Name: Aisha Muhammad Nawaz
Roll Number: 20L-0921
Section: BCS-4A
Course: Design & Analysis of Algorithms [Ms. Maryam Bashir]
Homework # 1
Due: 28-02-22 (monday)

Question 1

Part (a)

Time taken by each line of code

```
a = n;                (1)
while (a > 1) {         →  log2(n)
    b = 1;              ~> (log2 n)
    while (b < n) {     →  log2(n) . log2(n)
        k = 0;          ~> (log2(n))^2
        while (k < n) { →  (n/2 + 1)((log2(n))^2
            k = k+2;    ~> (n/2 + 1)((log2(n))^2
        }
        b = b*2;        ~> (log2(n))^2
    }
    a = a/2;            ~> log2(n)
}
```

3   3   5

WORKING :-

- for while loop 1

a

$\frac{a}{2}$

$\frac{a}{4}$

$\frac{a}{8}$

$a\left(\frac{1}{2}\right)^K$ * where K start with zero

$\frac{a}{2^K} > 1$ terminating point

$\frac{a}{2^K} = 1$

$2^K = a$

$\log_2 a = K$

$a = n$
so,
$\log_2 n = K$
$K = \log_2 n$

- for while loop2

b
$2b$
$4b$
$8b$

$b(2)^K$ * where K starts with zero

terminating point
$b(2)^K = n$
$b = 1$
$2^K = n$
$\log_2 n = K \rightarrow K = \log_2 n$

- for while loop 3

K
$K+2$
$K+4$
$K+6$

$a + (n-1)d$ arithmetic series

$K + (n-1)2$
$K + 2n - 2$
since $K = 0$,
$2n - 2$
$\hookrightarrow 2m - 2$

$2m - 2 = n$
$2m = n+2$
$m = \frac{n+2}{2}$
$\boxed{m = \frac{n}{2} + 1}$

Total T(n)

$= 1 + \log_2(n) + \log_2(n) + [\log_2(n)]^2 + [\log_2(n)]^2 + 2(n/2+1)(\log_2(n))^2 + (\log_2(n))^2 + \log_2(n)$

$T(n) = 1 + 3\log_2(n) + 3(\log_2(n))^2 + 2(n/2+1)(\log_2 n)^2$

$T(n) = 1 + 3\log_2 n + 3(\log_2 n) + 2(n/2+1)(\log_2 n)^2$

$= 1 + 3\log_2 n + (\log_2 n)^2 (3 + n + 2) = 1 + 3\log_2 n + (5+n)(\log$

$O(f(n)) = n(\log_2 n)^2$

$T(n) = 1 + 3\log_2 n + 5(\log_2(n))^2 + n(\log_2(n))^2$

$T(n) \le c \cdot f(n)$

$1 + 3\log_2 n + 5(\log_2 n)^2 + n(\log_2(n))^2 \le c \cdot n(\log_2 n)^2$

divide both sides by $n(\log_2 n)^2$

$\frac{1}{n(\log_2 n)^2} + \frac{3}{n \log_2 n} + \frac{5}{n} + 1 \le c$

This part will decrease as $n \uparrow$

$\boxed{\text{for } n > 1,\ c = 11/2 = 5.5}$ (Hence, shown).

Part (b)

```
a=1;                      (1)      -> log2(n/2)
while (a < n/2) {
    b=1;                           -> log2(n)
    while (b<n) {
        b= b*2;
    }                              . Log2(n/2)
    a= a*2;
}
```

Time taken by each line of code

WORKING

- for while loop 1

a

2a

4a

8a

$a(2)^k$ * where k starts with zero

since $a=1$

$(1)\ 2^k$

terminating point

$2^k = n/2$

$\log_2 n/2 = K$

$K = \log_2(n/2)$

- for while loop 2

b

2b

4b

8b

$b.(2)^k$ * when K starts with zero

since $b=1$

$(1)\ (2)^k$

terminating point

$2^k = n$

$k = \log_2 n$

- Total T(n)

$T(n) = 1 + 3\log_2(n/2) + 2\log_2(n) \cdot \log_2(n/2)$

$O(f(n)) = \log_2(n) \cdot \log_2(n/2)$

$T(n) \le c \cdot f(n)$

$$1 + 3\log_2(n/2) + 2\log_2(n)\cdot\log_2(n/2) \le c\cdot\log_2(n)\cdot\log_2(n/2)$$

dividing both sides by $\log_2(n)\cdot\log_2(n/2)$

$$\boxed{\frac{1}{\log_2(n)\cdot\log_2(n/2)} + \frac{3}{\log_2(n)}} + 2 \le c$$

↳ This part decreases as $n \uparrow$

4.971...

$$\frac{1}{\log_2(3)\cdot\log_2(3/2)} + \frac{3}{\log_2(3)} + 2 = 4.97\ldots$$

(hence shown).

$\boxed{\text{for } n > 2,\ c = 4.97}$

Part (c).

Time taken by each line of code

```
a = 1                      (1)
while (a < n) {            1/4 (n+3)
    b = n
    while (b > 1) {        1/4 (n+3)((n+2)·1/3)
        b = b - 3;
                           3
    a = a + 4;
                           3
```

WORKING

for while loop 1

a	$a+(n-1)d$	arithmetic series.
$a+4$	$a+(n-1)4$	since $a=1$
$a+8$	$a+4n-4$	
$a+12$	$1+4n-4$	termination point
	$4n-3$	$4m-3=n$
	↳ $4m-3$	$4m-3=n$
		$4m=n+3$
		$m=\frac{n}{4}+3/4$
		$\boxed{m=\frac{1}{4}(n+3)}$

for while loop 2

b	$a+(n-1)(-3)$	
$b-3$	$a+(n-1)(-3)$	
$b-6$	$a-3n+3$	
$b-9$	$a-3m+3$	
	since $a=b=n$	termination point
	$n-3m+3$	$n-3m+3=1$
		$n+3-1=3m$
		$m=\frac{1}{3}(n+2)$

Total

$$T(n) = 1 + \frac{3}{4}(n+3) + \frac{2}{4}(n+3)\cancel{(n+2)\left(\tfrac{1}{3}\right)}$$

$$T(n) = 1 + \frac{3}{4}(n+3) + \frac{2}{12}(n+3)(n+2)$$

$$T(n) = 1 + 3/4(n+3) + \frac{1}{6}(n+3)(n+2)$$

$T(n) = 1 + 3/4\,(n+3) + \frac{1}{6}(n+3)(n+2)$

$T(n) = 1 + \frac{3}{4}n + \frac{9}{4} + \frac{1}{6}(n^2 + 2n + 3n + 6)$

$T(n) = 1 + \frac{3}{4}n + \frac{9}{4} + \frac{1}{6}n^2 + \frac{5}{6}n + 1$

$T(n) = 2 + \frac{9}{4} + \frac{19}{12}n + \frac{1}{6}n^2$

$T(n) = \frac{17}{4} + \frac{19}{12}n + \frac{1}{6}n^2$

$O(f(n)) = n^2$

$T(n) \leq c.f(n)$

$\frac{17}{4} + \frac{19}{12}n + \frac{1}{6}n^2 \leq c.n^2$

dividing both side by n^2

$\boxed{\frac{17}{4n^2} + \frac{19}{12n}} + \frac{1}{6} \leq c.$

↳ This part decrease as n↑

for $n=1$, $c=6$

$\boxed{\text{for } n \geq 1,\ c = 6}$ (here, shown).

part (d)

$a = 0;$ → (1) $\log_2 n$ → $n^{\log_2 n}$

for $(i=1;\ i<n;\ i=i*2)$

for $(j=1;\ j<n;\ j=j*2)$ → $(2\log_2(n))$ → $(\log_2 n)^2$

$a{+}{+}$

WORKING

for for loop 1

i
$2i$
$4i$

$i\,(2)^k = n$

$\log_2 n = K$

for for loop 2

$(\log_2 n)^2$

Total $T(n) =$

$3 + 2\log_2(n) + 3(\log_2(n))^2$

$O(f(n)) = ((\log_2(n))^2$

$T(n) \leq c.f(n)$

$3 + 2\log_2(n) + 3(\log_2(n))^2 \leq c.(\log_2(n))^2$

dividing both sides by $(\log_2(n))^2$

This part decreas as n↑

$\boxed{\frac{3}{\log_2 n} + \frac{2}{\log_2 n}} + 3 \leq c$

$n=2$ $\frac{3}{\log_2(2)} + \frac{2}{\log_2(2)} + 3 \leq c$

$8 \leq c$ $c=8$

$$\boxed{\text{for } n \geq 2, \quad c=8}$$ (hence, shown)

Question 2 →

(a). $2^{2n} = O(2^n)$

$2^{2n} \leq c \cdot 2^n$

$2^n \cdot 2^n \leq c \cdot 2^n$

(dividing both sides by 2^n)

$2^n \leq c$

as $n \uparrow$ there is no such c that will always be greater than 2^n as we can not restrict the upper value of n

Disproved

(b). $8^n = O(4^n)$

$8^n \leq c \cdot 4^n$

$2^{3n} \leq c \cdot 2^{2n}$

$2^n \cdot 2^{2n} \leq c \cdot 2^{2n}$

dividing both sides by 2^{2n}

Disproved $2^n \leq c$ Same case as above not possible

(c). $3^{n+5} = O(3^n)$

$3^n \cdot 3^5 \leq c \cdot 3^n$

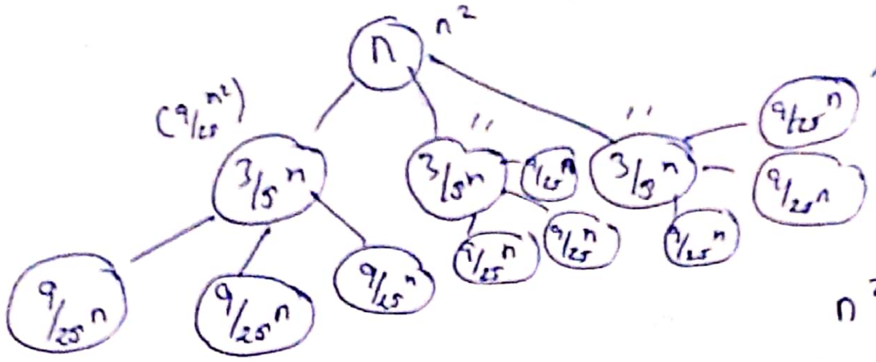
dividing both sides by 3^n

$3^5 \leq c$ $\boxed{c = 3^5}$ Proved

Question 3

Bubble sort time complexity = $O(n^2)$

(a). $T(n) = 3T(3/5\, n) + n^2$ } is running recurrence



n^2

$3\left(\frac{9}{25} n^2\right)$

pattern $= n^2, \frac{27}{25}n^2, \frac{81}{625}n^2 \quad 9\left(\frac{9}{25}\right)^2 n^2 = \frac{81}{625} n^2$

pattern $3^k\left(\frac{9}{25}\right)^k n^2$

total # of levels $= \log_{5/3} n$

$\frac{n}{(5/3)^k} = 1$

$n = (5/3)^k$

$\log_{5/3} n = k$

Geometric Series

$= \frac{1}{1 - 9/25} = \frac{25}{16}$

for $3^k = \frac{3^{k+1} - 1}{3-1} = \frac{1}{2}\left(3^{k+1} - 1\right)$

$$\boxed{\text{So, } \frac{1}{2}\left(3^{k+1} - 1\right) \cdot \frac{25}{16} \cdot \log_{5/3} n}$$

Asymptotic running time

(b).

Eg. 2, 1, 4, 5, 3

1st recursive sort 1, 2, 4, 5, 3

2nd recursive sort 1, 2, 4, 3, 5

3rd recursive sort 1, 2, 4, 3, 5

Question 4

$$T(n) = \frac{1}{6}n^4 - 4n^2$$

$$\Theta(n^4)$$

$$c_2\,\Theta(n^4) \le T(n) = \frac{1}{6}n^4 - 4n^2 \le \Theta(n^4)\cdot c$$

$$K_2 n^4 \le \frac{1}{6}n^4 - 4n^2 \le K_1 n^4$$

dividing both sides by n^4

$$K_2 \le \frac{1}{6} - \frac{4}{n^2} \le K_1$$

$$K_2 \le \frac{1}{6} - \frac{4}{n^2}$$

↳ This part decreases as $n\uparrow$

for $n_0 = 5$ $\quad n \ge n_0$

$$K_2 \le \frac{1}{6} - \frac{4}{5^2}$$

$$\boxed{K_2 = \frac{1}{150}}$$

$$\frac{1}{6} - \frac{4}{n^2} \le K_1.$$

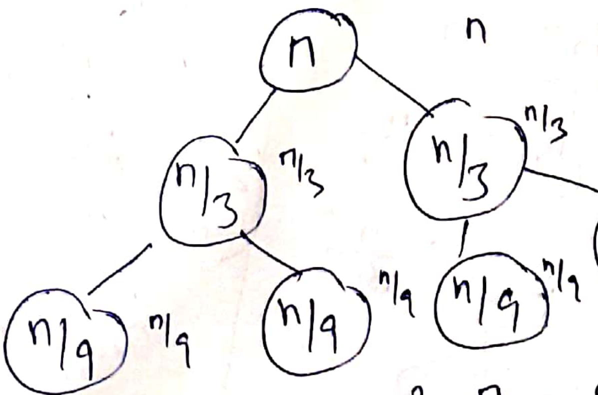
for $n_0 = 1$

$$\boxed{K_1 = \frac{1}{6}}$$

as $4/n^2$ decreases as n increases and less amount is subtracted from $1/6$

Question 5

$$T(n) = 2T(n/3) + c\cdot n$$



n $= cn$

$= 2/3\, n(c)$

$n/9 = \frac{4}{9}cn$

pattern $cn,\ c\,2/3\,n,\ c\,4/9\,n$

$\frac{n}{3^K} = 1$

$n = 3^K$

$\boxed{\log_3 n = K}$ total # levels $= \log_3 n$

$$3n \log_3 n$$

$$= O(n \log_3 n)$$ ↳ final Answer

$$\left(2/3\right)^K n$$

↳ Geometric Series with $r = 2/3$

$$= \frac{1}{1-2/3} = 3$$

$$= 3n$$

PS 7

Question 6 →

Using Guess / Trial and error me

$$50 n^2 \leq 2^n$$

$$50 n^2 \leq 2^n$$

$$50 \leq \frac{2^n}{n^2}$$

When $n = 13$

$$50 \leq \frac{2^{13}}{(13)^2}$$

$$50 \leq 48 \quad \times$$

When $n = 14$

$$50 \leq \frac{2^{14}}{(14)^2}$$

$$50 \leq 83.59 \quad \checkmark$$

So $n = 14$

Smallest value of n is 14

Question 7 →

(a). $T(n) = T(n/5) + O(n)^2$

n — n^2

$n/5$ — $\frac{n^2}{25}$

$\frac{n}{25}$ — $\frac{n^2}{625}$

$\frac{n}{125}$ — $\frac{n^2}{15625}$

$\vdots$

1

Total # of levels $\left(\frac{n}{5^K}\right)^* = 1$

$n = 5^K$

$5^K = n$

$\log_5 n = K$

$K = \log_5 n$

pattern →

$n^2, \frac{n^2}{25}, \frac{n^2}{625}$

$\sum_{K=0}^{n} \left(\frac{1}{25}\right)^K n^2$ where K starts with zero

↳ Geometric Series with $r = 1/25$

$= \frac{1}{1 - 1/25} = \frac{25}{24}$

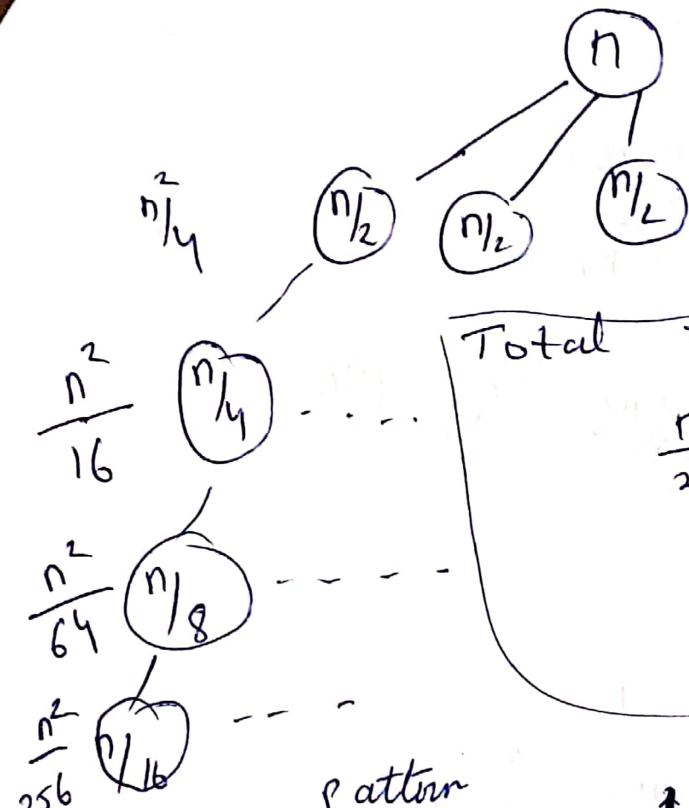
$= \frac{25}{24} n^2$

$$\boxed{\frac{25}{24} n^2 \log_5 n}$$

$O(n^2 \log_5 n)$

↳ final Answer.

b) $T(n) = 10\,T(n/2) + O(n)^2$



n^2

$\frac{10n^2}{4}$ up till 10 childen

$10\frac{n^2}{16}$

Total # of levels.

$\frac{n}{2^k} = 1$

$n = 2^k$

$\boxed{\log_2 n = k}$

pattern: $n^2, \frac{10^1}{4}n^2, \frac{10^2}{16}n^2, \frac{10^3}{64}n^2, \frac{10^4}{256}n^2$

$\doteq n^2\left(\frac{10}{4}\right)^k = n^2\sum_{k=0}^{n}\left(\frac{10}{4}\right)^k$ ↳ Geometric series with $r = 2.5$

$= \frac{(10/4)^{n+1} - 1}{10/4 - 1} = \frac{2}{3}\left((10/4)^{n+1} - 1\right)$

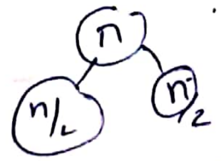
$$\frac{2}{3}\left((10/4)^{n+1} - 1\right)\cdot \log_2 n$$
$$= \frac{2}{3}(10/4)^{n+1}\log_2 n - \frac{2}{3}\log_2 n$$

Hence,

$$\boxed{O\left((10/4)^n \log_2 n\right)}$$

↳ final Answer..

(c). $T(n) = 10\,T(n/2) + \Theta(1)$



1

10 ... up till 10 childen

10^2

Total # of levels $= \log_2 n$

pattern = $1, 10, 10^2, 10^3$

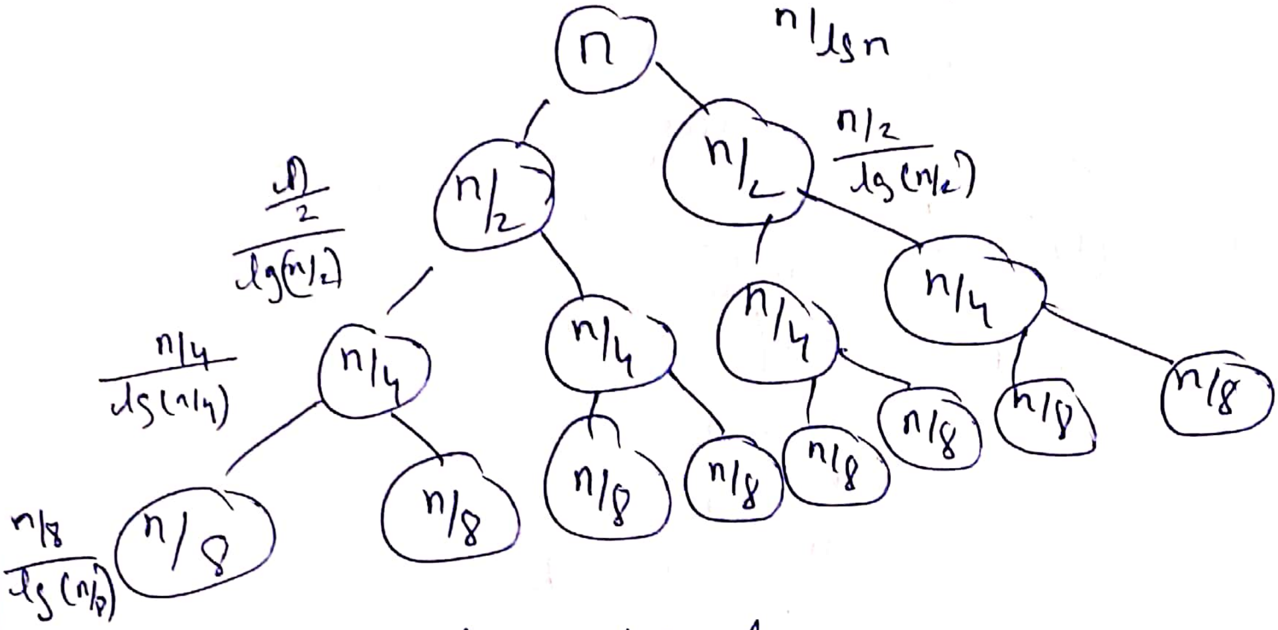
Geometric series $= \frac{10^{n+1} - 1}{9}$

$T = \left(10^{n+1} - 1\right)\frac{1}{9} \times \log_2(n) = \frac{(10)^{n+1}}{9}\log_2(n) - \frac{1}{9}\log_2(n)$

$= \frac{(10)}{9}(10)^n\log_2 n - \frac{1}{9}\log_2(n)$

$O\left(10^n \log_2(n)\right)$

(d). $T(n) = 2T(n/2) + n/\lg n$



Total # of levels

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log_2 n = k$$

Pattern

$$= \frac{n}{\lg n}, \frac{2(n/2)}{\lg(n/2)}, \frac{4(n/4)}{\lg(n/4)}, \frac{8(n/8)}{\lg(n/8)}$$

$$= \frac{n}{\lg n}, \frac{n}{\lg(n/2)}, \frac{n}{\lg(n/4)}, \frac{n}{\lg(n/8)} \ldots$$

$$n, n/2, n/4 = n\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right)$$

$$\hookrightarrow \frac{1}{2^k} = \frac{\frac{1}{2^{n+1}-1}}{2-1}$$

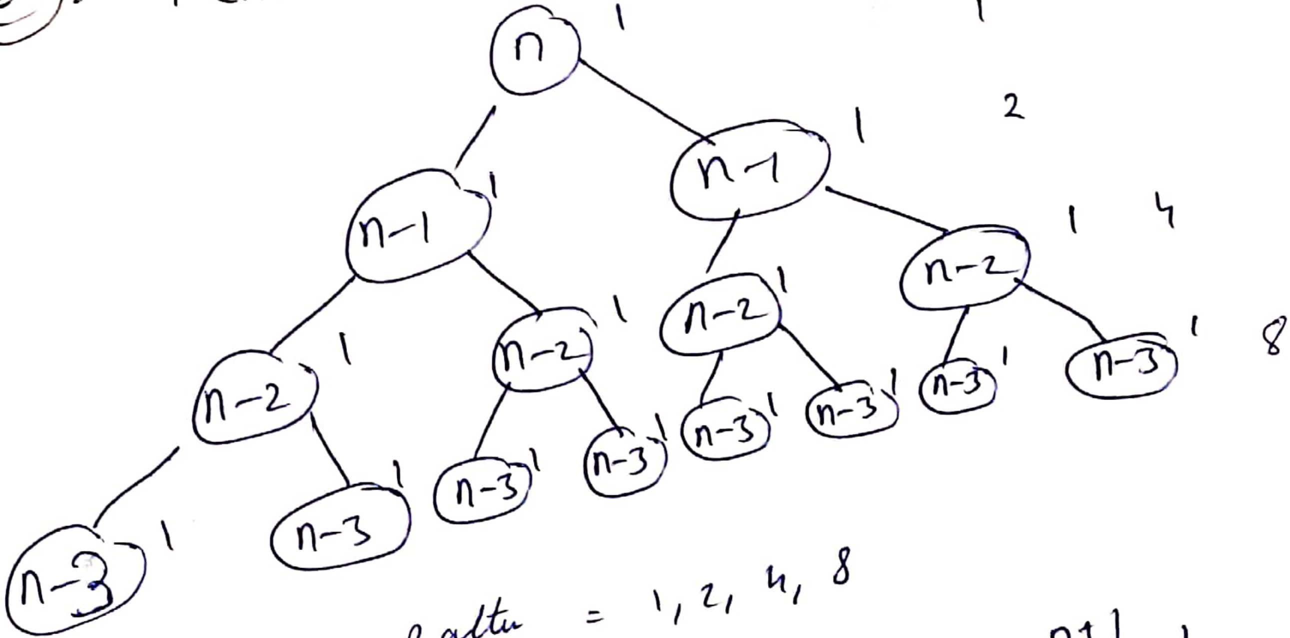
$$= \frac{2-1}{2^{n+1}-1} = \frac{1}{2^{n+1}-1}$$

$$\boxed{\frac{n}{\lg\left(n \cdot \frac{1}{2^{n+1}-1}\right)}}$$

$$\hookrightarrow = \frac{n}{\lg\left(\frac{n}{2^{n+1}-1}\right)} \log_2 n$$

$$O\left(\frac{n \log_2 n}{\log\left(\frac{n}{2^{n+1}-1}\right)}\right)$$ ↳ final Answer

(e). $T(n) = 2T(n-1) + \Theta(1)$



$$\text{pattern} = 1, 2, 4, 8$$

$$= \frac{2^{n+1} - 1}{1} = 2^{n+1} - 1$$

$$\boxed{\text{Total \# of levels} = \log_1 n}$$

$$= (2^{n+1} - 1) \cdot \log_1 n$$

$$= 2^{n+1} \log_1 n - \log_1 n$$

$$= (2)\, 2^n \log_1 n - \log_1 n$$

$$= O(2^n \log_1 n)$$

↳ final Answer

Scanned with CamScanner