Aishwarye Omer

EE 456 **Homework - 06** September 25, 2020

Question: Design and train an ADALINE network to implement a 2-input OR gate using error based method

Solution : The network will work on Bipolar Input, the function and the parameters are as follows:

- $f(0) = 1$ and $\alpha = 0.4$
- $Y_{in} = \sum_{1}^{n} W_i \times X_i + b$
- $W = \begin{bmatrix} 1 \\ -0.5 \\ 0.25 \end{bmatrix}$ initially, where $w_1 = 1, w_2 = -0.5$ and $b = 0.25$
- The truth table for the 2-input OR gate is as follows:

X_i	1	2	$(\vee)T_i$
1	-1	-1	-1
2	-1	1	1
3	1	-1	1
4	1	1	1

- Each X_i and T_i is a column vector.
- Error is calculated as: $E = 0.5 * \sum_{i=1}^{n}(T_i - Y_i)^2$. Error should decrease after every cycle of input is applied.
- $W_{new} = W_{old} + \alpha \times X_i \times (T_i - Y_{in})$
- $b_{new} = b_{old} + \alpha \times (T_i - Y_{in})$

Calculations:

- First calculate error E.

  1. $Y_{1_{in}}$ is as follows:

     $y_{1_{in}} = f(\ w_1 * X_{11} + w_2 * X_{12} + b\ )$

     $y_{1_{in}} = f(\ 1 * -1 + -0.5 * -1 + 0.25) = f(-0.25) = -1 = T_1 \rightarrow$ **ok**

2. $Y_{2_{in}}$ is as follows:

   $y_{2_{in}} = \ f(\ w_1 * X_{21} \ + \ w_2 * X_{22} \ + \ b\ )$

   $y_{2_{in}} = \ f(\ 1 * -1 \ + \ -0.5 * 1 \ + \ 0.25) \ = \ f(-1.25) \ = \ -1 \neq T_2 \rightarrow$ not ok

3. $Y_{3_{in}}$ is as follows:

   $y_{3_{in}} = \ f(\ w_1 * X_{31} \ + \ w_2 * X_{32} \ + \ b\ )$

   $y_{3_{in}} = \ f(\ 1 * 1 \ + \ -0.5 * -1 \ + \ 0.25) \ = \ f(1.75) \ = \ 1 = T_3 \rightarrow$ **ok**

4. $Y_{4_{in}}$ is as follows:

   $y_{4_{in}} = \ f(\ w_1 * X_{41} \ + \ w_2 * X_{42} \ + \ b\ )$

   $y_{4_{in}} = \ f(\ 1 * 1 \ + \ -0.5 * 1 \ + \ 0.25) \ = \ f(0.75) \ = \ 1 = T_4 \rightarrow$ **ok**

- Therefore error E is :

  $$E = 0.5 \ * \ \sum_{i=1}^{n} (T_i - Y_i)^2.$$

  $$E = 0.5 \ * \ [\ (T_1 - Y_1)^2 \ + \ (T_2 - Y_2)^2 \ + \ (T_3 - Y_3)^2 \ + \ (T_4 - Y_4)^2\ ]$$

  $$E = 0.5 \ * \ [\ (-1 - (-1))^2 \ + \ (1 - (-1))^2 \ + \ (1 - 1)^2 \ + \ (1 - 1)^2)\ ]$$

  $$E = 0.5 \ * \ 4 \ = \ 2$$

- We start from X_2 since Y_1 is ok according to current weight and bias. Since function on $Y_{2_{in}}$ is not equal to T_2, we need to train the network

- Let's train:

  1. $w_{1\ new} = w_{1\ old} \ + \ \alpha \times X_{21} \times (T_2 - Y_{2_{in}}) = 1 \ + \ 0.4 \times -1 \times 2.25 = \ 0.1$
  2. $w_{2\ new} = w_{2\ old} \ + \ \alpha \times X_{22} \times (T_2 - Y_{2_{in}}) = -0.5 \ + \ 0.4 \times -1 \times 2.25 = \ 0.4$
  3. $b_{new} = b_{old} \ + \ \alpha \times (T_2 - Y_{2_{in}}) = \ 0.25 \ + \ 0.5 \times 2.25 \ = \ 1.15$
  4. Therefore, $W = \begin{bmatrix} 0.1 \\ 0.4 \\ 1.15 \end{bmatrix}$

- Apply X_3, $\ y_{3_{in}} = f(\ 0.1 - 0.4 + 1.15\ ) \ = \ f(0.85) \ = \ 1 \rightarrow$ **ok**

- Apply X_4, $\ y_{4_{in}} = f(\ 0.1 + 0.4 + 1.15\ ) \ = \ f(1.65) \ = \ 1 \rightarrow$ **ok**

- Again start from X_1. Apply X_1, $\ y_{1_{in}} = f(\ -0.1 - 0.4 + 1.15\ ) \ = \ f(0.65) \ = \ 1 \rightarrow$ not ok.

- Let's train:

  1. $w_{1\ new} = w_{1\ old}\ +\ \alpha \times X_{11} \times (T_1 - Y_{1_{in}}) = 0.1\ +\ 0.4 \times -1 \times -1.65 =\ 0.76$
  2. $w_{2\ new} = w_{2\ old}\ +\ \alpha \times X_{12} \times (T_1 - Y_{1_{in}}) = 0.4\ +\ 0.4 \times -1 \times -1.65 =\ 1.06$
  3. $b_{new} = b_{old}\ +\ \alpha \times (T_1 - Y_{1_{in}}) =\ 1.15\ +\ 0.4 \times -1.65\ =\ 0.49$
  4. Therefore , $W = \begin{bmatrix} 0.76 \\ 1.06 \\ 0.49 \end{bmatrix}$

- Apply X_2, $y_{2_{in}} = f(\ -0.76 + 1.06 + 0.49\ )\ =\ f(0.79)\ =\ 1 \rightarrow$ **ok**
- Apply X_3, $y_{3_{in}} = f(\ 0.76 - 1.06 + 0.49\ )\ =\ f(0.19)\ =\ 1 \rightarrow$ **ok**
- Apply X_4, $y_{4_{in}} = f(\ 0.76 + 1.06 + 0.49\ )\ =\ f(2.31)\ =\ 1 \rightarrow$ **ok**
- Finally apply X_1, $y_{1_{in}} = f(\ -0.76 - 1.06 + 0.49\ )\ =\ f(-1.33)\ =\ -1 \rightarrow$ **ok**

Therefore the network is set for 2-input OR gate and we do not need to train it further.