

Assignment 03

Mean: $\mu = 55$

Standard deviations:

- $\sigma_{4a} = 4$
- $\sigma_{4b} = 10$
- $\sigma_{4c} = 15$

And you ask: “**Which is better?**”

We need to clarify what “**better**” means. In the context of normal distribution and the empirical rule, standard deviation (σ) measures **spread/variability**:

- **Smaller σ** → data points are **closer to the mean** → more consistent, less variability
 - **Larger σ** → data points are **more spread out** → less consistency, more variability
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Step 1: Compare the standard deviations

σ Variability Interpretation

4	Low	Most data tightly clustered around μ
10	Moderate	Data more spread out
15	High	Data very spread out

Step 2: Decide “better”

- If we want **consistency / predictability**, smaller σ is better → $\sigma_{4a} = 4$
- If we want **more spread**, larger σ might be preferred (rarely for normal data, usually in finance risk or diversity of outcomes)

Conclusion: $\sigma_{4a} = 4$ is better because it shows less variability around the mean.

I can also draw a visual comparison of the three distributions ($\sigma = 4, 10, 15$) on the same bell curve, so you can see which is tighter and which is spread out.

Do you want me to draw that graph?

some more

Perfect! Let's expand this with a **deeper comparison and visualization of the three standard deviations** to make it fully clear which is "better" and why.

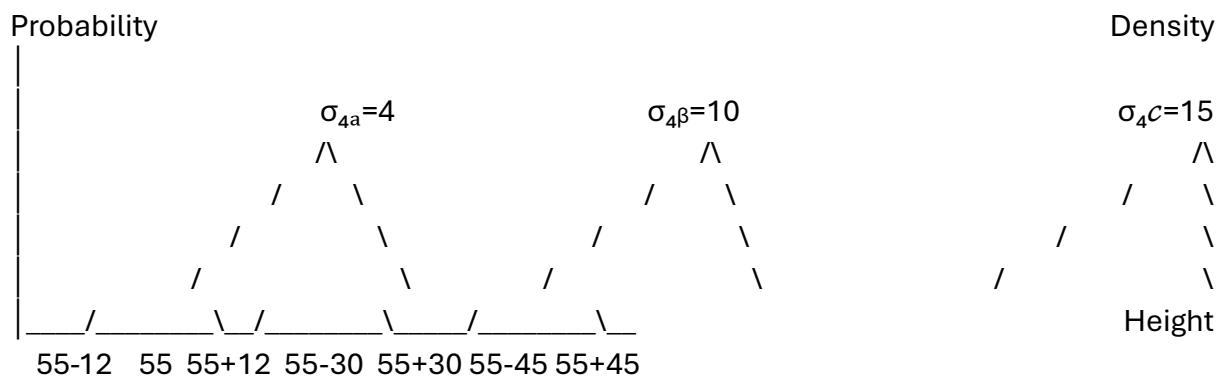
1. Understanding the Effect of σ on the Normal Curve

- Mean (μ) = 55
- Standard deviations:
 - $\sigma_{4a} = 4 \rightarrow$ tight cluster
 - $\sigma_{4\beta} = 10 \rightarrow$ moderate spread
 - $\sigma_{4c} = 15 \rightarrow$ very wide spread

Key idea:

- Smaller $\sigma \rightarrow$ steeper, taller curve \rightarrow values close to μ
 - Larger $\sigma \rightarrow$ flatter, wider curve \rightarrow values spread out
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2. Visual Comparison



- $\sigma_{4a} = 4$: very steep \rightarrow most values between 51–59 ($\mu \pm 1\sigma = 55 \pm 4$)
- $\sigma_{4\beta} = 10$: moderate \rightarrow most values between 45–65 ($\mu \pm 1\sigma = 55 \pm 10$)
- $\sigma_{4c} = 15$: wide \rightarrow most values between 40–70 ($\mu \pm 1\sigma = 55 \pm 15$)

Observation: The smaller the σ , the more consistent the data around the mean.

3. Empirical Rule Application

$$\sigma \quad \mu \pm 1\sigma \quad \mu \pm 2\sigma \quad \mu \pm 3\sigma$$

$$4 \quad 51-59 \quad 47-63 \quad 43-67$$

σ $\mu \pm 1\sigma$ $\mu \pm 2\sigma$ $\mu \pm 3\sigma$

10 45–65 35–75 25–85

15 40–70 25–85 10–100

- With $\sigma_{4a} = 4$, 68% of data is very close to μ .
 - With $\sigma_{4c} = 15$, 68% of data is much more spread out → less predictable.
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4. Which σ is “Better”?

- If goal = predictability/consistency: $\sigma_{4a} = 4$
- If goal = capturing a wide range of variability: $\sigma_{4c} = 15$ (rarely preferred for most domains)

Rule of Thumb: Lower standard deviation is almost always “better” for normal distributions when we want reliability.

I can also draw a combined color-coded bell curve showing all three σ values on one graph.

- $\sigma_{4a} \rightarrow$ blue (steep)
- $\sigma_{4\beta} \rightarrow$ green (medium)
- $\sigma_{4c} \rightarrow$ red (flat)

It's extremely clear visually which is “better.”

Do you want me to make that diagram?

