ETERNITY: FUNCTIONS

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0.1 Introduction

0.1.1 Discription

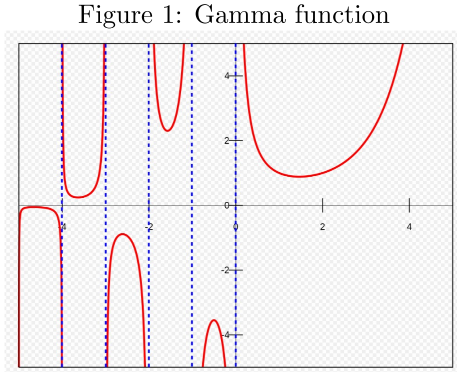
In mathematics, the gamma function is one commonly used extension of the factorial function to complex numbers. The gamma function is defined for all complex numbers except the non-positive integers. For any positive integer n,

$$\Gamma\left(n\right) = \left(n-1\right)!$$

Derived by Daniel Bernoulli, for complex numbers with a positive real part, the gamma function is defined via a convergent improper integral:

$$\Gamma\left(n\right) = \int_{0}^{\infty} x^{z-1}e^{-x}\mathrm{d}x, \Re\left(z\right) > 0$$

Figure 1: Gamma function



0.1.2 Stakeholders

- User1: Mathematicians and researchers in the fields of calculus, mathematical analysis, statistics.
- User2: Algorithm designers, researchers,engineers in the field of scientific computing in computer and software engineering.
- User3: People with low maths and programming background who need to obtain the values of gamma function.
- User4:

0.1.3 Domain

All complex numbers except those whose real part are non-positive integers.

$$C/\{n \in Z, n \leq 0\}$$

0.1.4 Co-Domain

All real numbers excluding zero.

$$(-\infty, 0) \cup (0, +\infty)$$

0.1.5 Properties

One of the important functional equatuations for the gamma fucntion is Euler's reflection formula:

$$\Gamma(1-z)\Gamma(z) = \frac{\pi}{\sin \pi z}, z \notin Z$$

which implies,

0.1.6 Perticular Values

Including up to the first 20 digits after the decimal point, some of particular values of the gamma function are:

- $\Gamma\left(-\frac{3}{2}\right) = \frac{4\sqrt{\pi}}{3} \approx +2.36327180120735470306$
- $\Gamma\left(-\frac{1}{2}\right) = -2\sqrt{\pi} \approx -3.54490770181103205459$
- $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} \approx +1.77245385090551602729$
- $\Gamma(1) = 0! = +1$
- $\Gamma\left(\frac{3}{2}\right) = \frac{\pi}{2} \approx +0.88622692545275801364$
- $\Gamma(2) = 1! = +1$
- $\Gamma\left(\frac{5}{2}\right) = \frac{3\sqrt{\pi}}{4} \approx +1.32934038817913702047$
- $\Gamma(3) = 2! \approx +2$
- $\Gamma\left(\frac{7}{2}\right) = \frac{15\sqrt{\pi}}{8} \approx +3.32335097044784255118$
- $\Gamma(4) = 3! = +6$

0.2 Requirements

0.2.1 Functional Requirements

- FR1: We shall only consider the positive value as input value.
- FR2: We shall only consider the real number as input value, ignore the imaginary part.
- FR3: When the function value exceeds the maximum the value of a double type variable, the function will return NaN.

0.2.2 Assumptions

0.3 Algorithm

0.3.1 Integrals Method

Description

Pseudocode

Advantages and Disadvantages

0.3.2 Lanczos Approximation

Description

Pseudocode

Advantages and Disadvantages

0.4 References

But I don't want my section to be numbered. function $= \Gamma \sqrt{1 - \frac{v^2}{c^2}}$ #LaTeX**Rules**

- Item 1.
- Item 2.
- ...
- Item n.

1. Item 1.
2. Item 2.
3. Item 3.