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EE 456 **Homework - 04** September 21, 2020

Question: Calculate the energy value of the Bi-directional Hetero-Associator as given in the lecture

Solution : The energy function is given as $E = -0.5 \times \sum_{i,j} X_i \times W_{ij} \times Y_j - \sum_i X_i \times Y_i + \sum_i \theta_i \times Y_i$.

The third term in the above formula for BAM network is 0 i.e. $\sum_i \theta_i \times Y_i = 0$

For a BAM network the appropriate energy function is the average of the signal energy for a forward and backward pass which is $E = -0.5 \times \sum_{i,j} X_i \times W_{ij} \times Y_j - \sum_i X_i \times Y_i$

a . Following are the inputs and synaptic weight matrix:

$$Y_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, Y_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, W = \begin{bmatrix} 0 & -2 \\ 0 & 2 \\ 2 & 0 \\ 0 & 2 \\ 0 & -2 \\ -2 & 0 \\ 0 & 2 \\ -2 & 0 \\ -2 & 0 \\ 0 & 2 \\ 0 & -2 \\ -2 & 0 \\ -2 & 0 \\ 2 & 0 \\ 0 & 2 \end{bmatrix}, X_1 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \text{ and } X_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

b . The first term is $-0.5 \times \sum_{i,j}^{j \neq i} X_i \times W_{ij} \times Y_j$. According to the above values

- $-0.5 \times [\quad (X_1 \times W \times Y_1) + (X_2 \times W \times Y_2) + (X_1 \times W \times Y_2) + (X_2 \times W \times Y_1) \quad]$
- $= -0.5 \times (16 + 14 + 16 + 14)$
- $= -30$

c . The second term is $(\sum_i X_i \times Y_i)$

- $[\quad (X_1 \times Y_1) + (X_2 \times Y_2) + (X_1 \times Y_2) + (X_2 \times Y_1) \quad]$ (multiplication element wise)
- $= (-5 + 5 - 1 - 1)$
- $= -2$

d . Therefore energy function for the above BAM network evaluates to

- $E = -30 - (-2) + 0$
- $E = -30 + 2$
- $E = -28$