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EE 456 # Homework - 11 September 25, 2020

Question: Design a two-input X-NOR gate around RBF centers at [1 0] and [0 1], as was done in class. Be sure to include the output cell and the weights and bias inputs.

Solution : The network will work on Unipolar Input, the function and the parameters are as follows:

- $Y_{in} = \sum_{1}^{n} W_i \times \phi_i + b$ for the final output after hidden layer
- $\phi_i \ = \ e^{-\|x_i - t_i\|^2}$
- $t_1 \ = \ [0\ 1]$ and $t_2 \ = \ [1\ 0]$
- The truth table for the 2-input X-NOR gate is as follows:

X_i	1	2	A_i
1	0	0	1
2	0	1	0
3	1	0	0
4	1	1	1

- Each X_i and A_i is a column vector where A is target.

Calculations:

- For $X_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

  1. $\phi_1 \ = \ e^{-\|x_1 - t_1\|^2} \ = \ e^{-1} \ = \ 0.368$
  2. $\phi_2 \ = \ e^{-\|x_1 - t_2\|^2} \ = \ e^{-1} \ = \ 0.368$
  3. Therefore, $y_1 = w_1 \times \phi_1 \ + \ w_1 \times \phi_1 + b$

- For $X_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

  1. $\phi_1 \ = \ e^{-\|x_2 - t_1\|^2} \ = \ e^{0} \ = \ 1$
  2. $\phi_2 \ = \ e^{-\|x_2 - t_2\|^2} \ = \ e^{-2} \ = \ 0.135$
  3. Therefore, $y_2 = w_1 \times \phi_1 \ + \ w_1 \times \phi_1 + b$

- For $X_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

  1. $\phi_1 \;=\; e^{-\|x_3 - t_1\|^2} \;=\; e^{-2} \;=\; 0.135$
  2. $\phi_2 \;=\; e^{-\|x_3 - t_2\|^2} \;=\; e^{0} \;=\; 1$
  3. Therefore, $y_1 = w_1 \times \phi_1 \;+\; w_1 \times \phi_1 + b$

- For $X_4 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

  1. $\phi_1 \;=\; e^{-\|x_4 - t_1\|^2} \;=\; e^{-1} \;=\; 0.368$
  2. $\phi_2 \;=\; e^{-\|x_4 - t_2\|^2} \;=\; e^{-1} \;=\; 0.368$
  3. Therefore, $y_1 = w_1 \times \phi_1 \;+\; w_1 \times \phi_1 + b$

- Therefore the matrix for ϕ_1 and ϕ_2 along with inputs and target looks like:

X_i	1	2	ϕ_1	ϕ_2	A_i
1	0	0	0.368	0.368	1
2	0	1	1	0.135	0
3	1	0	0.135	1	0
4	1	1	0.368	0.368	1

- According to our calculation, one set of possible weights and bias we get is

  $W_{=} \begin{bmatrix} -2.4 \\ -2.4 \\ 0.5 \end{bmatrix}$ initially, where $w_1 = -2.4, w_2 = -2.4$ and $b = 0.5$

- Let the threshold for the output cell be -2.0 i.e. $y = 1$ if $f(x) \geq 2.0$ and 0 otherwise.

- We test the output for each input including weights and bias:

  1. Apply $X_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

     $\rightarrow f(y_{in}) \;=\; w_1 \times \phi_{11} \;+\; w_2 \times \phi_{12} \;+\; b$
     $\rightarrow f(y_{in}) \;=\; -2.4 \times 0.368 + (-2.4) \times 0.368 \;+\; 0.5 \;=\; f(-1.2664) \;=\; 1$ and target $= 1 \rightarrow$ **ok**

2. Apply $X_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

   $\rightarrow\ f(y_{in}) \ = \ w_1 \times \phi_{21} \ + \ w_2 \times \phi_{22} \ + \ b$
   $\rightarrow\ f(y_{in}) \ = \ -2.4 \times 1 + (-2.4) \times 0.135 \ + \ 0.5 \ = \ f(-2.224) \ = \ 0$ and target $= 0 \rightarrow$ **ok**

3. Apply $X_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

   $\rightarrow\ f(y_{in}) \ = \ w_1 \times \phi_{31} \ + \ w_2 \times \phi_{32} \ + \ b$
   $\rightarrow\ f(y_{in}) \ = \ -2.4 \times 0.135 + (-2.4) \times 1 \ + \ 0.5 \ = \ f(-2.224) \ = \ 0$ and target $= 0 \rightarrow$ **ok**

4. Apply $X_4 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   $\rightarrow\ f(y_{in}) \ = \ w_1 \times \phi_{41} \ + \ w_2 \times \phi_{42} \ + \ b$
   $\rightarrow\ f(y_{in}) \ = \ -2.4 \times 0.368 + (-2.4) \times 0.368 \ + \ 0.5 \ = \ f(-1.2664) \ = \ 1$ and target $= 1 \rightarrow$ **ok**

5. $w_2/w_1 \ = \ 1$
6. $b/w1 \ = -0.2083$
7. Equation or decision boundary is $y \ = \ w_1 \times x_1 \ + \ w_2 \times x_2 \ + \ b \ = \ -2.4 \times x_1 \ - 2.4 \times x_2 \ + \ 0.5$

Therefore the network is set for 2-input X-NOR gate.