We know that,

$$P_K(x) = E(Y=k \mid X=x)$$

$$= \frac{\pi_k f_k(x)}{\sum_{l=1}^{k} \pi_l f_l(x)}$$

Here our distribution is normal

$$f_k(x) = \frac{1}{\sqrt{2\pi}\,\sigma_k} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right)$$

we get,

$$P_K(x) = \frac{\pi_k \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-1}{2\sigma^2}(x-\mu_k)^2\right)}{\sum_{l=1}^{K} \pi_l \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-1}{2\sigma^2}(x-\mu_l)^2\right)}$$

$$P_k(x) = \frac{\pi_k \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}(x-\mu_k)^2\right)}{\sum_{l=1}^{k} \pi_l \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}(x-\mu_l)^2\right)}$$

Because σ is dependent on k, here σ would be σ_k.

$$P_k(x) = \frac{\pi_k \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right)}{\sum_{l=1}^{k} \pi_l \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_l)^2\right)}$$

Let
$$a = \frac{\frac{1}{\sqrt{2\pi}}}{\sum_{l=1}^{k} \pi_l \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_l)^2\right)}$$

because a is independent of k i.e. it does not vary with k

Taking log on both sides

$$\log(P_k(x)) = \log(a) + \log(\pi_k) - \log(\sigma_k) + \left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right)$$

$$\log(P_k(k)) = \left(-\frac{1}{2\sigma_k^2}(x^2+\mu_k^2-2x\mu_k\right) + \log(\pi_k) - \log(\sigma_k) + \log(a)$$