Estimation of a TVP-VAR with MSV via Efficient Importance Sampling

Project Proposal

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> UFSC - PPGECO Seminários de Dissertação

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Using VAR models to describe macroeconomic relations

"It should be feasible to estimate large-scale macromodels as unrestricted reduced forms, treating all variables as endogenous."

Christopher Sims, Macroeconomics and Reality (1980).

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- ► RBC models x VAR approach
- Strong evidence of parameter instability in macroeconomic series:
 [Sims, 1999], [Bernanke and Mihov, 1998a], [Bernanke and Mihov, 1998b],
 [Mumtaz and Zanetti, 2015], [Galí and Gambetti, 2015],
 [Mumtaz and Theophilopoulou, 2015],
 [Mumtaz and Theophilopoulou, 2017], [Cogley and Sargent, 2001],
 [Cogley and Sargent, 2005], etc.

Cogley and Sargent (2005)

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Bad Luck vs Bad Policy

Changes in the inflation persistence

Changes in volatility structure

Bad luck:

It was not monetary strategy but increase/decrease of the shocks' volatility

Changes in monetary policy rules

Drifting coefficients

Bad policy:

The FED's view negates the natural rate theory and believes in an exploitable trade-off between unemployment and inflation

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Economic Motivation

TVP-VAR + MSV

Cogley and Sargent (2001, 2005)

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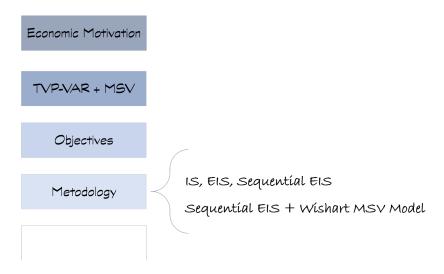
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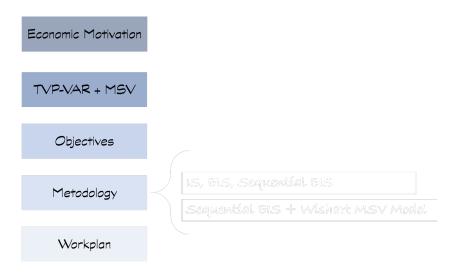


IS, FIS, Sequential FIS Sequential FIS + Wishart MSV Model

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Literature Review: TVP-VAR w/ MSV

Consider the following state-space representation of a TVP-VAR with MSV:

$$y_t = Z_t \alpha_t + \epsilon_t$$
 $\epsilon_t \sim \mathcal{N}_k(0_k, \Omega_t^{-1})$ (measure eq.) (1)
 $\alpha_t = \alpha_{t-1} + \nu_t$ $\nu_t \sim \mathcal{N}_p(0_p, Q)$ (state transition eq.) (2)

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- We don't observe α_t directly (**latent variable**);
- ▶ If Ω_t^{-1} were non-stochastic, the Kalman Filter could be used to obtain the ML function;
 - Since it is not the case, we have an analytically intractable high-dimensional integral.

Literature Review: Cogley and Sargent (2001, 2005)

Their first work was a TVP-VAR of the form:

$$y_t = Z_t \alpha_t + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}_k(0_k, R)$$
 (3)

$$\alpha_t = \alpha_{t-1} + u_t \qquad u_t \sim \mathcal{N}_p(0_p, Q) \tag{4}$$

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After being criticized in (2001) for the lack of MVS in their model, [Cogley and Sargent, 2005] proposed changing ϵ_t in (3) for $\epsilon_t = R_t^{1/2} \xi_t$, where ξ_t follows a standard normal distribution and $R_t = B^{-1}H_tB^{-1}$ with:

$$B = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \beta_{21} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{k1} & \beta_{k2} & \cdots & 1 \end{bmatrix}, \text{ and } H_t = \begin{bmatrix} h_{1t} & 0 & \cdots & 0 \\ 0 & h_{2t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{kt} \end{bmatrix}$$

where
$$ln(h_{it}) = ln(h_{it-1}) + \sigma_i \eta_{it}$$
, $\eta_{it} \sim \mathcal{N}(0, 1)$.

Literature Review: Primiceri (2005)

[Primiceri, 2005] proposed the same specification as [Cogley and Sargent, 2005] but allowed the covariances in *B* to evolve over time:

$$B_{t} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \beta_{21,t} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{k1,t} & \beta_{k2,t} & \cdots & 1 \end{bmatrix} \quad \text{with} \quad \beta_{t} = \beta_{t-1} + \nu_{t}$$
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where v_t are mean-zero normal errors with constant covariance matrix.

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Drawback: (5) is not invariant to the observable variables y_t order! The problem lies in the MSV structure!

Literature Review: Phillipov and Glickman (2006a, 2006b)

[Philipov and Glickman, 2006b] and [Philipov and Glickman, 2006a] proposes the following model:

$$y_t = \epsilon_t, \qquad \epsilon \sim \mathcal{N}_k(0_k, \Omega_t^{-1})$$
 (6)

$$\Omega_t | \Omega_{t-1} \sim W(\nu, S_{t-1}), \quad \text{with } S_t = \frac{1}{\nu} A^{1/2} \Omega_t^d A^{1/2'}$$
 (7)

where

- ► A is a positive definite symmetric matrix containing parameters determining the intertemporal sensitivity of each element of the precision matrix and
- ▶ $d \in [0, 1)$ is a scalar that accounts for overall persistence of the process.

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This specification is more flexible than Primiceri's one, and it is invariant to ordering!

Literature Review: Phillipov and Glickman (2006a, 2006b)

► [Philipov and Glickman, 2006a] had trouble estimating simultaneously the elements of the matrix A and persistence parameter *d* and proposed to use a fixed *d*;

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- ► [Philipov and Glickman, 2006a] had trouble estimating simultaneously the elements of the matrix A and persistence parameter *d* and proposed to use a fixed *d*;
- ► [Asai and McAleer, 2009] reported convergence problems in the Gibbs sampler.

Where are we?

Phillipov and

Primiceri (2005)

- The order of the variables y_t matters;
- Simple random walk structure for covariances;
- Missing steps in Gibbs sampler.

Estimation problems

Where are we?

Primiceri (2005)

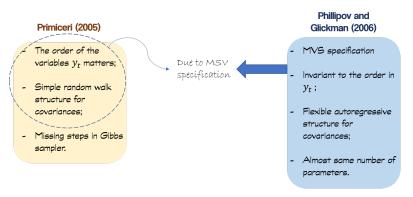
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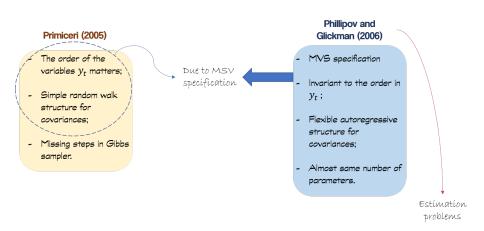
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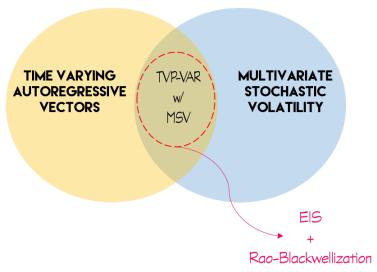
Where are we?



Estimation of a TVP-VAR with MSV via EIS Objective

Develop and implement efficient statistical inference procedures in TVP-VAR with MSV models.

Proposal



Proposal

Build a **TVP-VAR** with **Wishart MSV** model based on [Philipov and Glickman, 2006b] and obtain **maximum likelihood** estimates using **EIS** [Richard and Zhang, 2007] combined with a **Rao-Blackellization** step [Moura and Turatti, 2014].

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Specific Objectives:

 Obtain the ML estimates for the MSV Wishart model from [Philipov and Glickman, 2006b] using EIS as well develop diagnostic tools to study numerical properties;

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- Generalize the previous model to a TVP-VAR with drifting coefficients plus MSV and implement EIS algorithm combined with a Rao-Blackwellization step, as well as develop diagnostic tools;
- 3. Develop an empirical application using the model developed in (2), in line with [Mumtaz and Theophilopoulou, 2017].

Methodology: Monte Carlo Methods and Importance Sampling

- ► Monte Carlo (MC) Methods are an alternative way to solve complex integrals;
 - Specially used for high-dimension problems, where non-stochastic algorithms are way too slow;
- ► The idea is to resample values from a probability density (thus, is a stochastic method), using a pseudo-random number generator and the inverting theorem.

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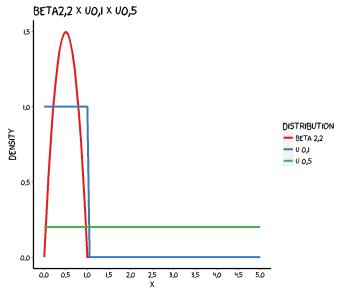
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In cases where the function f is known, using the law of large numbers we can use the sample mean to approximate (8):

$$I \approx \frac{1}{N} \sum_{i=1}^{N} g(x_i) \tag{10}$$

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Methodology: Monte Carlo Methods and Importance Sampling

Consider the following integral:

$$I = \int_{\mathcal{X}} g(x) \frac{f(x)}{m(x)} m(x) dx$$
 (11)

Now we have an expectation with respect to the density $m(\cdot)$:

$$I = \mathbb{E}_{m} \left[\frac{g(x)f(x)}{m(x)} \right]$$
 (12)

In cases where the function m is known, using the law of large numbers we can use the sample mean to approximate (11):

$$I \approx I_N^{IS} = \frac{1}{N} \sum_{i=1}^{N} \omega(x_i) g(x_i), \quad \text{with} \quad \omega_i = \frac{f(x_i)}{m(x_i)}$$
 (13)

Methodology: Efficient Importance Sampling

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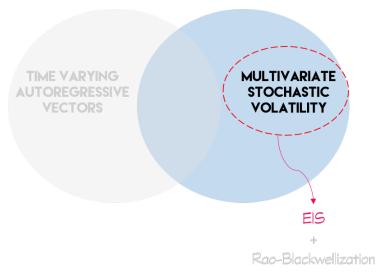
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 Sequential EIS is used in high-dimensional problems and consists in "breaking" the EIS algorithm in a sequence of low-dimension optimizations.

Methodology: Wishart MSV model



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Recall the model given in (6)-(7). Defining $f(\underline{y}, \underline{\Omega}, \theta)$ as the joint density of $\underline{y} = \{y_t\}_{t=1}^T$ and $\underline{\Omega} = \{\Omega_t\}_{t=1}^T$, where θ is the vector of unknown parameters, the likelihood associated to \underline{y}_t is given by:

$$L(\theta|\underline{y}) = \int f(\underline{y}.\Omega,\theta)d\Omega$$

$$= \int \cdots \int \prod_{t=1}^{T} f(y_t,\Omega_t|\Omega_{t-1},\underline{y}_{t-1};\theta)d\Omega_1 \ldots d\Omega_T$$
(14)

with \underline{y}_t e $\underline{\Omega}_t$ given by $\{y_t\}_{t=1}^T$ e $\{\Omega_t\}_{t=1}^T$, respectively.

Methodology: Wishart MSV model

The respective space-state representation is:

$$f(y_t, \Omega_t | \underline{\Omega}_{t-1}, \underline{y}_{t-1}; \theta) = g(y_t | \Omega_t, \underline{y}_{t-1}; \theta) \cdot p(\Omega_t | \underline{\Omega}_{t-1}, \underline{y}_{t-1}; \theta)$$
 (15)

where

$$g(y_t|\Omega_t, \underline{y}_{t-1}; \theta) \propto |\Omega_t|^{1/2} \exp\left\{-\frac{1}{2}y_t'\Omega_t y_t\right\}$$
 (16)

$$p(\Omega_t | \underline{\Omega}_{t-1}, \underline{y}_{t-1}; \theta) \propto |\Omega_t|^{\frac{\nu-k-1}{2}} \exp\left\{ -\frac{1}{2} tr \left[S_{t-1}^{-1} \cdot \Omega_t \right] \right\}$$
 (17)

Methodology: Wishart MSV model

Likelihood:

$$L(\theta|\underline{y}) = \int \prod_{t=1}^{T} \left[\frac{f(y_t, \Omega_t | \underline{\Omega}_{t-1}, \underline{y}_{t-1}; \theta)}{m(\Omega_t | \underline{\Omega}_{t-1}; \gamma_t)} \right] \prod_{t=1}^{T} m(\Omega_t | \underline{\Omega}_{t-1}; \gamma_t) d\underline{\Omega}$$

(18)

with $m(\Omega_t | \Omega_{t-1;\gamma_t}) = \frac{k(\Omega_t;\gamma_t)}{\chi(\Omega_{t-1};\gamma_t)}$ where $\chi(\Omega_{t-1};\gamma_t) = \int k(\Omega_{t-1};\gamma_t) d\Omega_t$.

MC estimate:

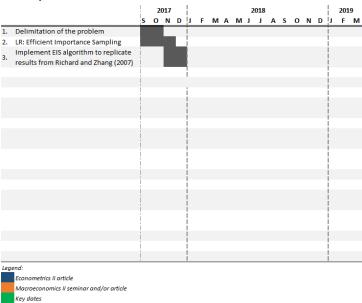
$$\tilde{L}(\theta|\underline{y}) = \frac{1}{N} \sum_{i=1}^{N} \left[\prod_{\ell=1}^{L} \frac{f(y_t, \tilde{\Omega}_t^i | \tilde{\Omega}_{t-1}^i, \underline{y}_{t-1}; \theta) \cdot \mathcal{X}(\tilde{\Omega}_{t-1}^i; \gamma_t)}{k(\tilde{\Omega}_t^i; \gamma_t)} \right]$$

Minimization problem:

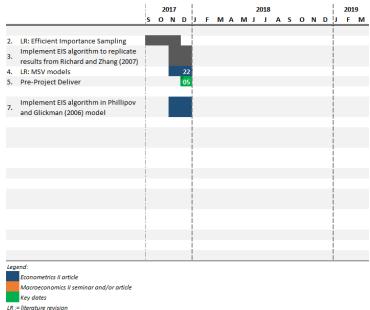
$$\hat{\gamma}_{t}(\theta) = \min_{\gamma_{t}} \sum_{i=1}^{N} \left\{ \ln \left[f(y_{t}, \tilde{\Omega}_{t}^{i} | \tilde{\Omega}_{t-1}^{i}, \underline{y}_{t-1}; \theta) \cdot \mathcal{X}(\tilde{\Omega}_{t-1}^{i}; \gamma_{t}) \right] - c_{t} - \ln k(\underline{\Omega}_{t}; \gamma_{t}) \right\}$$
(20)

Workplan - Completed tasks

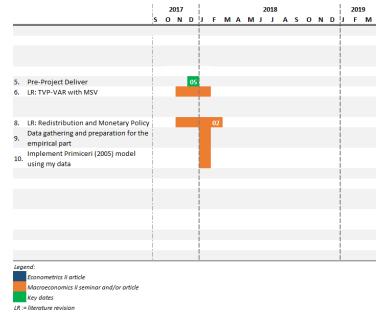
IR := literature revision



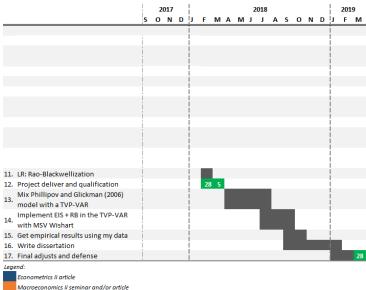
Workplan - Econometrics II related tasks



Workplan - Macroeconomics II related tasks

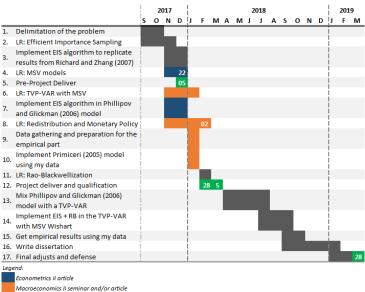


Workplan - What future holds



Key dates

Workplan - General Overview



Key dates

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