simplicity, but there are some studies, such as the one used in this thesis, where there is not a “population” being studied. However, this does not invalidate our arguments here.

3 Ideally, when conducting this type of study, one needs to make a full sample design, calculating sample size to obtain enough test power (for the frequentist approach) and ensuring that the observations respect the assumptions needed to perform the analysis. For the sake of conciseness, we are not going to extend in this sort of detail in here.

4 A complete research on gender wage inequality should consider other factors as age, experience, education, among others, and probably a more sophisticated model than simply comparing the group's wages directly. But, as a toy example, this simplification is unharmful.

X_i or Y_j, with $i,\ j \in \{1, \dots, 40\}$, can assume any wage from the existing men or women in the population.

One function of interest that depends on the sample is the *likelihood function.* It is widely used in both Bayesian and frequentist approaches and is defined as follows.

Definition A.1. - Likelihood function
Let $f(\boldsymbol{x}|\boldsymbol{\mu})$ be the joint density of a sample $\boldsymbol{X} = (X_1, \dots, X_n)$, with μ being a parameter (either a scalar or a vector). Then, given that $\boldsymbol{X} = \boldsymbol{x}$ was observed, the function of $\boldsymbol{\mu}$ defined as

$$\mathcal{L}(\mu|\boldsymbol{x}) \coloneqq f(\boldsymbol{x}|\mu)$$

is called the *likelihood function.*

At first glance, it may seem that $\mathcal{L}(\mu|\boldsymbol{x})$ is just the joint density. The detail is noticing that while $f(\boldsymbol{X}|\mu)$ can vary for any given sample $(x_1, \dots, x_n)$ and μ is kept the same, $\mathcal{L}(\mu|\boldsymbol{x})$ is defined for a specific sample $(\underset{\sim}{x}_1, \dots, \underset{\sim}{x}_n)$ and will vary for different values of μ. Even under the assumption that μ is a random variable, the likelihood does not satisfy the required conditions to be called a probability density function - in fact, only in very few cases the area under the curve or the sum of all possible values of $\mathcal{L}(\mu|\boldsymbol{x})$ will be equal to 1. Nevertheless, the likelihood function contains important information regarding μ, as we shall see below.

From this point onward, Bayesians and frequentists will follow different paths.

For the frequentist (or classical) investigator, the parameters are unknown *fixed* quantities that she has no access to, i.e., they represent a population's characteristic of interest that cannot be observed or directly measured. The inference process, then, relies on using estimators, which are functions of the sample, to infer about the parameters. The subtlety is in acknowledging that all uncertainty lies in the sample information only. In our example, under the frequentist hat, both μ_F and μ_M are considered fixed and this implies that they cannot be treated as random variables[5].

An *estimator* (or statistic) is any function of the sample that does not depends on the parameters. A very common estimator is the sample mean, that in our case can be computed for each group, and will depend only on the information from the data. For the men's group, the sample mean will be denoted by $\bar{X}$, whose formula is:

$$\bar{X} = \frac{\sum_{i=1}^{40} X_i}{40}. \tag{A.1}$$

[5] Ignoring the trivial case where the random variable is a constant with probability 1.

Analogously, the sample mean for women's wages will be denoted by $\bar{Y}$ and its calculation will be similar to (A.1). Potentially, $\bar{X}$ may have several distinct values, one for each selected sample - but, on average, we expect that $\bar{X}$ will be equal to μ_M. It is important to notice that since estimators are functions of (only) the sample and this is, before being observed, a random variable, estimators are also random variables[6]. This implies that we can compute moments and other quantities such as confidence intervals using the estimators. The value obtained when applying a given sample to an estimator is called *estimate* and since estimates will vary accordingly to the selected sample, it is of interest to pursue estimators with good properties, such as unbiasedness, minimum variance, consistency, among others.

Computing the sample mean like (A.1) is not the only way to make inference for μ_M: another possibility would be to maximize the likelihood function with respect to μ. Intuitively, this process would give us the value of μ that would be the most likely to have produced the observed sample $(\underset{\sim}{x}_1, \ldots, \underset{\sim}{x}_n)$. This estimation method is known as the *maximum likelihood method* (ML). By computing an estimate using an estimator like (A.1) or via other methods such as the ML, method of moments and similar, one gets a *point estimate* for μ, which is a single real value (or vector of values in the multiparametric case). We can, at most, compute confidence intervals, but their interpretation is not straightforward: a $(1 - \alpha)\%$ confidence interval for μ means that, on average, if one produces independent random 100 samples from the same population, in $(1 - \alpha)\%$ of the generated intervals will contain μ. Note that if we consider μ, the probability that it belongs to a given interval is either 0 (it does not belong) or 1 (it belongs), reinforcing the idea that all uncertainty is with the sample and not the parameter.

In addition to rely on a single estimate, all this frequentist procedure does not take into account previous information that we may have regarding wages in the population, including previous studies, real-world constraints or other information based on economic theory. Another limitation could arise when the likelihood function is too complex because optimization procedures to calculate the ML estimator may fail us - a common situation when dealing with TVP-VAR models with stochastic volatility. Furthermore, frequentist methods can only make inference based on what is observed. For example, if tails is not observed in four draws, the ML estimate for the probability of heads would be equal to one. That is, the ML estimator would suggest an one-sided coin, a fairly unrealistic assumption, specially considering the small sample size. Finally, if we obtain more data, such as a new survey, we cannot update our previous result. The only two options would be gathering all information (old and new) combined and recompute the estimates or completely discard the first sample and proceed to a new estimation procedure with the new sampled values.

To go Bayesian, on the other hand, is to embrace the idea of using probabilities

[6] Any function of a random variable is a random variable itself.

to express our degree of uncertainty, including the one associated with μ. This way, we can treat unknown parameters as random variables, allowing the inference procedure to occur in a more straightforward manner: we now can ask ourselves about the probability of μ assuming certain values, being contained in a given interval, among others. For the Bayesian investigator, a degree of uncertainty regarding μ will always exist and the inference procedure consists in finding a probability density or distribution that will help us to quantify this uncertainty (HAMILTON, 1994).

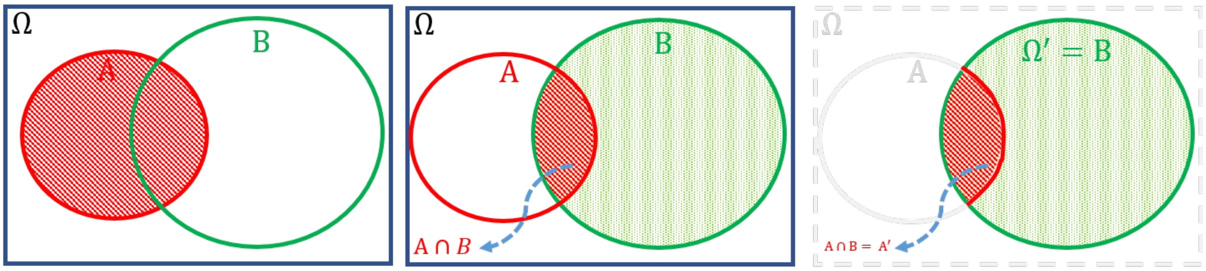
The foundation of the Bayesian inference is the Bayes Theorem, whose simplest version for two events is presented in equation (A.2). The theorem establishes the conditional probability of an event A to an event B as being equal to the ratio between the probability of A and B occurring at the same time and the probability of event B only.

Theorem A.2. ***Bayes Theorem***
If A and B are events defined in the same sample space Ω with $\mathbb{P}(B) \neq 0$, then

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B|A)\mathbb{P}(A) + \mathbb{P}(B|A^c)\mathbb{P}(A^c)} = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}. \tag{A.2}$$

Although simple, theorem (A.2) has an important underlying meaning on how we can use prior knowledge to reduce our uncertainty. Figure 19 has a representation of a sample space Ω with two events, A and B. Of course, there are other possible situations, for example, A and B are disjoint, $A \subseteq B$, or others, but they are analogous. In our example, A and B have an intersection, represented by $A \cap B$. If we do not have any information regarding B, then the probability of A occurring is equal to its "size" with respect to Ω (first panel, left). However, if we know that B has occurred (middle panel), we can update our knowledge and calculate the probability of A just considering the size of $A \cap B$ with respect to B. In some sense, this is equivalent to impose a restriction in our original sample space and look to only events that have a non-null intersection with B (last panel, right). If the area of $A \cap B$ as a proportion of B is greater or smaller than the area of A in comparison to Ω, the information that B has happened is also giving us a piece of information about A.

Figure 19 – Example of a Bayes Theorem application.



If nothing is known regarding B, then the probability of A will be computing considering the whole sample space Ω (left). However, if it is known that B has occurred (middle), we can reduce our sample space for just B and then evaluate the probability of A within this new Ω' (right), which is A'.
Source – Own elaboration.

We can see the term $\mathbb{P}(A|B)$ of (A.2) as the *posterior probability* of A given our knowledge about B, and this posterior is equal to our knowledge about B given what we already know about A plus the probability of A occurring (*prior* to observing B), normalized (divided) by the probability of B happening alone. We can shift the notation and consider a vector of sample values x that comes from a probability distribution with parameter μ as follows.

In the Bayesian inference, we will be interested in finding the posterior distribution of μ, denoted by $\mathbb{P}(\mu|\boldsymbol{x})$, which will represent our knowledge about μ after observing the sample data. To obtain this posterior, we will use the Bayes formula from (A.2), which can be rewritten as:

$$\underbrace{\mathbb{P}(\mu|\boldsymbol{x})}_{\text{posterior}} = \frac{\overbrace{\mathbb{P}(\boldsymbol{x}|\mu)}^{\text{likelihood}} \;\; \overbrace{\mathbb{P}(\mu)}^{\text{prior for } \mu}}{\underbrace{\mathbb{P}(\boldsymbol{x})}_{\substack{\text{marginal density of } \boldsymbol{x} \\ \text{(constant)}}}}. \tag{A.3}$$

It is common to suppress the denominator of (A.3) (since it is a constant) and change the equality for a proportional relation, obtaining te following form:

$$\mathbb{P}(\mu|\boldsymbol{x}) \propto \mathbb{P}(\boldsymbol{x}|\mu)\mathbb{P}(\mu). \tag{A.4}$$

The challenge for the Bayesian investigator lies in finding $\mathbb{P}(\mu|\boldsymbol{x})$. Most textbook applications will result in examples where this posterior has a closed formula and is a known probability distribution. However, in complex models with many parameters, such as the case of TVP-VARs, many complexities arise. In general terms, the posterior distribution for the multidimensional case is a joint density for all parameters and it is not always straightforward to find it. For such situations, numerical and simulation methods, such as the Markov Chain Monte Carlo (MCMC) methods are employed. In this thesis, we used an MCMC method called *Gibbs Sampler*, which is described in Chapter 3.

APPENDIX B – DATA TREATMENT AND TRANSFORMATIONS

The series used in our VAR model were treated to be considered stationary. All the cleaning and treatment process is detailed in this appendix. Information about the data sources can be found in Table (1).

CAPITAL-LABOR RATIO

We define the capital-labor ratio ($^K/_L$) as the quotient between the share of capital income and labor income, both with respect of GDP. After computing this quotient, we divided the series in two distinct pieces, marked by the structural break[1] present in January 2004. The confidence interval for this structural break covers a 4-year period that goes from October 2001 to October 2005. It is possible that the change in the series was motivated by the change to inflation target regime, implemented at the end of the 90s, however we found no evidence in the literature confirming or rebutting this hypothesis.

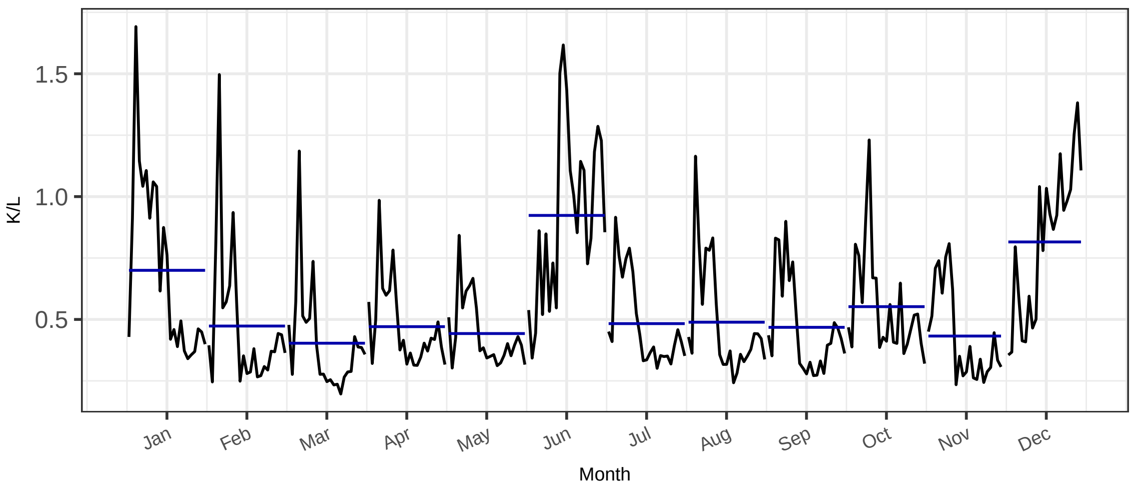
Since the capital-labor ratio presents a seasonal behavior with peaks in July, December and January (Figure 20a), the pieces of the series were individually treated and then recombined to form the series that was used in the model. Figure (21) contains the original series and the filtered one (after using the X13-ARIMA filter). The QS statistics of the filtered series favours the null hypothesis of no seasonal pattern (p-values less than 0.01 for both filtered series and residuals).

INTEREST RATE

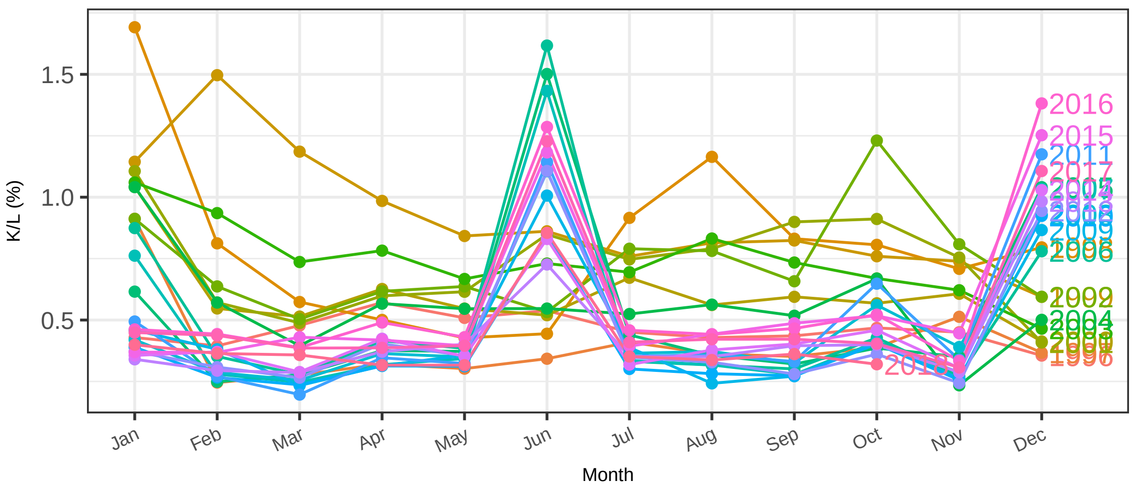
Selic is the short term interest rate in Brazil, defined by the Central Bank and it is used as the main instrument of monetary policy. More details can be found on Chapter 4. Although it is common in the empirical literature for other economies to use as the short term rate the 3-month treasury bill, due to the smoother behavior of this series in comparison to what would be the equivalent of the Selic series, in Brazil the 3-month treasury bill series, which is the Swap reference rate in 90-day term (Swap) is virtually the same as the Selic series, as can be seen in Figure 22. Moreover, the Swap series starts only in the year of 1999 and, in the case of being used in the model, this would imply finding a proxy for this missing data. A model using the Swap series with the 44 observations

[1] The endogenous test for structural breaks used in this work is based on the work from Bai & Perron (1998; 2003).

Figure 20 – Graph of the capital-labor ratio, organized by selected periods. Brazil, 1996-2018.



(a) Grouped monthly values.



(b) Monthly values grouped by year.

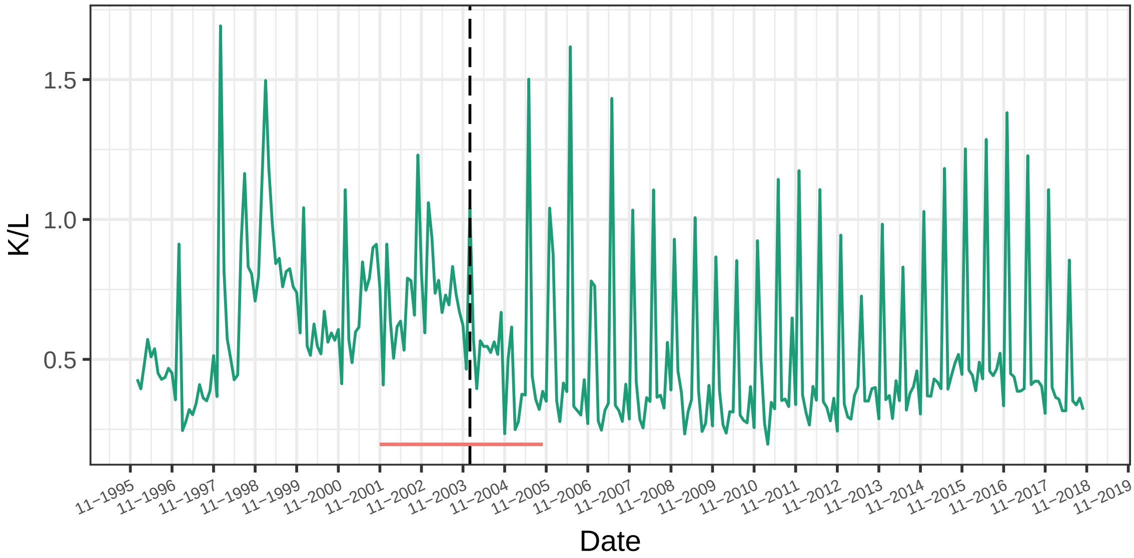
Notes: The capital-labor ratio is obtained through the quotient between the capital income series and labor income series. Both graph of the monthly averages (Figure 20a) as well as the graph of the monthly averages organized by years show that June, December and January present peaks that distinguish them from the other values. This cyclic behavior is considered statistically significant using the QS test with 95% of confidence.
Source – Own elaboration using data from BCB-DSTAT.

imputed with Selic values was estimated and the results were very similar to the ones using only the Selic series as interest rate in our model.

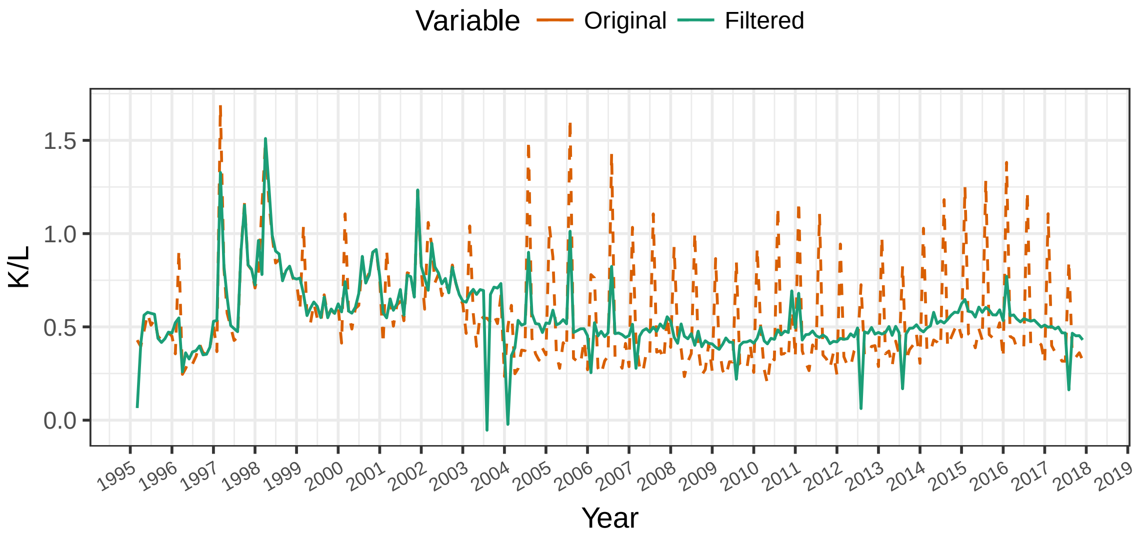
The SELIC rate was converted from monthly to annual rate using the following formula:

$$\text{annual rate} = \Big((1 + \text{monthly rate}/100)^{12} - 1\Big) * 100.$$

Figure 21 – Monthly values of the capital-labor ratio. Brazil, 1996-2018.



(a) Monthly values of the original data. The dashed line denotes que structural break at January 2004 and the horizontal red line is the respective 95% confidence interval, ranging from October 2001 to October 2005.

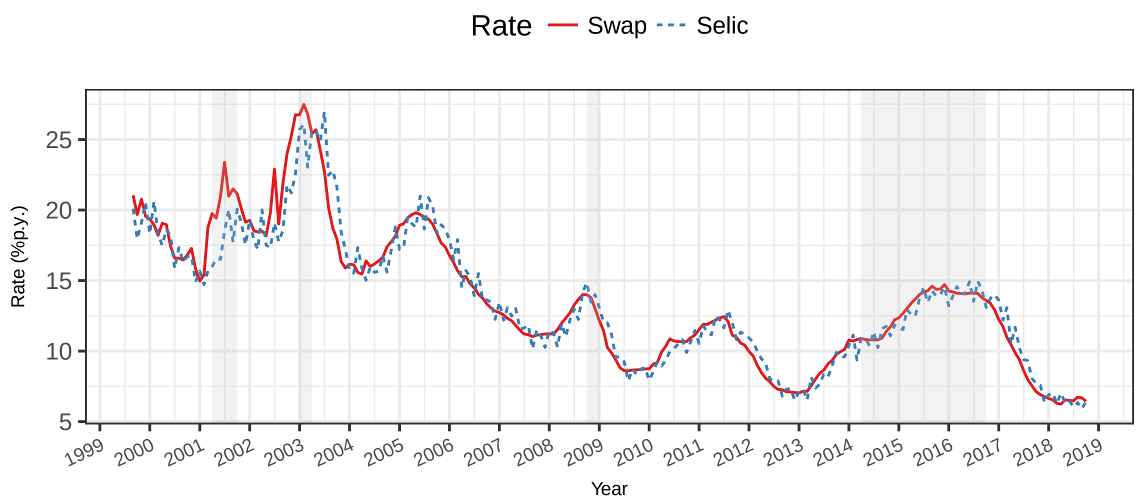


(b) Original series (orange - dashed) and series after seasonal adjustment (green - continuous).

Notes: The vertical gridlines indicates the November month of the informed year on the x-axis. The seasonal adjustment were made using the X13-ARIMA algorithm without data transformation. After the filter, it was not possible to reject the null hypothesis (lack of seasonality), considering the QS test with 95% of confidence. This result holds when looking for the whole series, its residuals and only the last 8 more recent years. The seasonal adjustment were made dividing the original series in two pieces, separated by the structural break indicated in Figure (21a).
Source – Own elaboration using data from BCB-DSTAT.

This series presents significant seasonality at a 5% significance level (Figure 23). Moreover, the series presents two significant structural breaks, the first in May 1999 (95% confidence interval from April 1999 to December 1999) and the second in August 2006 (95% confidence interval ranging from July to October 2006), which are shown in Figure (28a). Therefore, we divided the SELIC series in three consecutive pieces by the two structural breaks and

Figure 22 – Swap and Selic reference rates, accumulated per year. Brazil, 1999-2018.



Comparison between Selic (dashed blue line, after seasonal adjustment) and Swap (continuous red line, without seasonal adjustment) reference rates, per annum. Shaded areas indicate recession periods, accordingly to the Economic Cycle Dating Committee (CODACE). The vertical gridlines indicates the November month of the informed year on the x-axis. The Spearman correlation coefficient between the two series was equal to 0.9607 (p-value < 0.001).
Source – Own elaboration using data from BCB-Demab and B3.

treated the seasonality for each piece individually before merging back the data.

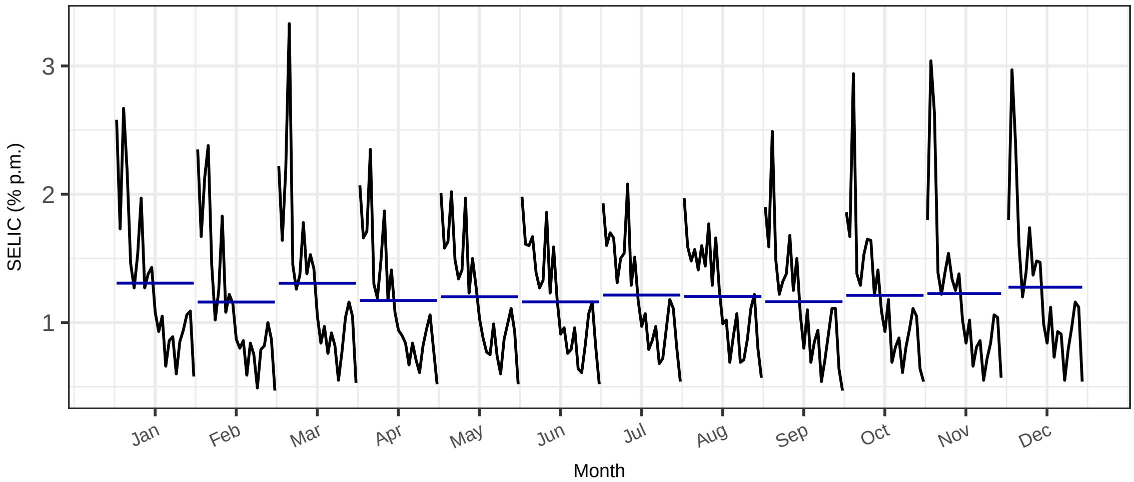
GDP

It is common to use the Economic Activity Index from the Central Bank (IBC-Br) as monthly approximation of Brazil's GDP, however, this series starts in January 2003, which would leave us with fewer data points to estimate the model. As an alternative, we used the monthly GDP estimate from the Central Bank.

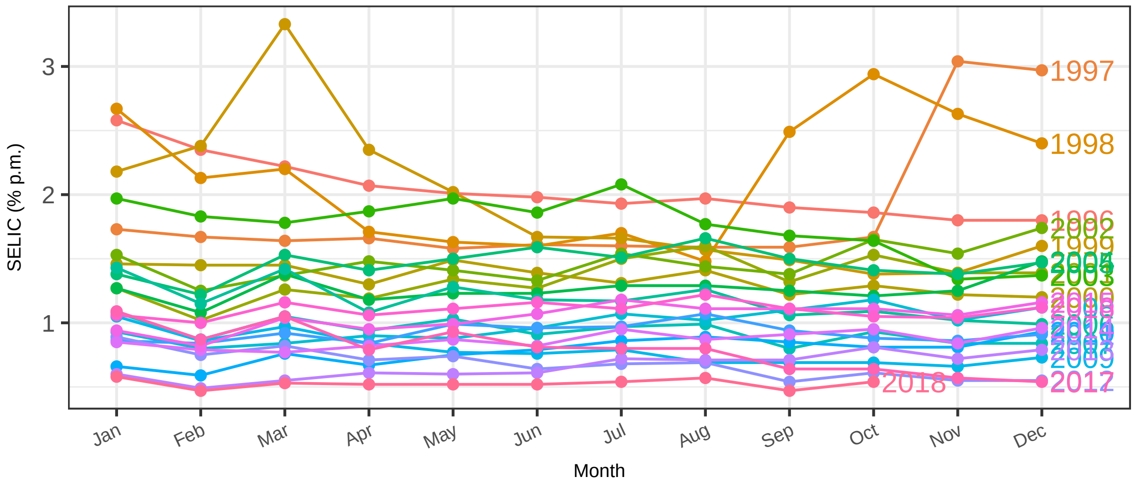
The monthly estimate of the Brazilian GDP series is an interpolation of the quarterly official GDP (calculated by IBGE) and it is made by the Brazil's Central Bank with the aim of using the monthly series with other macroeconomic aggregates in their models. Since the series is in nominal terms, we used the March 2018 IPCA consume price index to calculate the GDP in real values.

The GDP series exhibits an upward trend, that initially was removed by calculating the 12-month variation. However, by doing this procedure we artificially insert an autocorrelation of lag 12 in the series. As an alternative, we could use the HP filter combined with a methodology such as the X-13 ARIMA to treat for seasonality. However, Hamilton (2018) advises against the HP filter, stating that this procedure artificially includes spurious relations in the data. His suggestion, then, is to make regressions of the variable at time t against its most recent past observations and using the residuals from this procedure instead the original variable.

Figure 23 – Basic interest rate - SELIC - in annual percentage, grouped by periods. Brazil, 1996-2018.



(a) Values grouped by month.



(b) Monthly values organized by year.

The SELIC rate series shows a discrete, but significative to a 95% confidence level, seasonal behavior: every January there is a rise followed by a reduction in February and this pattern repeats over the following months. In 1998 occurred a major international crisis that led to changes in the Brazilian Central Bank administration and a sharp rise in the interest rate.
Source – Own elaboration using data from BCB-Demab.

The population estimate is calculated by IBGE based on the institution's surveys and the national Census. However, before 1999, the population estimates were per year and only after that there is a monthly series (see Table 1 for more information). So, for the values before 1999, we imputed the same year value for all months. Since this period accounts for the data used in the prior, the impact on the posterior results due to this approximation ought to be small.

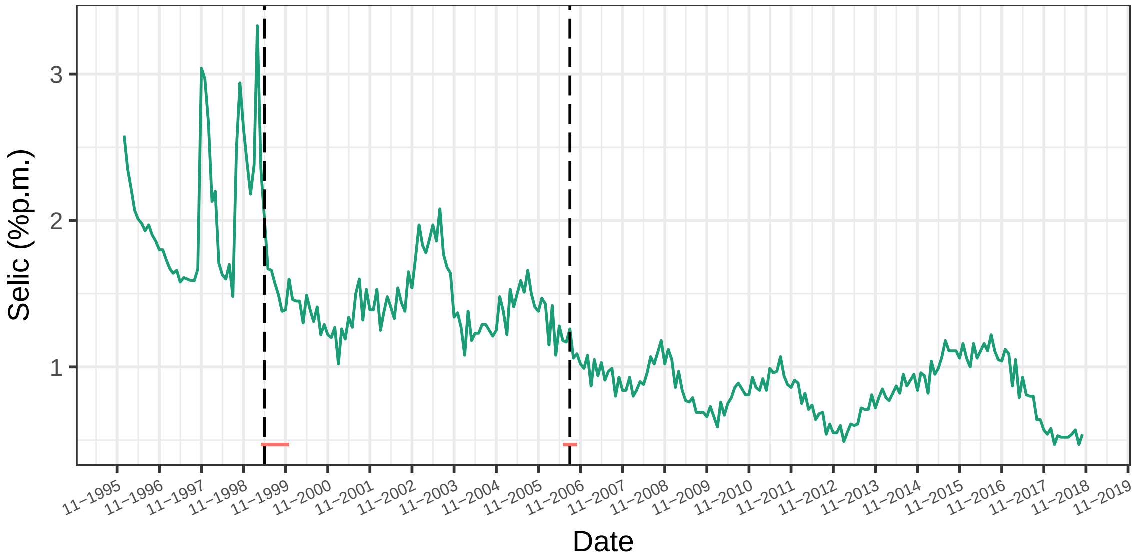
INFLATION RATE

As inflation rate, we used the monthly values of the Broad National Consumer Price Index (IPCA), which is the inflation index used by the Bacen when making decisions regarding the Monetary Policy (Banco Central do Brasil, 2019a). To avoid inserting an autocorrelation of order 12 between the values, we opted by using the values of the monthly inflation, without compounding the series. The analysis of seasonal pattern is exhibited in Figure 27 and although discrete, it seems that the series has an U-shape, with an outlier at the end of 2002. This outlier represents the flow of incomming dollars caused by the uncertainty regarding the presidential elections that ocurred in October 2002. Later on, at the beginning of 2003, the Central Bank raised the interest rates in order to control the inflation.

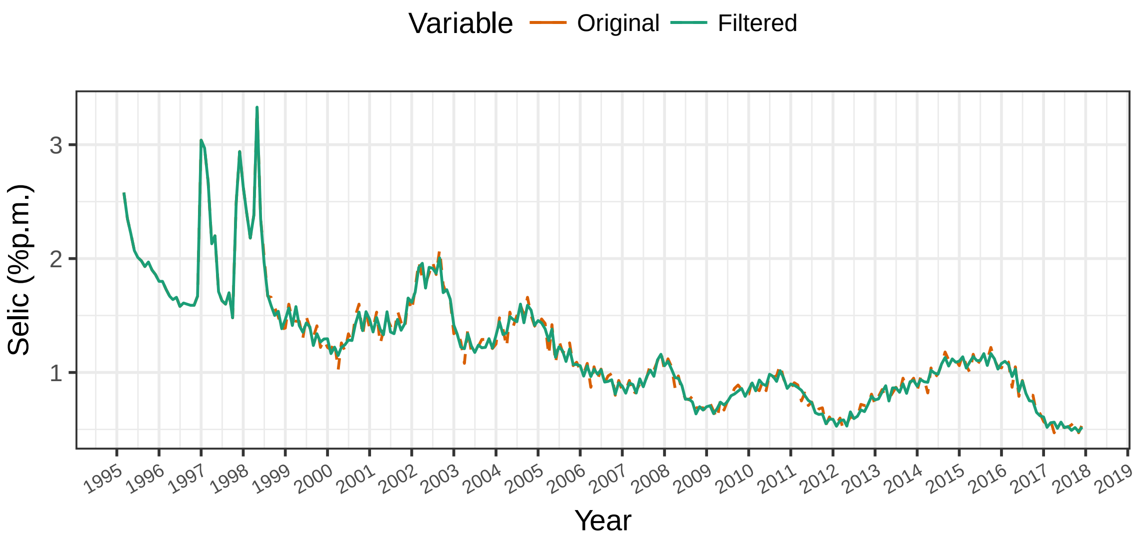
REAL EFFECTIVE EXCHANGE RATE INDEX

The real effective exchange rate is calculated considering a bundle of real exchange rates among Brazil and other countries, chosen by their respective importance in the Brazilian trade balance and weighted by trade volume. The real index is obtained using the June 1994 consumer index price. Although the series does not present a significant seasonal pattern, we calculated the first difference of the logarithm, in order to assure stationarity.

Figure 24 – Monthly values of SELIC interest rate. Brazil, 1996-2018.



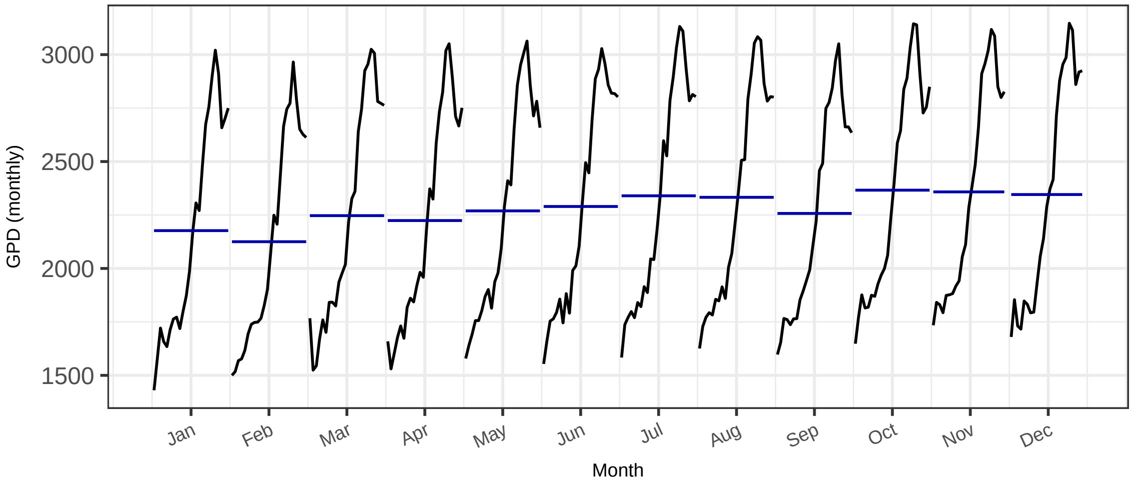
(a) Monthly values of the series. The vertical dashed lines denote the structural breaks in May 1999 and August 2006, whilst the horizontal red lines are their 95% confidence intervals: April to December 1999 and July to October 2006, respectively.



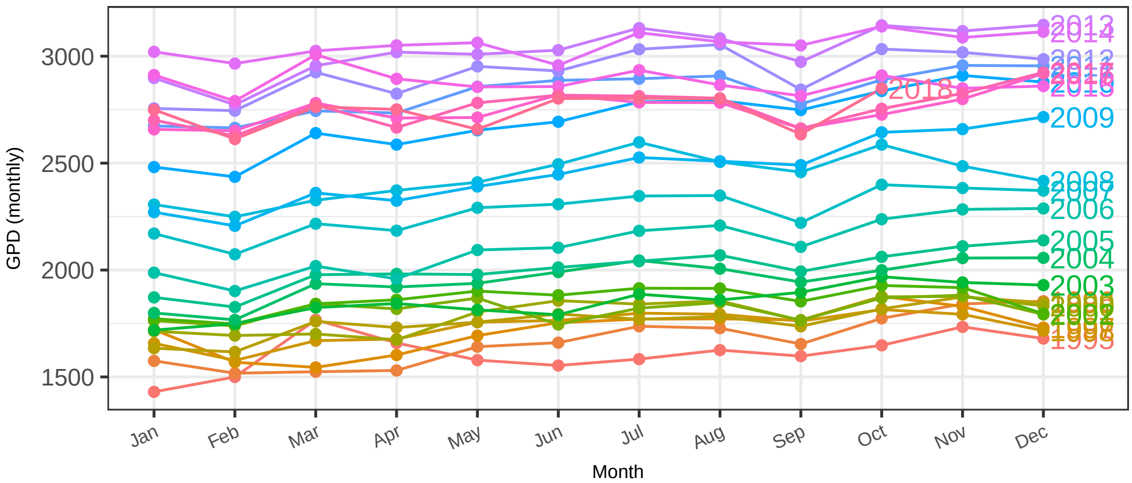
(b) Original series (orange - dashed) and series after seasonal adjustment (green - continuous).

(28a) - The structural break in 1999 is possibly related with the international crisis that culminated in Russia's debt default in August 1998 and a sharp rise in the Brazilian interest rates in the following year, after the replacement of the Central Bank board (BOGDANSKI; TOMBINI; WERLANG, 2000). As for the year of 2006, the economic scenario was more favorable than the year before and the (Brazilian) Monetary Policy Council - COPOM - opted by successive lowering in the Selic rate (COPOM, 2006).
(28b) - The vertical gridlines indicates the November month of the informed year on the x-axis. The seasonal adjustment were made using the X13-ARIMA algorithm without data transformation. After the filter, it was not possible to reject the null hypothesis (lack of seasonality), considering the QS test with 95% of confidence. This result holds when looking for the whole series, its residuals and only the last 8 more recent years. The seasonal adjustment were made dividing the original series in two pieces, separated by the structural break indicated in Figure (28a). The differences between the original and filtered series are very discrete because the seasonal behavior of the original series was subtle (see Figure 23).
Source – Own elaboration using data from BCB-Demab.

Figure 25 – Monthly GDP, in millions of Brazilian Reals using March 2018 IPCA as deflator, grouped by periods. Brazil, 1996-2018.



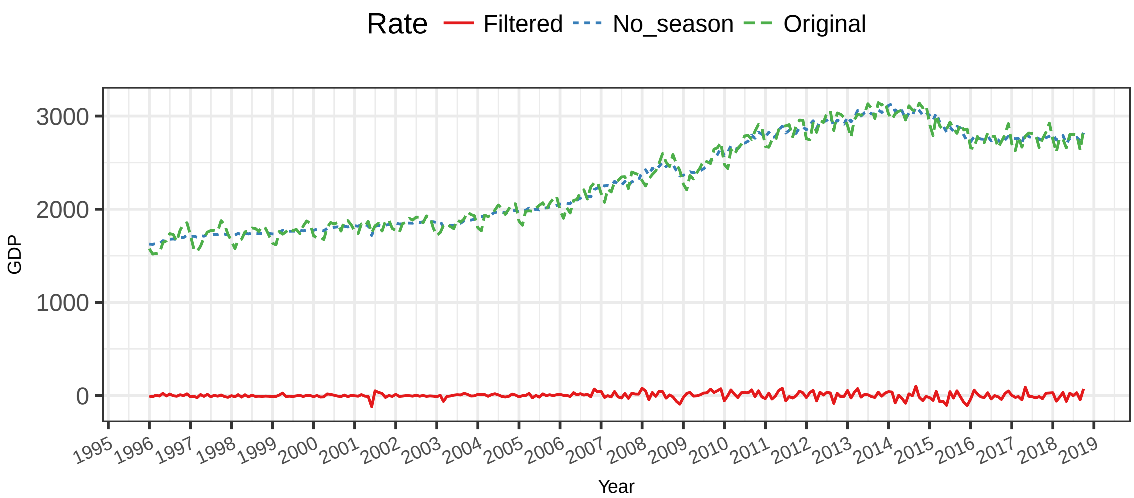
(a) Values grouped by month.



(b) Monthly values organized by year.

The monthly GDP has a tenuous seasonal pattern. Nevertheless, this behavior is statistically significant considering a 95% confidence level in the QS test. On average, the values are higher in the second semester than in the first (Figure 25a) and there is a consistently fall in September (Figure 25b). The deflator used was the IPCA from March 2018.
Source – Own elaboration using data from BCB-Depec.

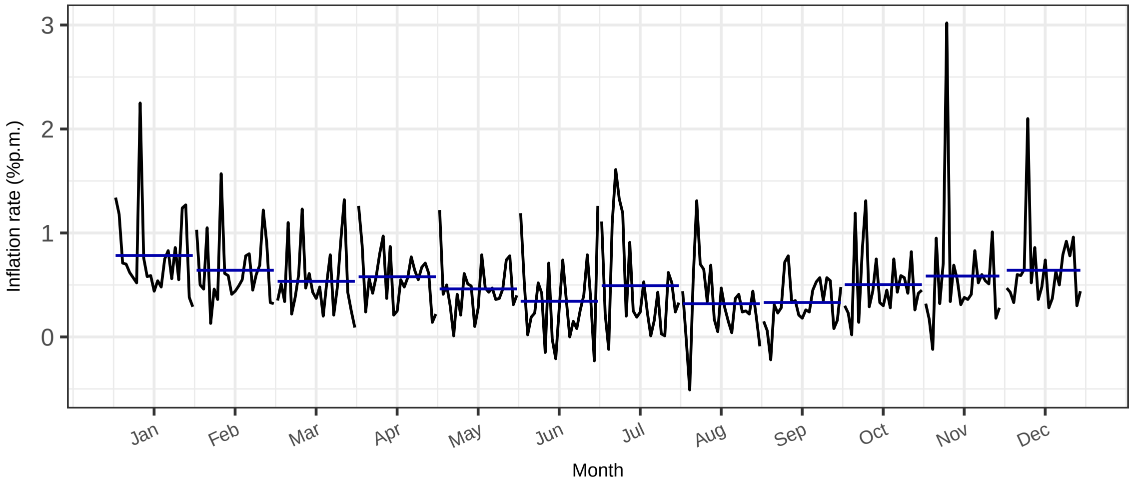
Figure 26 – Per Capita GDP. Brazil, 1995-2018.



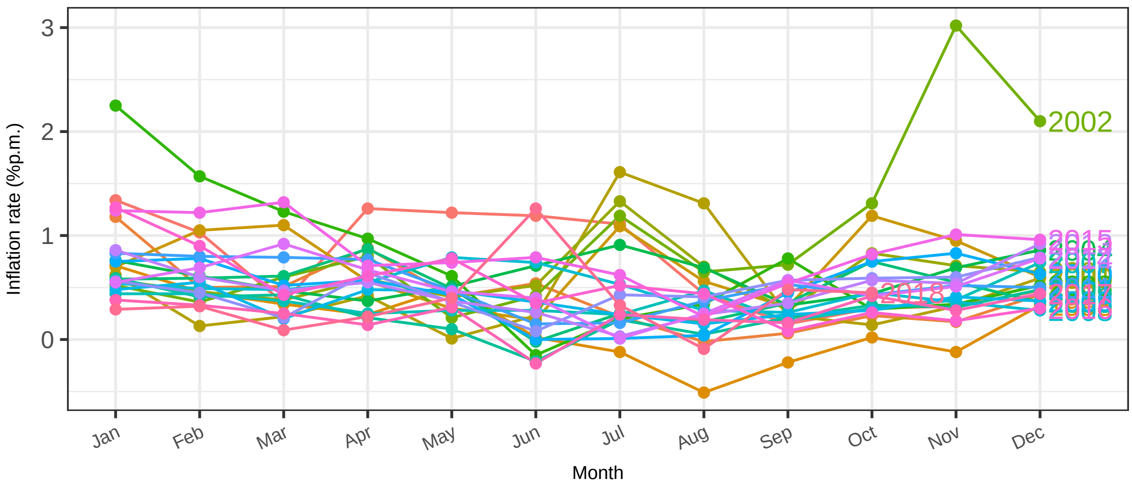
Comparison of the per capita GDP: (i) raw series (Original label, dashed green line); (ii) raw series without seasonality (No season label, dotted blue series) and (iii) the residuals from the regression of y_t against its four immediately past values (filtered label, continuous red line), as indicated by Hamilton (2018).

Source – Own elaboration using data from BCB-Demab.

Figure 27 – Broad National Consumer Price Index (IPCA), grouped by periods. Brazil, 1996-2018.



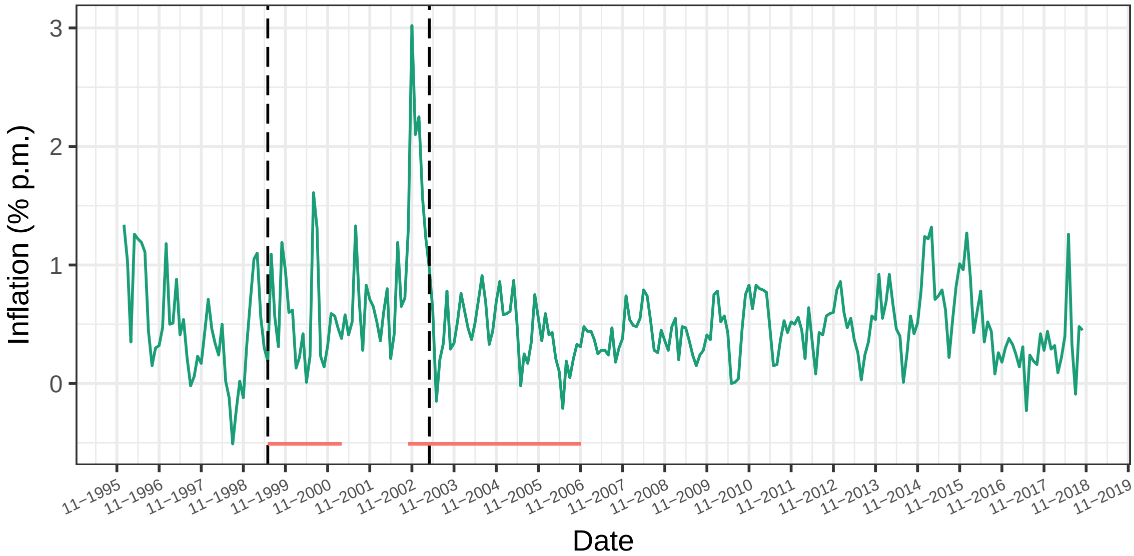
(a) Values grouped by month.



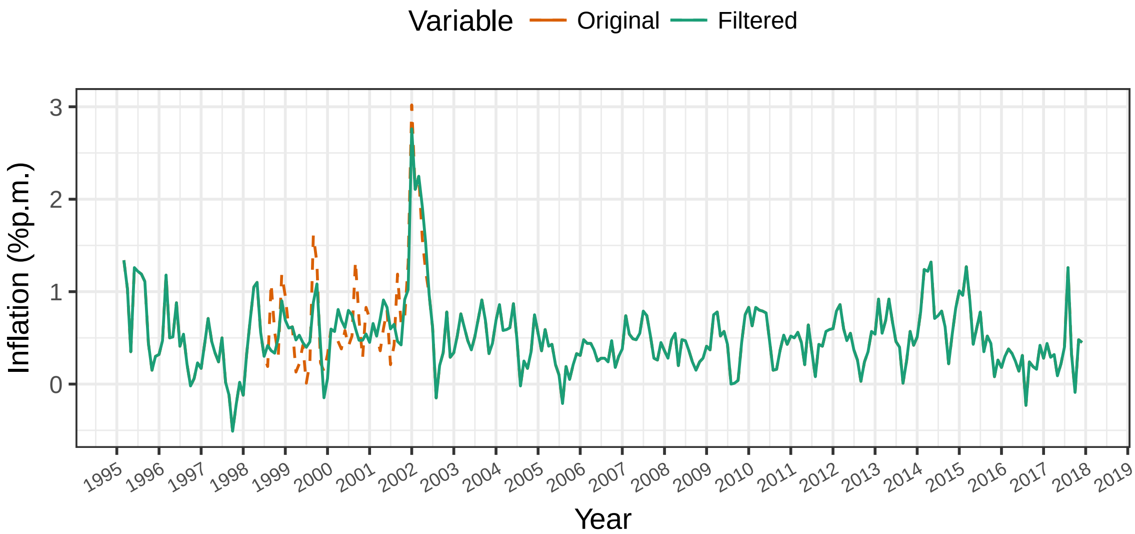
(b) Monthly values organized by year.

The IPCA series exhibits a clear pattern of decreasing inflation along the year, which is likely due to some price rigidity in the economy (Figure 27a), but looking at Figure (27b), it appears that this pattern is motivated by the presence of outliers in 1996 and 2003. However, the QS test shows no evidence of statistically significance of the seasonal pattern, considering a 95% confidence level. The base month is March 2018.
Source – Own elaboration using data from IBGE.

Figure 28 – Monthly values of the IPCA inflation index. Brazil, 1996-2018.



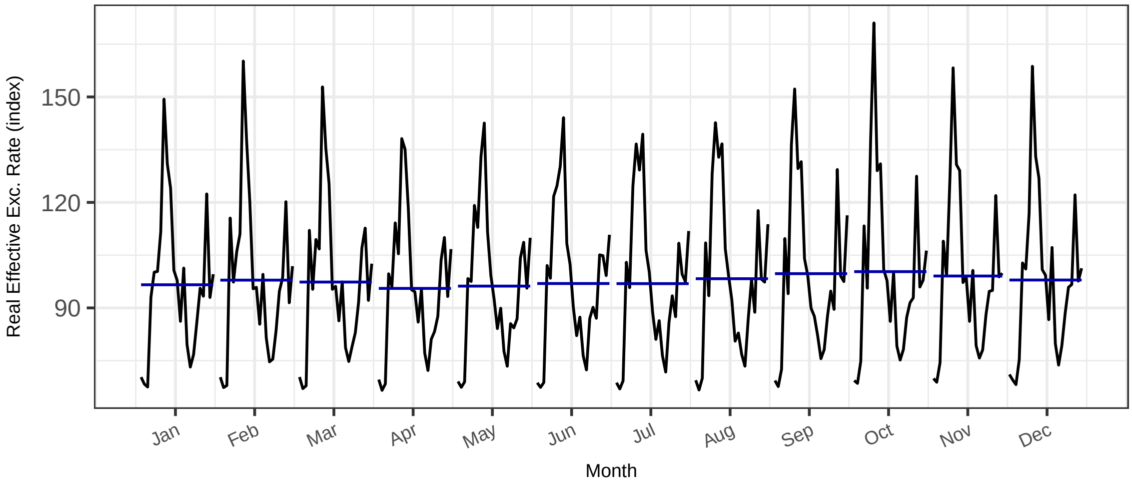
(a) Monthly values of the series. The vertical dashed lines denote the structural breaks in June 1999 and April 2003, whilst the horizontal red lines are their 95% confidence intervals: October 1995 to March 2003 and October 2002 to November 2006, respectively.



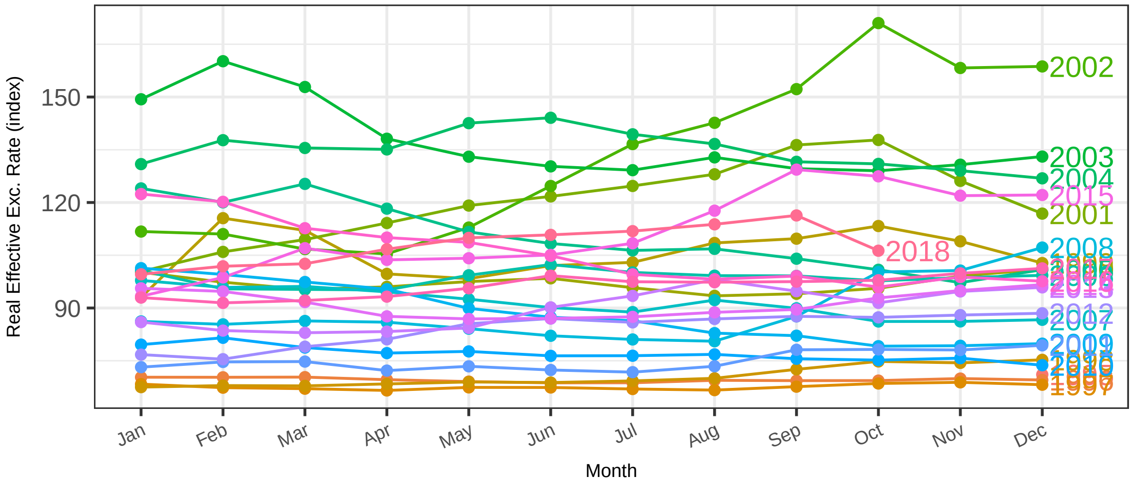
(b) Original series (orange - dashed) and series after seasonal adjustment (green - continuous).

Source – Own elaboration using data from BCB-Demab.

Figure 29 – Real effective exchange rate index, grouped by periods.
Brazil, 1996-2018.



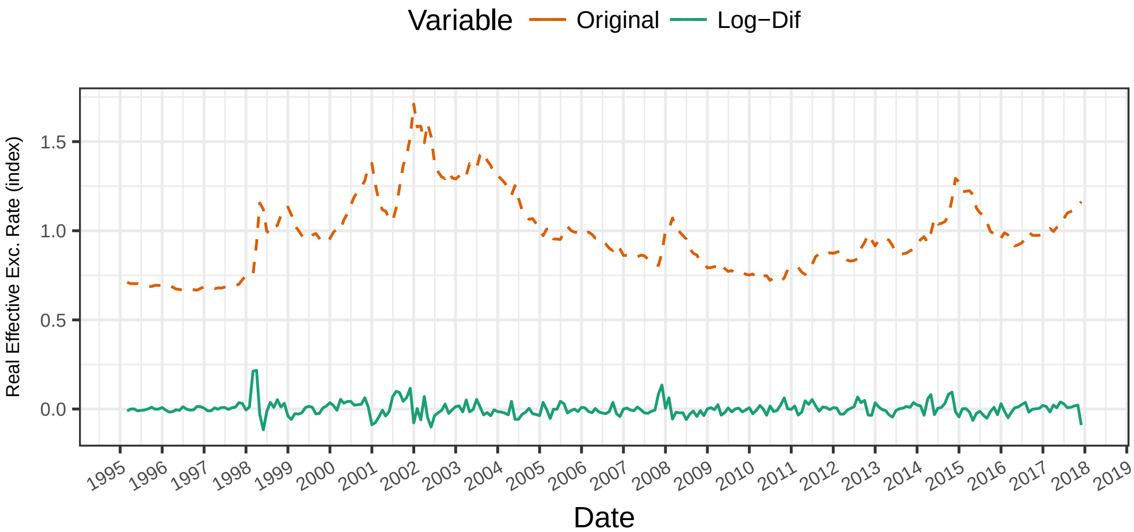
(a) Values grouped by month.



(b) Monthly values organized by year.

The real effective exchange rate is calculated considering a bundle of real exchange rates among Brazil and other countries, chosen by their respective importance in the Brazilian trade balance and weighted by trade volume. The deflator is the IPCA from June 1994. The QS test shows no evidence of statistically significance of the seasonal pattern, considering a 95% confidence level.
Source – Own elaboration using data from BCB-Depec.

Figure 30 – Monthly values of Real Exchange Interest Rate index. Brazil, 1999-2018.



The vertical gridlines indicates the November month of the informed year on the x-axis. There was no need to make seasonal adjustment for the series, but in order to obtain stationarity we took the first difference of the logarithm of the original data.
Source – Own elaboration using data from BCB-Depec.

APPENDIX C – CONVERGENCE DIAGNOSTICS

Since we are using the Gibbs Sampler as method to obtain posterior draws, it is advised to run some convergence diagnostics that help to evaluate if the Markov Chain has or not converged to its stationary distribution. In theory, this should happen for every case, but since this is an assymptotic result, but in practice we use some tools to decide, based on the finite sequence of values generated by the algorithm, if they behave as having the same distribution or not.

We obtained ten thousand posterior draws for each parameter after a burn-in of 200 thousand iterations and kept one in four draws. In other words, we ran the model 200.000 times and discarted these initial values. After that, we generated more 10.000 values for each parameter, but saved only 2500, equally spacing the saves. The burn-in procedure is necessary to enforce that the draws come from the real full conditional posterior, which in theory happens after after the convergency of the Markov Chain, whilst the discard of one in four values helps to prevent autocorrelation between draws.

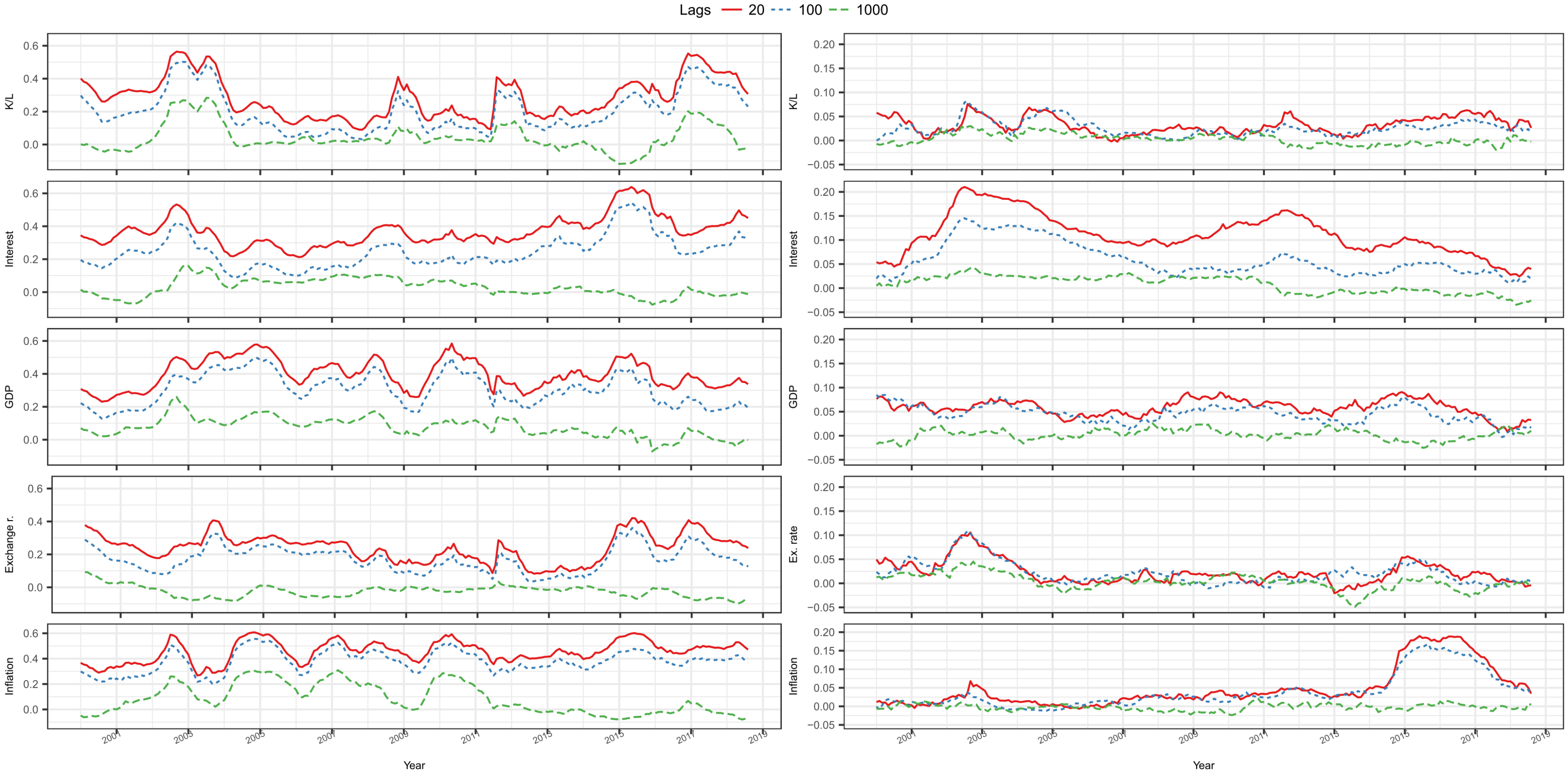
Chain autocorrelation

Since the models is highly parametrized, diagnosis poses a challenge. Here we are following Primiceri (2005) and exhibiting the graph for the partial autocorrelation function (paf) considering the first difference parameters of the variable in its own equation (for example, the β upfront the K/L value at $t-1$ in the K/L equation) and the volatility of each VAR equation. The lags considered for the ACFwere 20, 100 and 1000. The graphic in figure fig:correlation shows the ACF evaluated for these three lags, for each selected parameter.

Multivariate diagnostics

An additional convergence test is made simulating two distinct models (with different random seeds but with the same data) to calculate a potential scale reduction factor for each parameter and one multivariate factor considering the global convergence of the chain (BROOKS; GELMAN, 1998). In this stage, we replicated the same setup: 5 series, burn-in equal to 200.000 followed by 10.000 draws where one in four was kept. We calculated the Gelman factor for each first lag coefficients on the measurement equations and for the standar deviations at each time t ($t \in \{1, \ldots, 226\}$). Figure (32) contains the Gelman coefficients with their respective superior values of the 95% confidence interval. It

Figure 31 – Autocorrelation function for selected parameters

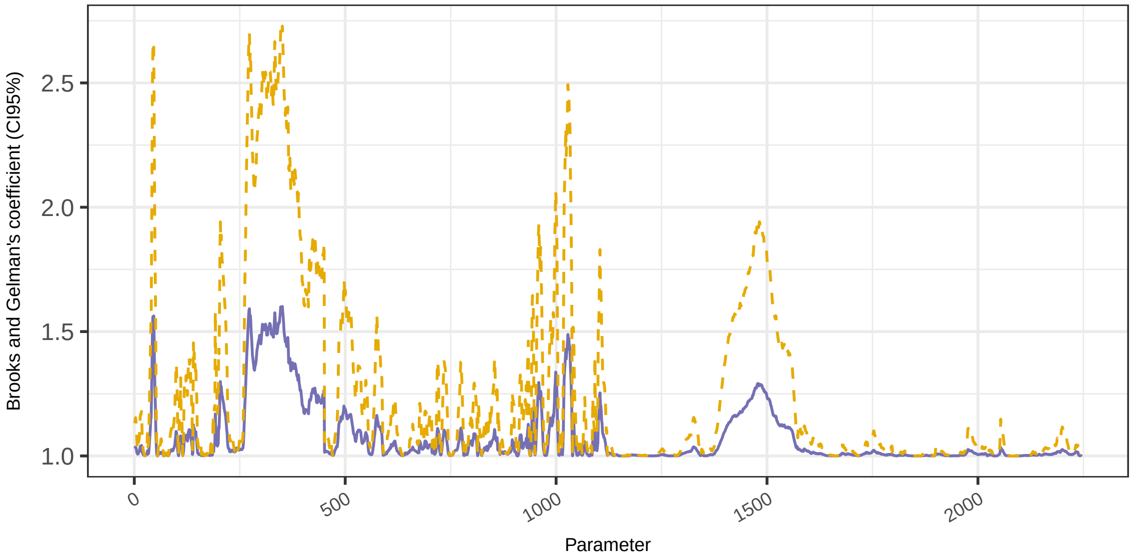


The autocorrelation function (y axis) is based on 2500 values of each parameter. We calculated the ACF the first lag parameter of each variable in its own equation (left side) and volatilities of each variable (right side), considering the lags 20 (continuous - red line), 100 (dotted - blue line) and 1000 (dashed - green line). Each parameter was evaluated for all 226 posterior time periods, which runs along the x axis. This was based on 2.500 values taken in equally spaced intervals from 10.000 posterior draws, using a burn-in equal to 200 thousand iterations.

Source – Own elaboration based on the model results.

is possible to observe that the right part of the graph, which corresponds to the standard deviations, is higher than the first part, which could suggest convergence problems.

Figure 32 – Brooks-Gelman Diagnostic Statistics for selected coefficients and volatilities of the TVP-VAR



Computed values of the Brooks and Gelman's statistics for 2.140 selected model parameters: coefficients of the first and second lag of each variable in its own equation and volatilities of each equation, all evaluated for each one of the 226 periods considered in the posterior analysis. The Brooks and Gelman's statistics consider two independent chains and computes a positive number for each one of the parameters and values more close to 1, are considered better. The overall multivariate statistics for this two samples was equal to 3.4. Values close to one indicate that the chain converged. In the graph, the statistic is represented by the purple continuous line while the superior value of the 95% confidence interval is represented by the dashed yellow line. Roughly the first half values (in the left) are the measurement equation coefficients and the second half on the right are the volatilities. This was based on two 10k simulations using a burn-in equal to 300k iterations.
Source – Own elaboration based on the model results.