# Conditional Probability Distribution (CPD) in Bayesian Networks

A Bayesian Network is a graphical model that represents probabilistic relationships among variables using Conditional Probability Distributions (CPDs). Each node in a Bayesian Network is associated with a CPD that describes how the node's probability distribution depends on its parent nodes.

## 1. Conditional Probability Table (CPT)

If a node is discrete, its CPD is often represented as a Conditional Probability Table (CPT). The CPT lists the probability of the node taking specific values given all possible combinations of its parent values.

### Example: Probability of Rain Given Cloudy Conditions

Let Rain (R) depend on Cloudy (C):

P(R | C)

|  |  |  |
| --- | --- | --- |
| Cloudy (C) | P(Rain = Yes) | P(Rain = No) |
| True | 0.8 | 0.2 |
| False | 0.2 | 0.8 |

## 2. Factorized Representation of CPD

For a node X with parents P1, P2, ..., Pn, the CPD is expressed as:

P(X | P1, P2, ..., Pn)

This representation ensures that the joint probability distribution of all variables in the network can be written as the product of their CPDs:

P(X1, X2, ..., Xn) = ∏ P(Xi | Parents(Xi))

## 3. Continuous CPD

If variables are continuous, CPDs can be modeled using Gaussian distributions or linear Gaussian models. The conditional probability in such cases can be expressed as:

P(X | P) = N(μ\_X + ∑ w\_i P\_i, σ\_X^2)

## 4. No Parent Case (Marginal Probability)

If a node has no parents, its CPD is simply its prior probability:

P(X)

## 5. Example: Bayesian Network with CPD

Consider a Bayesian Network with three nodes:  
- Cloudy (C) → Rain (R)  
- Rain (R) → Traffic (T)

The CPDs for this network are:

1. P(C) – Probability of cloudy weather.

2. P(R | C) – Probability of rain given whether it is cloudy.

3. P(T | R) – Probability of traffic given whether it is raining.

Using the chain rule, the joint probability of all variables in the network is:

P(C, R, T) = P(C) P(R | C) P(T | R)

## Conclusion

CPDs in Bayesian Networks play a crucial role in defining probabilistic dependencies between variables. Whether using a Conditional Probability Table (CPT) for discrete variables or a continuous Gaussian model, CPDs help in efficient inference and decision-making in probabilistic models.