

Lotus House  
Distribution problem

**Problem description**

The problem starts with a set of suppliers, a set of carpentry workshops, one construction site and one truck. Each workshop has a different distance (km) from each supplier and the construction site. It is assumed that the truck's capacity exceeds the total demand for raw materials and processed materials, so the truck needs to make only two shuttles. The first shuttle transports raw materials from the suppliers to the workshops. Some workshops require raw materials from all suppliers, while some others only require raw materials from one supplier. The truck can start from supplier 1, visit some workshops, then supplier 2 and the rest of the workshops. Alternatively, the truck can go to all the suppliers before visiting the workshops. Based on contract terms, the workshops will support transporting all the processed materials to one workshop. Therefore, the second shuttle, which happens on a different day, transports the processed materials from this particular workshop to the construction site. The cost of each shuttle is based on the driving distance (km). The base cost of a shuttle is 500 (Unit: 1,000 VND). Once the driving distance exceeds 10 km, it will start charging 12 per km. Total cost of the entire operations is the sum of cost of both shuttles. **Table 1** shows the parameters of this problem as math notation.

**TABLE 1: PARAMETERS OF THE DISTRIBUTION PROBLEM**

Notation	Definition
$s \in S$	Set of suppliers and index of suppliers
$w \in W$	Set of workshops and index of workshops
$c \in C$	Construction site and index of construction site
$d_{ss'}$	Distance from supplier $s$ to workshop $s'$
$d_{sw}$	Distance from supplier $s$ to workshop $w$
$d_{ww'}$	Distance from workshop $w$ to workshop $w'$
$d_{wc}$	Distance from workshop $w$ to construction site $c$
$k$	Index of shuttle
$x_s^k$	Binary variable equal to 1 if the truck visits supplier $s$ in shuttle $k$
$x_w^k$	Binary variable equal to 1 if the truck visits workshop $w$ in shuttle $k$
$x_c^k$	Binary variable equal to 1 if the truck visits construction site $c$ in shuttle $k$
$R$	Set of raw materials
$r_{ms}$	Binary variable equal to 1 if supplier $s$ has raw material $m$
$w_{ms}$	Binary variable equal to 1 if workshop $w$ requires raw material $m$
$y_{sj}^k$	Binary variable equal to 1 if shuttle $k$ transports raw material from supplier $s$ to workshop $j$

The model is built based on the parameters above.

**Objective:** Minimize total distance of both shuttles.

**Constraints:**

- Each supplier is visited once and only once in the first shuttle.
- Only the first shuttle visits suppliers.

$$\min \sum_{k=1}^2 \left( \sum_{s \in S} \sum_{w \in W} d_{sw} x_{sw}^k + \sum_{w \in W} \sum_{w' \in W} d_{ww'} x_{ww'}^k + \sum_{w \in W} \sum_{c \in C} d_{wc} x_{wc}^k \right)$$

$$\sum_{k=1}^1 x_s^k = 1, \forall s \in S$$

$$x_s^2 = 0, \forall s \in S$$

3. Each workshop is visited once and only once in the first shuttle.
4. Diagonal constraints.
5. Before visiting a workshop, the truck must have the required raw materials.
6. Only the second shuttle visits the construction site.
7. The construction site is the final destination of the second shuttle.
8. Subtour elimination (optional).
9. Revisit prevention (optional).

$$\sum_{k=1}^1 x_w^k = 1, \forall w \in W$$

$$x_{ww}^k = 0, \forall w \in W, \forall k$$

$$x_{ss}^k = 0, \forall s \in S, \forall k$$

$$\sum_{s \in S} r_{ms} x_s^1 \geq w_{mw} x_w^1, \forall m \in R, \forall w \in W$$

$$x_c^1 = 0, \forall c \in C$$

$$\sum_{w \in W} x_{wc}^2 = 1, \forall c \in C$$

$$x_{ss'}^k + x_{s's}^k \leq 1, \forall s, s' \in S, s \neq s', \forall k$$

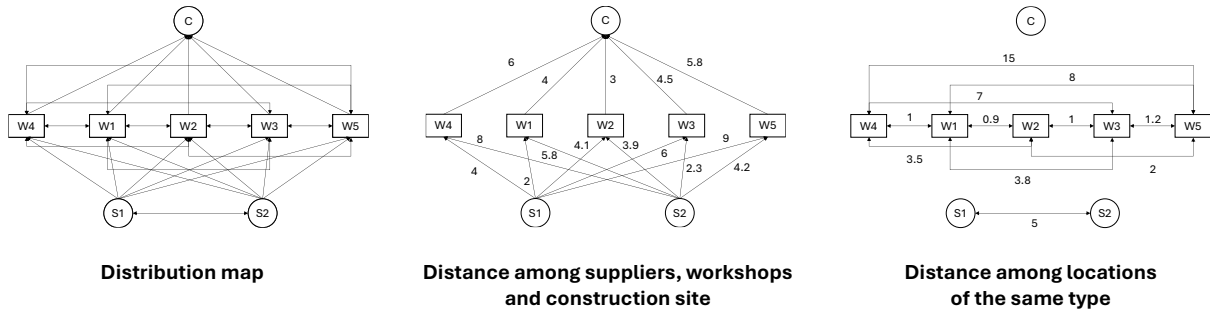
$$x_{ww'}^k + x_{w'w}^k \leq 1, \forall w, w' \in W, w \neq w', \forall k$$

$$x_{sw}^1 + x_{ws}^2 \leq 1, \forall s \in S, \forall w \in W$$

Error! Reference source not found. presents the Python code developed to solve this optimization problem. This model is based on knowledge from the Distribution Management course at KEDGE, which originally used Excel Solver to solve the problem. However, the problem description in this case is more complex, leading to a multitude of constraints and requiring Python.

### Inputs of the model:

There are two suppliers ( $S = 2$ ), five workshops ( $W = 5$ ) and two sets of raw materials ( $R = 2$ ). Supplier 1 has raw material type 1, which is required by workshop 4. Supplier 2 has raw material type 2, which is required by workshop 5. Workshops 1, 2 and 3 require all the raw materials from both stations. **Figure 1** shows the distance between each location.



**FIGURE 1: INPUTS OF THE DISTRIBUTION MODEL**

**Unit: km**

**Implementation and results:** The optimal route is as follows. This is a very different approach compared to the company's current decision-making process, which is based on expertise and experience. The routing is optimized, with minimal driving distance and transportation cost. In the future, the model may be extended for more complex scenarios.

#### Shuttle 1:

$S1 > S2 > W1 > W2 > W3 > W4 > W5$

Total distance: 68.6 km

Cost of shuttle:

$$500 + 12 \times (68.6 - 10) = 1204$$

#### Shuttle 2:

$W1 > \text{Construction site}$

Total distance: 4 km

Cost of shuttle:

$$500 + 12 \times 0 = 500$$

#### Both shuttles:

Total distance: 72.6 km

Total costs:

$$1204 + 500 = 1704$$