Lecture 4

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Outline

1. Phase transitions in network traffic.
2. Phase transitions in graphs.

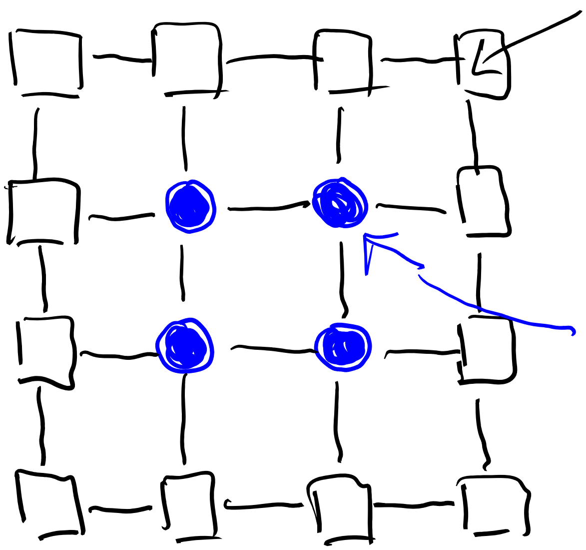
Note: The paper today addresses the first topic.

1. Phase transitions

- First, we consider phase transitions in network traffic (from a low to a highly congested traffic)

$$\frac{\lambda^{-k} e^{-\lambda}}{k!}$$

- Model



processor w/ unlimited queues; generates Poisson traffic to random dest.

router (only)

node w/ least # packets

use a fitness function

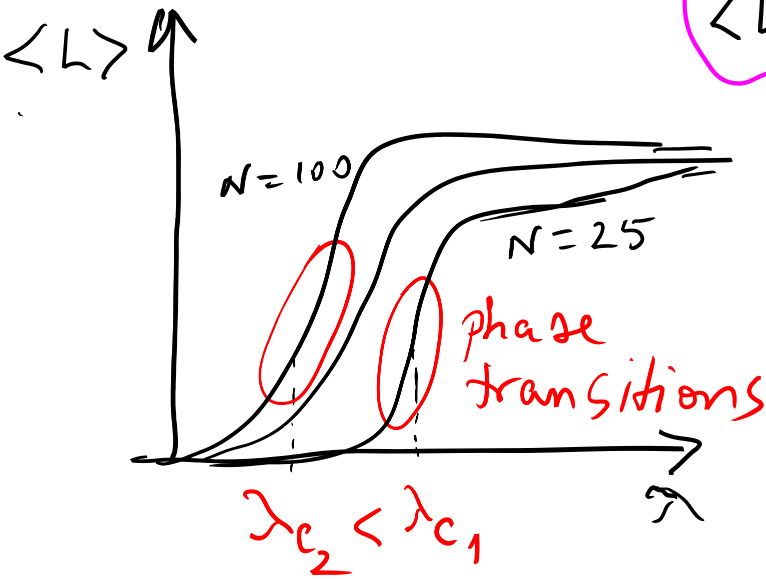
- Routing: Intermediate node selected based on shortest path following either a determ or prob. routing

$$P(A) = \frac{e^{-\beta x_A}}{e^{-\beta x_A} + e^{-\beta x_B}} \quad ; \quad P(B) = \frac{e^{-\beta x_B}}{e^{-\beta x_A} + e^{-\beta x_B}}$$

$$P(A) + P(B) = 1$$

- Results

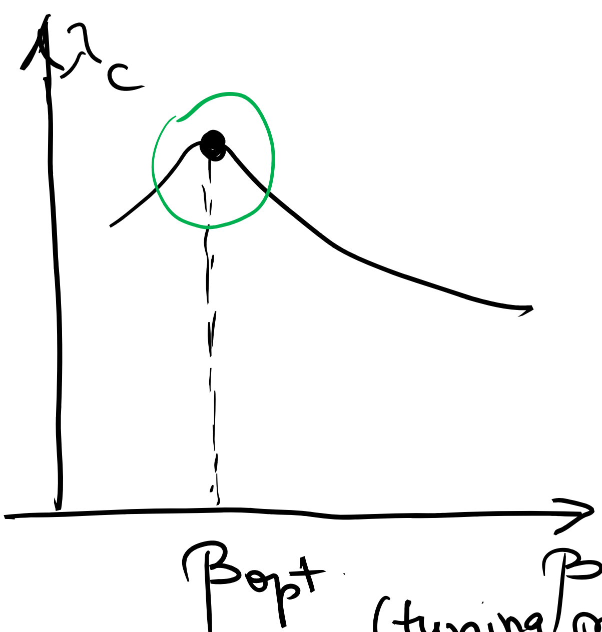
average lifetime of a packet (avg. time between sending and receiving a packet)

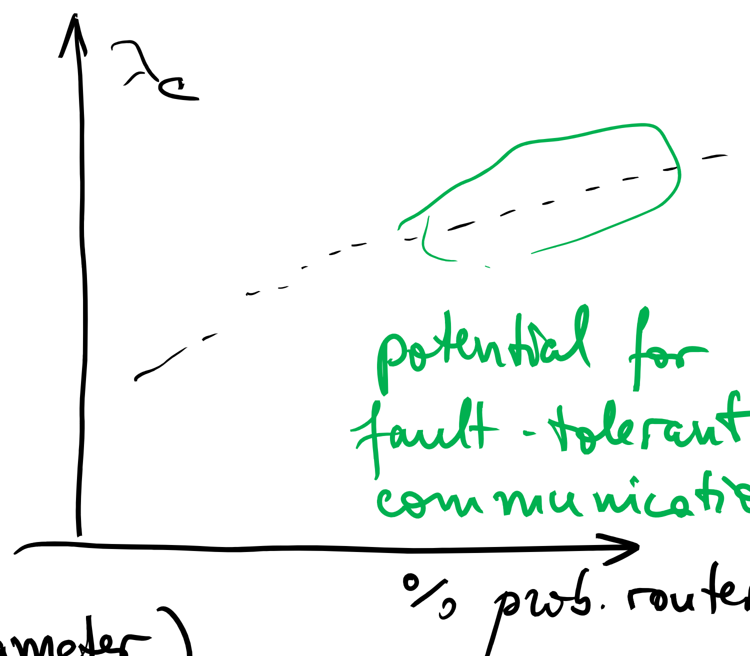


determ. routing

$N=25$; prob. vs. determ. routing

- An emergent (collective) behavior of routers decides the congestion in the network





potential for fault-tolerant communication

2. Random graphs

- Basic idea: Consider n disconnected vertices (nodes). Each pair of nodes gets connected with a certain probability $p > 0$. This produces a statistical ensemble of all possible graphs of n nodes.

- Typically, large n are considered while keeping the mean degree z constant:

each node gets potentially $(n-1)$ links

$$z = p(n-1) \quad \text{(mean degree)}$$

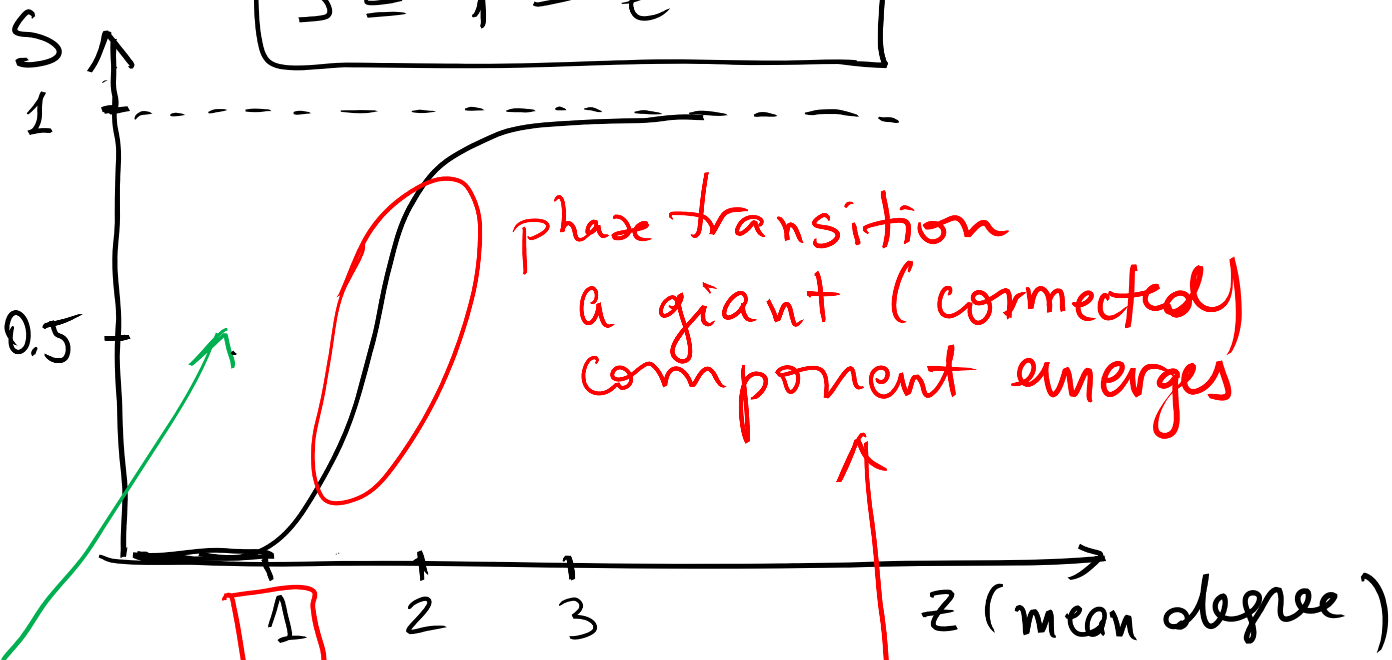
$$P_k = \binom{n}{k} p^k (1-p)^{n-k} \approx \frac{z^k e^{-z}}{k!} \quad \text{(random graphs)}$$

$\uparrow$ prob. of having a node of k degree

Note: This decreases very fast with k: prob. of highly connected nodes is very small!

- S is the size (fraction of connected nodes) of a giant component

$$\boxed{S = 1 - e^{-zS}}$$



many small components with an exponential size distrib.

$z=1$ is a critical value

$$\boxed{p > \frac{1}{n-1}}$$

- S plays the role of an order parameter in this phase transition:

$$\boxed{S \sim |z-1|^{\beta}}$$

Power law size distribution