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EE 456 # Homework - 05 September 21, 2020

Question: Design and train an ADALINE network to implement a 3-input OR gate

Solution : The network will work on Bipolar Input, the function and the parameters are as follows:

- $f(0) = 1$ and $\alpha = 0.5$
- $y = \sum_{1}^{n} W_i \times X_i + b$
- $W_= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ initially, where $w_1 = 0, w_2 = 0, w_3 = 0$ and $b = 0$
- The truth table for the 3-input OR gate is as follows:

X_i	1	2	3	$(\vee)T_i$
1	-1	-1	-1	-1
2	-1	-1	1	1
3	-1	1	-1	1
4	-1	1	1	1
5	1	-1	-1	1
6	1	-1	1	1
7	1	1	-1	1
8	1	1	1	1

- Each X_i and T_i is a column vector.

Calculations:

- Apply X_1, $y = f(\ 0 \times -1\ +\ 0 \times -1\ +\ 0 \times -1\ +\ 0\ ) = f(0) = 1 \rightarrow$ not ok
- Let's train:
  1. $w_{1\ new} = w_{1\ old}\ +\ \alpha \times X_{11} \times T_1 = 0\ +\ 0.5 \times -1 \times -1 =\ 0.5$
  2. $w_{2\ new} = w_{2\ old}\ +\ \alpha \times X_{12} \times T_1 = 0\ +\ 0.5 \times -1 \times -1 =\ 0.5$
  3. $w_{3\ new} = w_{3\ old}\ +\ \alpha \times X_{13} \times T_1 = 0\ +\ 0.5 \times -1 \times -1 =\ 0.5$

4. $b_{new} = b_{old} \ + \ \alpha \times T_1 = \ 0 \ + \ 0.5 \times -1 \ = \ -0.5$
5. Therefore , $W = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ -0.5 \end{bmatrix}$

- Apply X_2, $y = f(\ -0.5 - 0.5 + 0.5 - 0.5\ ) \ = \ f(-1) \ = \ -1 \rightarrow$ not ok
- Let's train:

1. $w_{1\ new} = w_{1\ old} \ + \ \alpha \times X_{21} \times T_2 = 0.5 \ + \ 0.5 \times -1 \times 1 = \ 0$
2. $w_{2\ new} = w_{2\ old} \ + \ \alpha \times X_{22} \times T_2 = 0.5 \ + \ 0.5 \times -1 \times 1 = \ 0$
3. $w_{3\ new} = w_{3\ old} \ + \ \alpha \times X_{23} \times T_2 = 0.5 \ + \ 0.5 \times 1 \times 1 = \ 1$
4. $b_{new} = b_{old} \ + \ \alpha \times T_2 = \ -0.5 \ + \ 0.5 \times 1 \ = \ 0$
5. Therefore , $W = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

- Apply X_3, $y = f(\ 0 + 0 - 1 + 0\ ) \ = \ f(-1) \ = \ -1 \rightarrow$ not ok
- Let's train:

1. $w_{1\ new} = w_{1\ old} \ + \ \alpha \times X_{31} \times T_3 = \ -0.5$
2. $w_{2\ new} = w_{2\ old} \ + \ \alpha \times X_{32} \times T_3 = \ 0.5$
3. $w_{3\ new} = w_{3\ old} \ + \ \alpha \times X_{33} \times T_3 = \ 0.5$
4. $b_{new} = b_{old} \ + \ \alpha \times T_3 = \ 0.5$
5. Therefore , $W = \begin{bmatrix} -0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$

- Apply X_4, $y = f(\ 0.5 + 0.5 + 0.5 + 0.5\ ) \ = \ f(2) \ = \ 1 \rightarrow$ **ok**
- Apply X_5, $y = f(\ -0.5 - 0.5 - 0.5 + 0.5\ ) \ = \ f(-1) \ = \ -1 \rightarrow$ not ok
- Let's train:

1. $w_{1\ new} = w_{1\ old} \ + \ \alpha \times X_{51} \times T_5 = \ 0$
2. $w_{2\ new} = w_{2\ old} \ + \ \alpha \times X_{52} \times T_5 = \ 0$
3. $w_{3\ new} = w_{3\ old} \ + \ \alpha \times X_{53} \times T_5 = \ 0$
4. $b_{new} = b_{old} \ + \ \alpha \times T_5 = \ 1$
5. Therefore , $W = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

- Apply X_6, $y = f(\ 0+0+0+1\ )\ =\ f(1)\ =\ 1 \rightarrow$ **ok**
- Apply X_7, $y = f(\ 0+0+0+1\ )\ =\ f(1)\ =\ 1 \rightarrow$ **ok**
- Apply X_8, $y = f(\ 0+0+0+1\ )\ =\ f(1)\ =\ 1 \rightarrow$ **ok**
- Now, start again from X_1
- Apply X_1, $y = f(\ 0+0+0+1\ )\ =\ f(1)\ =\ 1 \rightarrow$ not ok
- Let's train:
  1. $w_{1\ new} = w_{1\ old}\ +\ \alpha \times X_{11} \times T_1 =\ 0.5$
  2. $w_{2\ new} = w_{2\ old}\ +\ \alpha \times X_{12} \times T_1 =\ 0.5$
  3. $w_{3\ new} = w_{3\ old}\ +\ \alpha \times X_{13} \times T_1 =\ 0.5$
  4. $b_{new} = b_{old}\ +\ \alpha \times T_1 =\ 0.5$
  5. Therefore , $W = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$
- Apply X_2, $y = f(\ -0.5-0.5+0.5+0.5\ )\ =\ f(0)\ =\ 1 \rightarrow$ **ok**
- Apply X_3, $y = f(\ -0.5+0.5-0.5+0.5\ )\ =\ f(0)\ =\ 1 \rightarrow$ **ok**
- Apply X_4, $y = f(\ -0.5+0.5+0.5+0.5\ )\ =\ f(1)\ =\ 1 \rightarrow$ **ok**
- Apply X_5, $y = f(\ 0.5-0.5-0.5+0.5\ )\ =\ f(0)\ =\ 1 \rightarrow$ **ok**
- Apply X_6, $y = f(\ 0.5-0.5+0.5+0.5\ )\ =\ f(1)\ =\ 1 \rightarrow$ **ok**
- Apply X_7, $y = f(\ 0.5+0.5-0.5+0.5\ )\ =\ f(1)\ =\ 1 \rightarrow$ **ok**
- Apply X_8, $y = f(\ 0.5+0.5+0.5+0.5\ )\ =\ f(2)\ =\ 1 \rightarrow$ **ok**
- Finally Apply X_1, $y = f(\ -0.5-0.5-0.5+0.5\ )\ =\ f(-1)\ =\ -1 \rightarrow$ **ok**

Therefore the network is set for 3-input OR gate.