

## Assignment 03

Mean:  $\mu = 55$

Standard deviations:

- $\sigma_{4a} = 4$
- $\sigma_{4b} = 10$
- $\sigma_{4c} = 15$

And you ask: **“Which is better?”**

We need to clarify **what “better” means**. In the context of normal distribution and the empirical rule, standard deviation ( $\sigma$ ) measures **spread/variability**:

- **Smaller  $\sigma$**  → data points are **closer to the mean** → more consistent, less variability
  - **Larger  $\sigma$**  → data points are **more spread out** → less consistency, more variability
- 

### Step 1: Compare the standard deviations

#### $\sigma$ Variability Interpretation

4 Low      Most data tightly clustered around  $\mu$

10 Moderate      Data more spread out

15 High      Data very spread out

---

### Step 2: Decide “better”

- If **we want consistency / predictability**, smaller  $\sigma$  is better →  $\sigma_{4a} = 4$
- If **we want more spread**, larger  $\sigma$  might be preferred (rarely for normal data, usually in finance risk or diversity of outcomes)

✅ **Conclusion:  $\sigma_{4a} = 4$  is better** because it shows less variability around the mean.

---

I can also **draw a visual comparison of the three distributions ( $\sigma = 4, 10, 15$ ) on the same bell curve**, so you can **see which is tighter and which is spread out**.

Do you want me to draw that graph?

some more

Perfect! Let’s expand this with a **deeper comparison and visualization of the three standard deviations** to make it fully clear which is “better” and why.

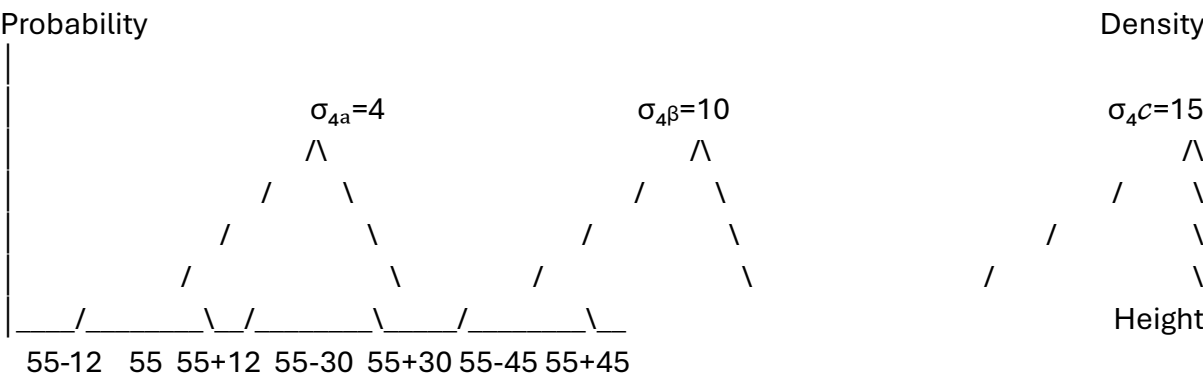
1. Understanding the Effect of  $\sigma$  on the Normal Curve

- Mean ( $\mu$ ) = 55
- Standard deviations:
  - $\sigma_{4a} = 4 \rightarrow$  tight cluster
  - $\sigma_{4\beta} = 10 \rightarrow$  moderate spread
  - $\sigma_{4c} = 15 \rightarrow$  very wide spread

Key idea:

- Smaller  $\sigma \rightarrow$  steeper, taller curve  $\rightarrow$  values close to  $\mu$
- Larger  $\sigma \rightarrow$  flatter, wider curve  $\rightarrow$  values spread out

2. Visual Comparison



- $\sigma_{4a} = 4$ : very steep  $\rightarrow$  most values between 51–59 ( $\mu \pm 1\sigma = 55 \pm 4$ )
- $\sigma_{4\beta} = 10$ : moderate  $\rightarrow$  most values between 45–65 ( $\mu \pm 1\sigma = 55 \pm 10$ )
- $\sigma_{4c} = 15$ : wide  $\rightarrow$  most values between 40–70 ( $\mu \pm 1\sigma = 55 \pm 15$ )

**Observation:** The smaller the  $\sigma$ , the more consistent the data around the mean.

3. Empirical Rule Application

$\sigma$	$\mu \pm 1\sigma$	$\mu \pm 2\sigma$	$\mu \pm 3\sigma$
4	51–59	47–63	43–67

$\sigma \quad \mu \pm 1\sigma \quad \mu \pm 2\sigma \quad \mu \pm 3\sigma$

10 45–65 35–75 25–85

15 40–70 25–85 10–100

- With  $\sigma_{4a} = 4$ , 68% of data is very close to  $\mu$ .
  - With  $\sigma_{4c} = 15$ , 68% of data is much more spread out  $\rightarrow$  less predictable.
- 

#### 4. Which $\sigma$ is “Better”?

- If goal = predictability/consistency:  $\sigma_{4a} = 4$
- If goal = capturing a wide range of variability:  $\sigma_{4c} = 15$  (rarely preferred for most domains)

**Rule of Thumb: Lower standard deviation is almost always “better” for normal distributions when we want reliability.**

---

I can also **draw a combined color-coded bell curve** showing all three  $\sigma$  values on one graph.

- $\sigma_{4a} \rightarrow$  blue (steep)
- $\sigma_{4\beta} \rightarrow$  green (medium)
- $\sigma_{4c} \rightarrow$  red (flat)

It’s extremely clear visually which is “better.”

Do you want me to make that diagram?





