

**CSE585/EE555: Digital Image Processing II**  
**Spring 2020**

**Project #2 — Homotopic Skeletonization and Shape Analysis**

**assigned:** 4 February 2020

**due:** Friday 21 February 2020 in CANVAS project drop box

**reading assignment:** Sections 6.7-6.12 of P&V (PitasCh6.pdf),  
 Maragos and Schafer paper (Maragos-Schafer.pdf),  
 Gonzalez and Woods Ch.9 excerpt on CANVAS.

1. *Homotopic Skeletonization* — Consider  $X \odot B_i$ , the **thinning of  $X$  by  $B_i$** , defined by P&V eq. (6.10.1) as

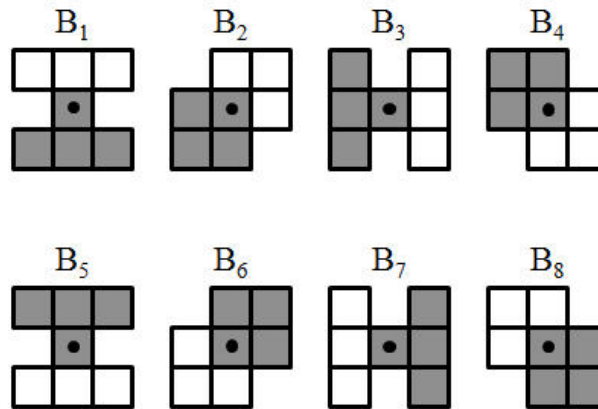
$$X \odot B_i = X - X * B_i, \quad (1)$$

where  $B_i = \{B_i^f, B_i^b\}$  is a set of two structuring elements and  $X * B_i$  is the hit-or-miss transform given by

$$X * B_i = (X \ominus B_i^f) \cap (X^c \ominus B_i^b) \quad (2)$$

$B_i^f$  probes for foreground, while  $B_i^b$  probes the background (complement). Note that (2), as defined by Gonzalez and Woods in Ch. 9 of their book (see CANVAS excerpt), is exactly the same as the hit-or-miss transform we considered earlier.

You will use the following set of  $B_i$ :



For each  $B_i$ ,

- The “•” pixel corresponds to the origin
- The 4 shaded pixels correspond to the “1” values for  $B_i^f$
- The 3 white pixels correspond to the “1” values for  $B_i^b$
- The 2 remaining pixels in the  $3 \times 3$  neighborhood are not considered!

You are to implement an algorithm for homotopic skeletonization, given by the following:

$i = 0$  ;

$X_i = X$  ;

**do**

$i = i + 1$  ;

$X_i = X_{i-1}$  ;

**for**  $j = 1, 2, \dots, 8$

$X_i = X_i \circ B_j$  ;

**while** {  $X_i \neq X_{i-1}$  }

Note that each structuring element pair  $B_j$  strips/thins off a layer of pixels in a particular direction; i.e.,  $B_1$  strips off top (north) pixels,  $B_2$  strips off top-right (northeast) pixels,  $B_3$  strips off right (east) pixels, etc. Thus, each thinning iteration through the complete set of 8  $B_j$  isotropically "burns off" a layer around  $X$ . Thus, the algorithm effectively implements the "grass fire" transform, while preserving the homotopy of the original image.

Also, note that  $B_1$  is the reflection of  $B_5$ ,  $B_2$  is the reflection of  $B_6$ , etc. Hence, the need for the symmetric/reflection " $(.)^s$ " operation is unnecessary for the structuring elements in the hit-or-miss transform.

Apply your algorithm to the "penn256" and "bear" images. Give the results of the algorithm for  $X_2$ ,  $X_5$ ,  $X_{10}$ , and the final skeletonized image. Show each result as a superposition of  $X_i$  on the original image.

2. *Shape Analysis* — For both parts below, let  $B$  be the  $3 \times 3$  square. Also, for each considered image, you will need to isolate distinct objects and find the minimum bounding rectangle (MBR) [also referred to as the bounding box] enclosing each distinct object. Feel free to use the MatLab connected-component labeling function to help with this.
  - (a) Consider the "match1" image, which contains 4 objects (clover, spade, steer, and airplane).
    - i. Compute the size distribution  $U(n)$ , spectrum  $f(n)$ , and complexity  $H(X|B)$  of each object. Give well-labeled plots and/or tables, as appropriate, for your results. Which object is the most complex? You must give quantitative results to back up your answer.
    - ii. Now, consider image "match3," which contains rotated versions of the objects in "match1." Use pecstral analysis, as discussed in P&V Section 6.11 and the Lecture 6 notes, to determine which object in "match3" best matches each of the objects in "match1." Be sure to give all necessary pecstral and distance calculations in your report, in addition to the specific algorithm you use.
  - (b) Consider the image "shadow1," which has four solid objects; the objects are characters from the "Peanuts" comic strip. Quantitatively (and automatically) match them to the proper objects depicted in the complementary image "shadow1rotated." Give sufficient results to make it perfectly clear how you arrived at your results.
3. Write a detailed report describing your results and implementations. Also, give a well-commented listing of your MATLAB code, abiding by the code specifications of the class project protocol.